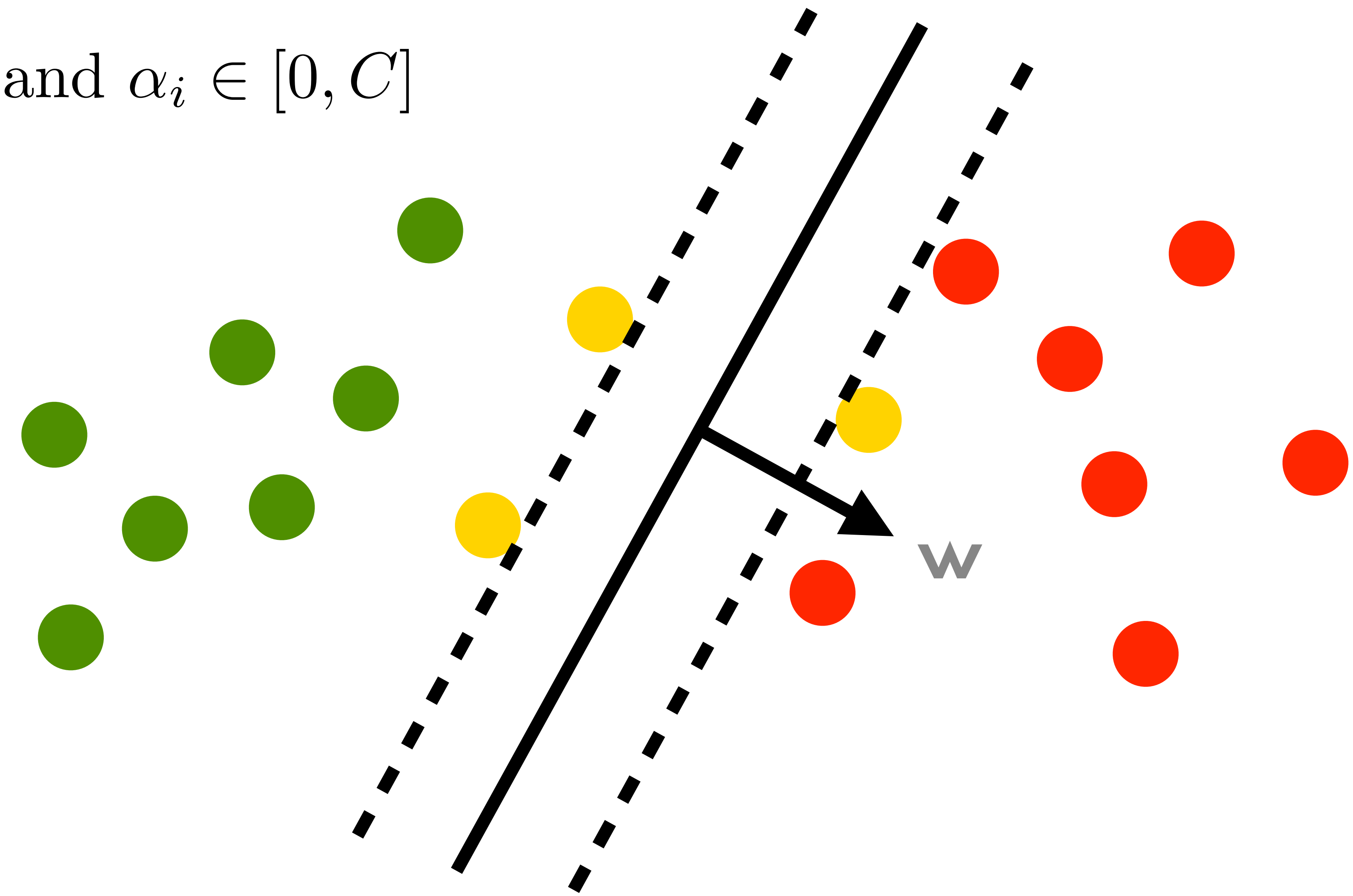
6.867
Nonlinear models,Kernels

Fall 2016

$$\underset{\alpha}{\text{maximize}} - \frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j \langle x_i, x_j\rangle + \sum_i \alpha_i$$

$$\text{subject to } \sum_i \alpha_i y_i = 0 \text{ and } \alpha_i \in [0, C]$$



$$w = \sum_i y_i \alpha_i x_i$$

$$\alpha_i \left[y_i \left[\langle w, x_i\rangle + b\right] + \xi_i - 1\right] = 0$$

$$\eta_i \xi_i = 0$$

$$\alpha_i = 0 \Longrightarrow y_i \left[\langle w, x_i\rangle + b\right] \geq 1$$

$$0 < \alpha_i < C \Longrightarrow y_i \left[\langle w, x_i\rangle + b\right] = 1$$

$$\alpha_i = C \Longrightarrow y_i \left[\langle w, x_i\rangle + b\right] \leq 1$$

courtesy: A. Smola (CMU, 2013)

The Kernel Trick

Linear soft margin problem

$$\underset{w,b}{\text{minimize}} \; \frac{1}{2}\|w\|^2 + C\sum_i \xi_i$$

$$\text{subject to } y_i\left[\langle w, x_i\rangle + b\right] \geq 1 - \xi_i \text{ and } \xi_i \geq 0$$

Dual problem

$$\underset{\alpha}{\text{maximize}} - \frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j \langle x_i, x_j\rangle + \sum_i \alpha_i$$

$$\text{subject to } \sum_i \alpha_i y_i = 0 \text{ and } \alpha_i \in [0, C]$$

Support vector expansion

$$f(x) = \sum_i \alpha_i y_i \langle x_i, x\rangle + b$$

courtesy: A. Smola (CMU, 2013)

The Kernel Trick

Linear soft margin problem

$$\underset{w,b}{\text{minimize}} \ \frac{1}{2} \|w\|^2 + C \sum_i \xi_i$$

$$\text{subject to } y_i \left[\langle w, \phi(x_i) \rangle + b \right] \geq 1 - \xi_i \text{ and } \xi_i \geq 0$$

Dual problem

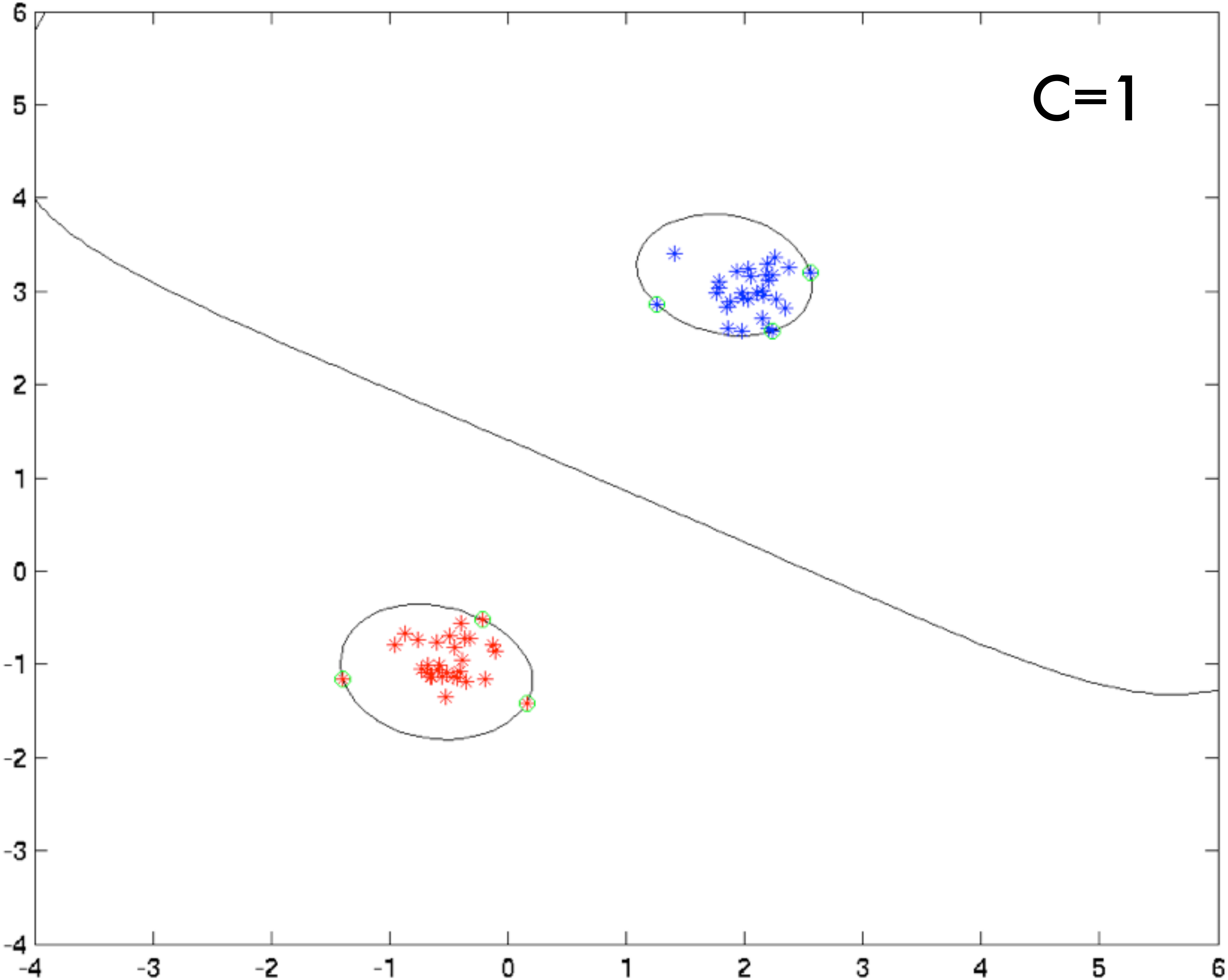
$$\underset{\alpha}{\text{maximize}} - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j k(x_i, x_j) + \sum_i \alpha_i$$

$$\text{subject to } \sum_i \alpha_i y_i = 0 \text{ and } \alpha_i \in [0, C]$$

Support vector expansion

$$f(x) = \sum_i \alpha_i y_i k(x_i, x) + b$$

courtesy: A. Smola (CMU, 2013)



