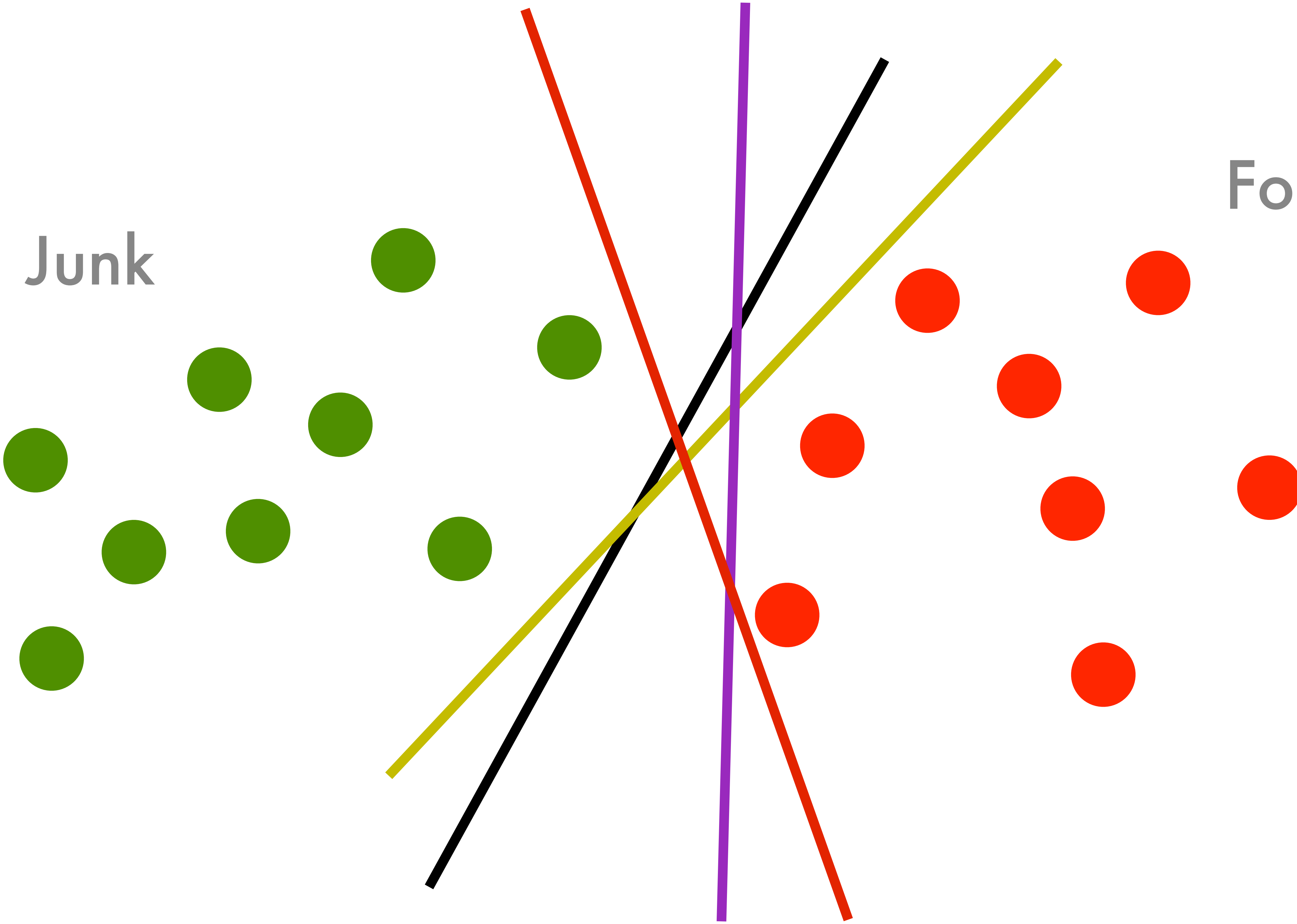
6.867
Support Vector Machines

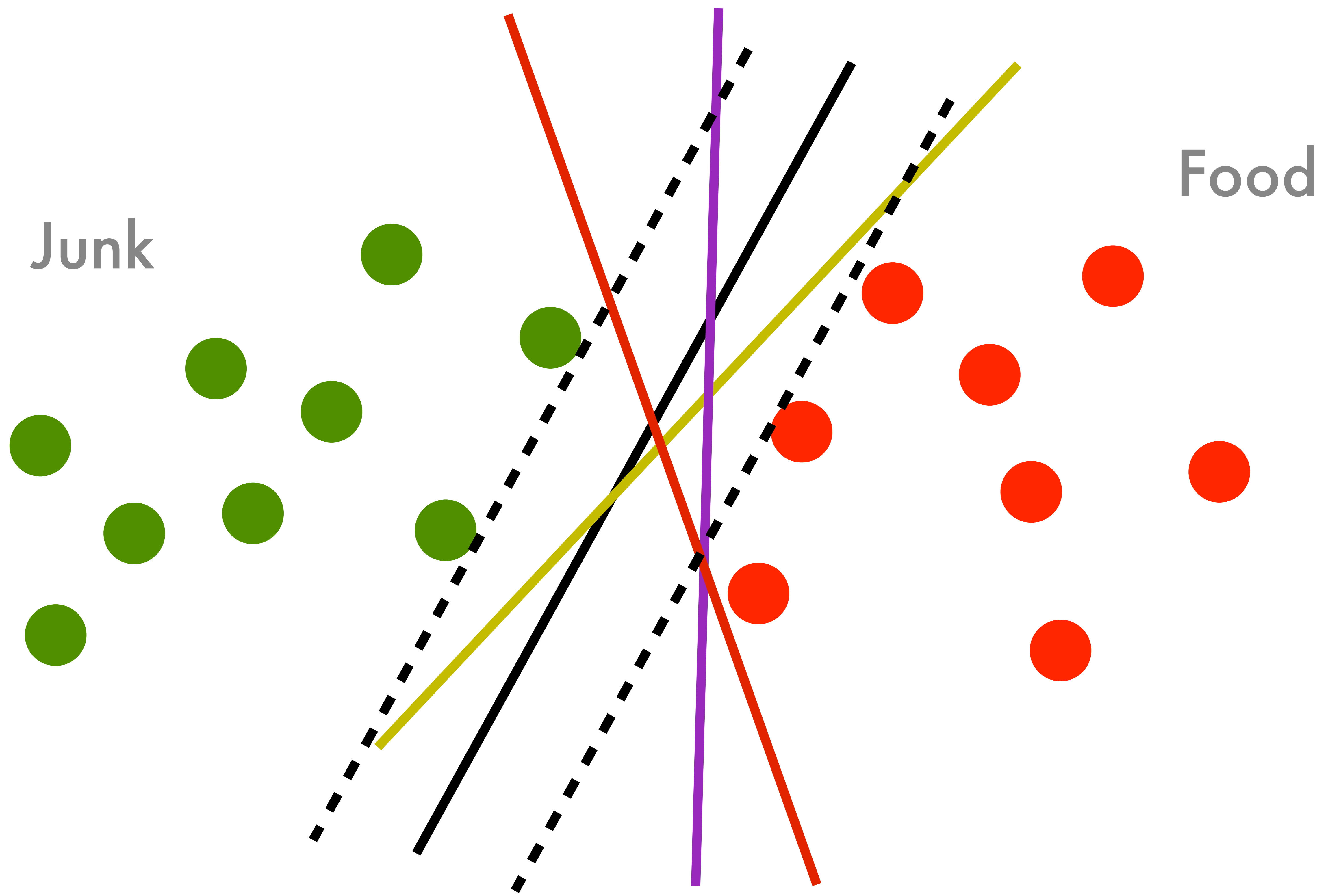
Fall 2016
Suvrit Sra

Linear Separator



credit: A. Smola (CMU, 2013)

Linear Separator

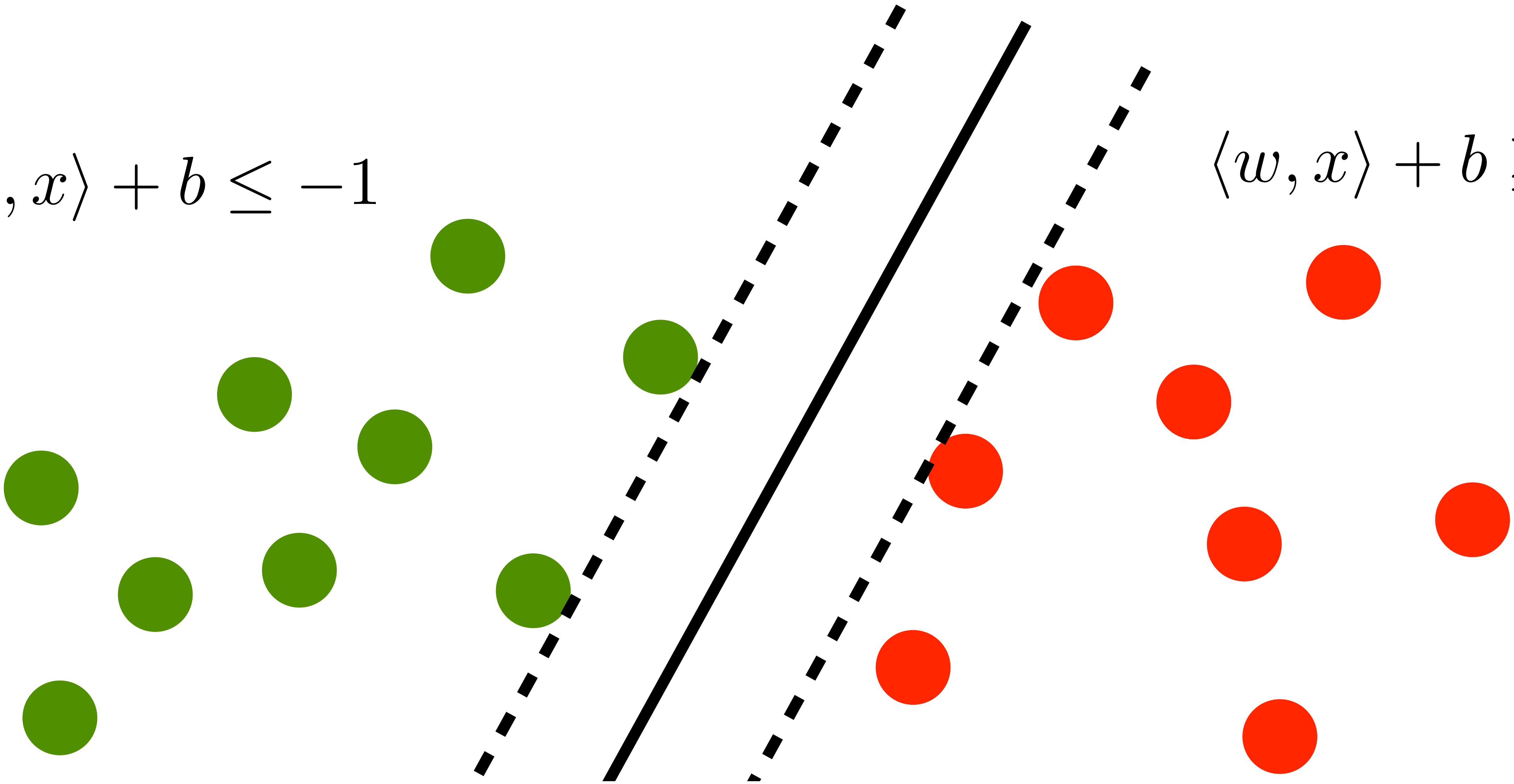


credit: A. Smola (CMU, 2013)

Margin

$$\langle w, x \rangle + b \leq -1$$

$$\langle w, x \rangle + b \geq 1$$

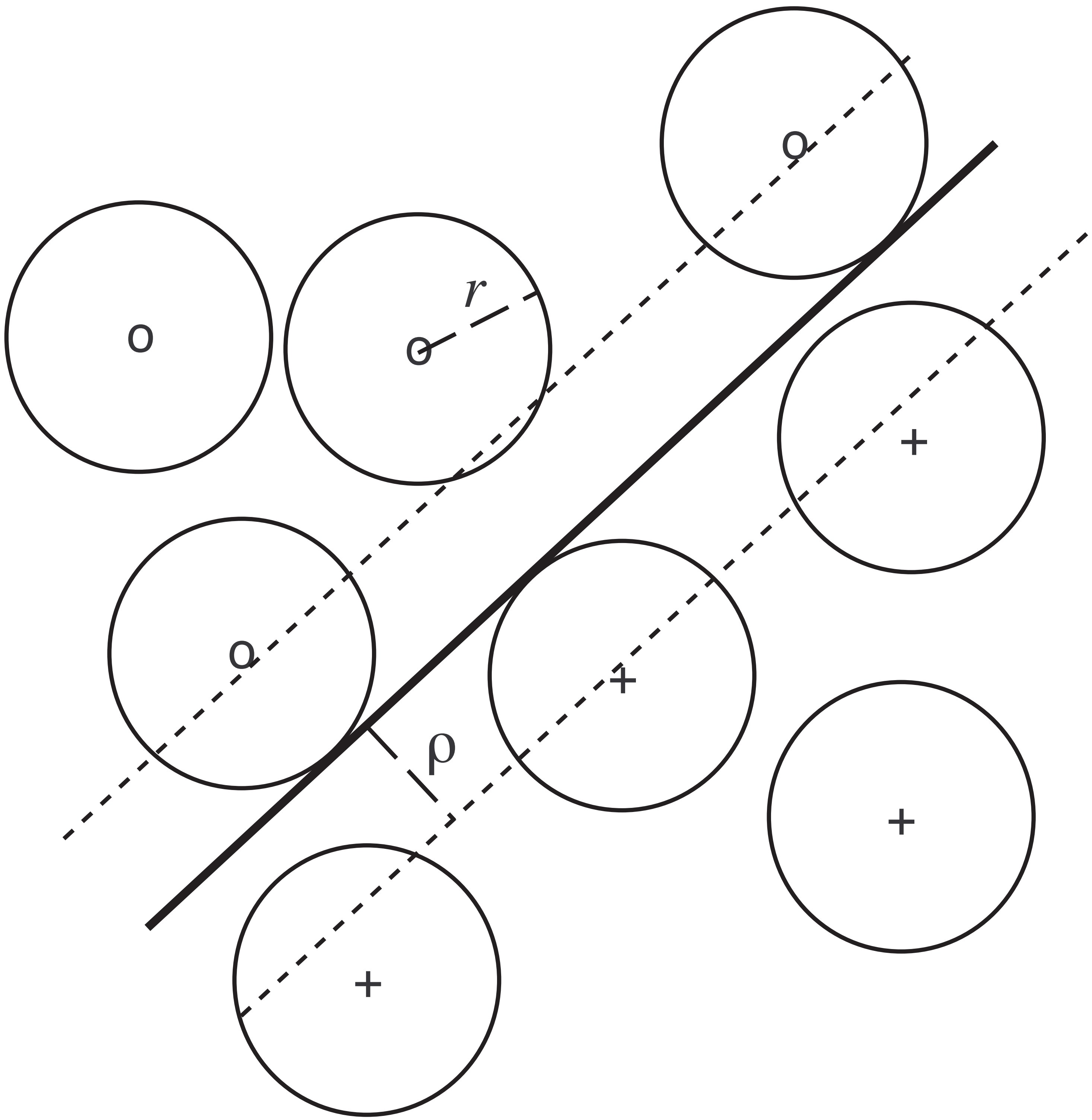


hyperplane

$$f(x) = \langle w, x \rangle + b$$

credit: A. Smola (CMU, 2013)

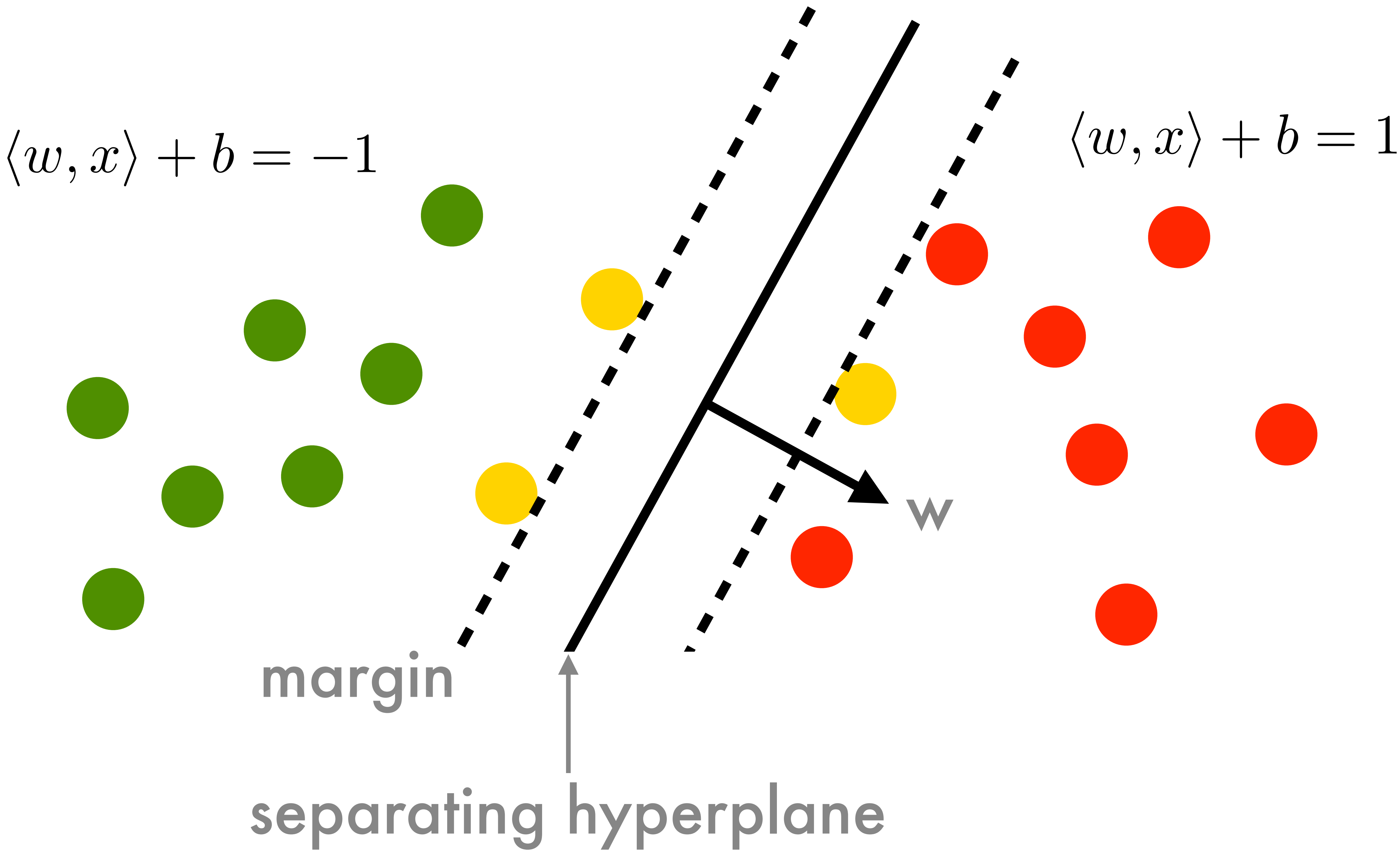
Why large margins?



