Digital System Design

Finite State Formulation, State Reduction, and Assignment

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Begin at the beginning

"Begin at the beginning," the King said, very gravely, "and go on till you come to the end: then stop."

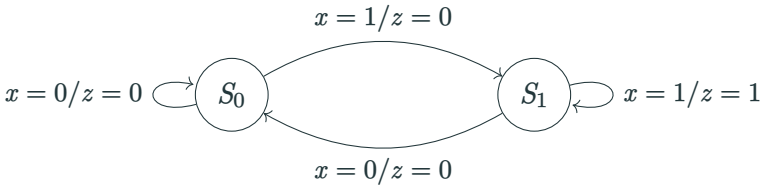
—Lewis Carroll, Alice in Wonderland

Mealy Formulation

Example 1: ‘11’ Sequence Detector

x	0	1	1	0	1	0	1	1	1	0
z	0	0	1	0	0	0	0	1	1	0

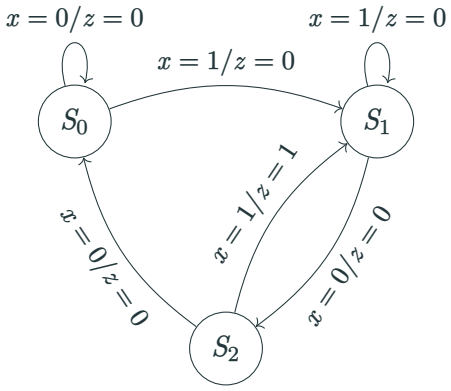
Example 1: Resulting State Diagram



Example 2: ‘101’ Sequence Detector

x	0	0	1	1	0	1	1	0	0	1	0	1	0	1	0	0
z	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0

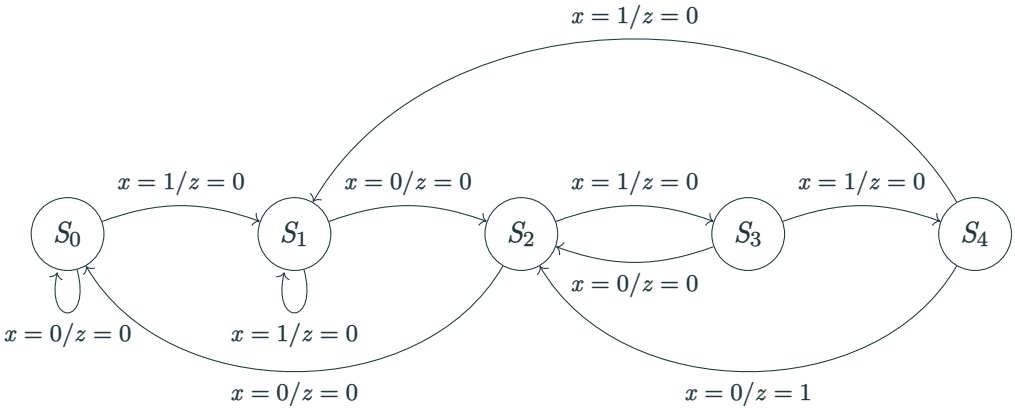
Example 2: Corresponding State Diagram



Example 3: ‘10110’ Sequence Detector

x	0	1	0	1	1	0	1	1	0	0	0	1	0	1	1	0	0	1	1	0
z	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0

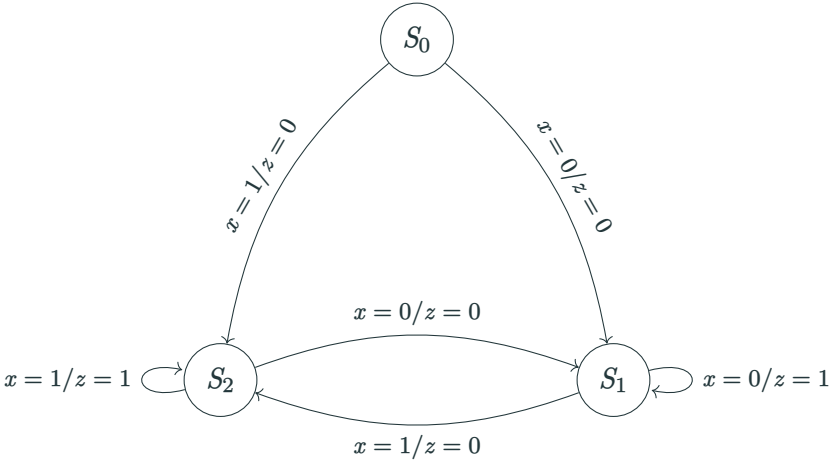
Example 3: State Diagram



Example 4: ‘00’/‘11’ Detector

x	0	1	0	0	0	1	0	1	1	1	1	0	0	1	0	1	1	0	0	0
z	0	0	0	1	1	0	0	0	1	1	1	0	1	0	0	0	1	0	1	1

