

CCDSTRU Project Specifications

Term 2, AY 2022–2023 Due: Apr 11, 2023 (T) 0800

Implement a computer program (either in C or Java) following the specifications of the system given below.

Applicable Sets

 $A : \{x \in \mathbf{Z}^+ \mid x < 4\}$

 $\mathbf{M}: \{2\} \times \mathbf{A}$

 $S: A \times A$

 $\mathbf{B}: \{3\} \times \mathbf{A}$

 $V : \{true, false\}$

 $L: A \times \{1\}$

 $P : \{x \in \mathbf{Z}^+ \mid x < 10\}$

 $C: A \times \{2\}$

 $T: \{1\} \times A$

 $\mathbf{R}: \mathbf{A} \times \{3\}$

$$\mathbf{F}: \{x \in \mathcal{P}(S) \mid |x| = 3\} - \{\{(1,1), (2,2), (3,3)\}, \{(1,3), (2,2), (3,1)\}\}\$$

System Variables

 $Occ, Free \subseteq \mathbf{S}$

 $One, Two, Three, Four, Five, Six \subseteq \mathbf{P}$

 $Peg \subseteq \mathbf{P}$

 $W \subseteq \mathcal{P}(\mathbf{P})$

 $ok \in \mathbf{V}$

 $turn \in \mathbf{V}$

 $over \in \mathbf{V}$

 $next \in \mathbf{V}$

System Facts

$$Free = \mathbf{S} - Occ$$

$$Peg = \mathbf{P} - (One \cup Two \cup Three \cup Four \cup Five \cup Six)$$

$$W = \{One\} \cup \{Two\} \cup \{Three\} \cup \{Four\} \cup \{Five\} \cup \{Six\}\}$$

$$over \leftrightarrow \left(\exists w \ \left(w \in \mathbf{W} \land |w| = 3 \land \sum_{a \in w} a < 15\right) \lor \forall w \ \left(w \in \mathbf{W} \land |w| = 3 \land \sum_{a \in w} a = 15\right)\right)$$

System Initialization

$$ok = false$$
 $Three = \emptyset$

$$next = false$$
 $Four = \emptyset$

$$turn = true$$
 $Five = \emptyset$

$$One = \varnothing$$
 $Six = \varnothing$

$$Two = \emptyset$$
 $Peg = \mathbf{P}$

System States and Behavior

 $NextPlayerMove(peg \in P, pos \in S)$

$$peg \in Peg \land pos \in Free \rightarrow ok = \neg ok$$

$$\land \; Occ = Occ \cup \{pos\}$$

$$ok \land pos \in \mathbf{T}$$
 \rightarrow $One = One \cup \{peg\}$

$$\wedge next = \neg next$$

$$ok \land pos \in \mathbf{M}$$
 $\rightarrow Two = Two \cup \{peg\}$

$$\wedge next = \neg next$$

$$ok \land pos \in \mathbf{B}$$
 $\rightarrow Three = Three \cup \{peg\}$

$$\land \ next = \neg next$$

$$ok \land next \land pos \in \mathbf{L}$$
 \rightarrow $Four = Four \cup \{peg\}$

$$\land \ next = \neg next$$

$$\wedge ok = \neg ok$$

$$ok \land next \land pos \in \mathbf{C} \qquad \rightarrow \quad Five = Five \cup \{peg\}$$

$$\land \ next = \neg next$$

$$\wedge ok = \neg ok$$

$$ok \land next \land pos \in \mathbf{R} \rightarrow Six = Six \cup \{peg\}$$

$$\wedge next = \neg next$$

$$\wedge \ ok = \neg ok$$

GameOver(over)

 $result \in \{B \text{ wins, A wins}\}\$

$$over \wedge turn \wedge \exists w \, (w \in \mathbf{W} \wedge |w| = 3 \wedge \sum_{a \in w} a < 15) \qquad \rightarrow \quad result = \mathbf{B} \text{ wins}$$

$$over \wedge \neg turn \wedge \exists w \, (w \in \mathbf{W} \wedge |w| = 3 \wedge \sum_{a \in w} a < 15) \quad \rightarrow \quad result = \mathbf{A} \text{ wins}$$

$$over \land turn \land \forall w, (w \in \mathbf{W} \land |w| = 3 \land \sum_{a \in w} a = 15) \rightarrow result = A wins$$

$$over \wedge \neg turn \wedge \forall w, (w \in \mathbf{W} \wedge |w| = 3 \wedge \sum_{a \in w} a = 15) \quad \rightarrow \quad result = \mathbf{B} \text{ wins}$$

$$\neg over \rightarrow turn = \neg turn$$