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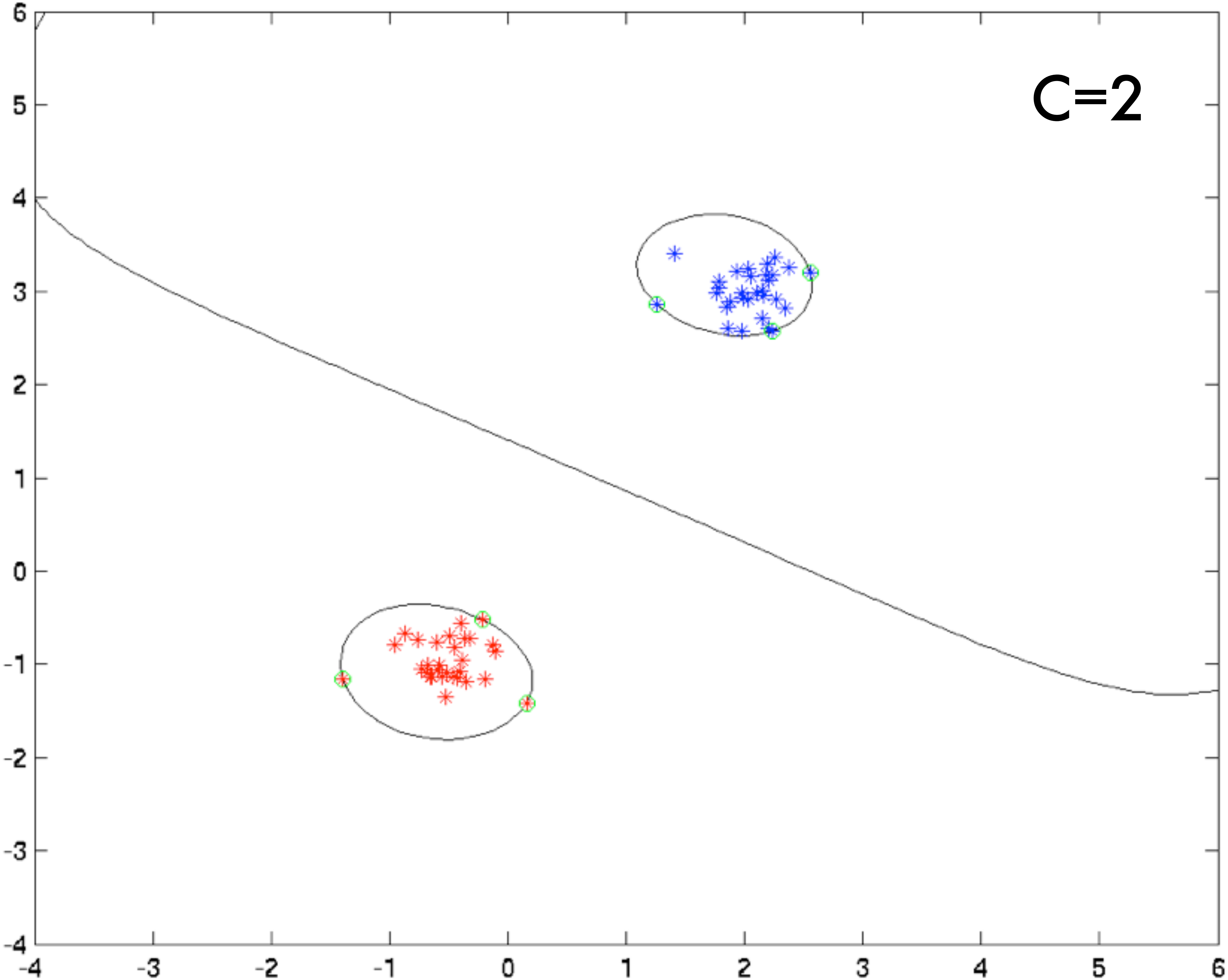
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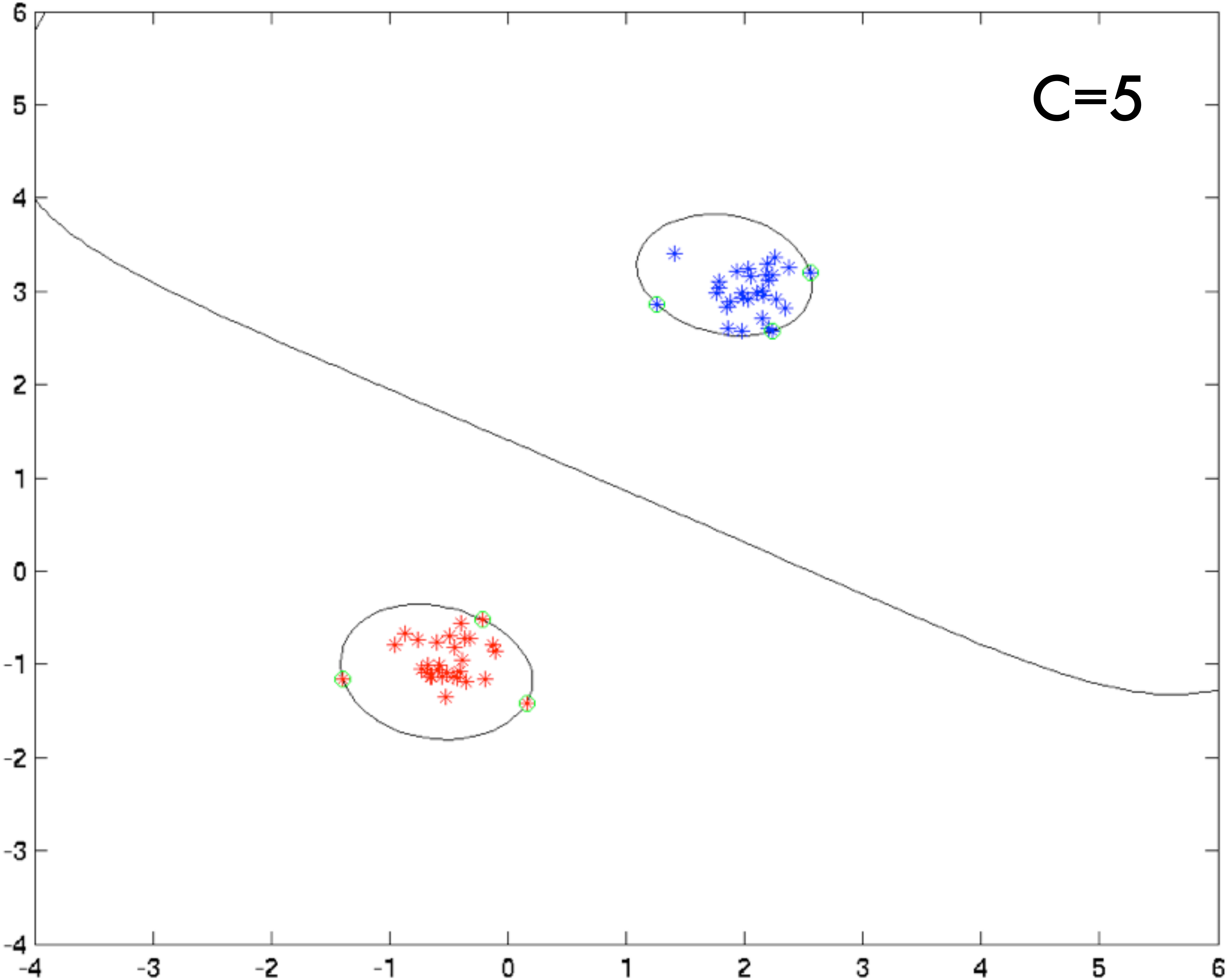

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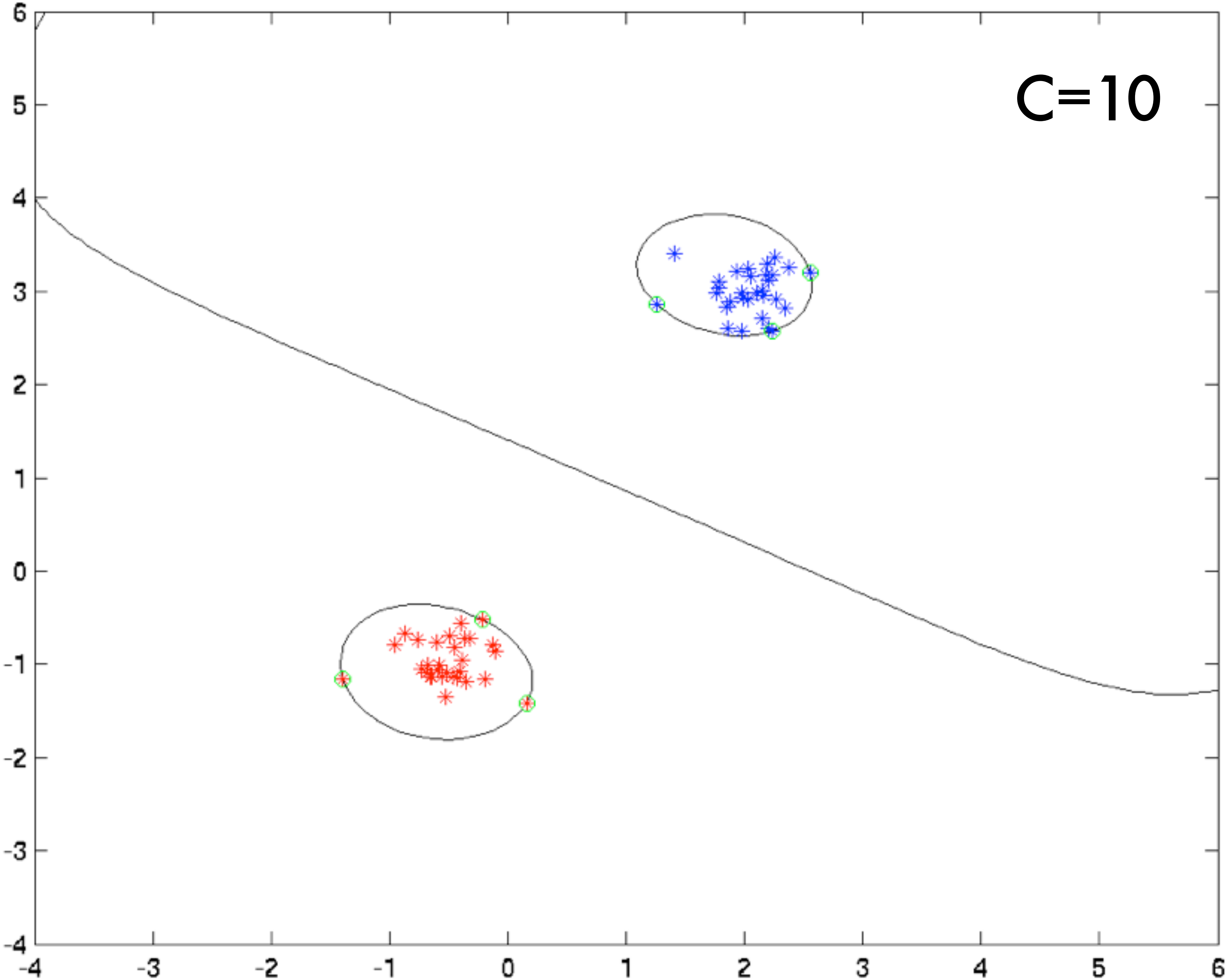
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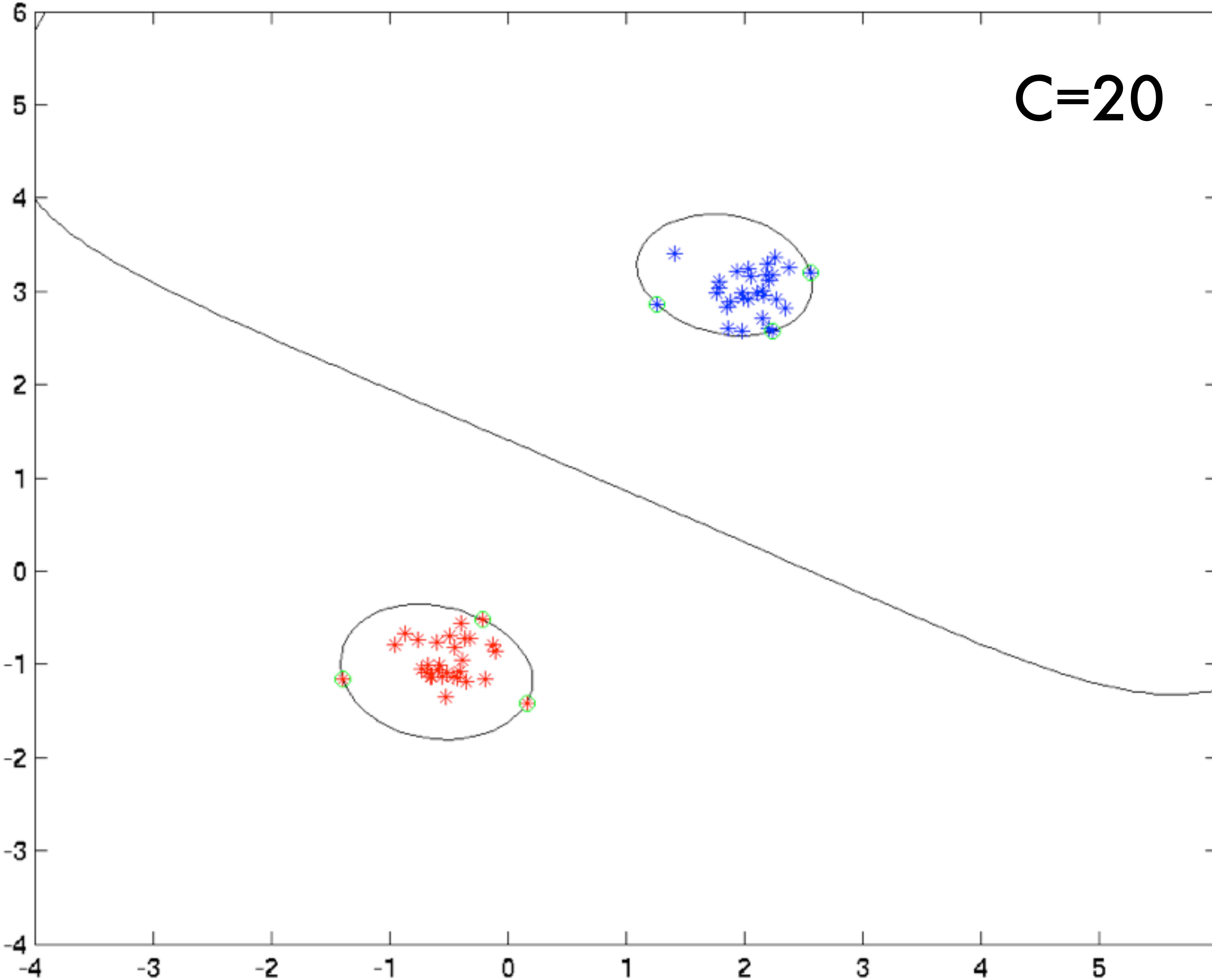
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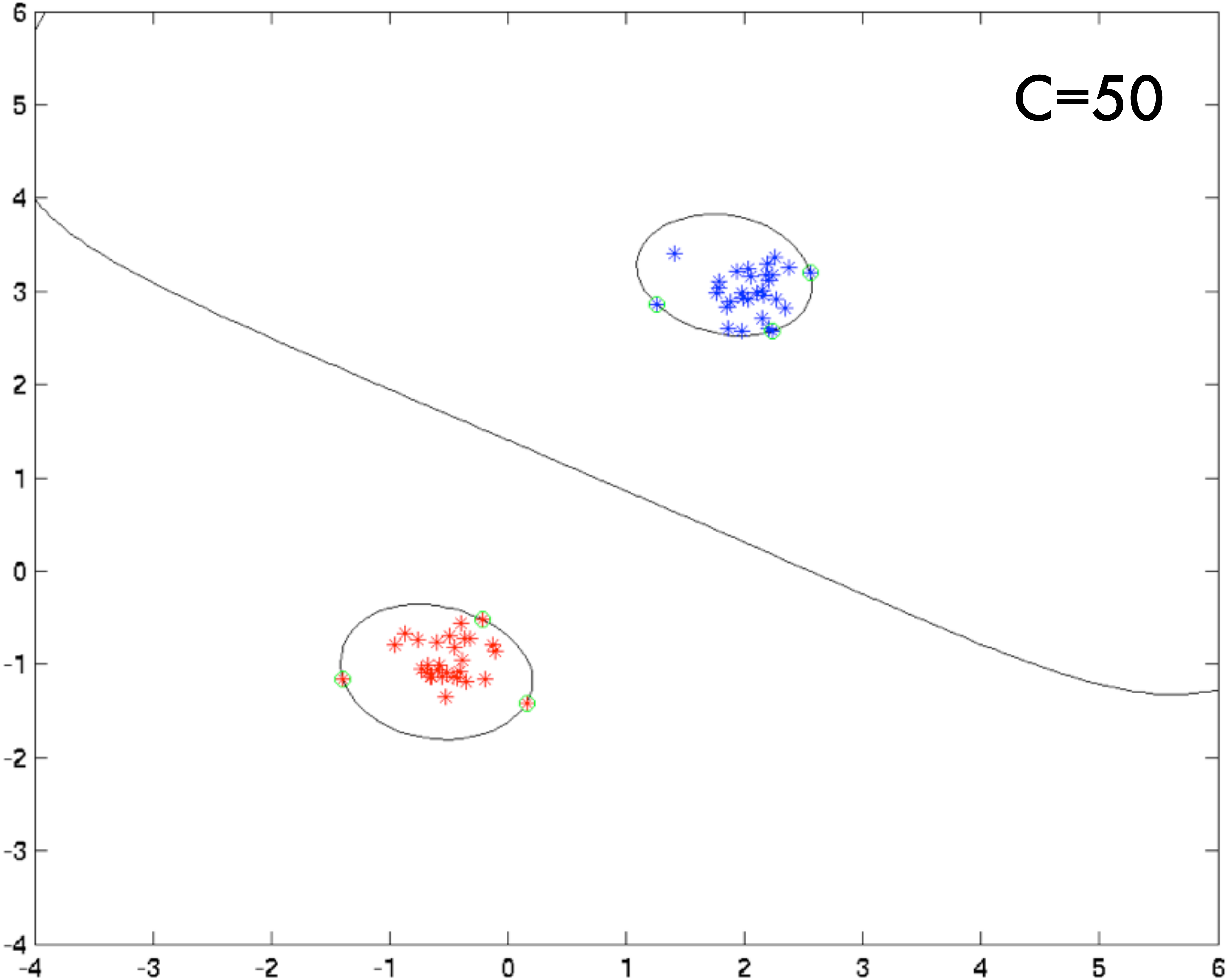
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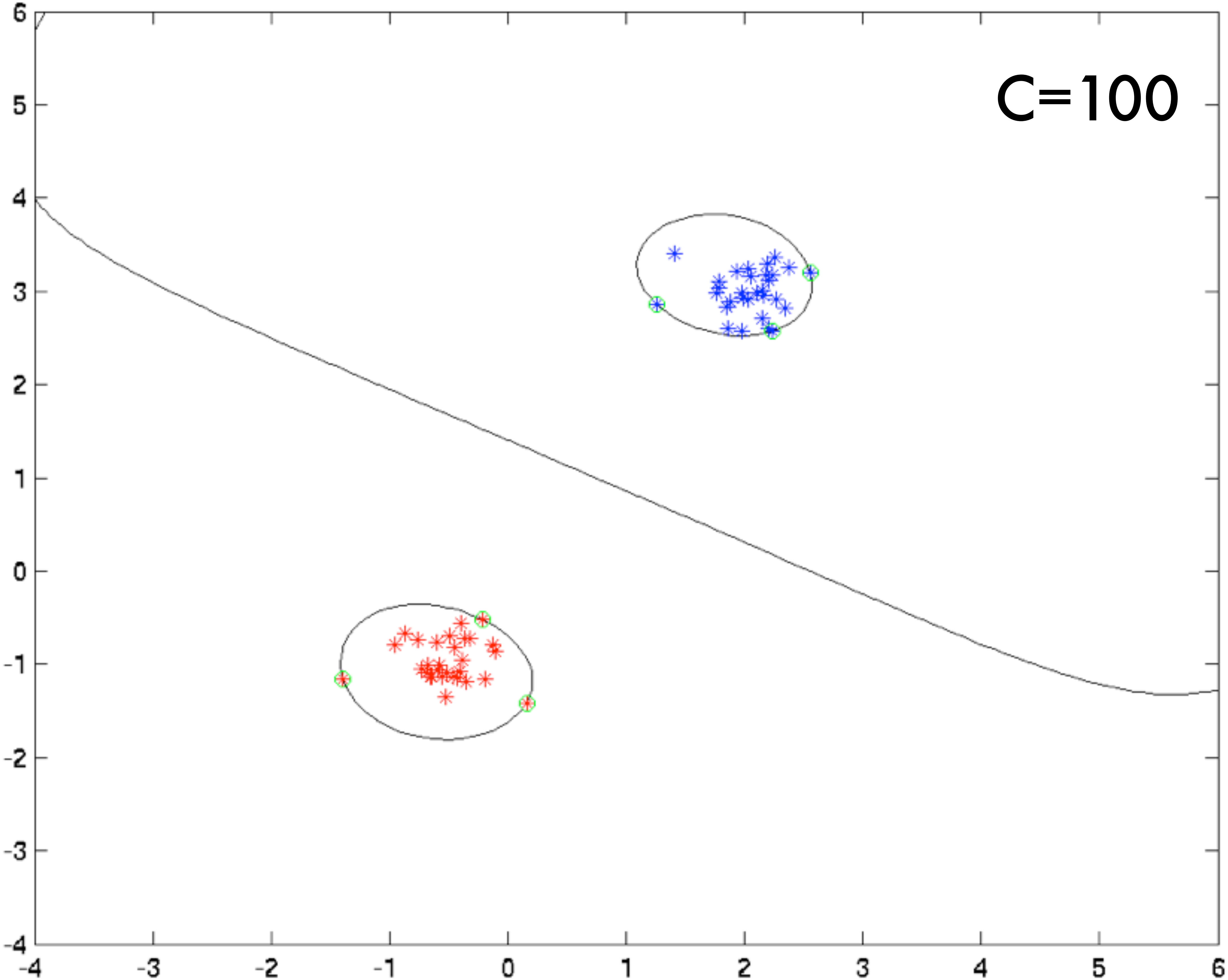