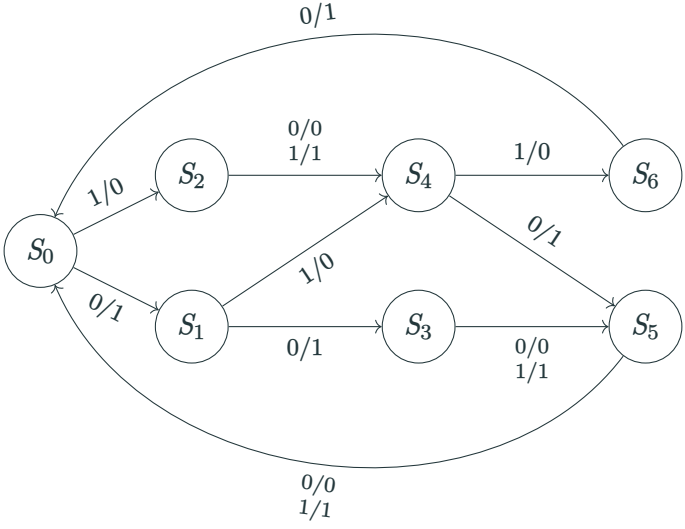
Example 4: State Diagram



Example 5: BCD to Excess-3 Code Converter

x, BCD				z, Excess-3			
t_3	t_2	t_1	t_0	t_3	t_2	t_1	t_0
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

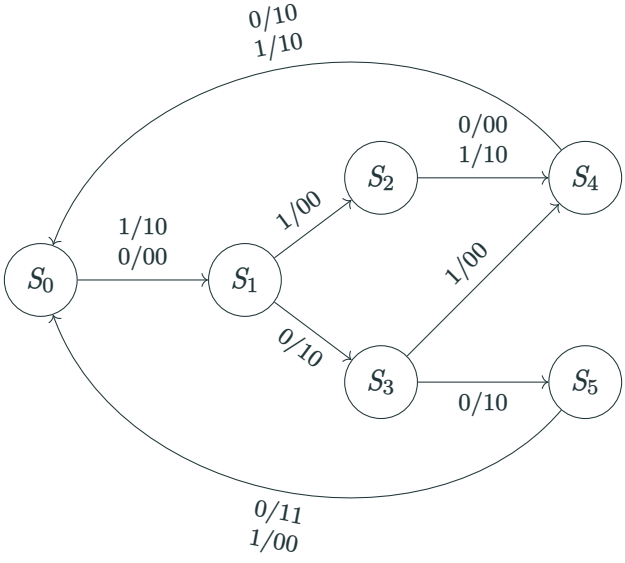
Example 5: State Diagram



Example 6: Decreased by 2 Converter

- A sequential circuit has one input, x, and two outputs, z and w.
- The input signal x represents a 4-bit binary number N, which is input least significant bit first.
- The ouput signal z represents a 4-bit binary number equal to $N - 2$, which is output least significant bit first.
- At the time the fourth input occurs, $w = 1$ if $N - 2$ is negative; otherwise, $w = 0$.
- The circuit always resets after the fourth bit of x is received.

Example 6: State Diagram

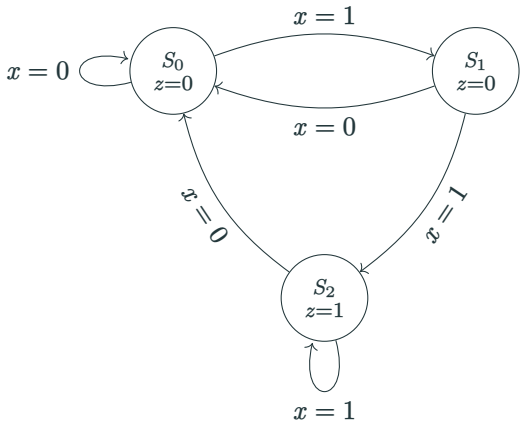


Moore Formulation

Example 1: ‘11’ Sequence Detector

x	0	1	1	0	1	0	1	1	1	0
z	0	0	0	1	0	0	0	0	1	1

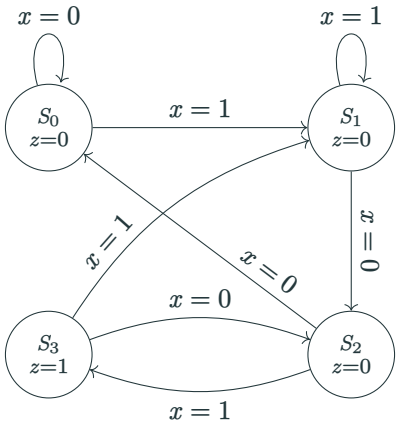
Example 1: Resulting State Diagram



Example 2: ‘101’ Sequence Detector

x	0	0	1	1	0	1	1	0	0	1	0	1	0	1	0	0
z	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0