- Maximum robustness relative to uncertainty
- Symmetry breaking
- Independent of correctly classified instances
- Easy to find for easy problems

credit: A. Smola (CMU, 2013)

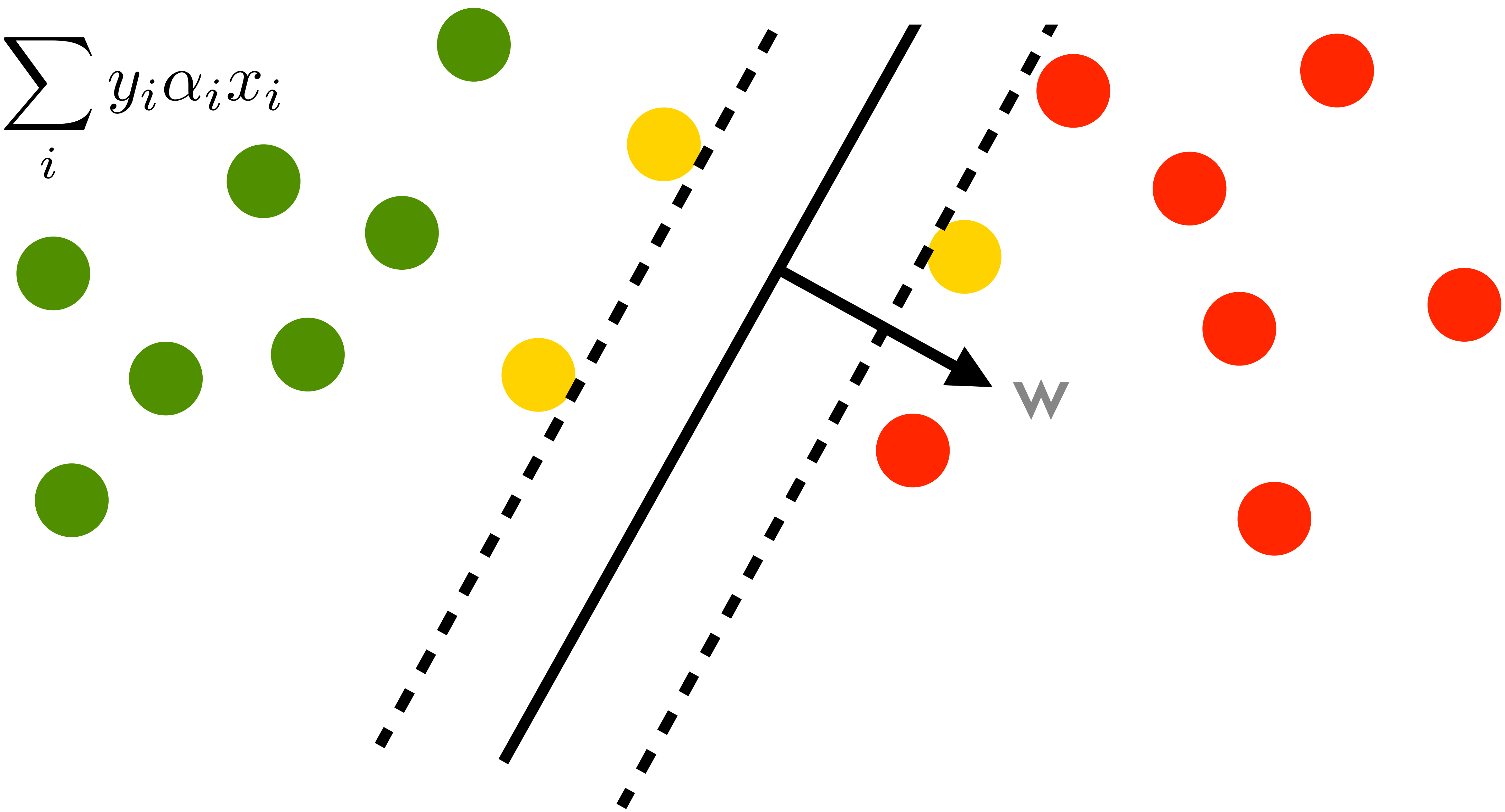
Large Margin Classifier



credit: A. Smola (CMU, 2013)

SVM Properties

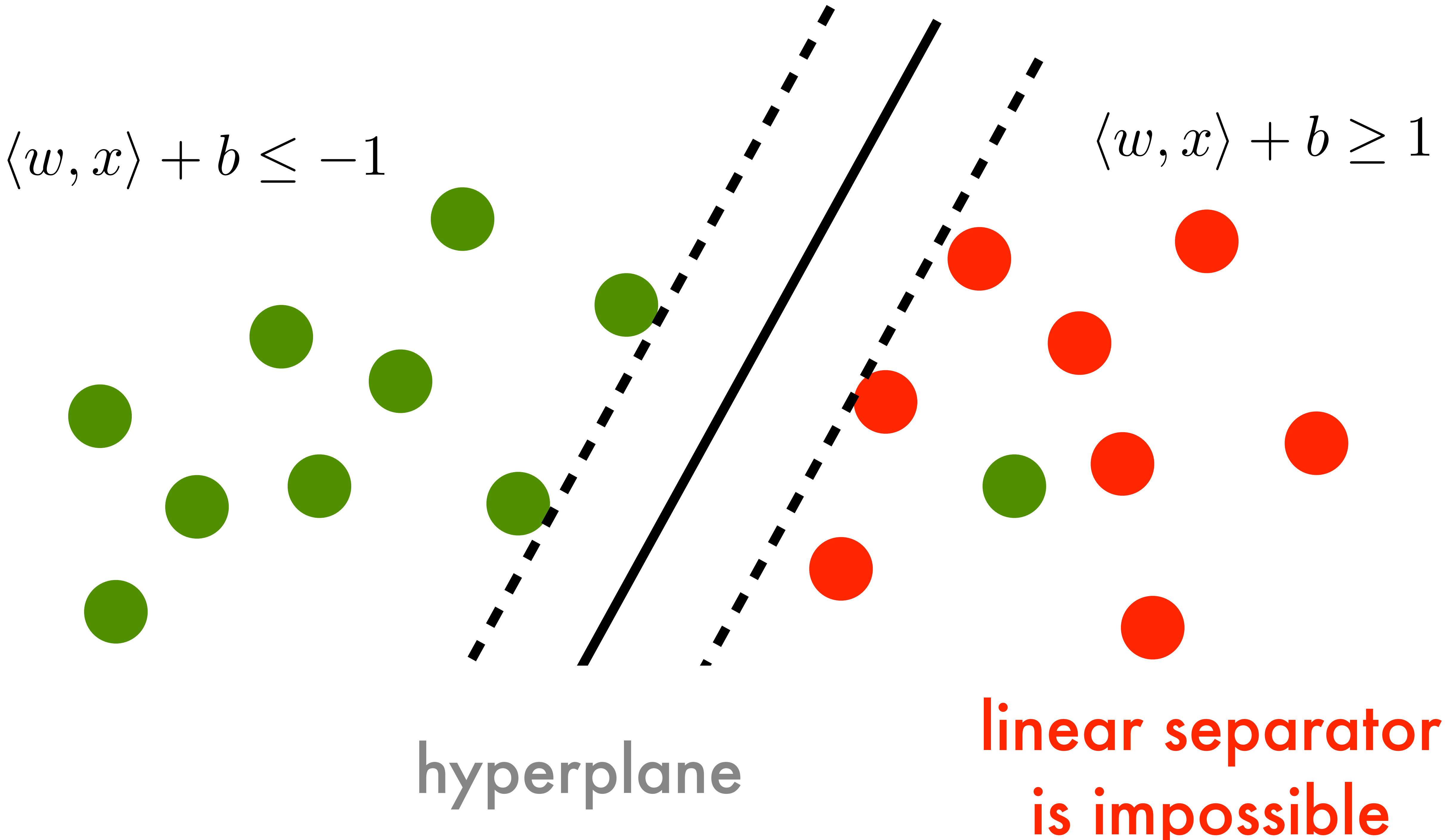
$$w = \sum_i y_i \alpha_i x_i$$



- Weight vector w as weighted linear combination of instances
- Only points on margin matter; ignore the rest, solution remains unchanged
- Keeps instances away from the margin

credit: A. Smola (CMU, 2013)

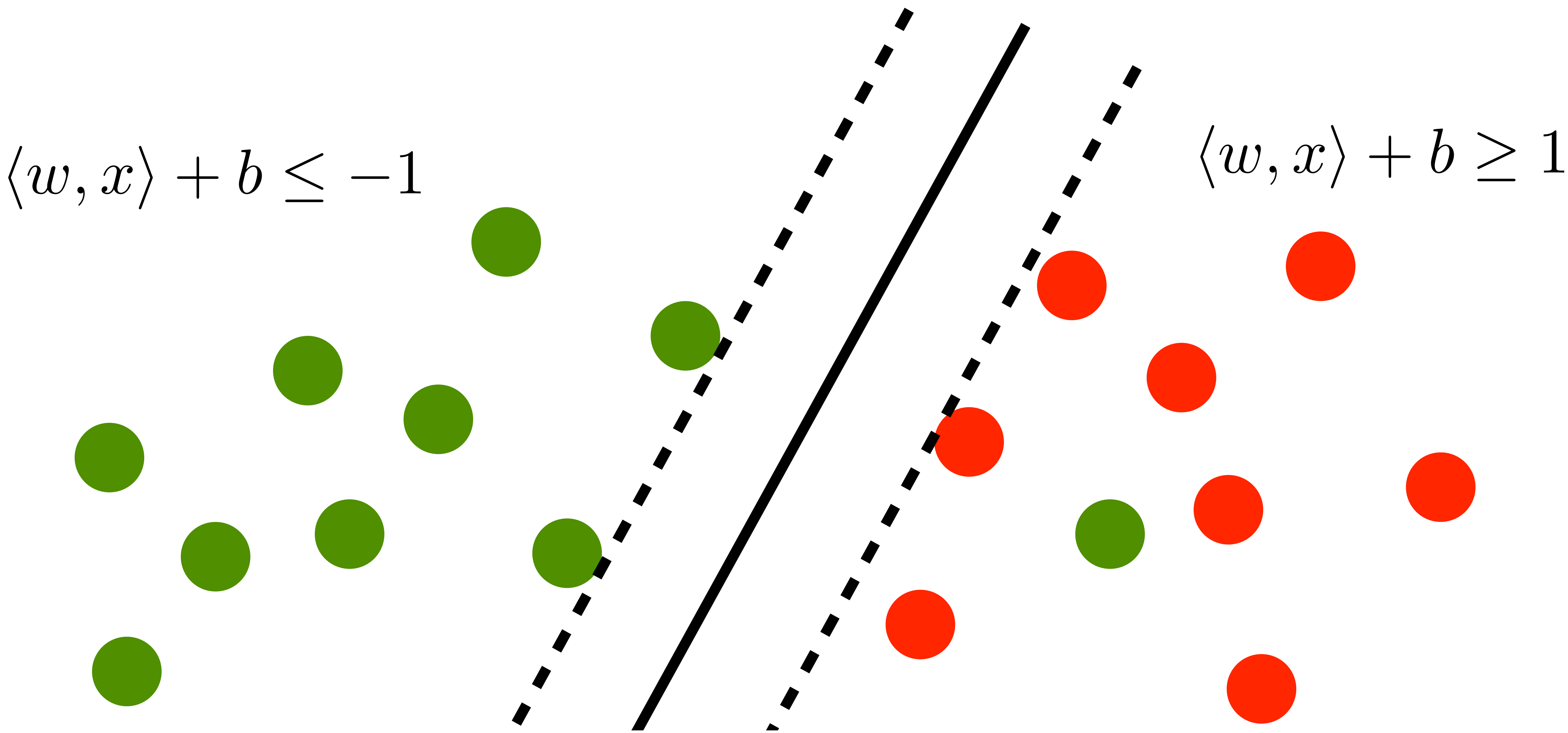
Inseparable data



$f(x) = \langle w, x \rangle + b$

credit: A. Smola (CMU, 2013)

Inseparable data



minimum error separator is impossible

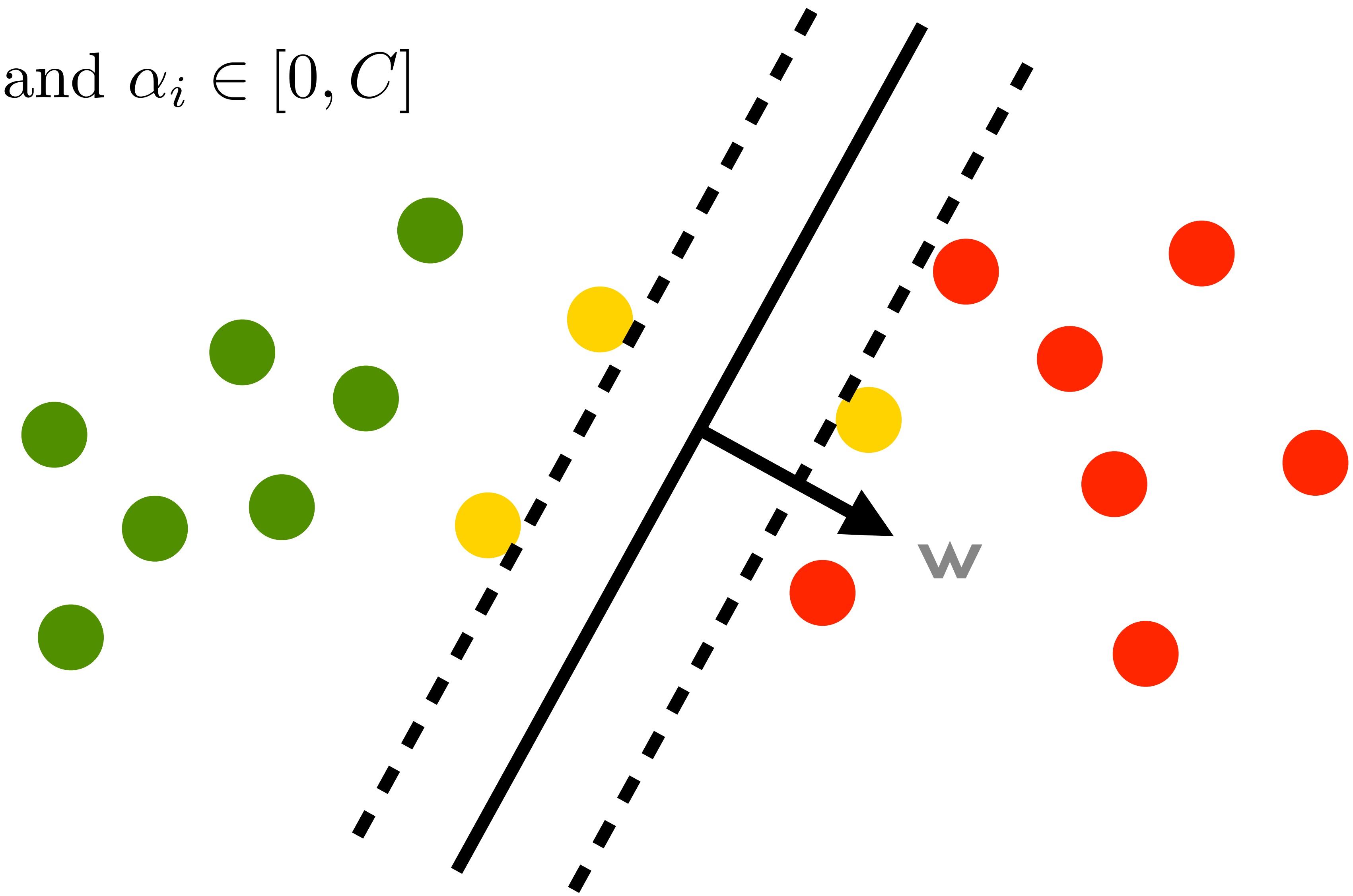
Theorem (Minsky & Papert)