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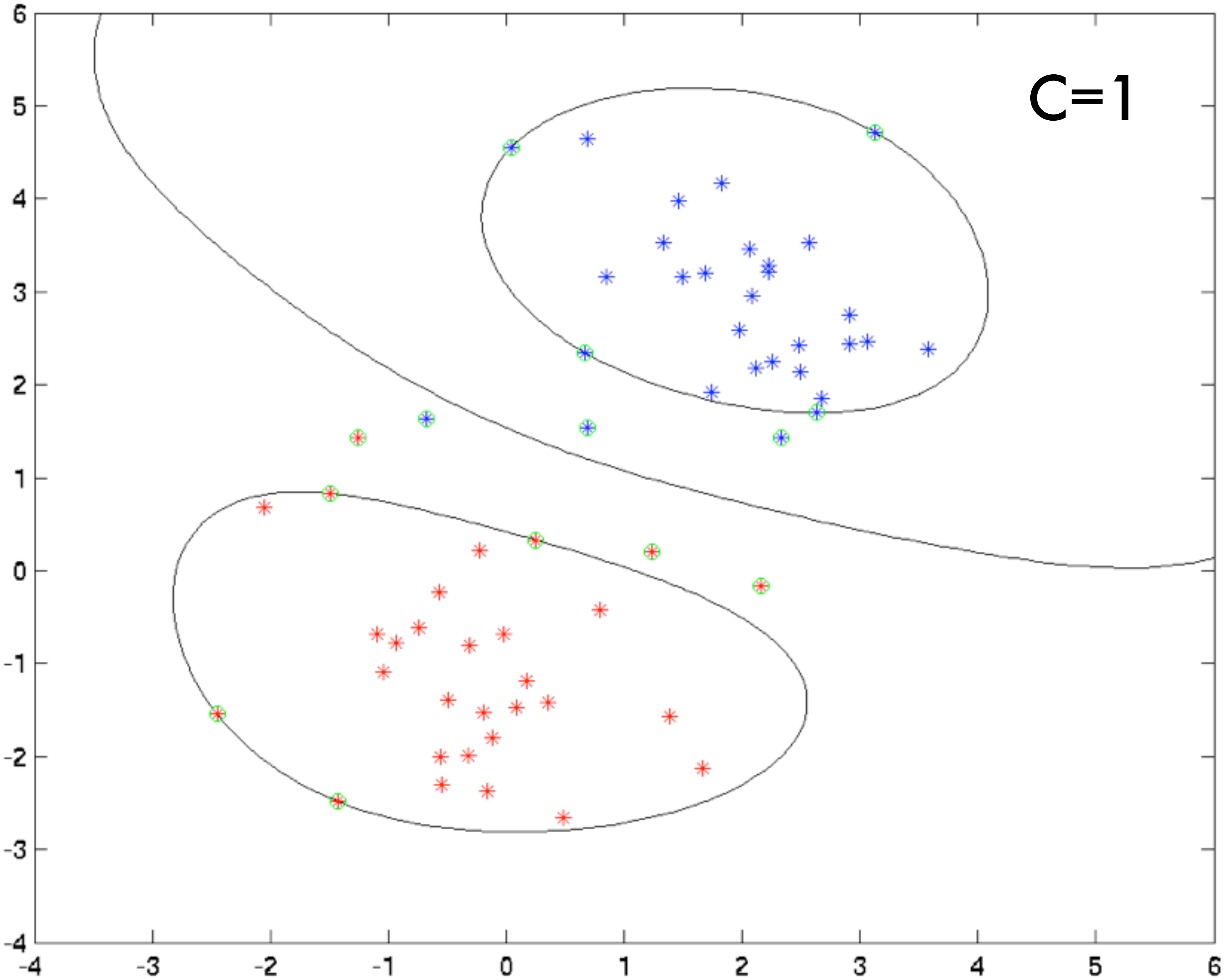
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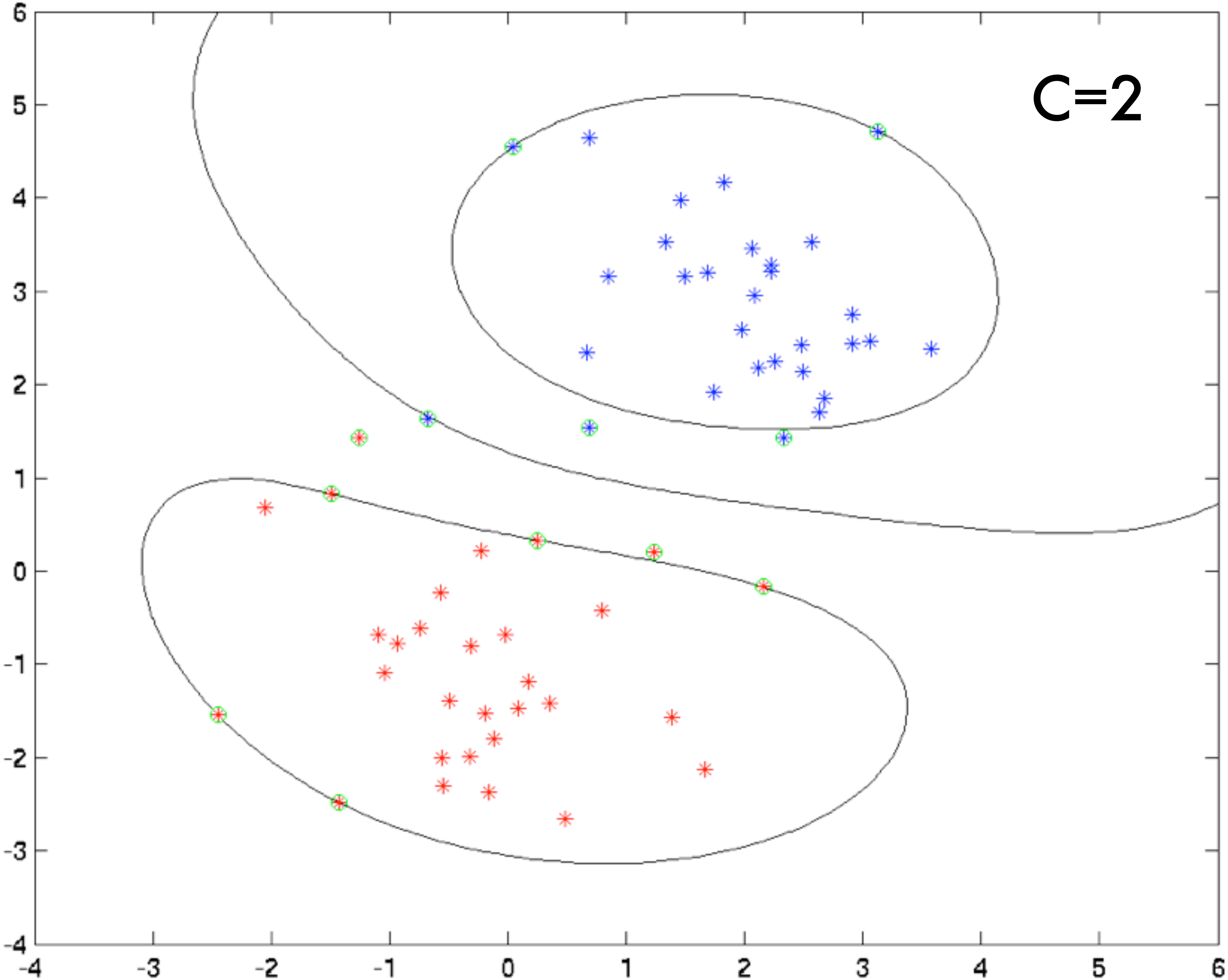
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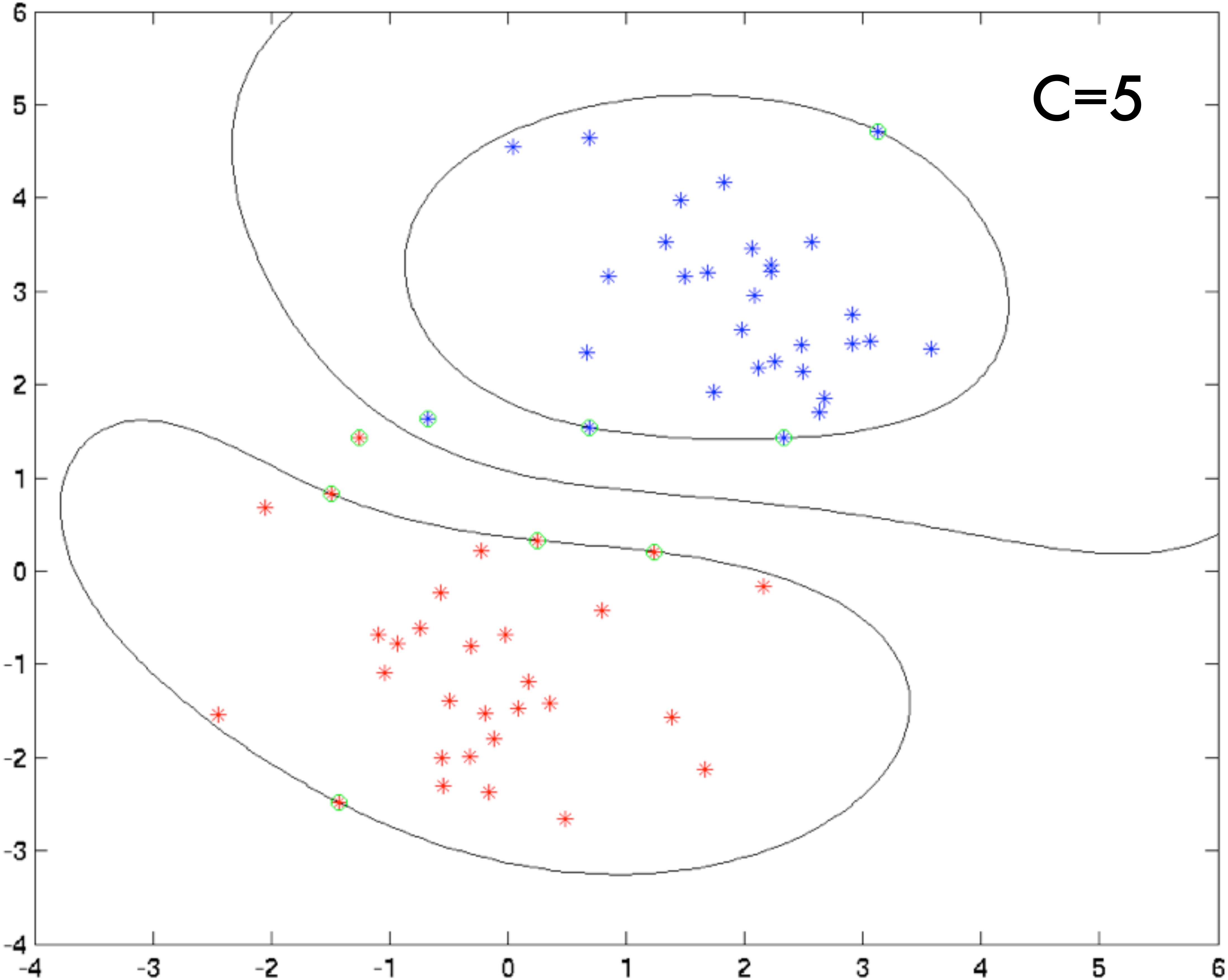

