Digital Signal Processing

Laboratory 5 Aliasing Caused by Sampling

5.1 Introduction

The purpose of this lab[1] is to investigate aliasing for sine waves and chirp signals.

5.2 Aliasing a Sinusoid

A continuous-time sinusoid is expressed as

$$x(t) = \sin(2\pi F_0 t + \theta)$$

The continuous-time signal can be sampled at a rate $F_s = 1/T$ to obtain a discrete-time signal

$$x(n) = x(t)|_{t=nT} = x(t)|_{t=n/F_s} = \sin(2\pi \frac{F_0}{F_s} n + \theta)$$

5.2.1 EXERCISE

The following exercise plots $x(n)$ for different combination of F_o and F_s to illustrate the aliasing problem. Set the sampling frequency to $F_s = 8$ kHz.

1. Make a single plot of a sampled sine wave having a frequency of 300 Hz sampled over an interval of 10 ms. Set the phase θ to an arbitrary value and plot using `stem`.
2. Make a plot of the same sinusoid using `plot`. In this case, the points are connected with straight lines. Connecting the signal samples with straight lines is a form of "signal reconstruction" that makes a continuous-time signal from discrete-time samples.
3. Make a series of plots of sinusoids with frequency from 100 Hz to 475 Hz in steps of 125 Hz. Use `subplot` to put the four plots in a single figure. Note that the apparent frequency of the sinusoid is *increasing*.
4. Make another series of plots of sinusoids with frequency from 7,525 Hz to 7,900 Hz in steps of 125 Hz. Use `subplot` to put the four plots in a single figure. Note that the apparent frequency of the sinusoid is now *decreasing*. Explain this phenomenon.

[1] S. Burrus, etal., *Computer-Based Exercises for Signal Processing using Matlab*, Prentice-Hall: Englewood Cliffs, NJ, 1994, pp.29-31

5. Make a similar series of plots of sinusoids but vary the frequency from 32,100 Hz to 32,475 Hz in steps of 125 Hz. Use `subplot` to put the four plots in a single figure. Note the apparent frequency of the sinusoid.

5.3 Aliasing a Chirp Signal

A linear frequency modulated signal is a good test for aliasing since the frequency changes over a range. The mathematical definition of a chirp is

$$c(t) = \cos(\pi \mu t^2 + 2\pi f_1 t + \psi)$$

The instantaneous frequency of the signal can be found by taking the time derivative of the argument of the cosine:

$$f_1(t) = \mu t + f_1$$

5.3.1 EXERCISE

1. Set the parameters of the chirp to $f_1 = 4$ kHz, $\mu = 600$ kHz/s, and ψ arbitrary. If the total time duration of the chirp is 50 ms, determine the frequency range that is covered by the *swept* frequency of the chirp.

2. Let the sampling frequency be $F_s = 8$ kHz. Plot the discrete-time samples of the chirp using both `stem` and `plot`. There will be aliasing since the swept bandwidth of the chirp exceeds the sampling frequency.

3. Notice that the chirp signal exhibits intervals in time where the apparent frequency gets very low. The instantaneous frequency is passing through zero at these points. Determine from the plot the times when this happens. Verify that these are the correct times by checking where the aliasing of the swept frequency occurs.