Finding the minimum error separating hyperplane is NP-hard

credit: A. Smola (CMU, 2013)

$$\underset{\alpha}{\text{maximize}} -\frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j \langle x_i, x_j\rangle + \sum_i \alpha_i$$

$$\text{subject to } \sum_i \alpha_i y_i = 0 \text{ and } \alpha_i \in [0, C]$$



$$w = \sum_i y_i \alpha_i x_i$$

$$\alpha_i \left[y_i \left[\langle w, x_i\rangle + b\right] + \xi_i - 1\right] = 0$$

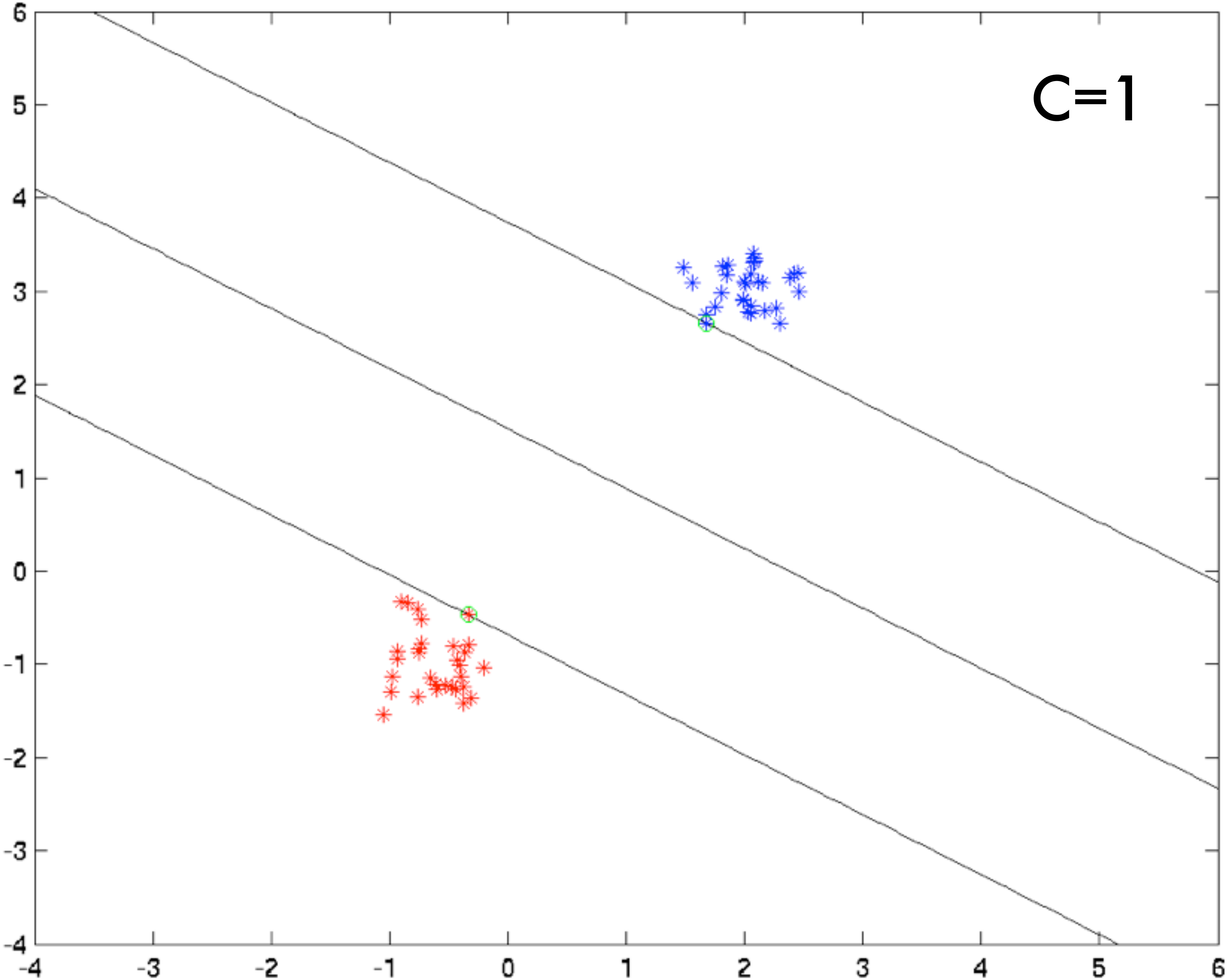
$$\eta_i \xi_i = 0$$

$$\alpha_i = 0 \Longrightarrow y_i \left[\langle w, x_i\rangle + b\right] \geq 1$$

$$0 < \alpha_i < C \Longrightarrow y_i \left[\langle w, x_i\rangle + b\right] = 1$$

$$\alpha_i = C \Longrightarrow y_i \left[\langle w, x_i\rangle + b\right] \leq 1$$

credit: A. Smola (CMU, 2013)



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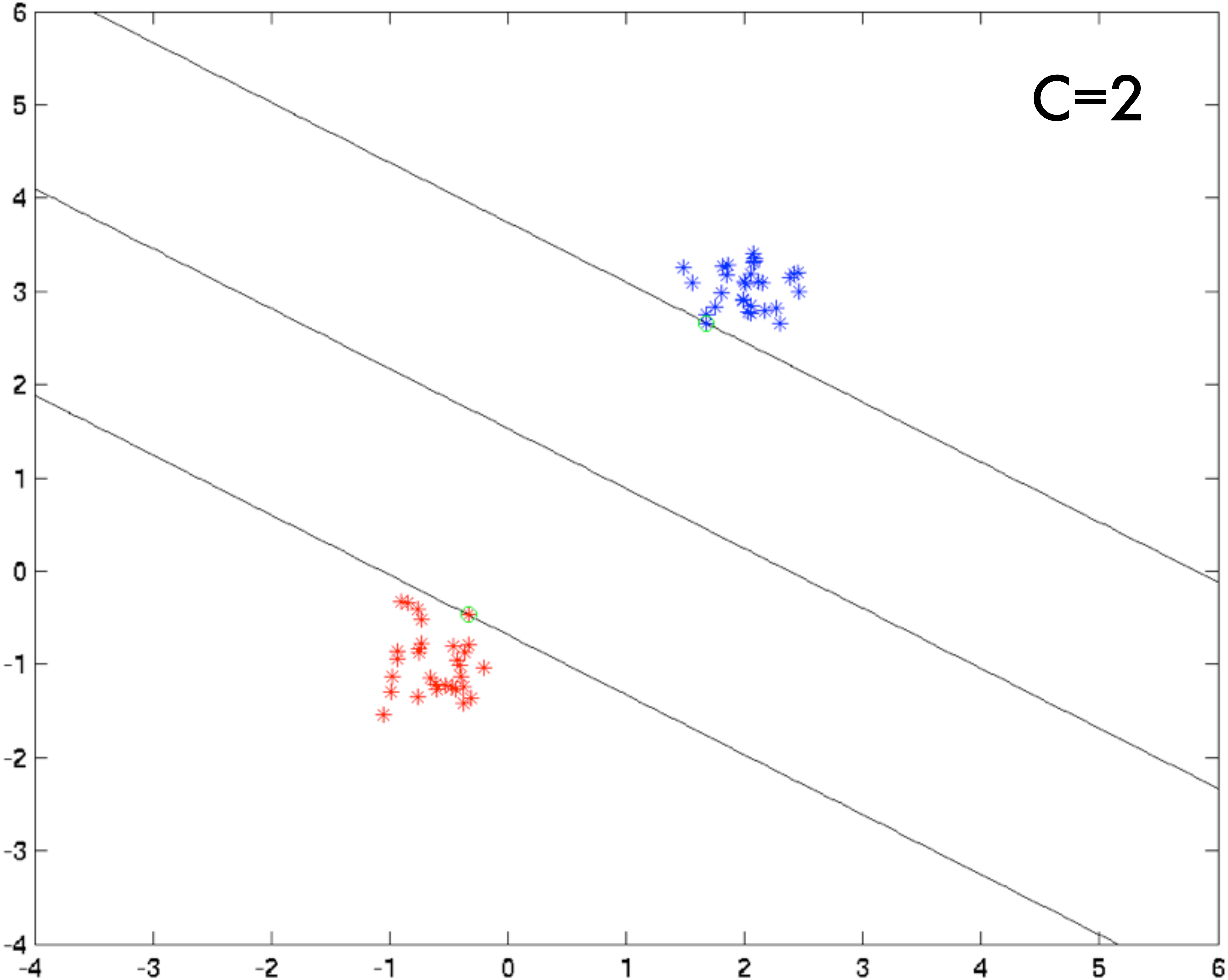
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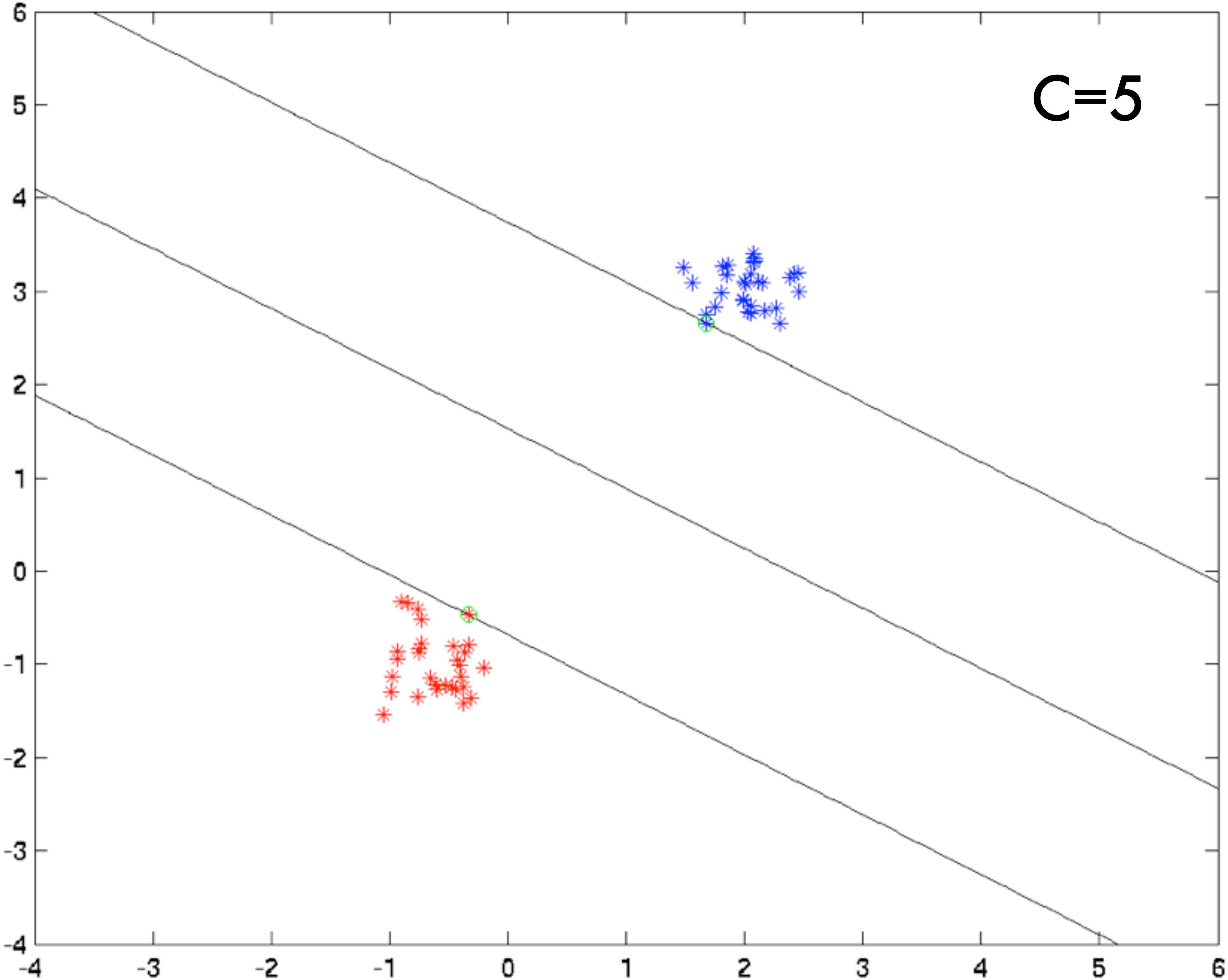

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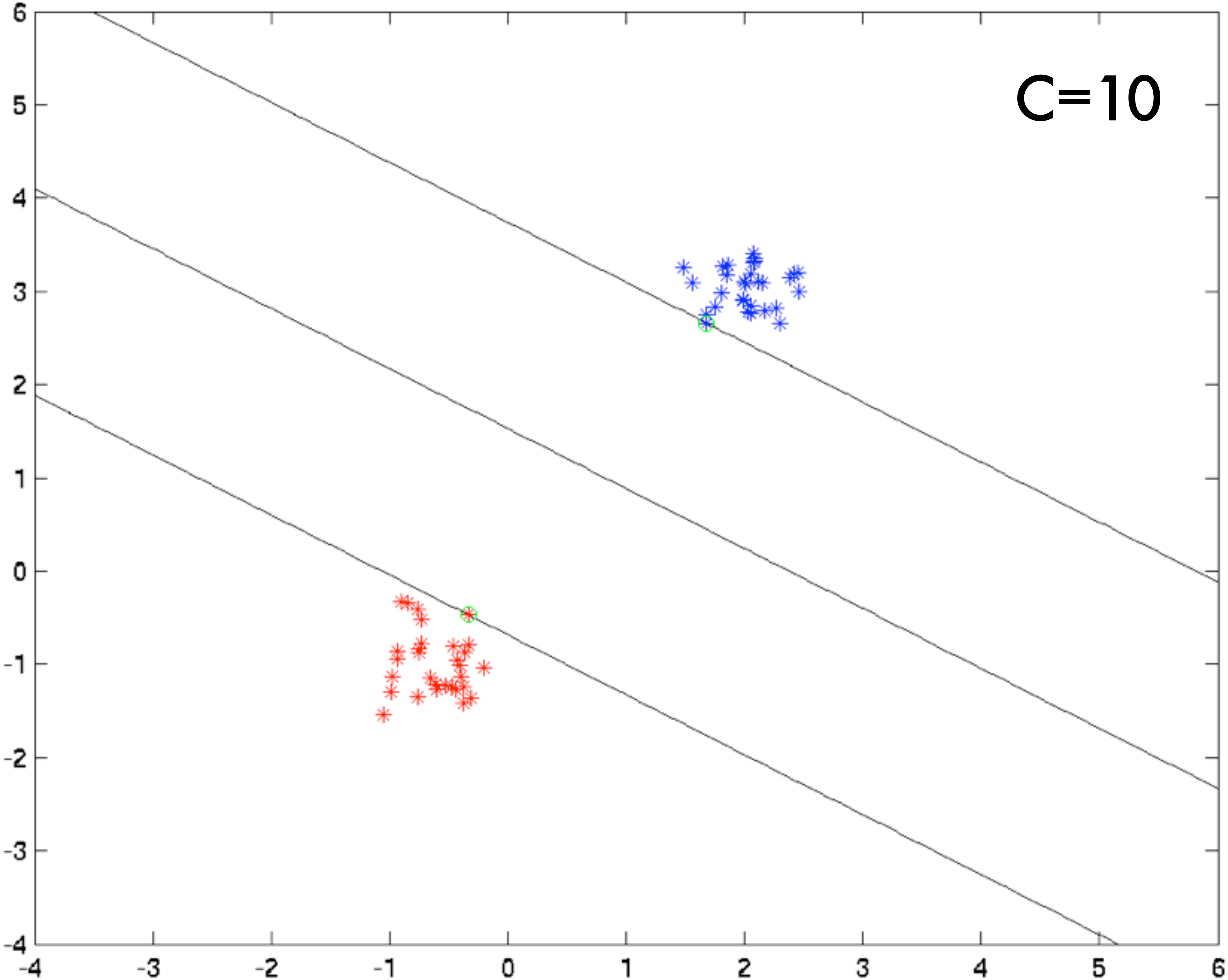