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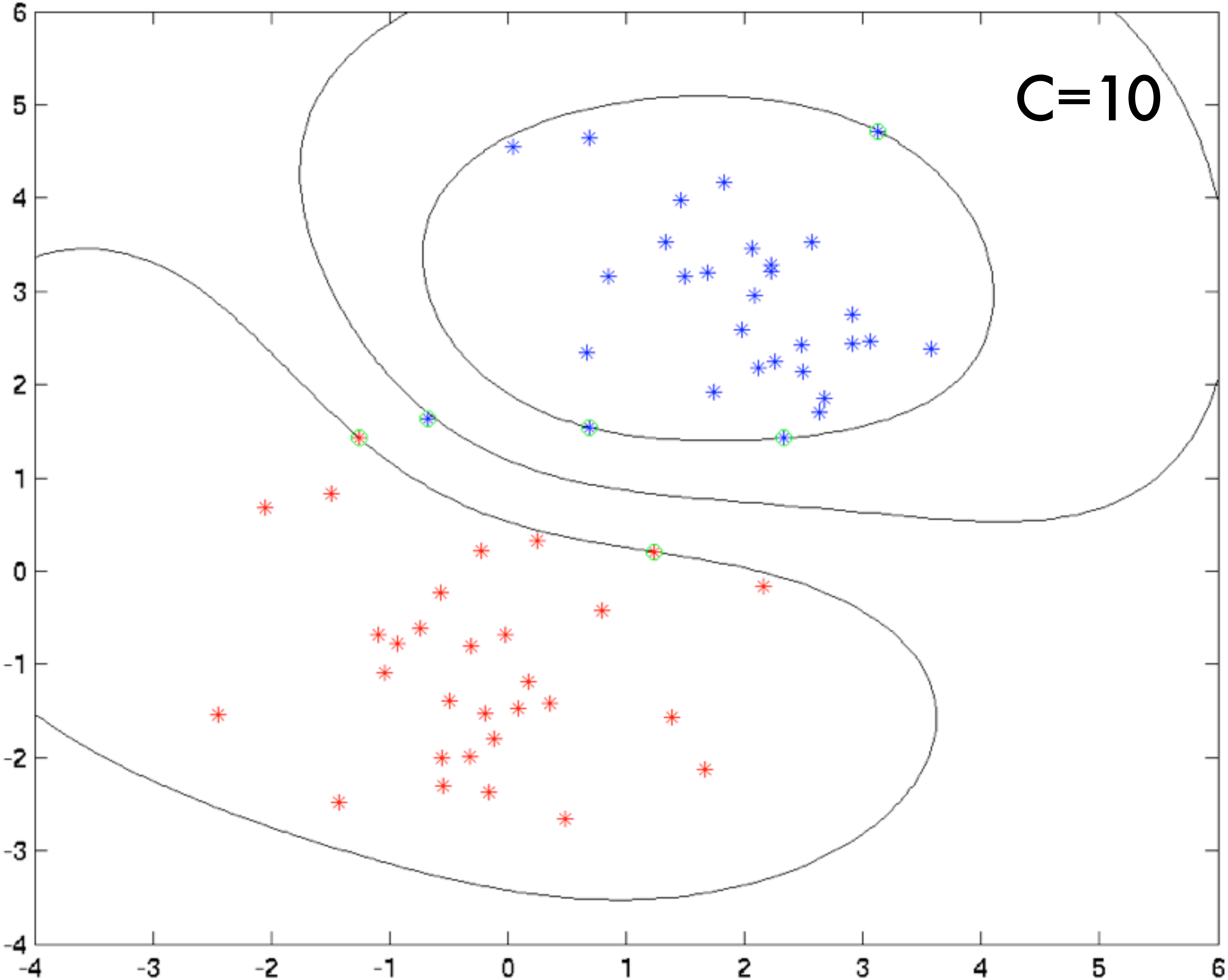
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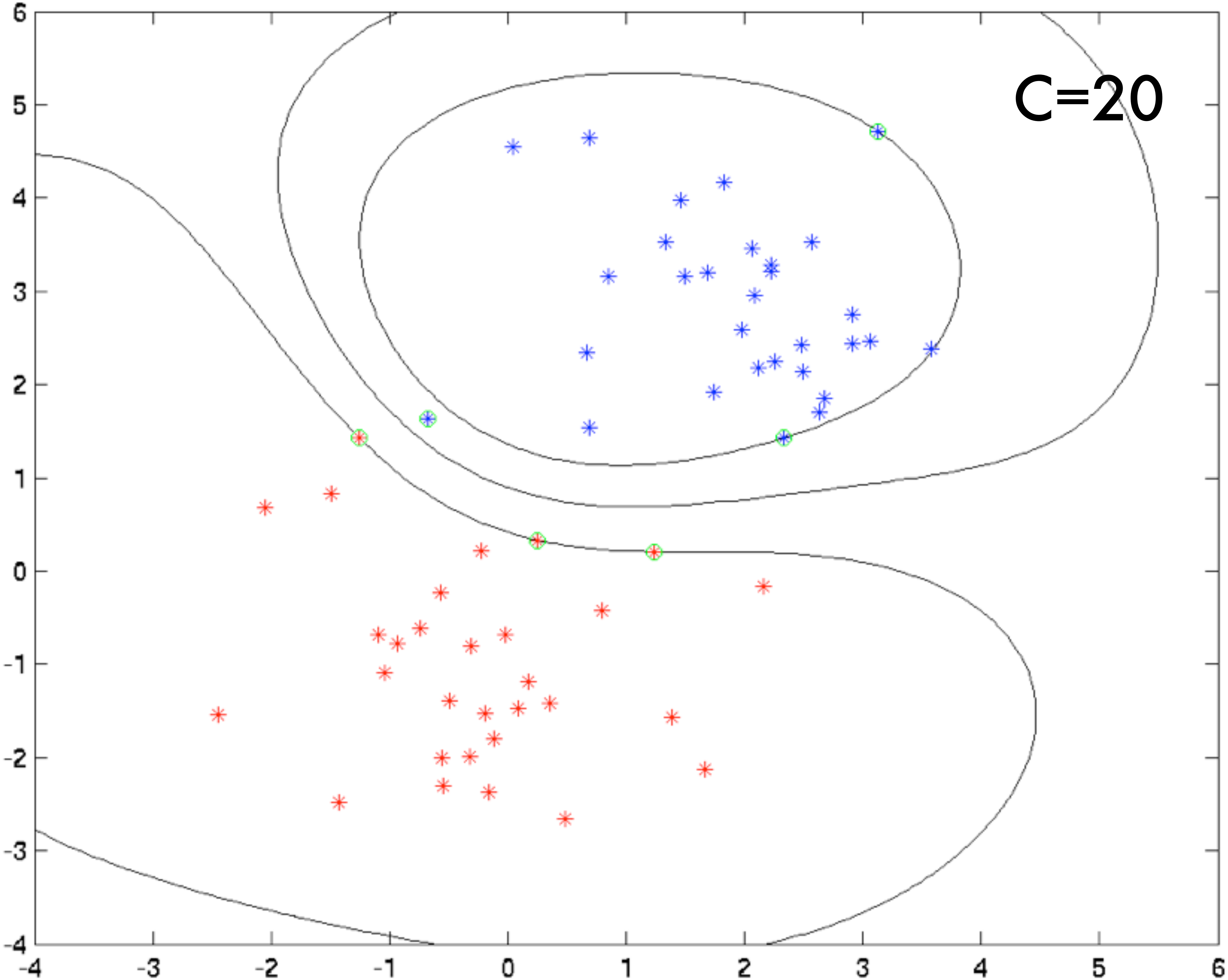
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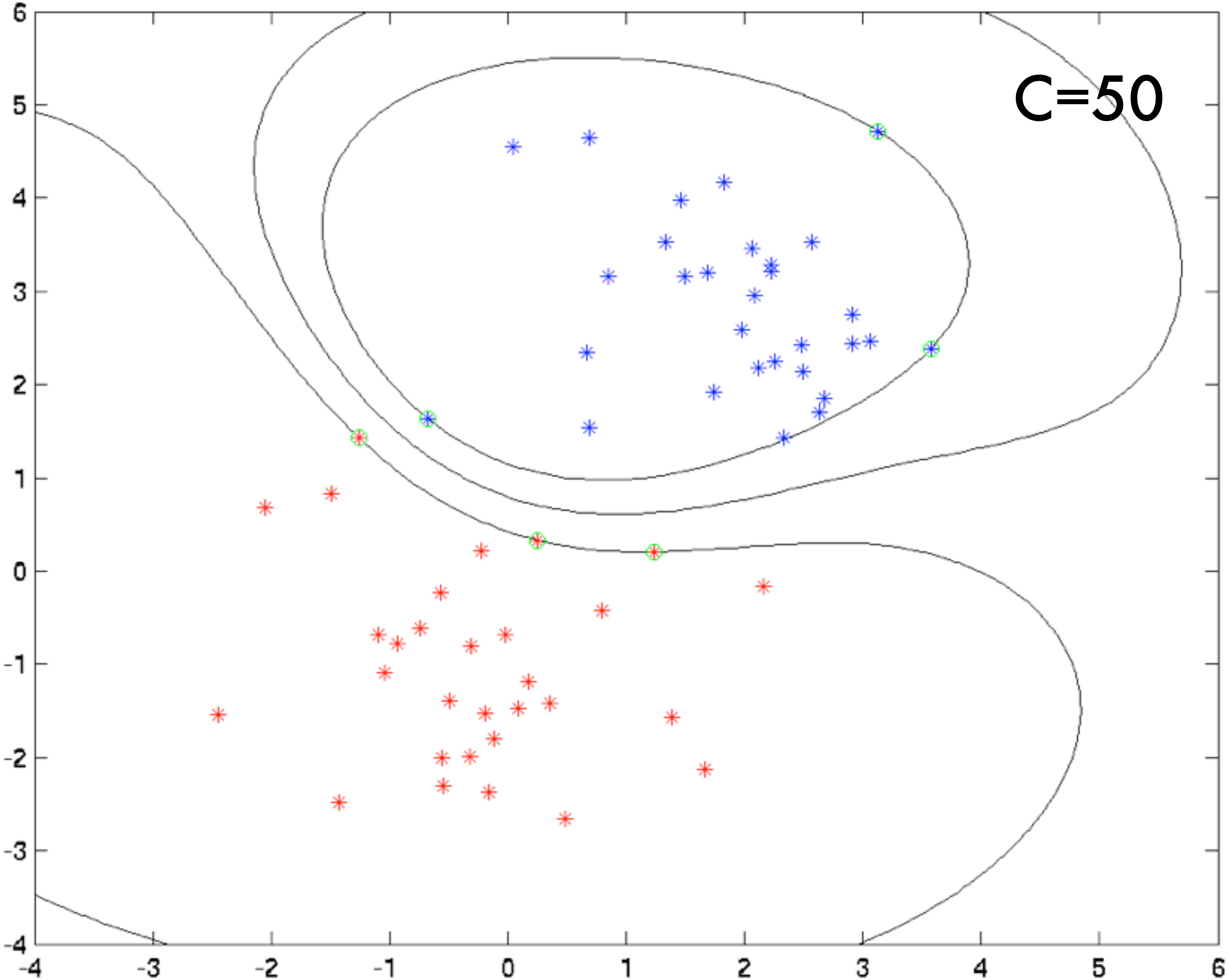
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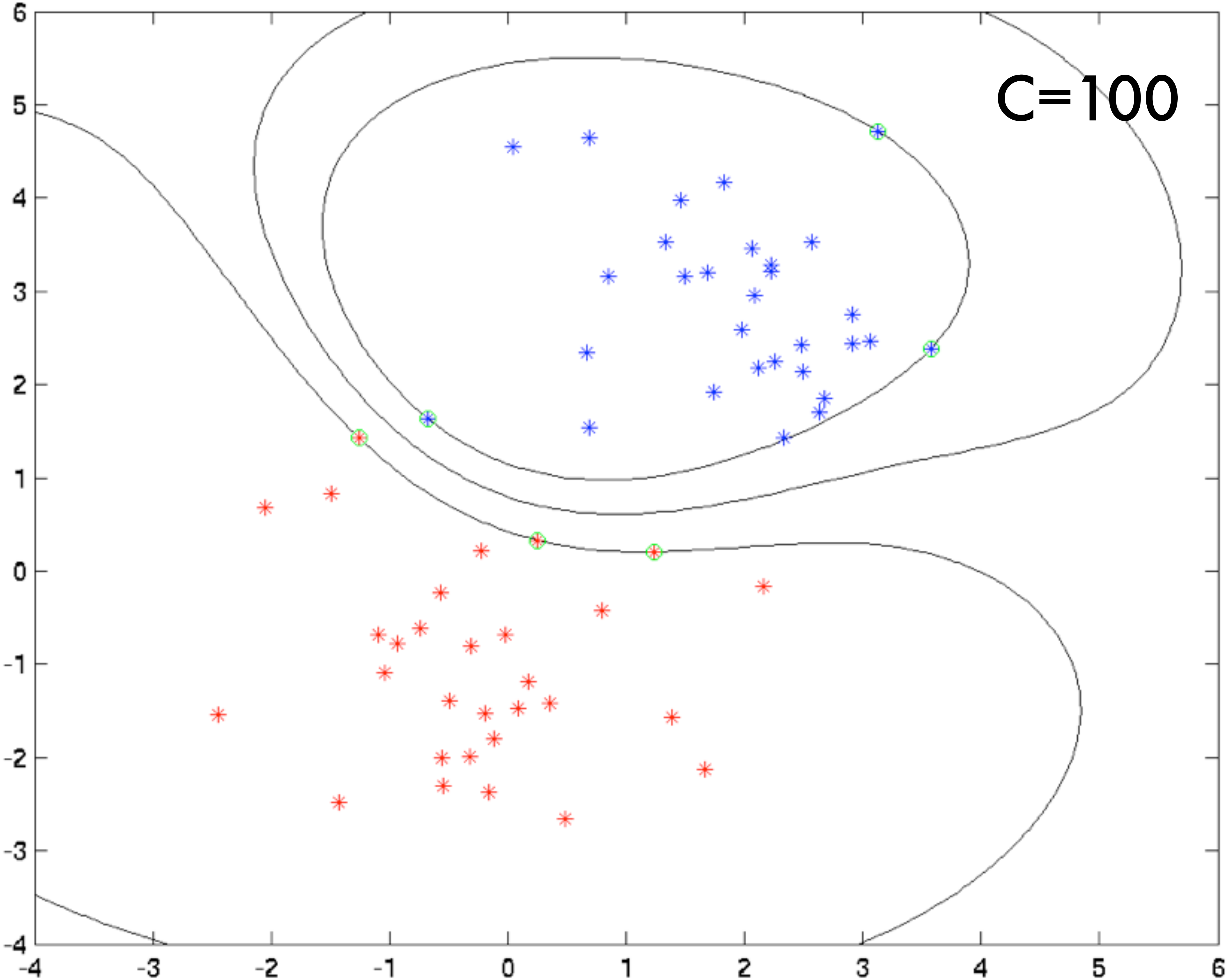
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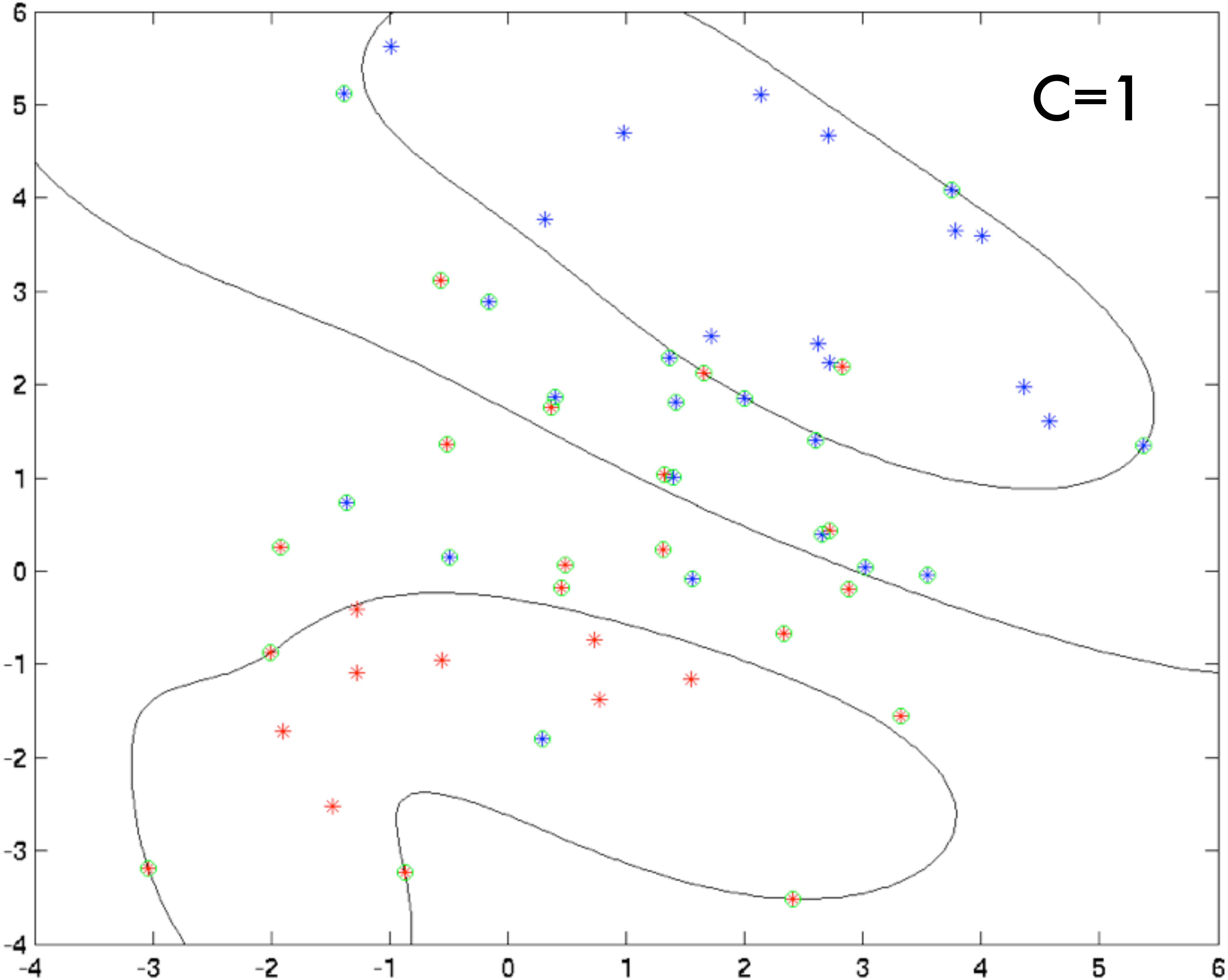