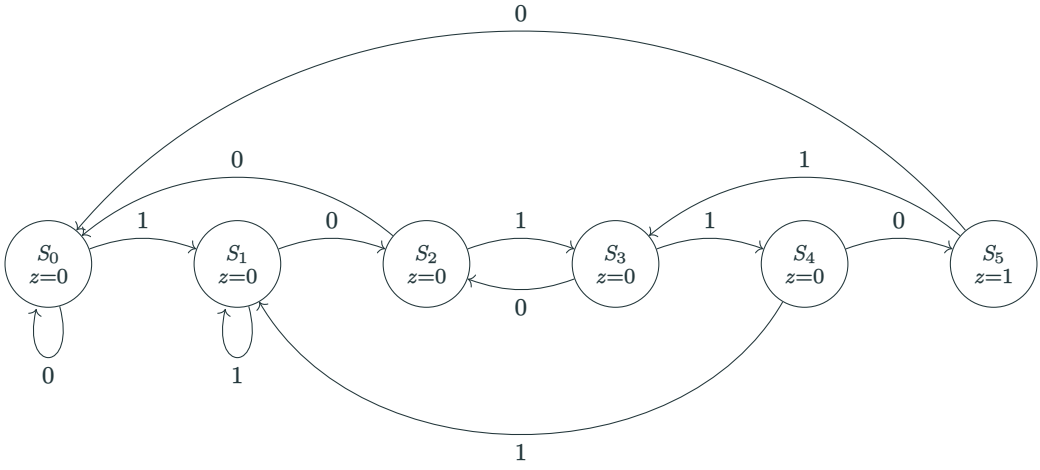
Example 2: Corresponding State Diagram



Example 3: ‘10110’ Sequence Detector

x	0	1	0	1	1	0	1	1	0	0	0	1	0	1	1	0	0	1	1	0
z	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0

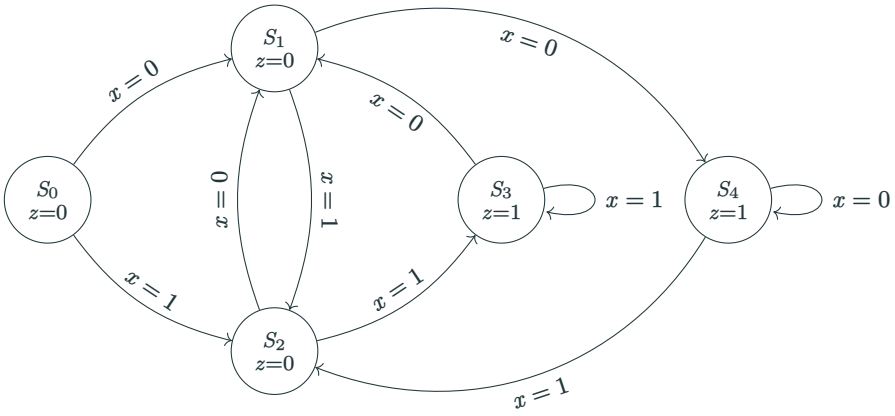
Example 3: State Diagram



Example 4: ‘00’/‘11’ Detector

x	0	1	0	0	0	1	0	1	1	1	1	0	0	1	0	1	1	0	0	0
z	0	0	0	0	1	1	0	0	0	1	1	1	0	1	0	0	0	1	0	1

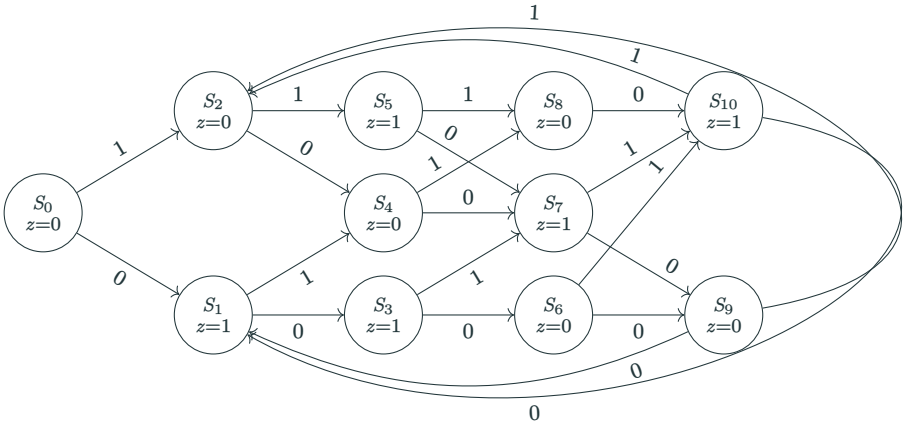
Example 4: State Diagram



Example 5: BCD to Excess-3 Code Converter

x, BCD				z, Excess-3			
t_3	t_2	t_1	t_0	t_4	t_3	t_2	t_1
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