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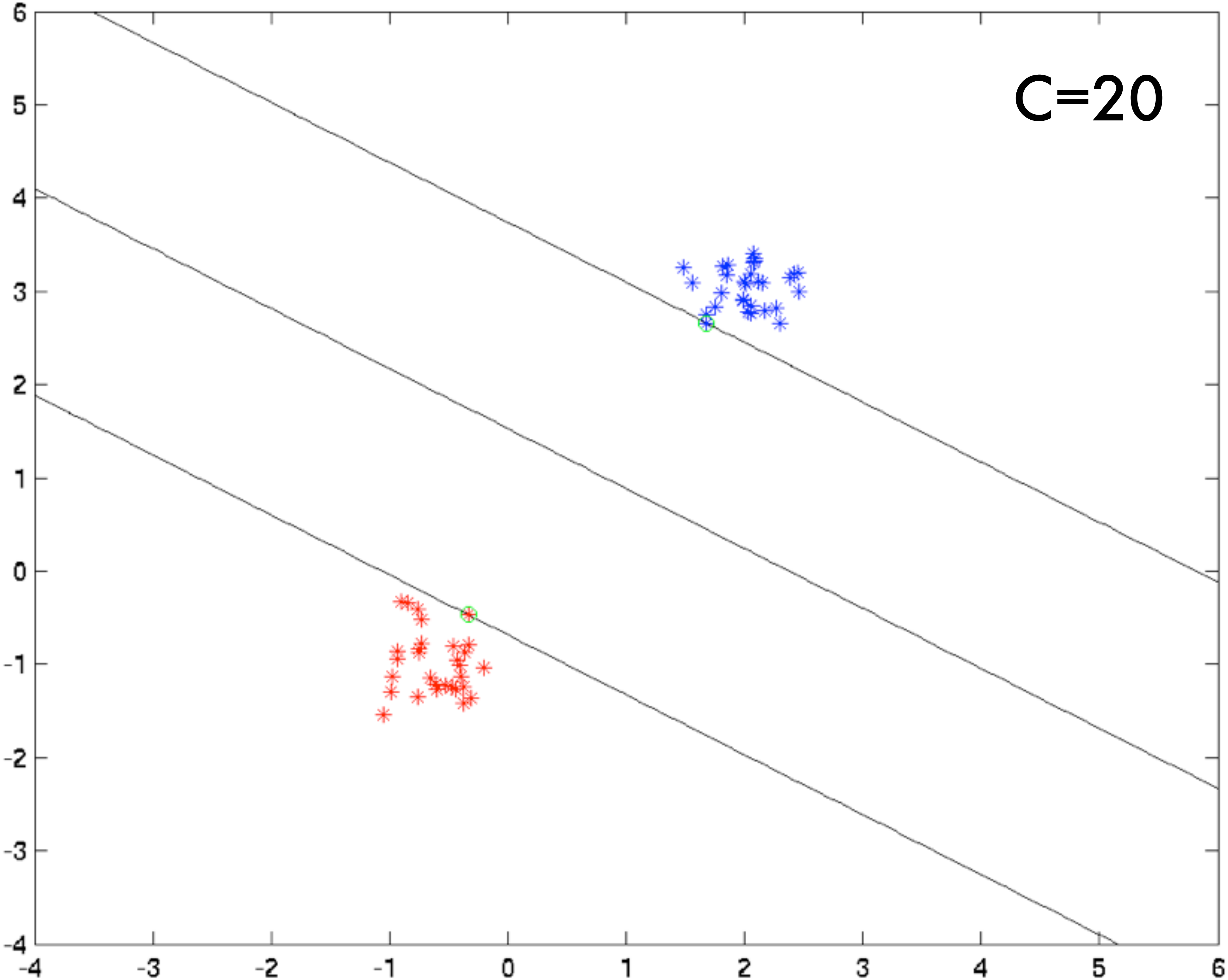
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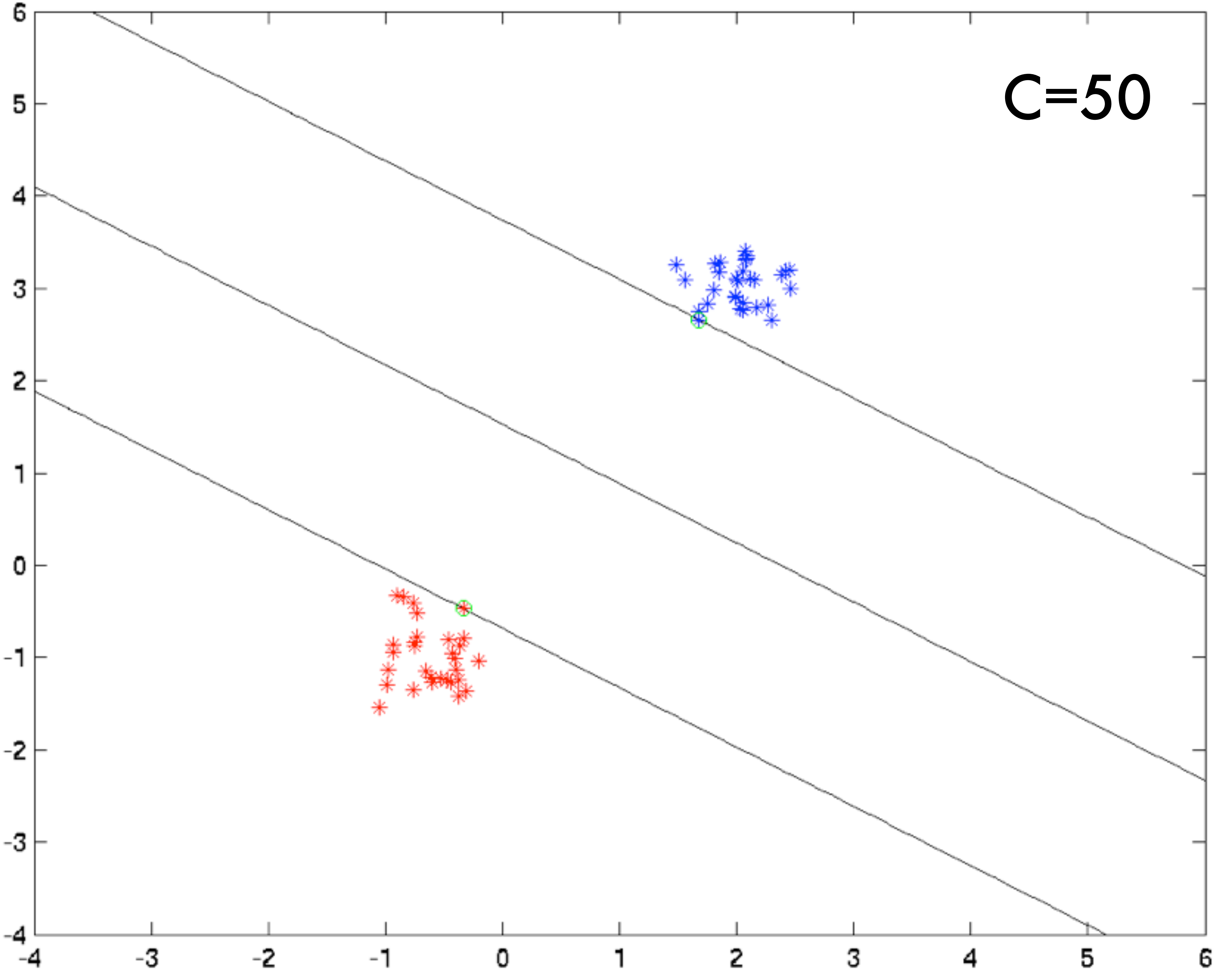

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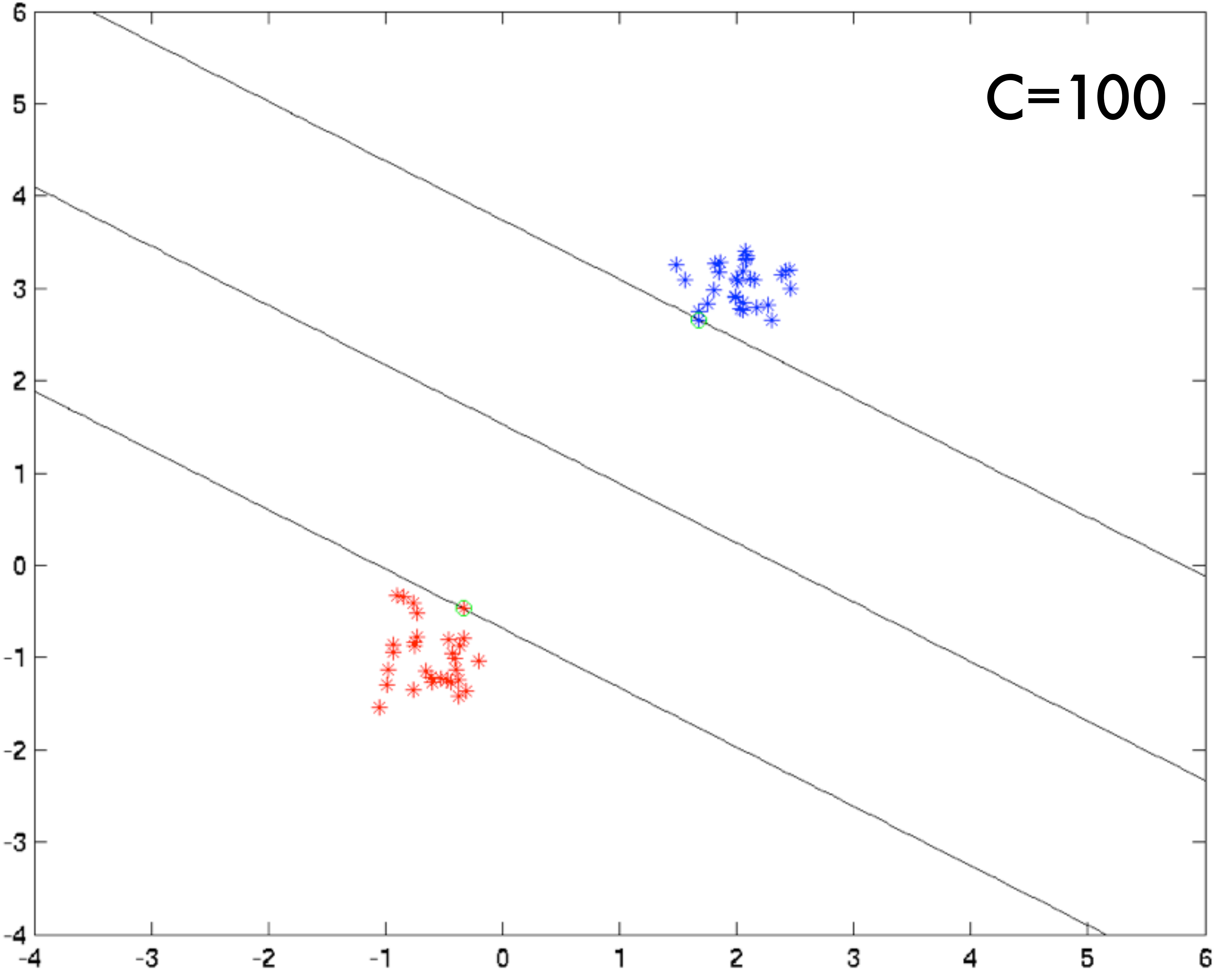
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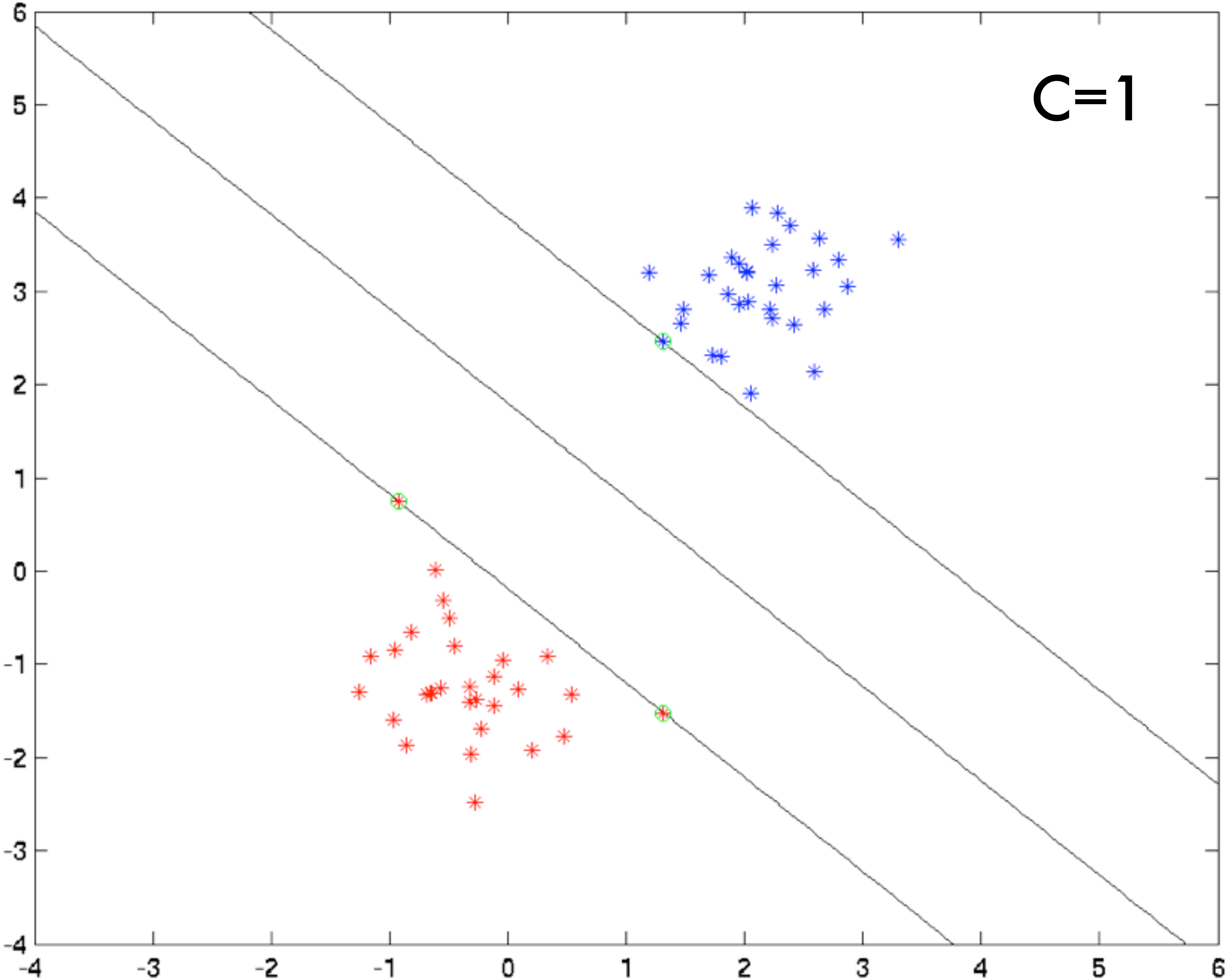
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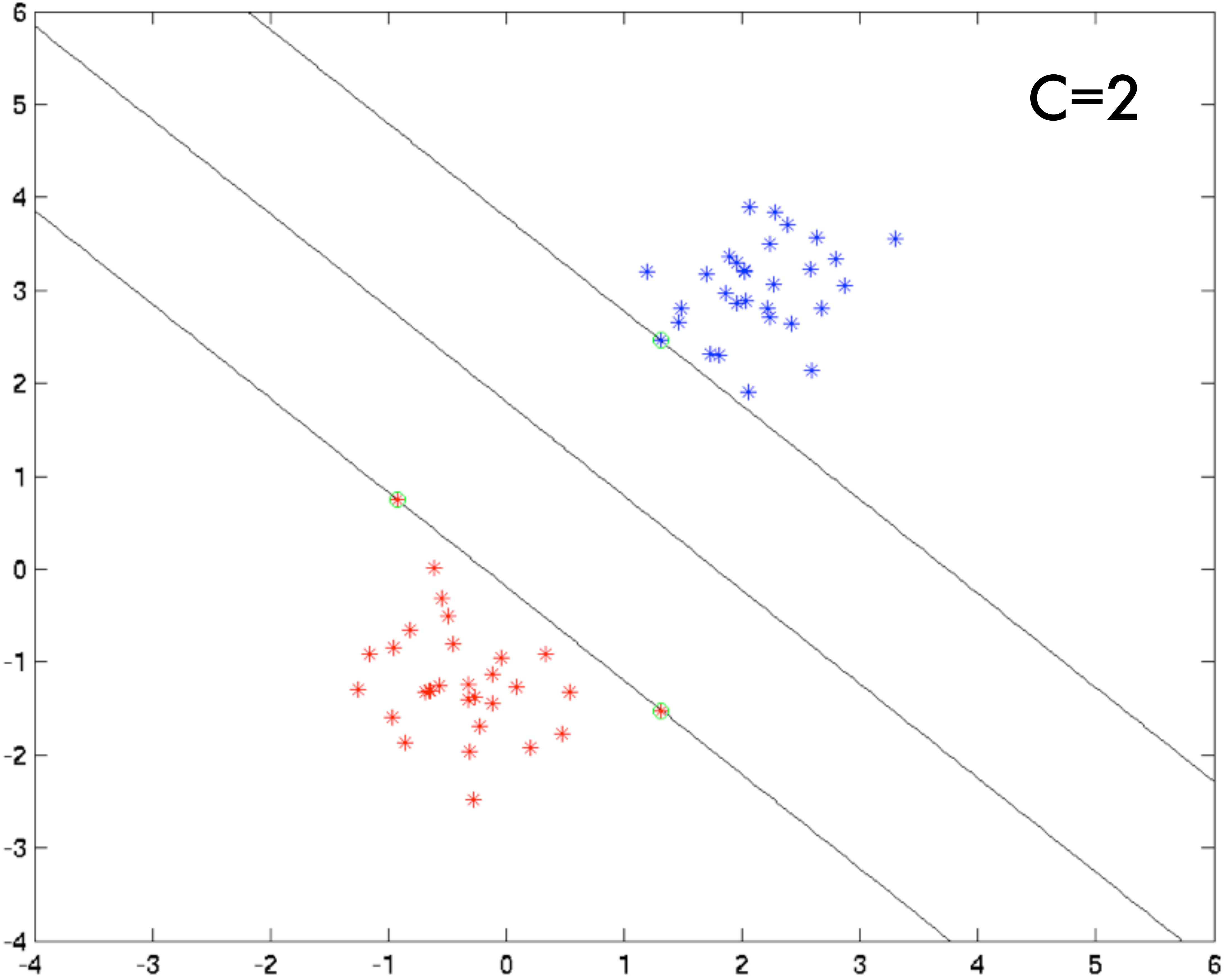
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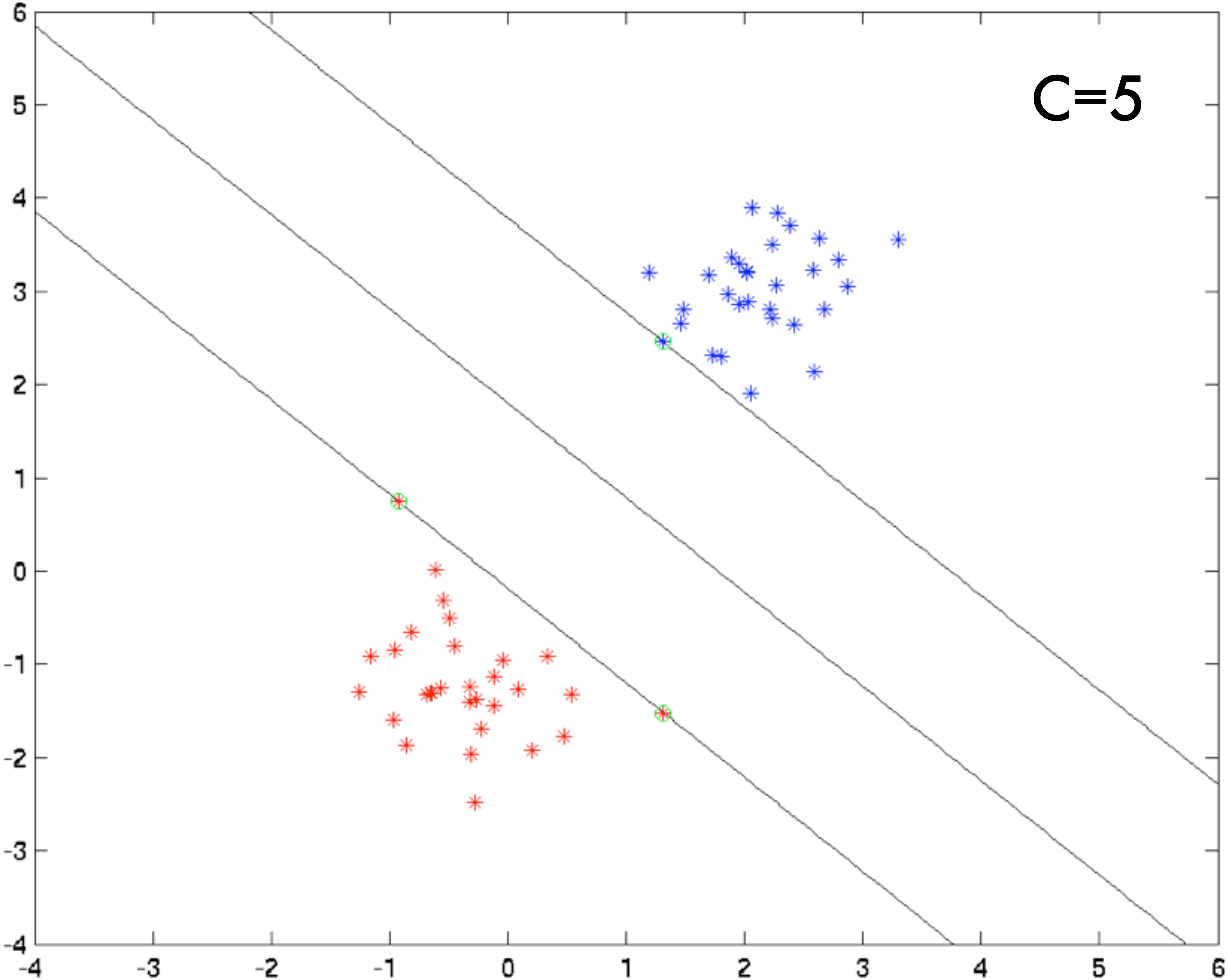
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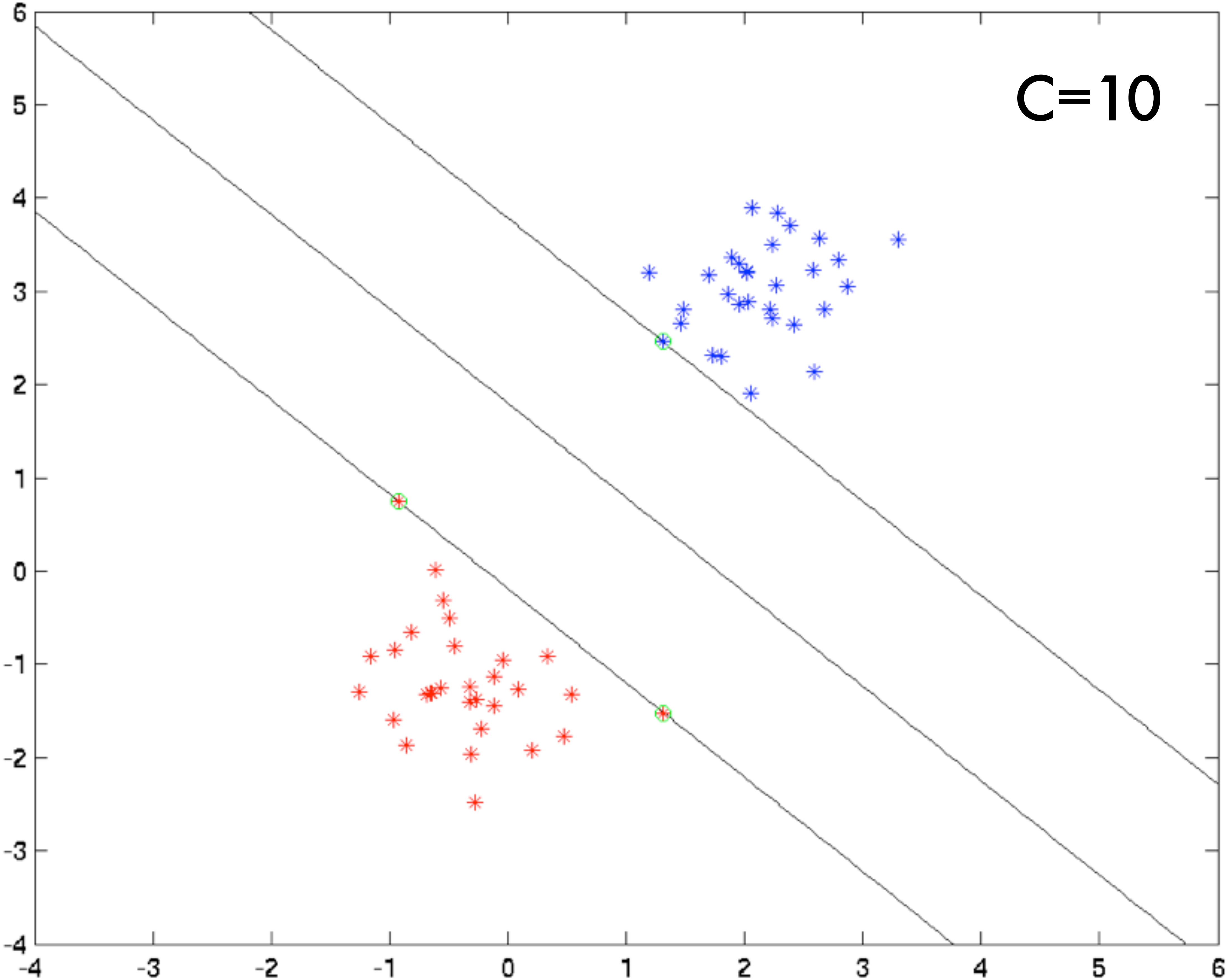