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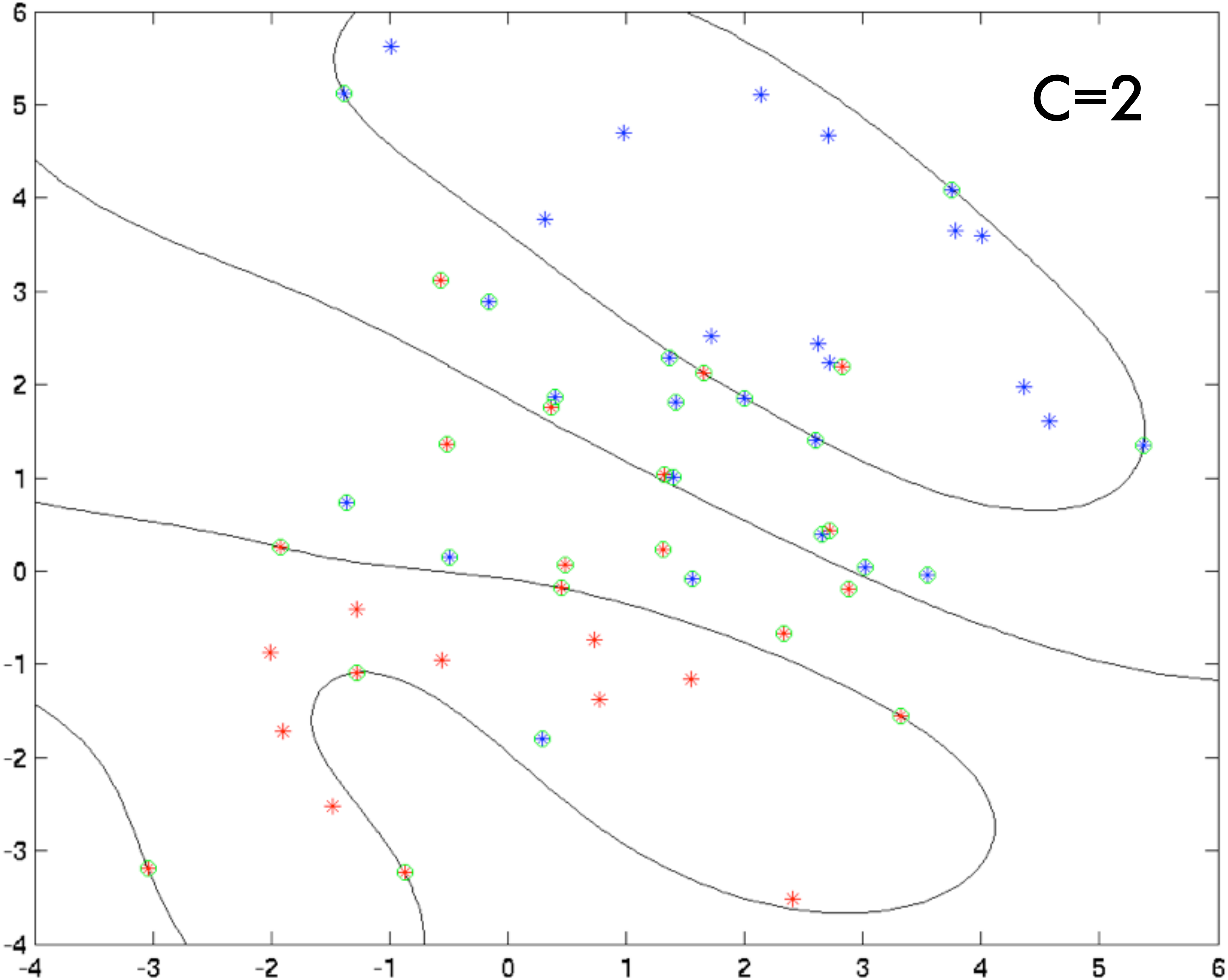
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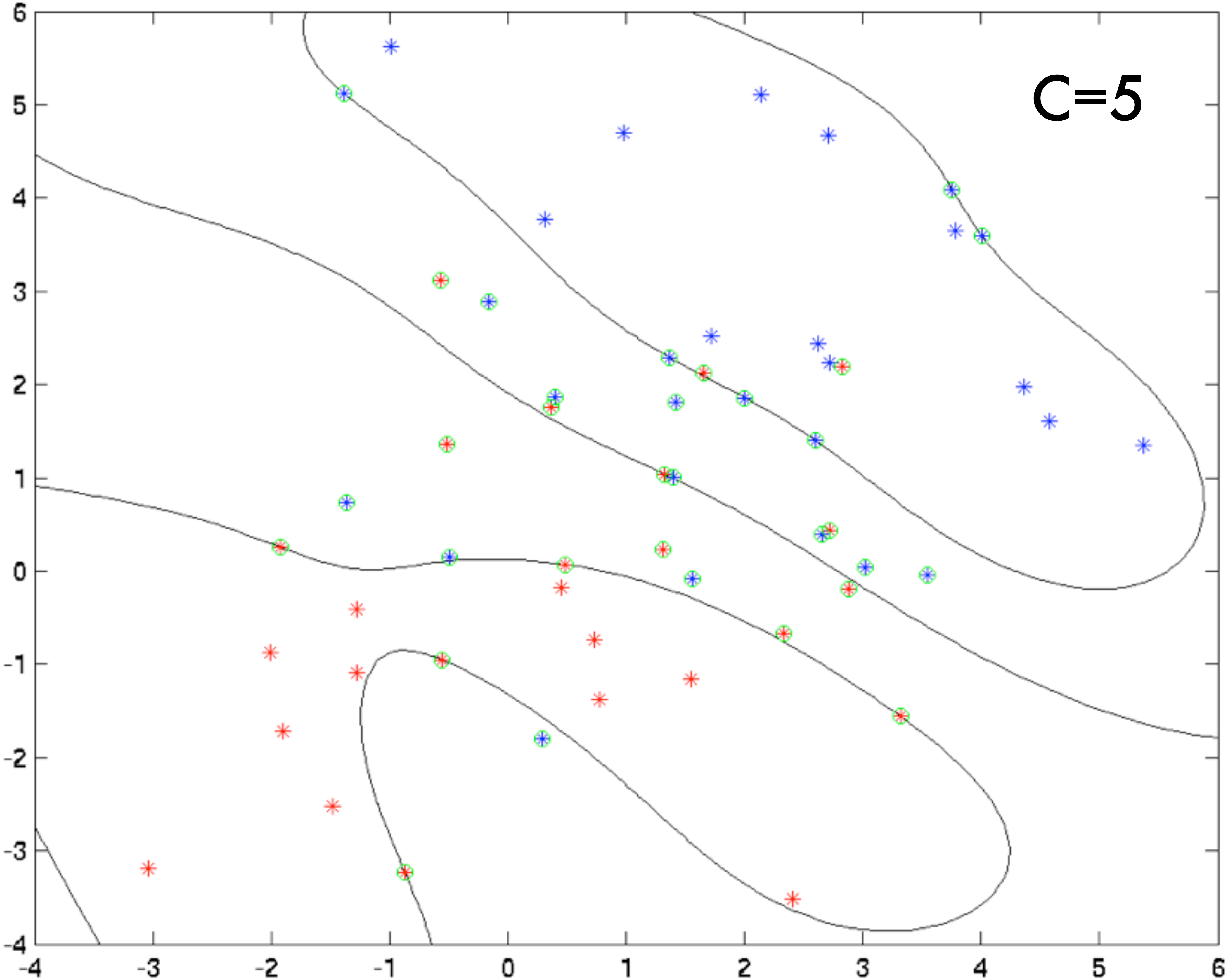

