ASTR 404 Fall 2016
Homework #7

Due online **Friday, October 14 at 5:00 pm**. Please remember to show all work on quantitative questions. No credit will be given without it, **even if the numerical result is correct**. Point values for each question are indicated in parenthesis after the question number.

For this assignment, you will examine the properties of a realistic stellar structure model produced using the MESA stellar evolution code. I have used MESA to generate five solar-metallicity ZAMS models with different masses. Each model file is named `##M_at_ZAMS.dat`, where `##` = the stellar mass in solar masses. Download and use the model file corresponding to the last digit in your University ID number, as listed in the table below.

UIN ends in	0, 1	2, 3	4, 5	6, 7	8, 9
Mass to use	0.5	0.9	2.0	5.0	15.0

Each model file is a text file containing profiles of a number of quantities at different radii, running from the surface of the star to its center. The quantities are listed at the top of the file along with vital statistics regarding the stellar model (mass, abundances, etc.); explanations are given on the next page. The questions below ask you to make a number of plots and interpret the results. You can write a program to make these plots, use a plotting package such as gnuplot, or break out your trusty spreadsheet.

In addition to your model file, you should also download the two polytrope data files, `n1.5polytrope.dat` and `n3.0polytrope.dat`. These contain ξ and $\theta(\xi)$ for $n = 1.5$ and $n = 3.0$ polytropes. To use them you will need to convert to radius and density using $r = \alpha\xi$ and $\rho = \rho_{\rm central}\theta^n$ as discussed below.

All plots must have clearly labeled axes and titles. You should also use log-log axis scaling when the range of variation of a quantity is such that the curve hugs an axis when linear scaling is used. For plots with multiple quantities, please use different line styles or colors and provide a legend.

1. (10) Plot the gas density for your model versus radius. Overplot the $n = 1.5$ and $n = 3.0$ polytropes, scaled to the central density and the radius of your model star. For $n = 1.5$, the first zero is at $\xi = R/\alpha = 3.65375 \implies \alpha = R/3.65375$. For $n = 3.0$, the first zero is at $R/\alpha = 6.89685$.
   a) Do either of the polytropes provide a good fit to the density profile in any parts of the star? If so, where?
2. (10) Plot the gas and radiation pressure versus radius.
   a) Where does radiation pressure dominate, if at all? Explain why the radiation pressure dominates in these locations in terms of the mixed ideal gas plus radiation equation of state.
   b) Considering the answer to (a), can you explain why either of the polytropes would be a good fit to the density profile in certain parts of the star?

3. (10) Plot the energy generation rate per unit mass due to the pp chain and CNO cycle, and the total of the two, versus radius.
   a) At what radius does the energy generation rate drop to 50% of its central value? Is it primarily due to the pp chain or the CNO cycle?
4. (10) Plot the adiabatic index versus radius.
   a) Where is the adiabatic index different from 5/3, if at all? Why does it vary or not vary from 5/3?
5. (10) Plot the radiative, convective, adiabatic, and actual temperature gradients vs. radius. In the MESA output files, temperature gradients are expressed in terms of

$$\nabla \equiv \frac{d \ln T}{d \ln P} = \frac{P}{T}\frac{dT}{dP},$$

so to compute $\frac{dT}{dr}$ note that $\frac{dT}{dP} = \frac{dT}{dr}\frac{dr}{dP}$, giving

$$\frac{dT}{dr} = \frac{T}{P}\frac{dP}{dr}\nabla = -\frac{Gm}{r^2}\frac{\rho T}{P}\nabla\ .$$

You can choose to plot ∇ or $\frac{dT}{dr}$, but keep the above in mind when interpreting the plot. In particular, notice that ∇ is positive while $\frac{dT}{dr}$ is negative.

   a) Are there any convective regions in the star? If so, where and why? (Refer to your plots in questions 3-4 for help in answering this question.) Is the convection efficient (ie. is the convective gradient close to the adiabatic gradient)?

The profile quantities and their explanations appear below.

zone	The zone number, starting with 1 at the surface
q	Fraction of star mass interior to outer boundary of this zone
logR	log10(radius/Rsun) at outer boundary of zone
logRho	log10(density/(g/cm3)) at center of zone
logT	log10(temperature/K) at center of zone
logP	log10(pressure/(dyn/cm2)) at center of zone
logPgas	log10(gas pressure/(dyn/cm2)) at center of zone
luminosity	Luminosity at outer boundary of zone (in Lsun units)
x_mass_fraction_H	Mass fraction of hydrogen
y_mass_fraction_He	Mass fraction of helium
z_mass_fraction_metals	Mass fraction of metals
gamma1	Adiabatic index, dlnP_dlnRho at constant S
opacity	Opacity measured at center of zone (cm2/g)
pp	PP energy generation rate (erg/sec/g)
cno	CNO energy generation rate (erg/sec/g)
gradr	dlnT/dlnP required for purely radiative transport
gradT	Mixing length theory value for required temperature gradient dlnT/dlnP
grada	Adiabatic temperature gradient, dlnT_dlnP at constant S
actual_gradT	Actual temperature gradient dlnT/dlnP in model