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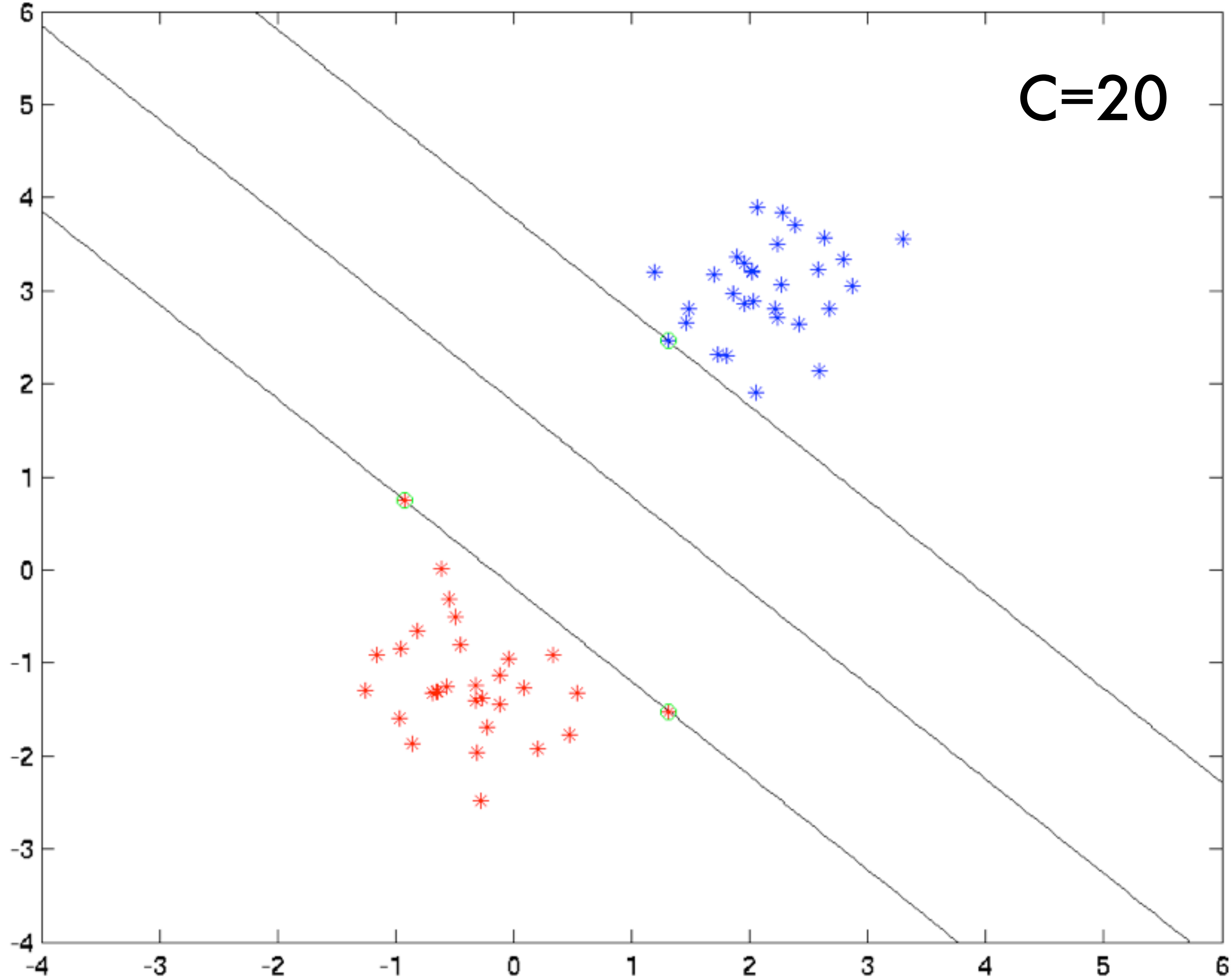