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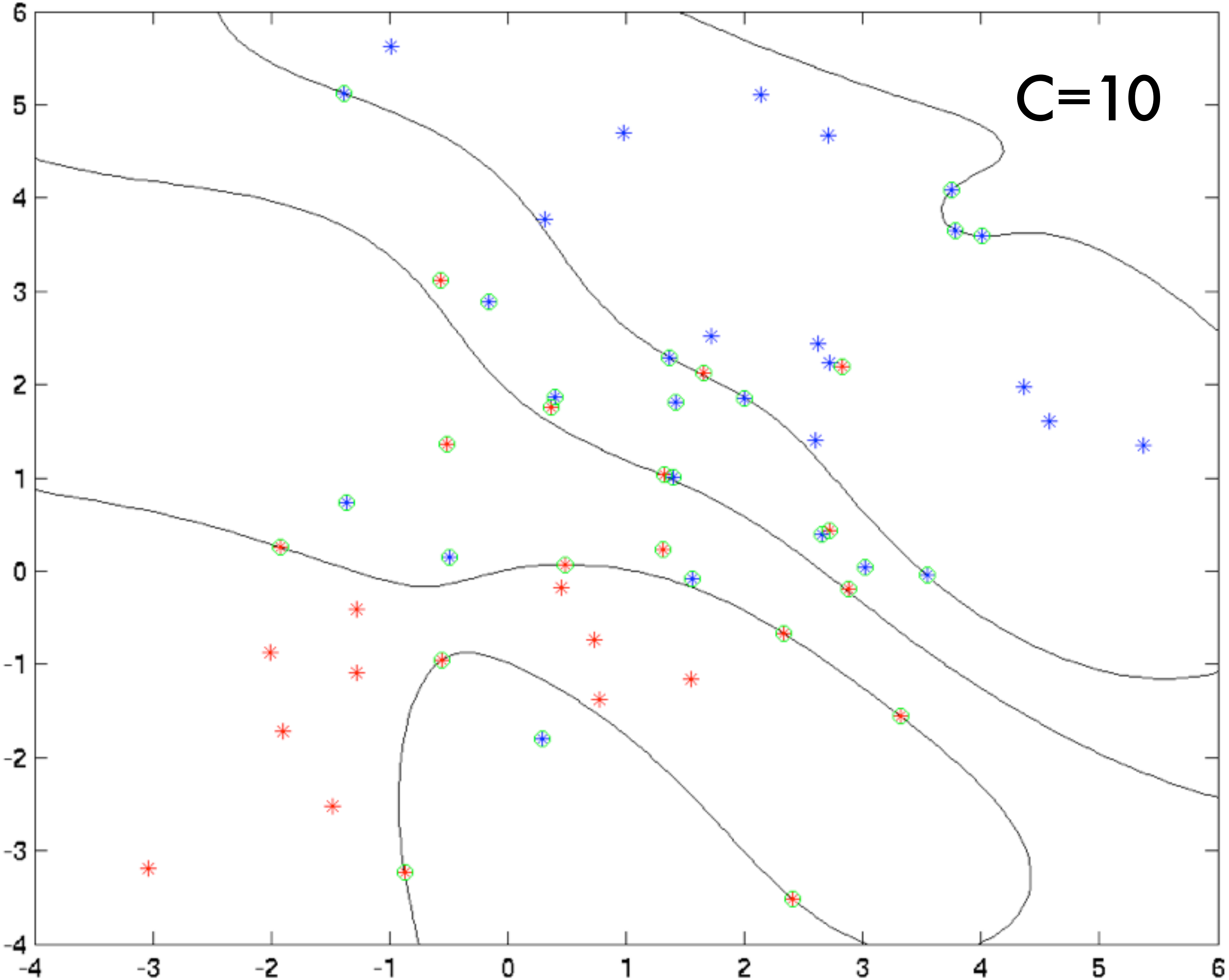

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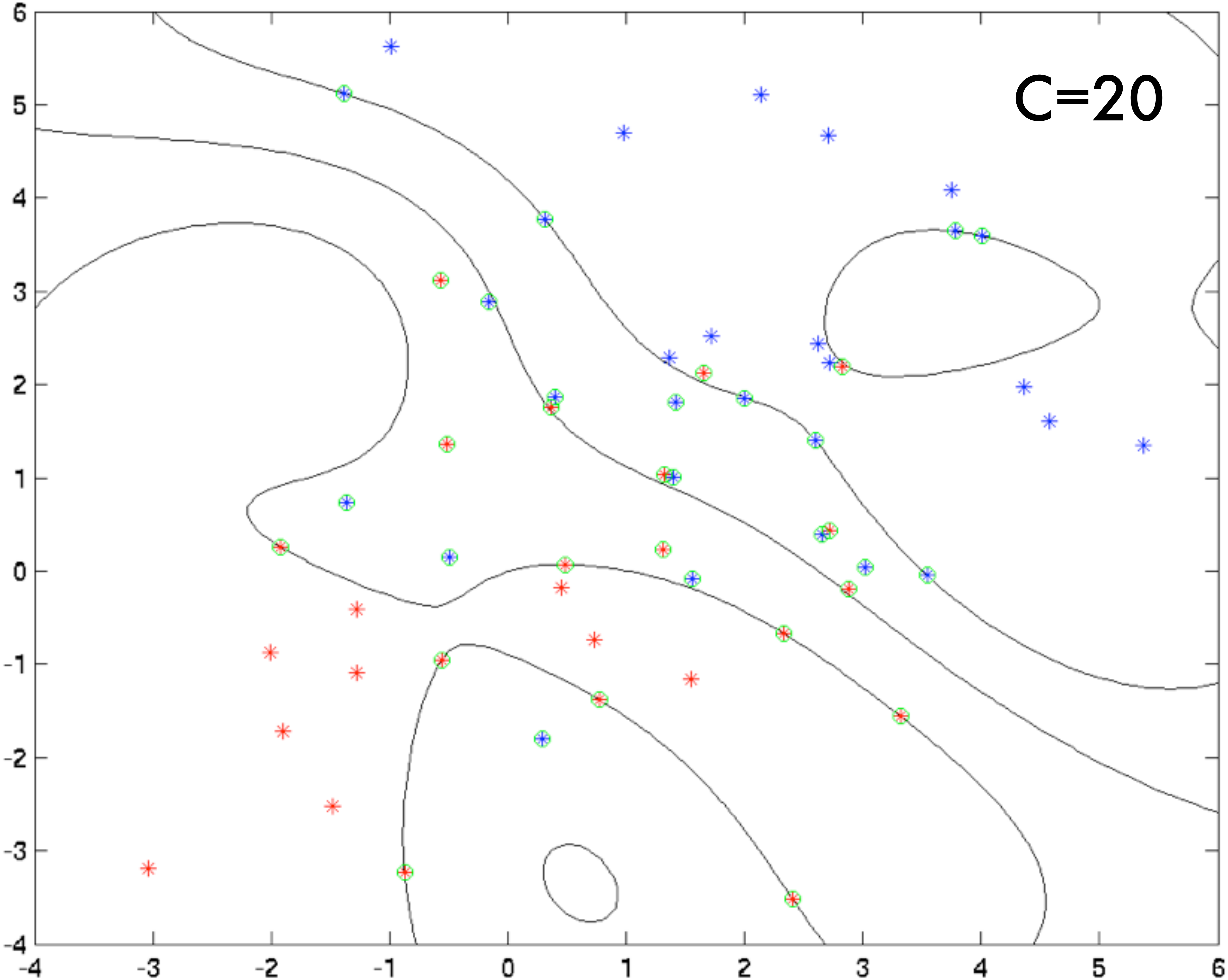
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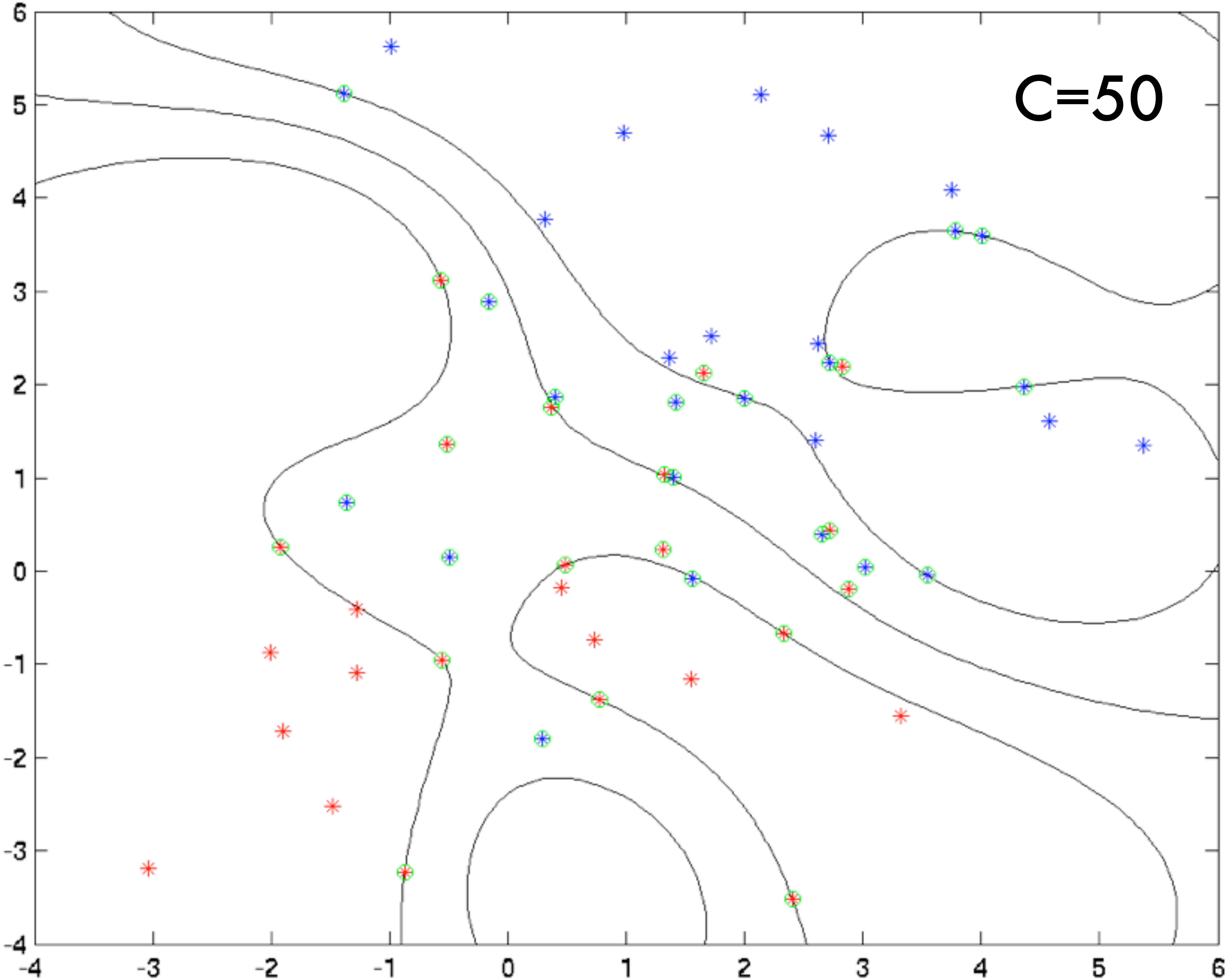
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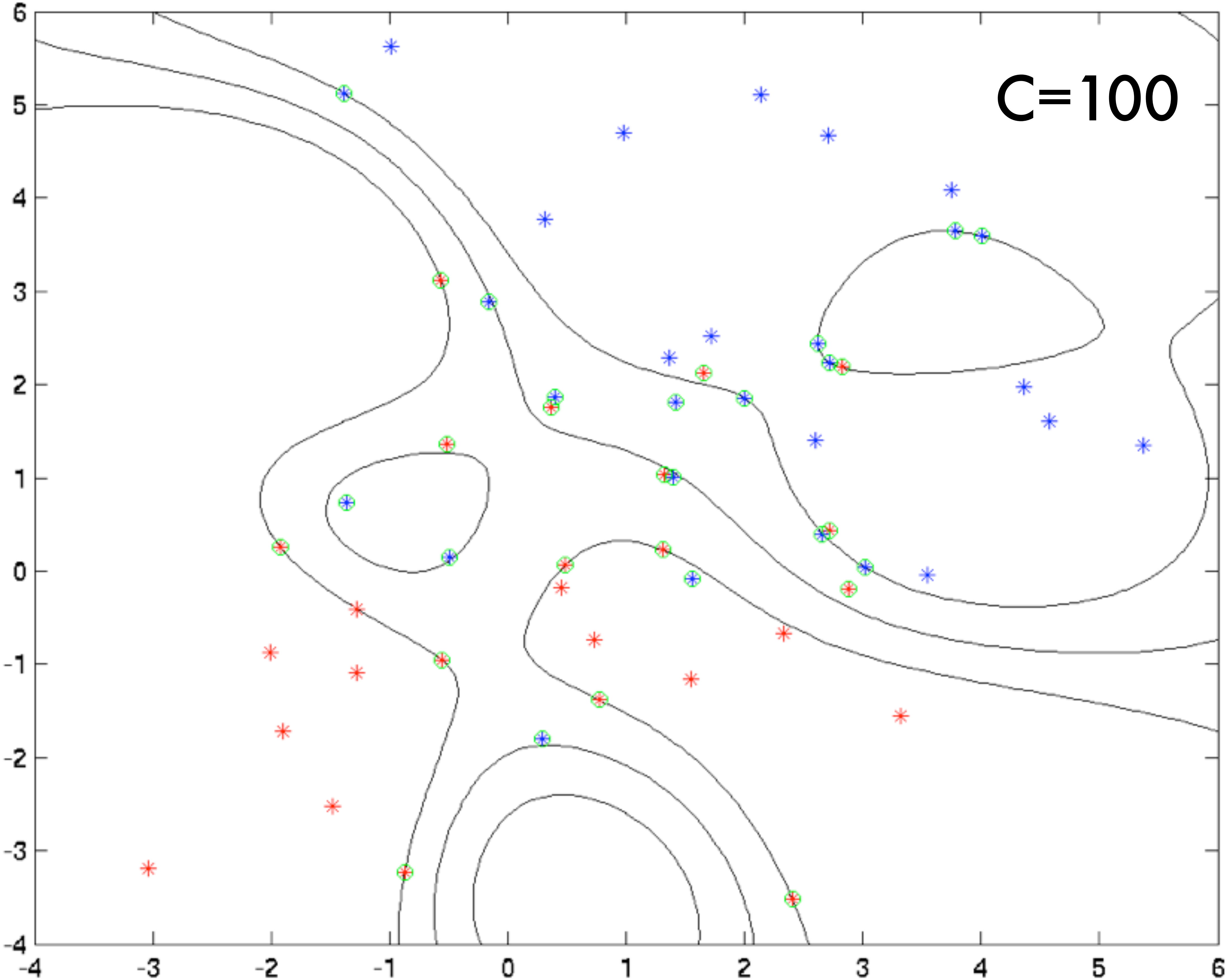