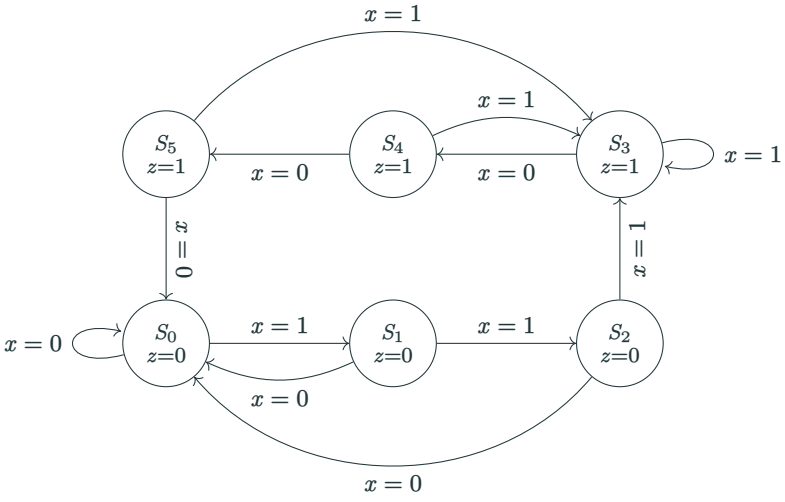
Example 5: State Diagram



Example 6: On/Off Circuit

x	0	1	0	1	1	1	0	1	0	0	0	1	1	1	0	0	1	0	0	0	0
z	0	0	0	0	0	0	1	1	1	1	1	0	0	0	1	1	1	1	1	1	0

Example 6: State Diagram



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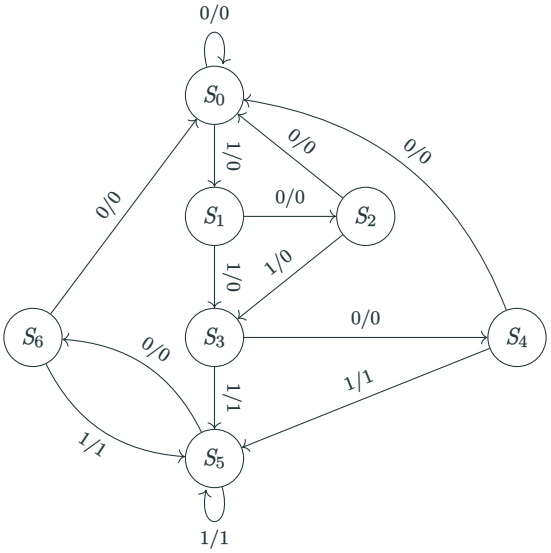
State Diagram



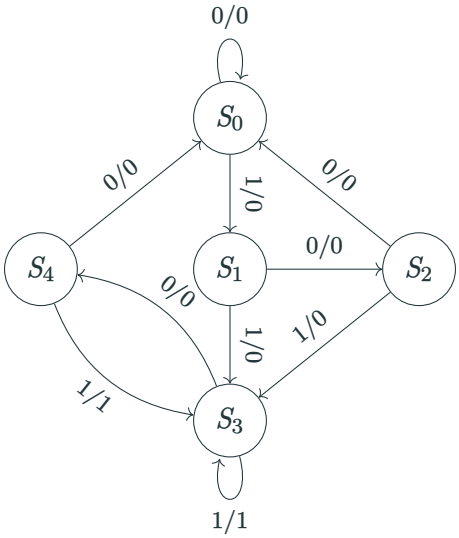
State Table

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
S_0	S_0	S_1	0	0
S_1	S_2	S_3	0	0
S_2	S_0	S_3	0	0
S_3	S_4	S_5	0	1
S_4	S_0	S_5	0	1
S_5	S_6	S_5	0	1
S_6	S_0	S_5	0	1

Reduced State Table

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
S_0	S_0	S_1	0	0
S_1	S_2	S_3	0	0
S_2	S_0	S_3	0	0
S_3	S_4	S_3	0	1
S_4	S_0	S_3	0	1

Reduced State Diagram



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Binary State Assignments

State	Assignment 1 Binary	Assignment 2 Gray Code	Assignment 3 Johnson	Assignment 4 One-Hot
S_0	000	000	000	00001
S_1	001	001	100	00010
S_2	010	011	110	00100
S_3	011	010	111	01000
S_4	100	110	011	10000