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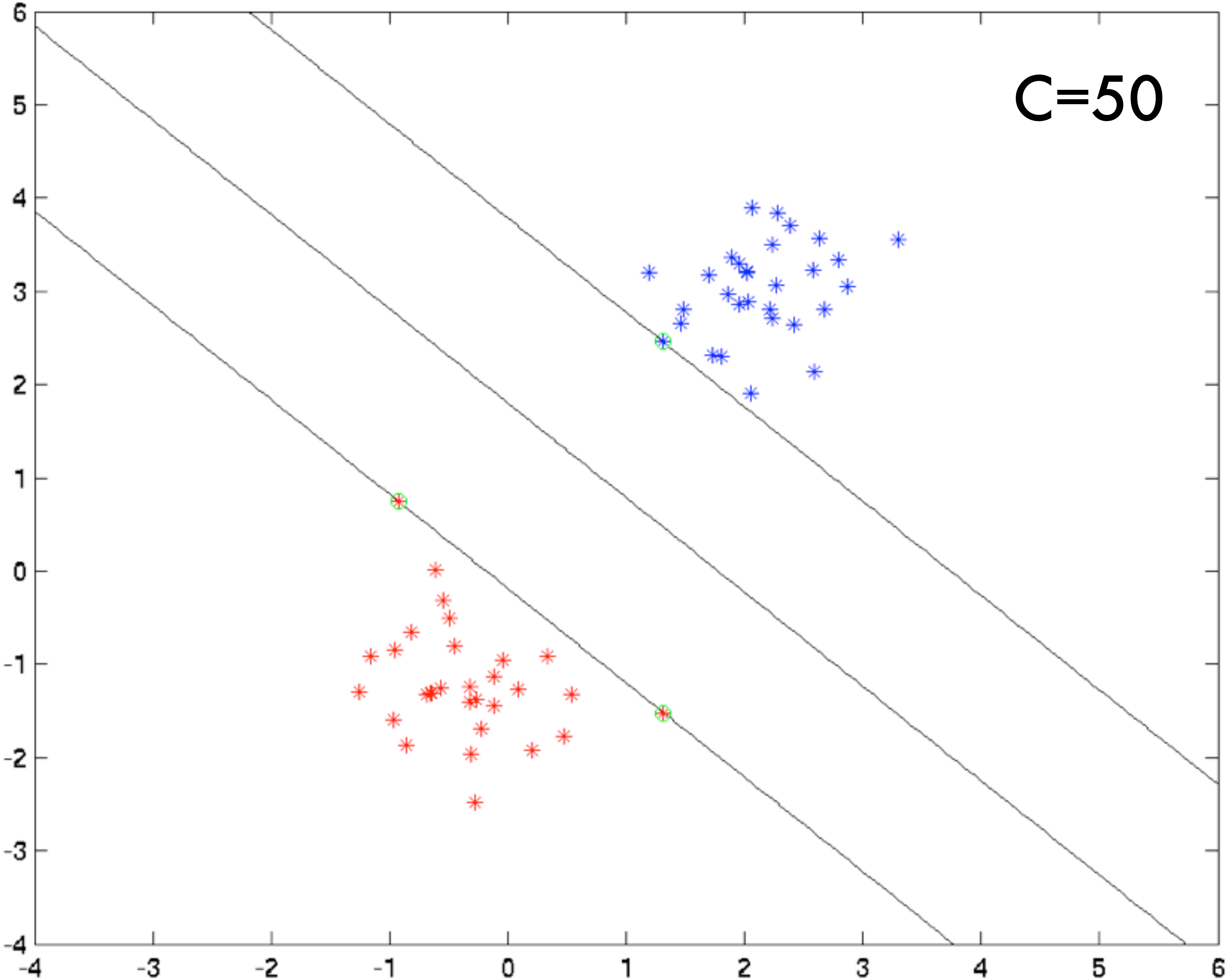
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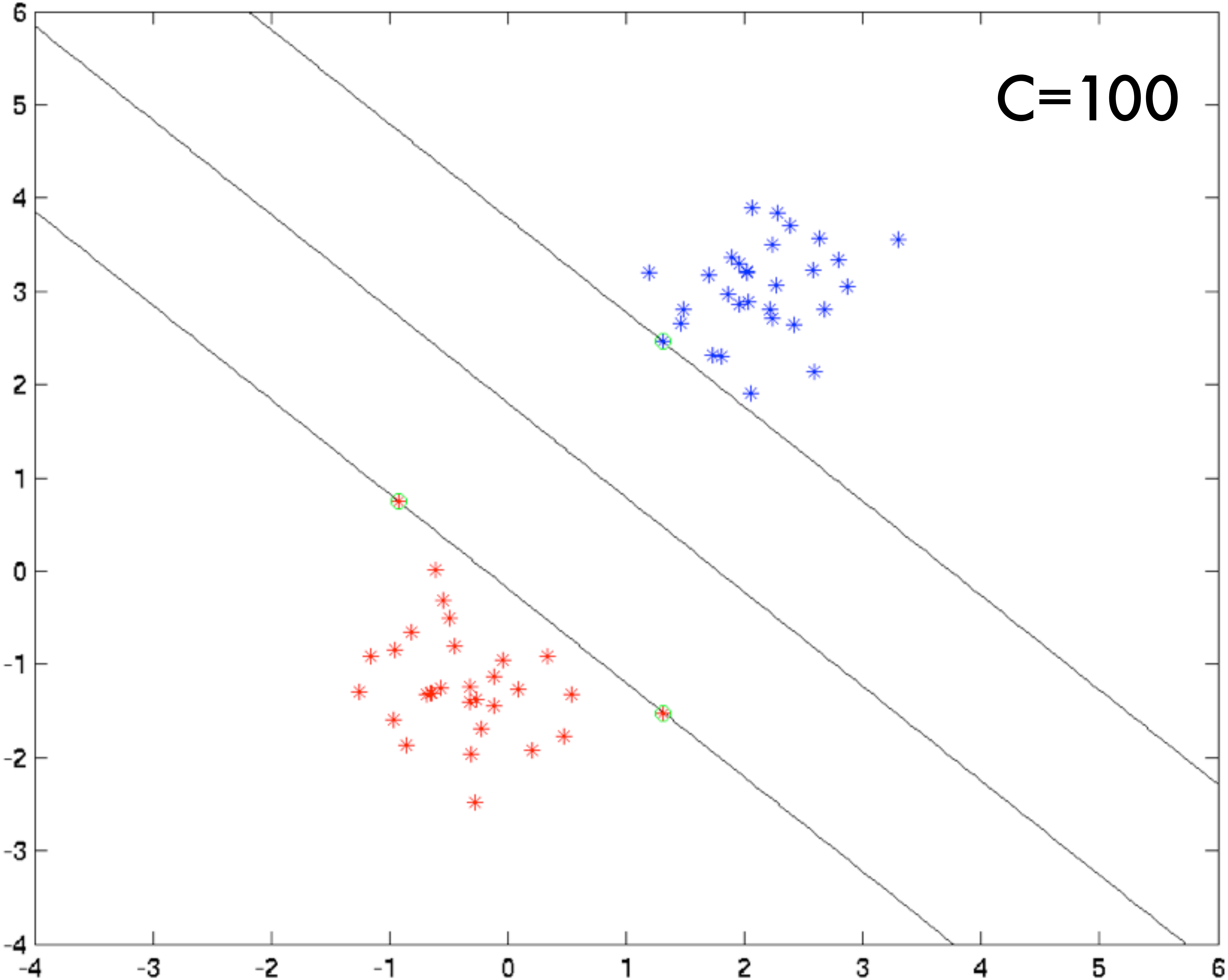
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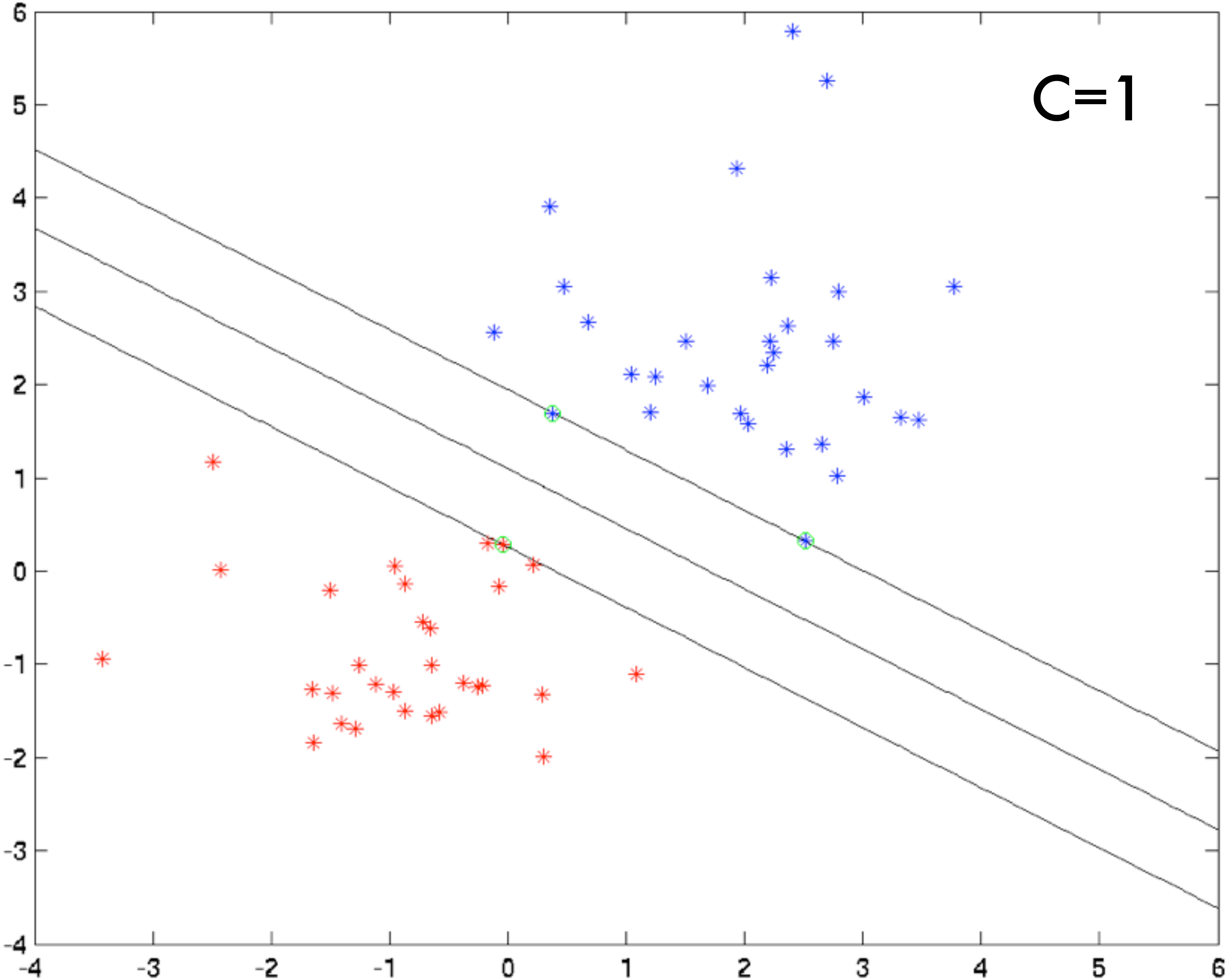
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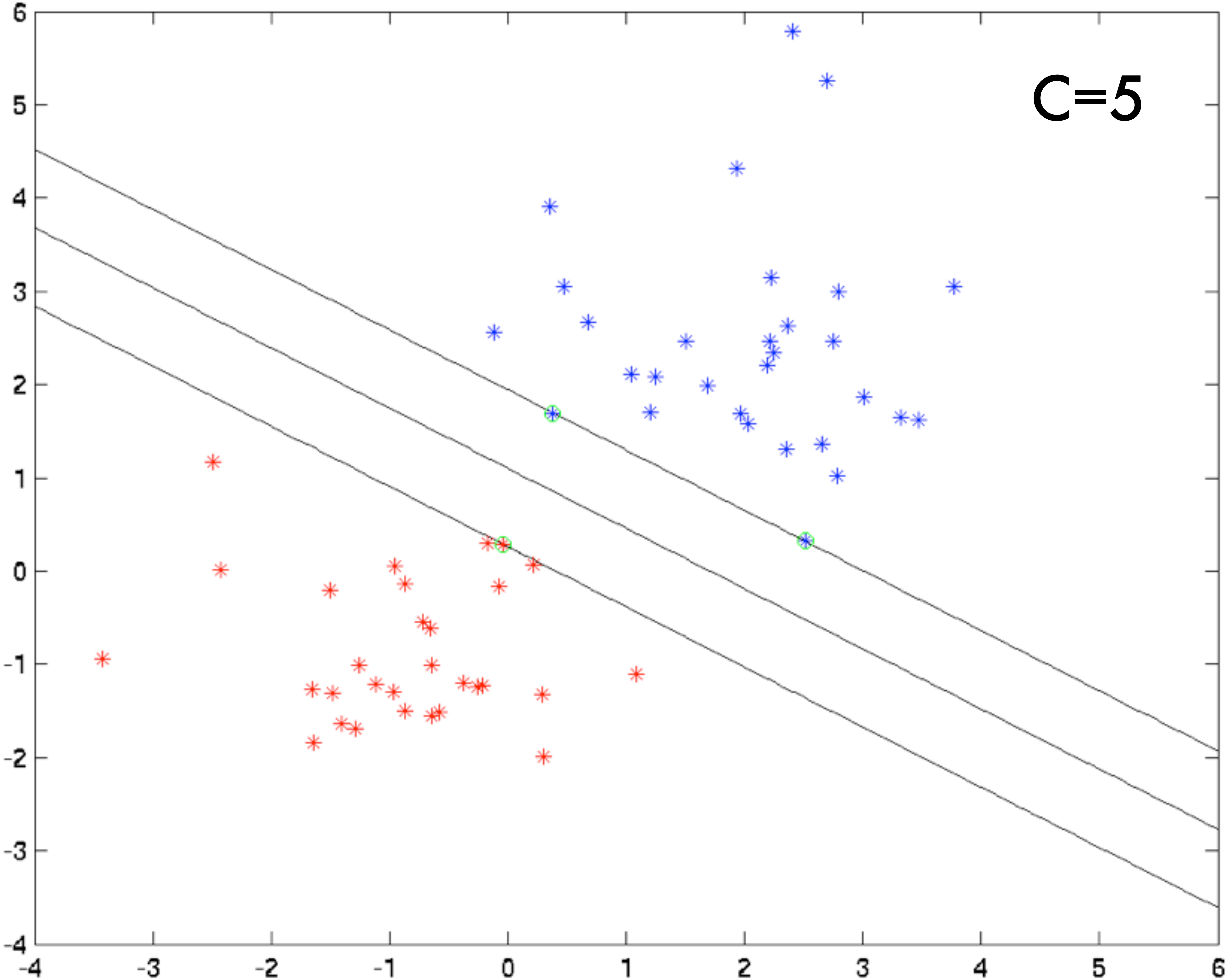