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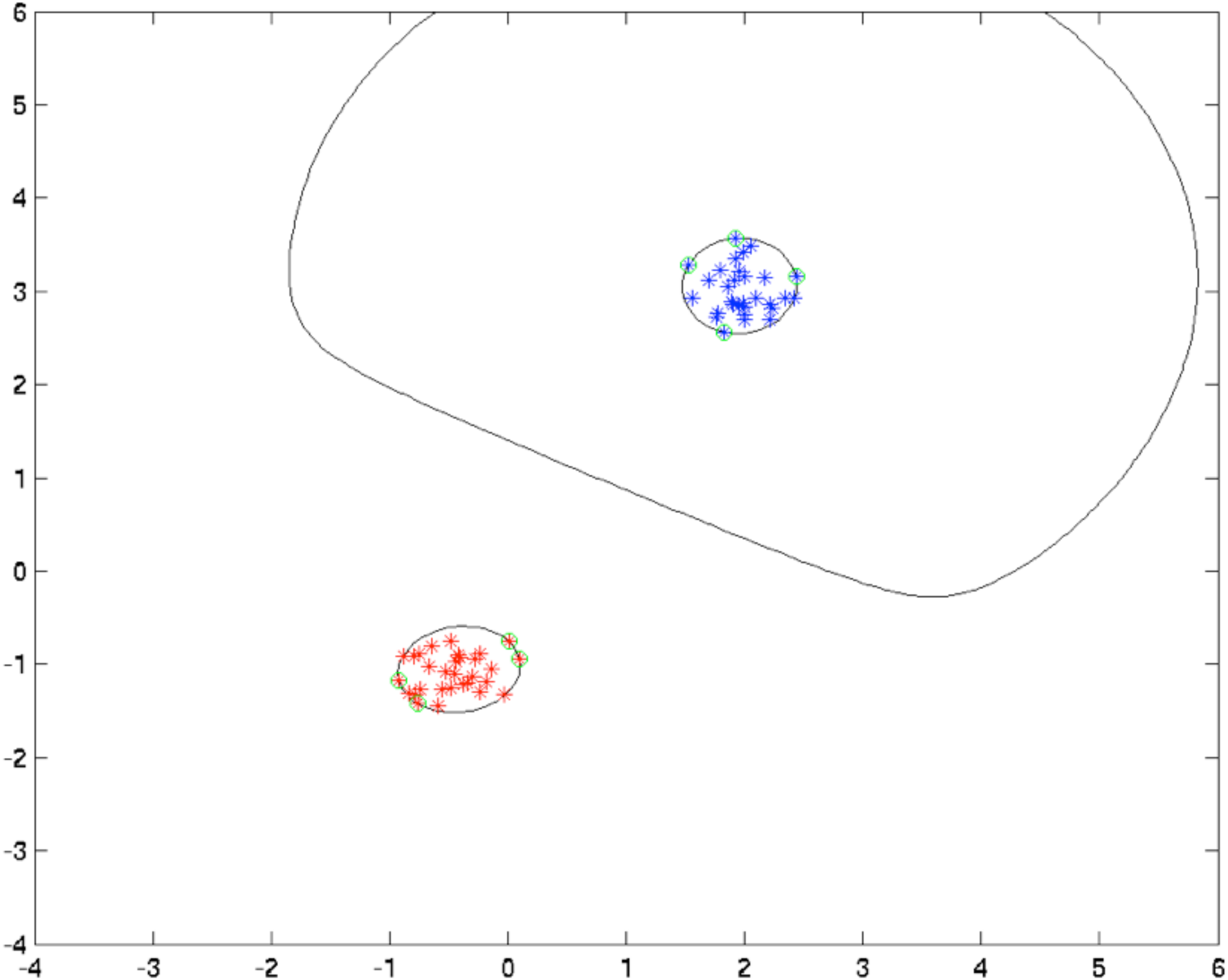
now with a narrower kernel

courtesy: A. Smola (CMU, 2013)