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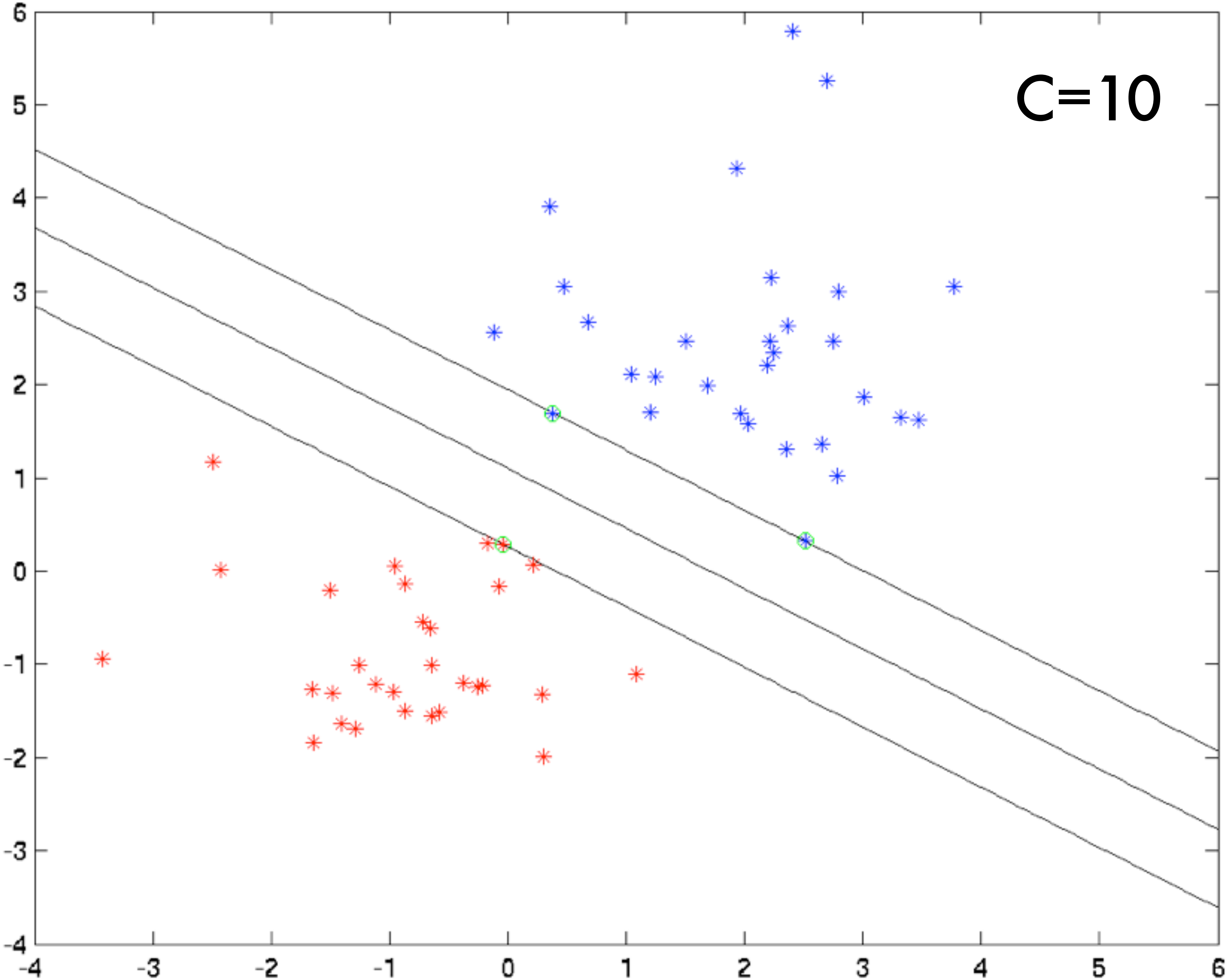
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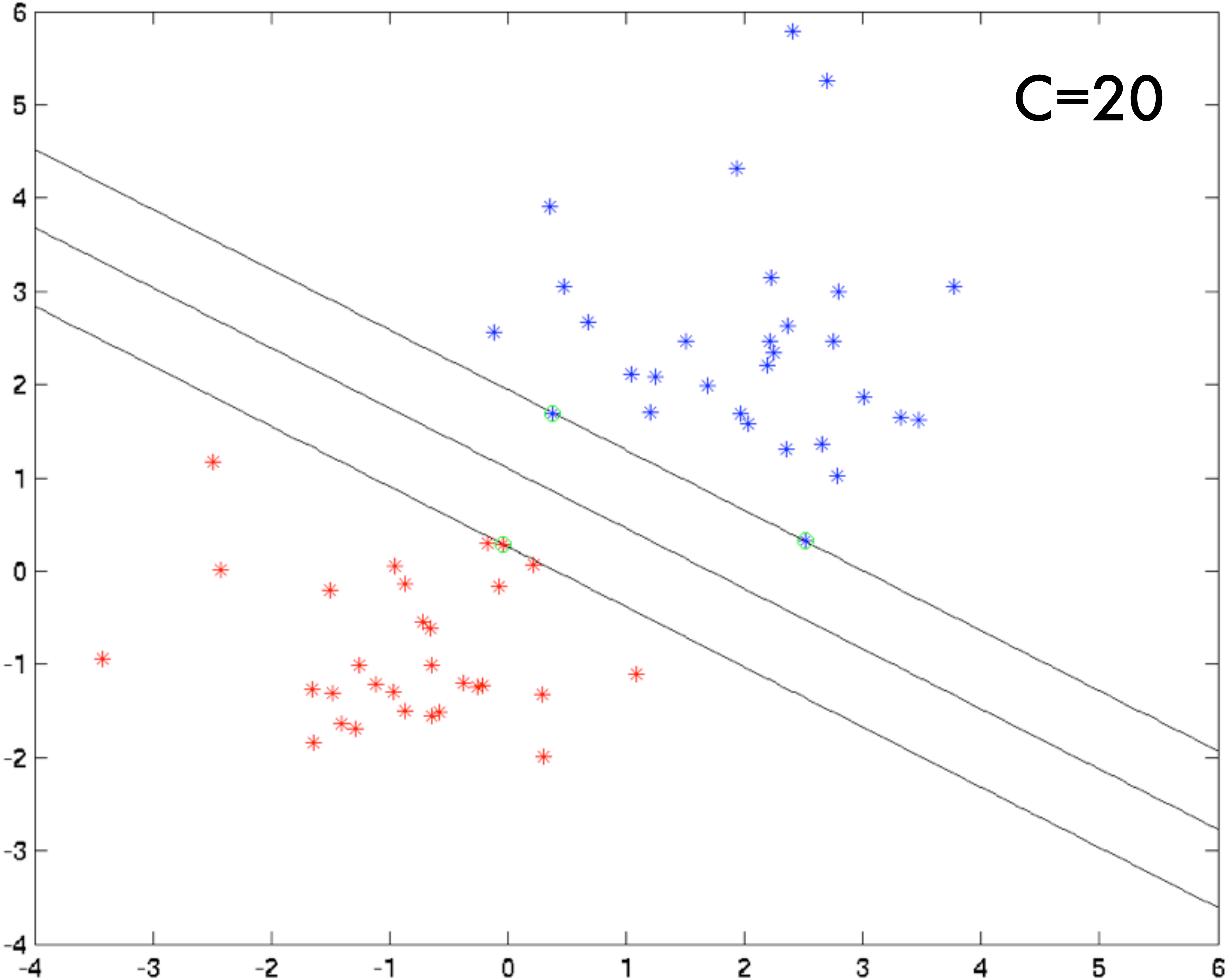
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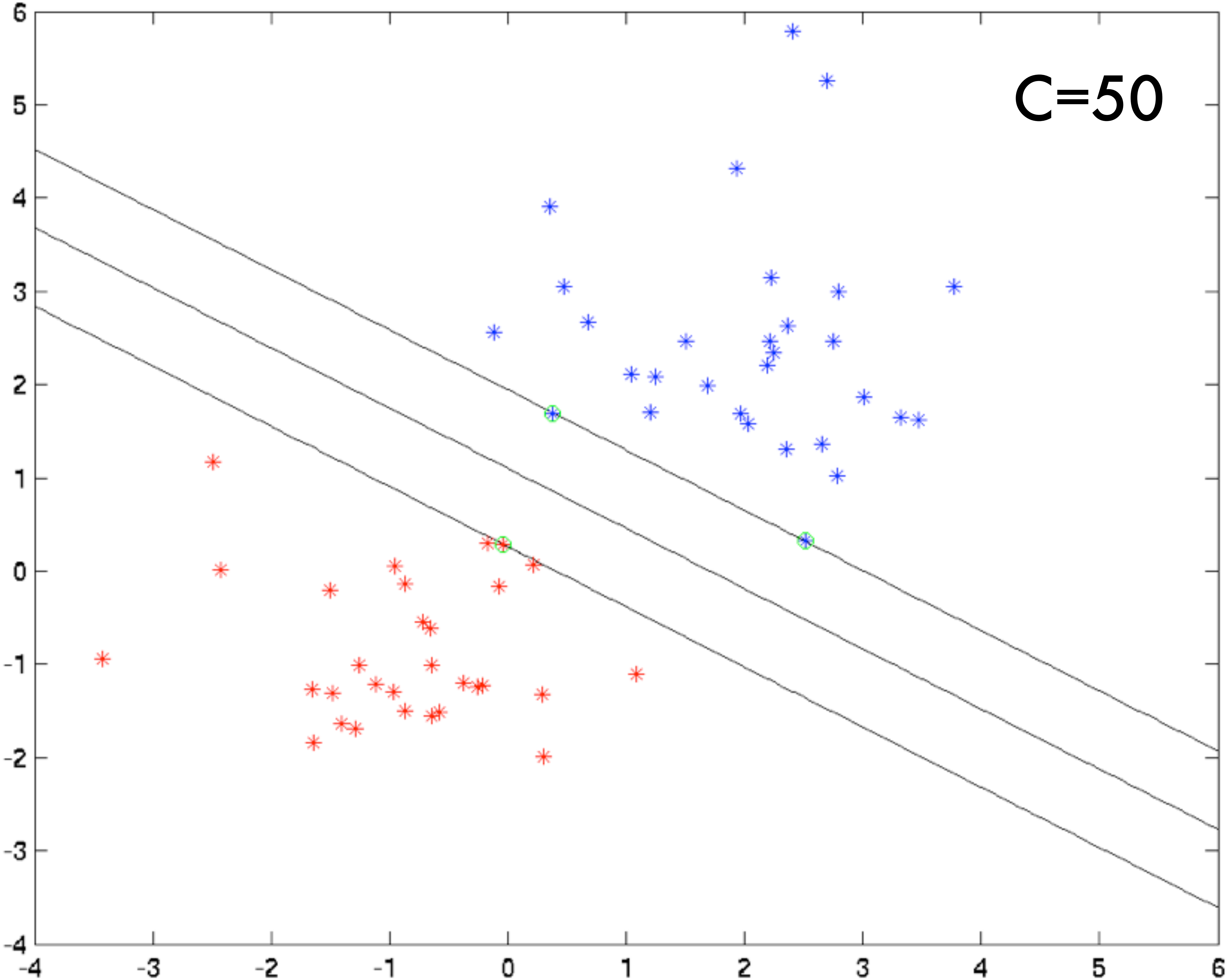
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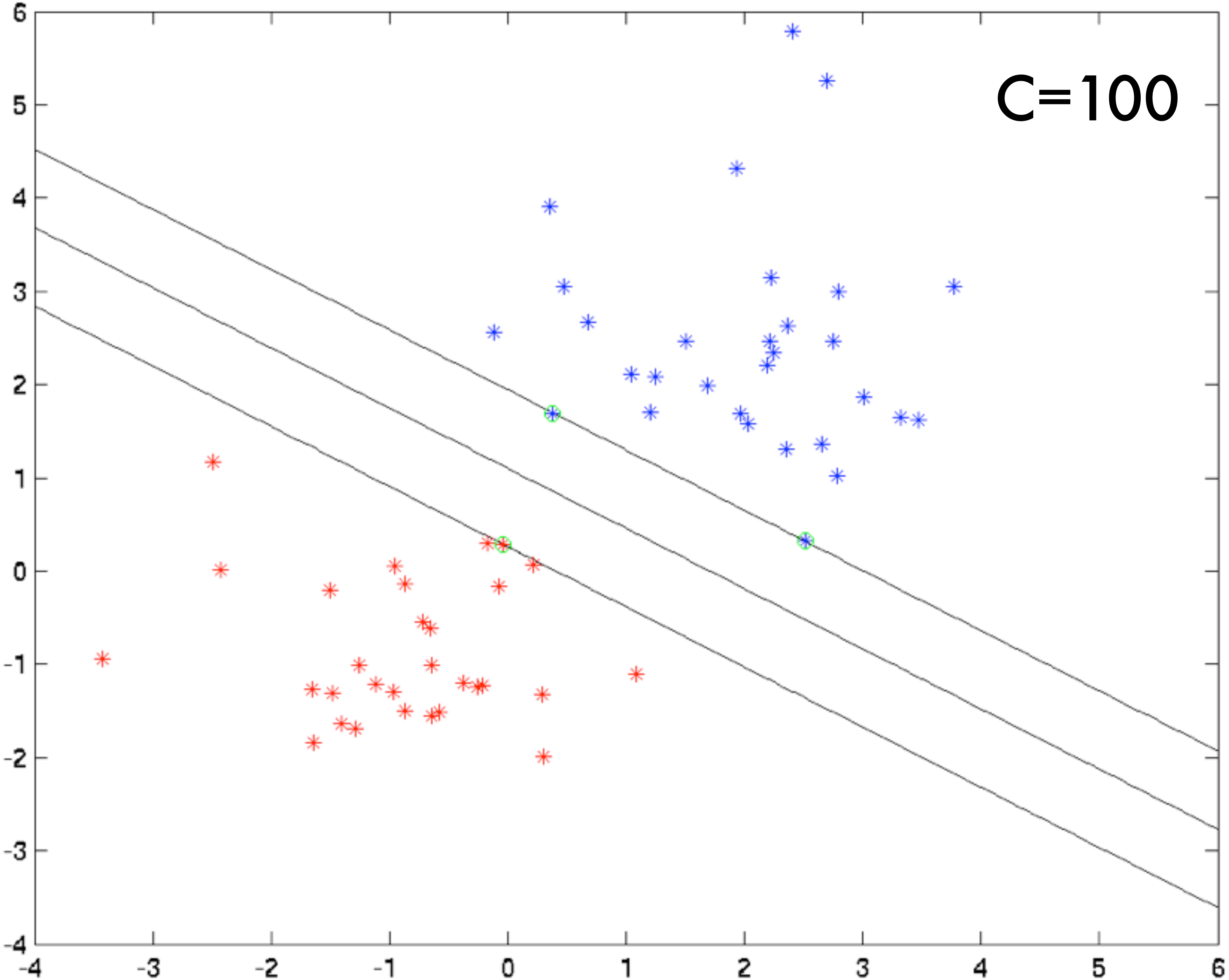
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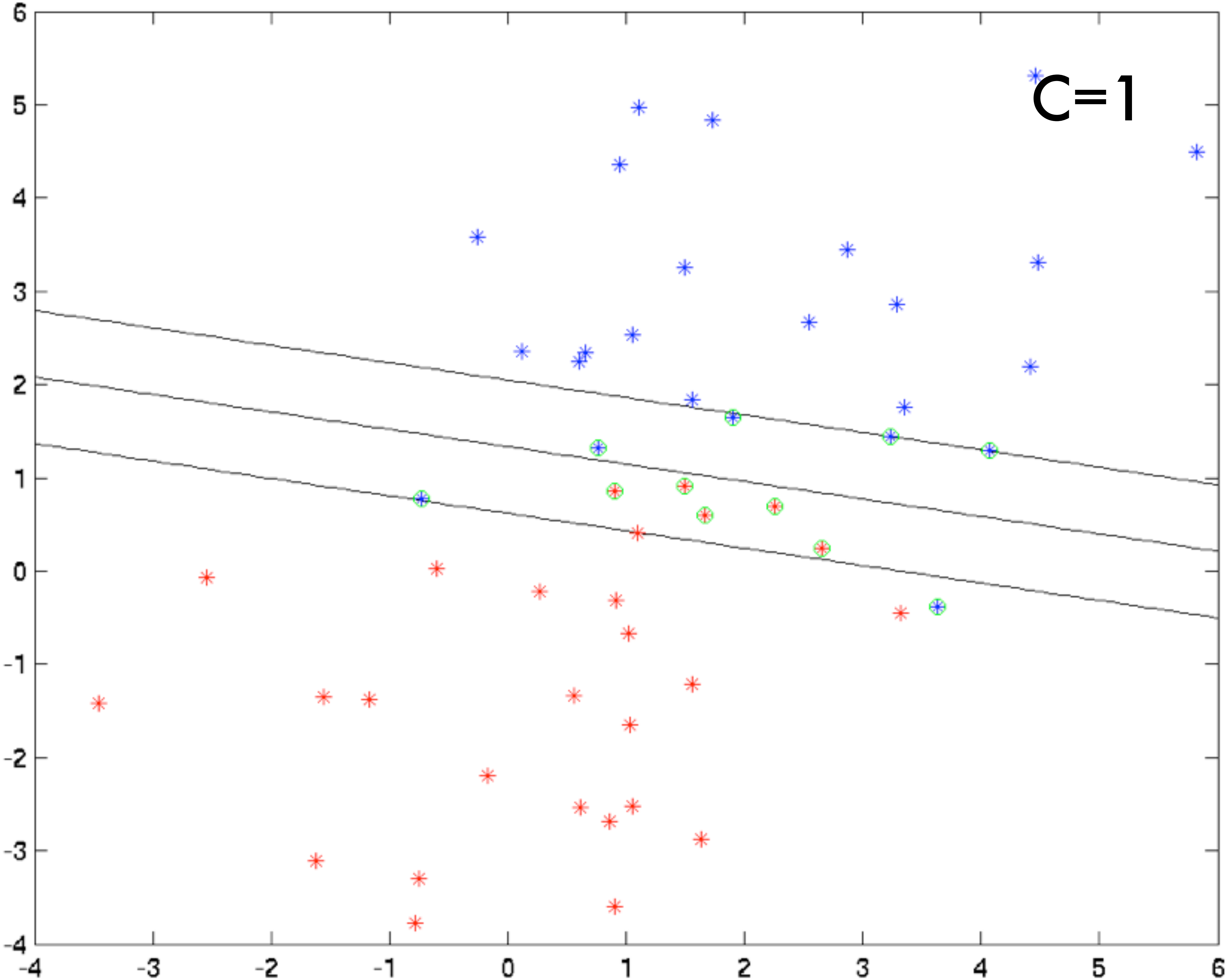

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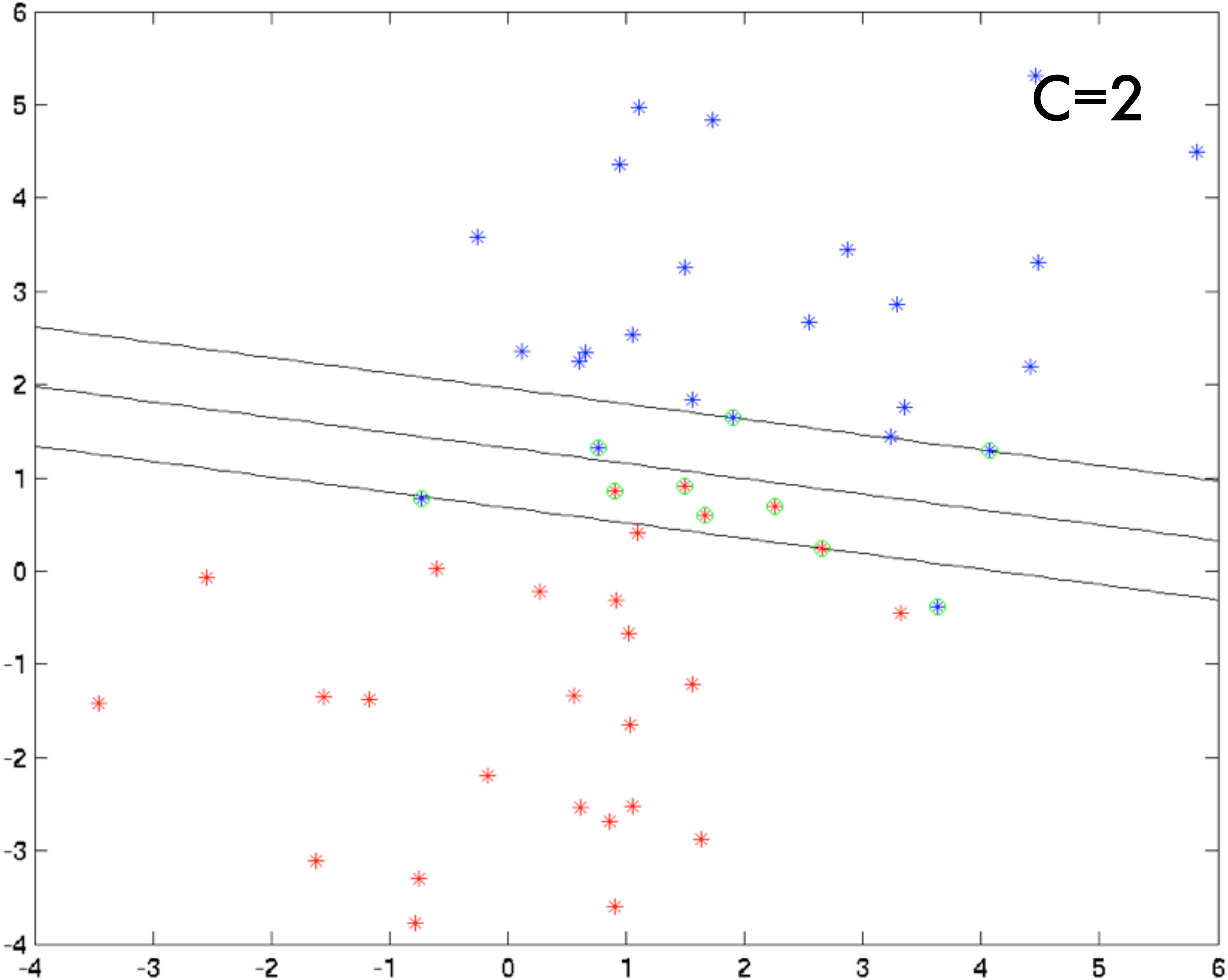
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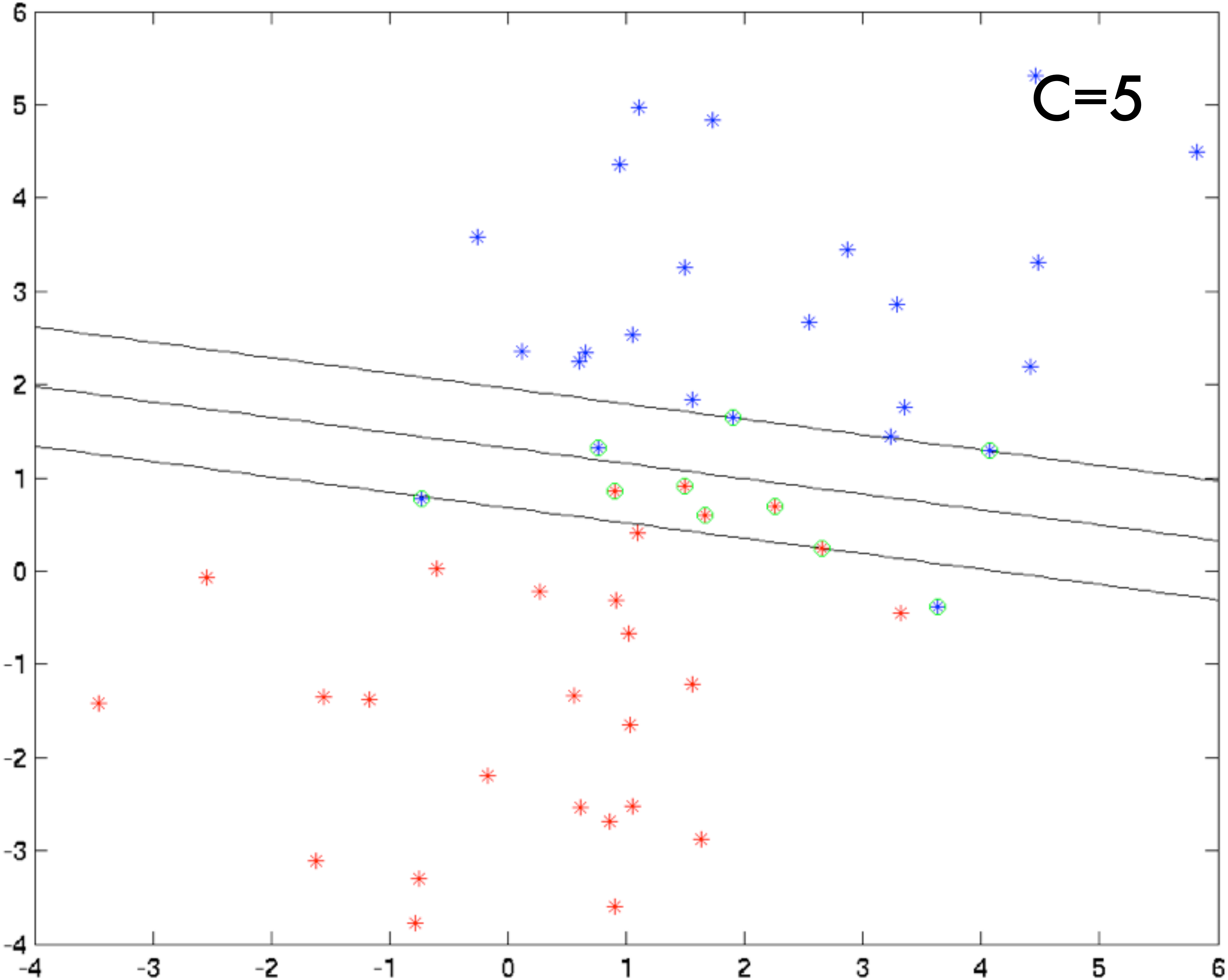

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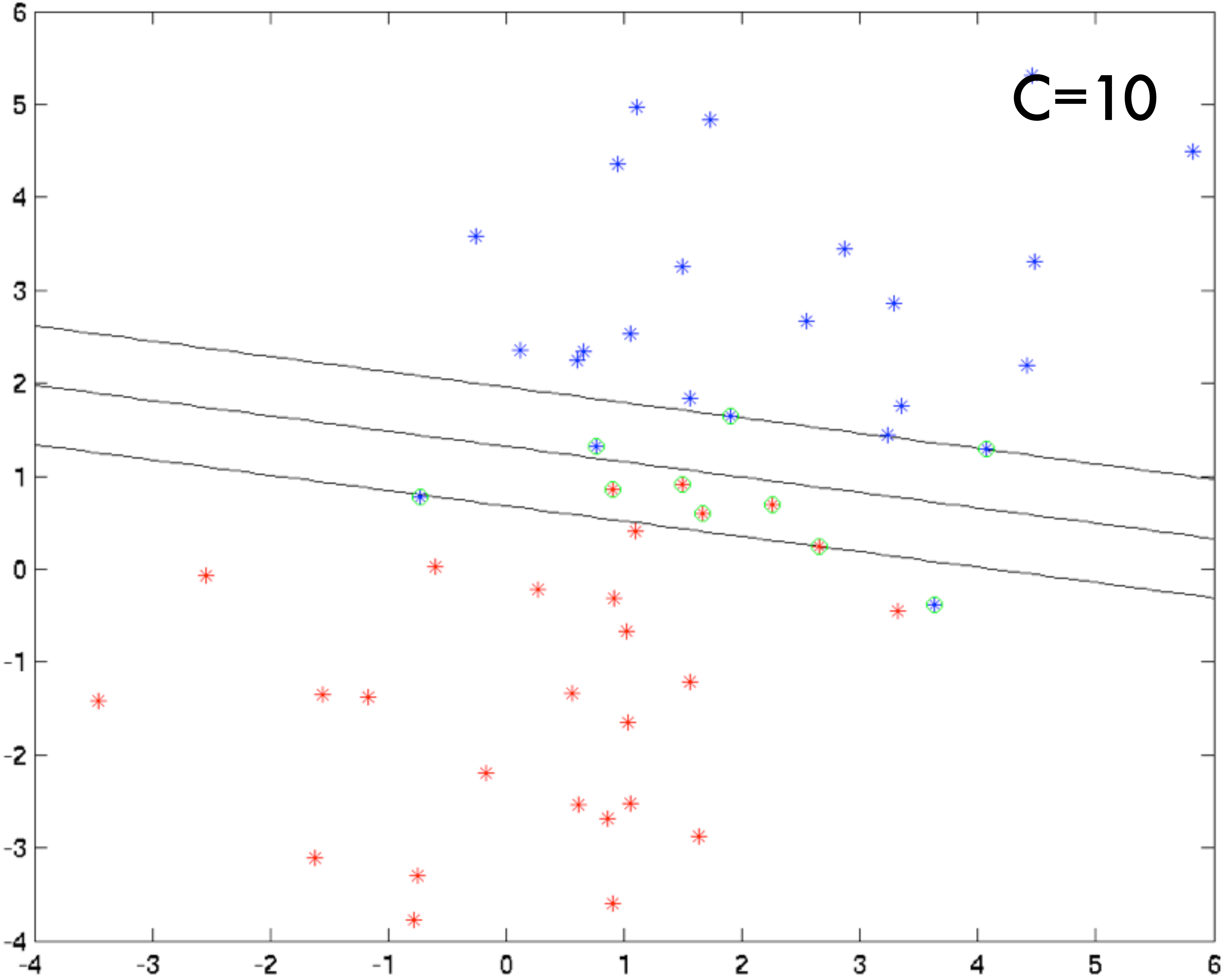

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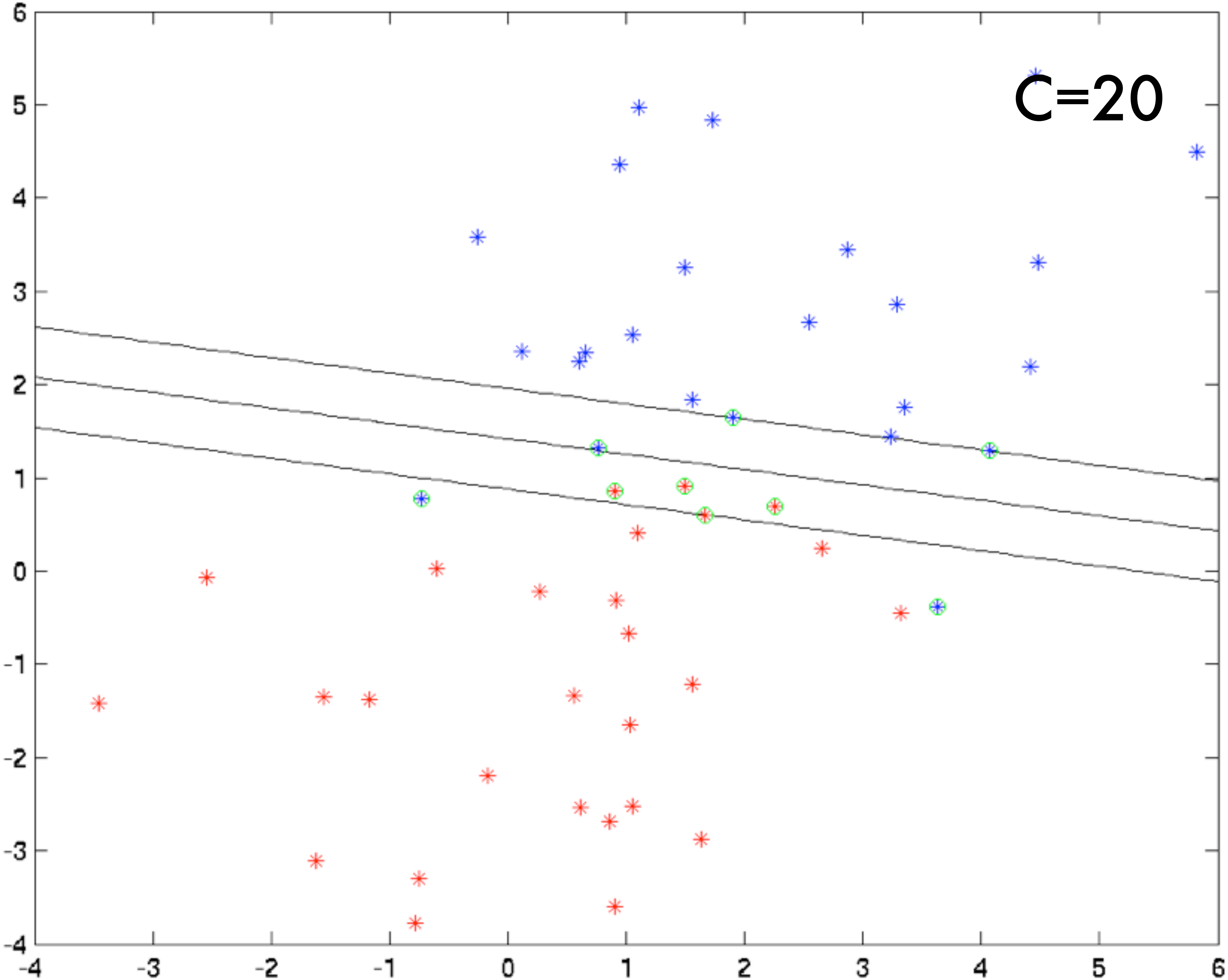

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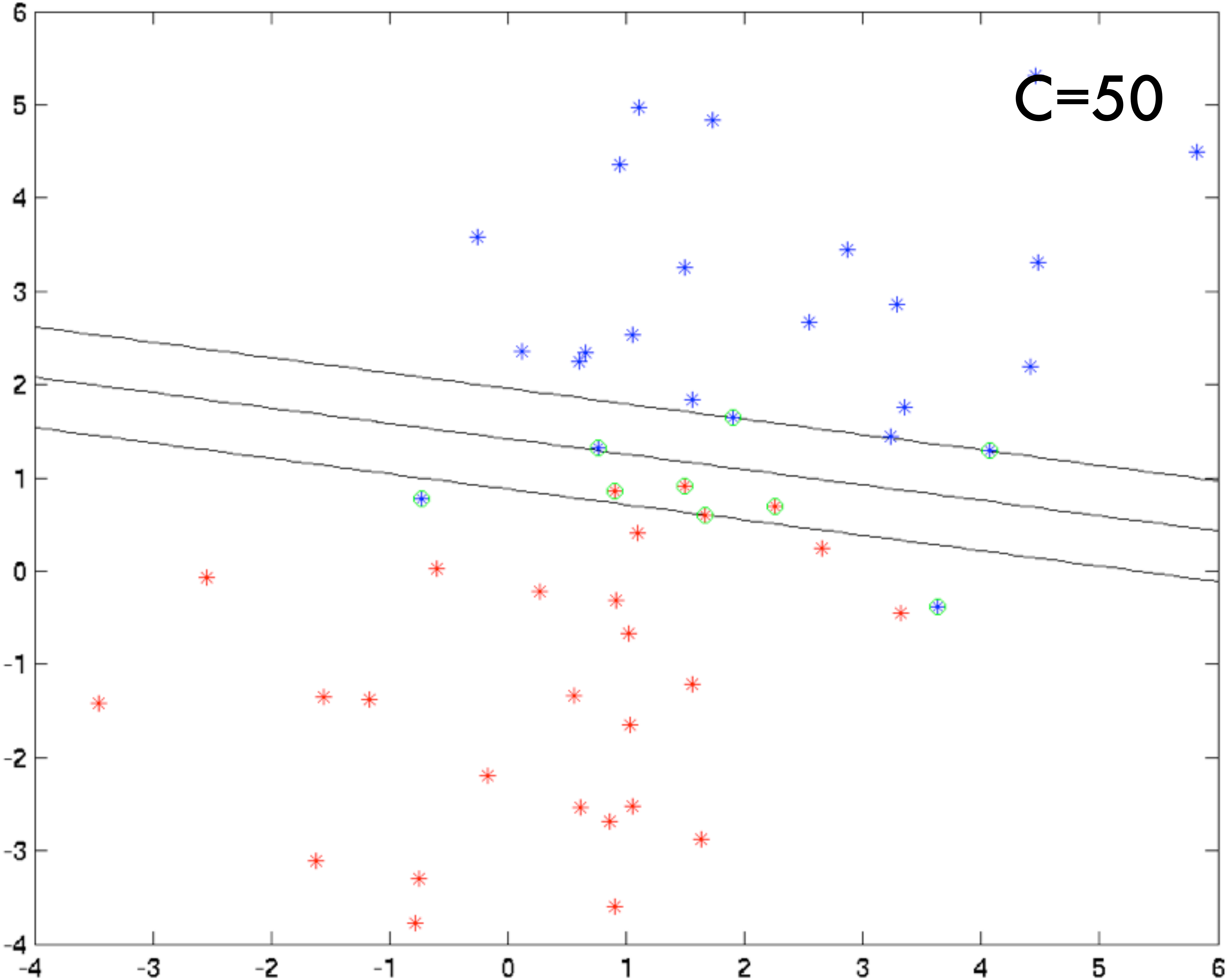
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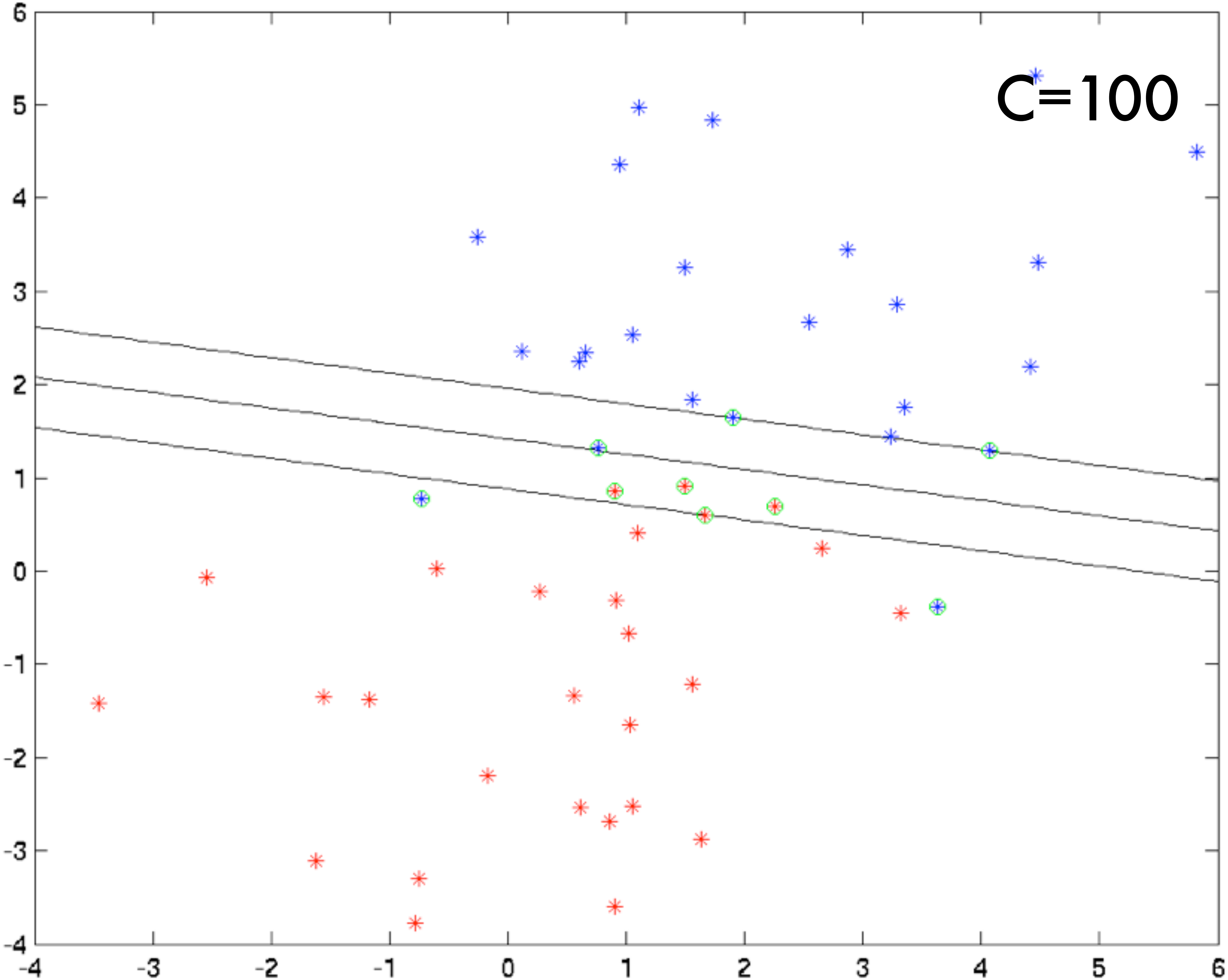
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