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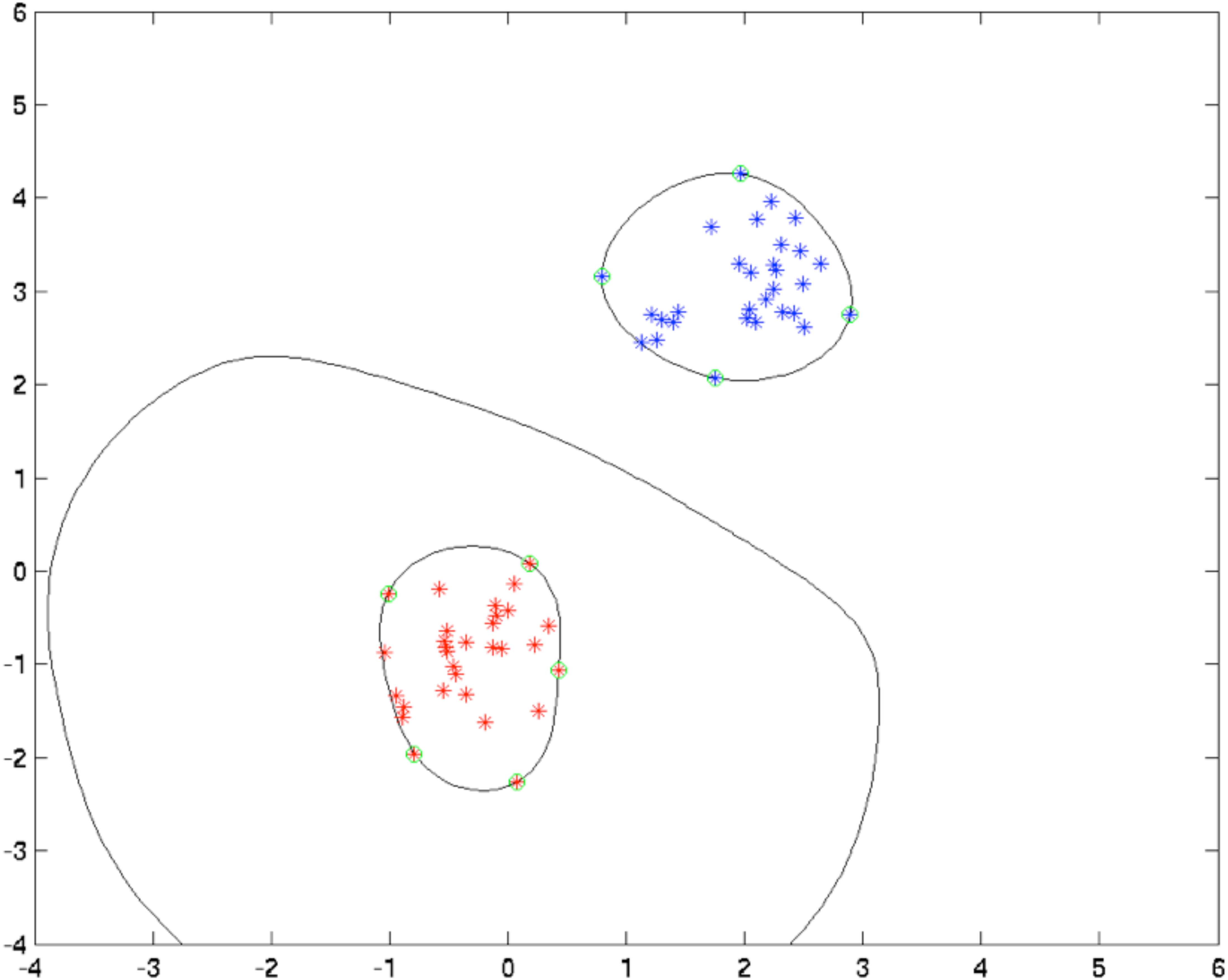
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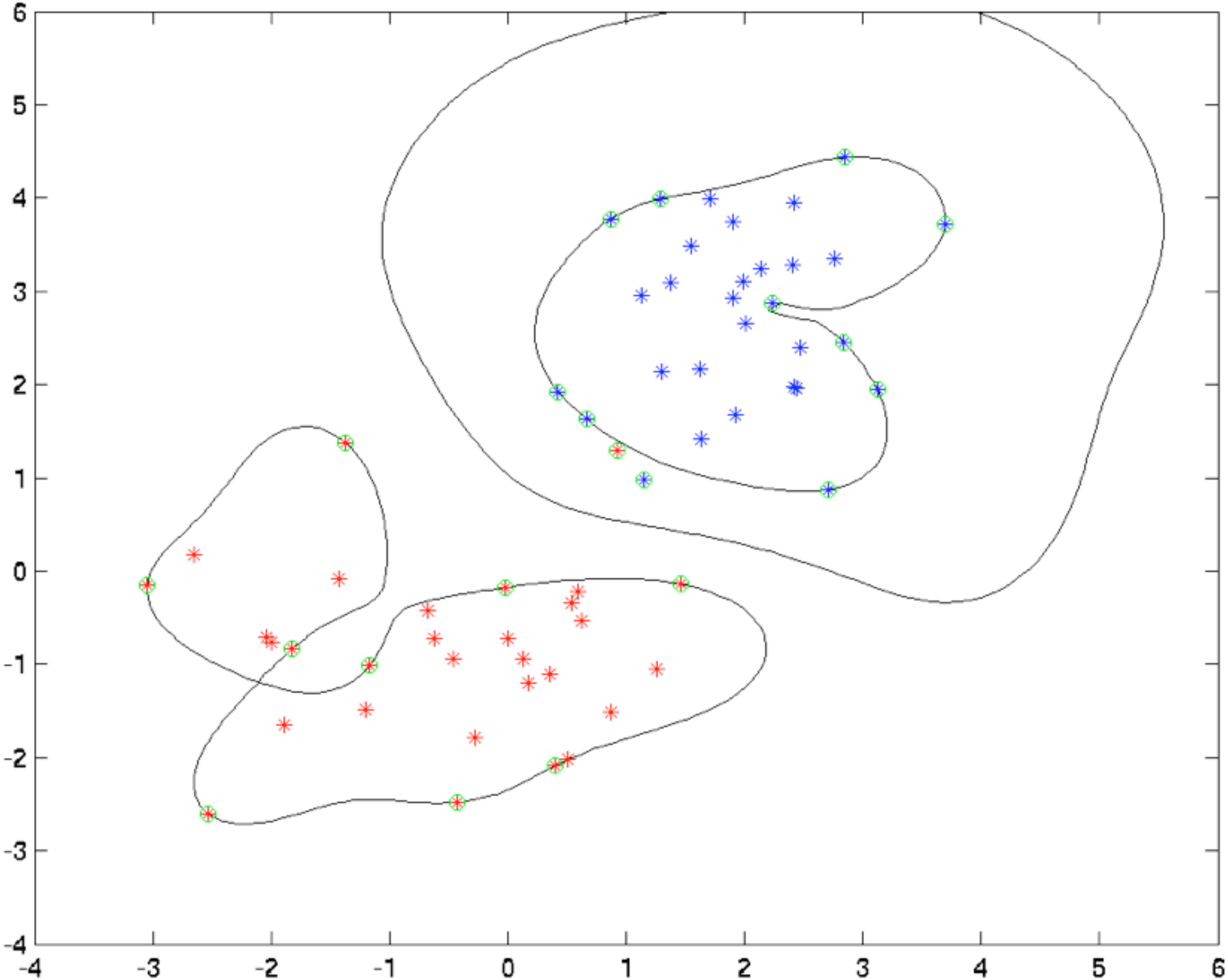
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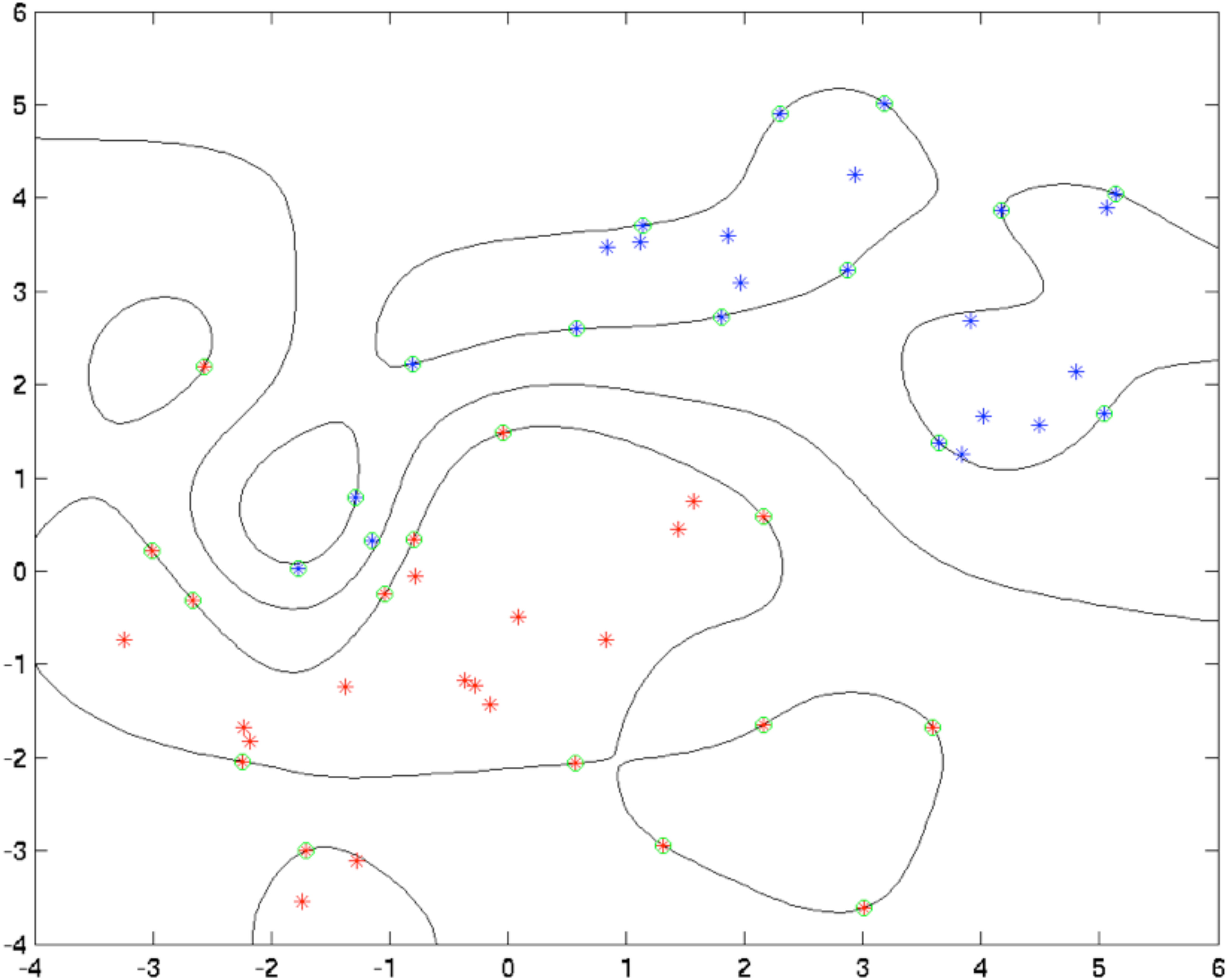

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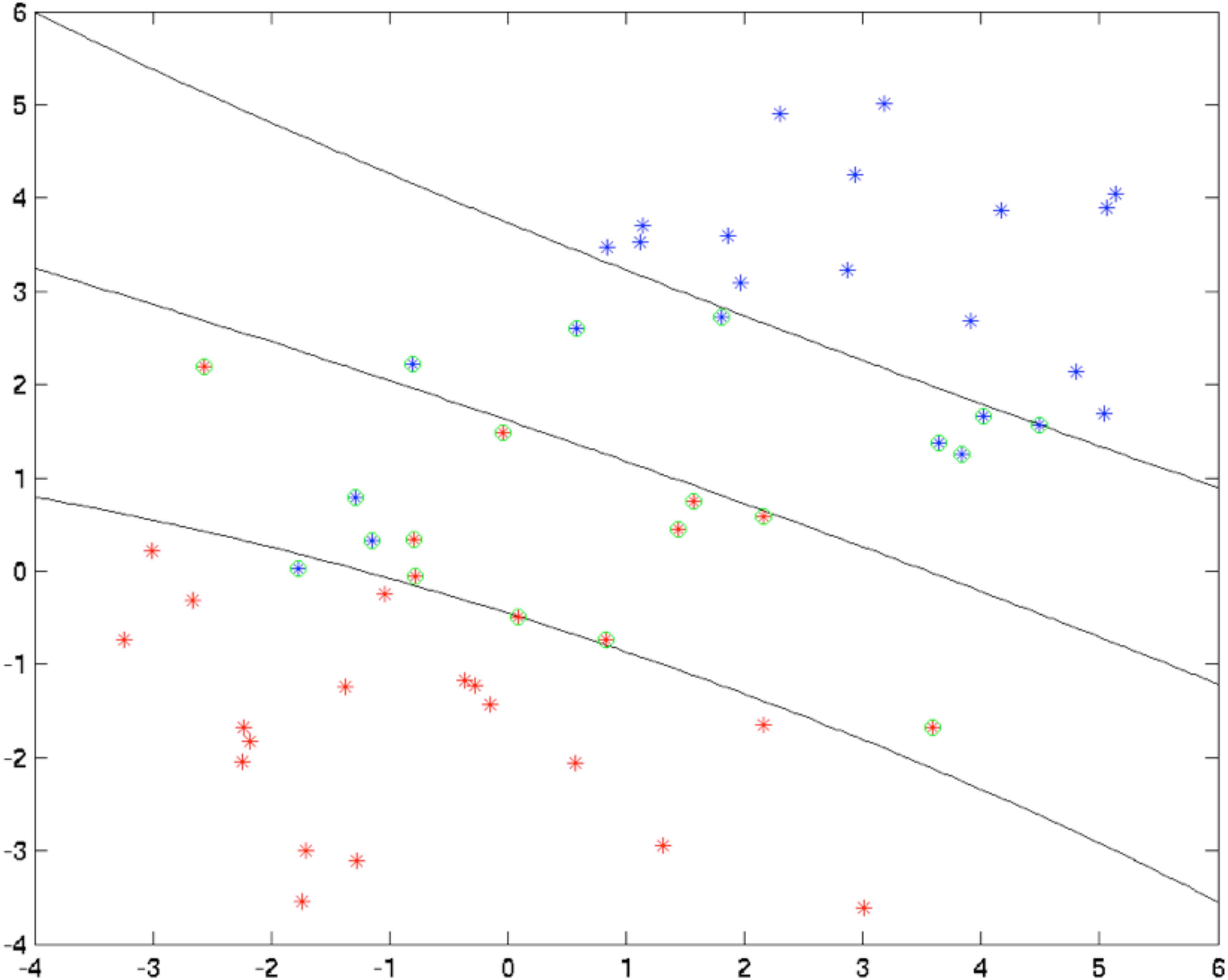
now with a very wide kernel

courtesy: A. Smola (CMU, 2013)



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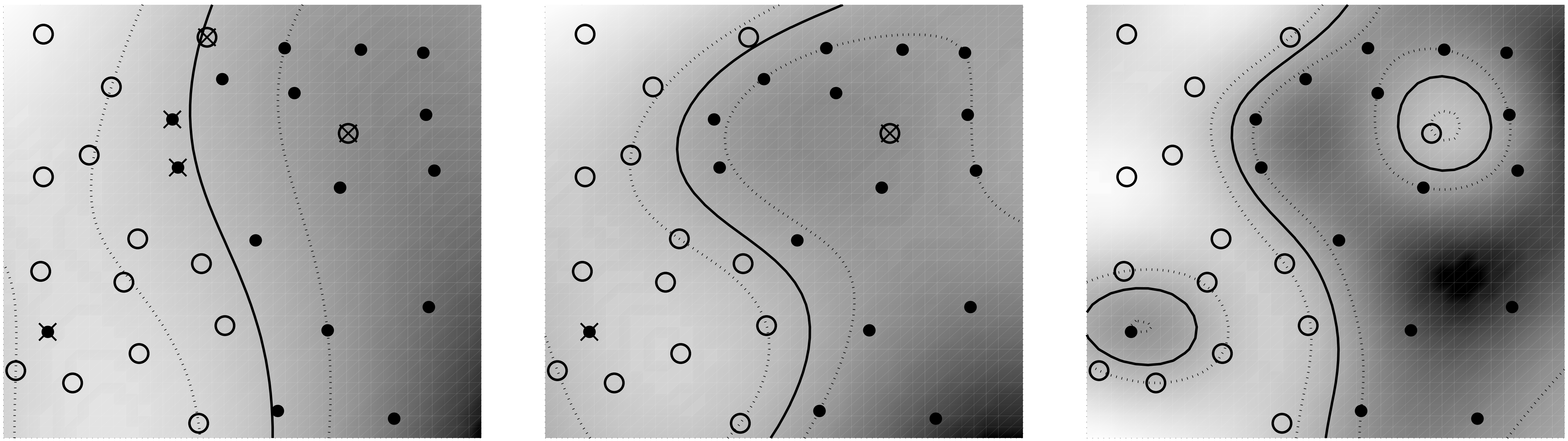
Nonlinear separation



- Increasing C allows for more nonlinearities
- Decreases number of errors
- SV boundary need not be contiguous
- Kernel width adjusts function class

courtesy: A. Smola (CMU, 2013)