Crystal Characterization: solution

INTRODUCTION TO DIGITAL LOW-LEVEL RADIO FREQUENCY CONTROLS IN ACCELERATORS

Lab 1

Shreeharshini Dharanesh Murthy, Dan Wang

US PARTICLE ACCELERATOR SCHOOL
JANURARY 23 – 27, 2023

Contents

1 Solutions

1. Assuming the Nano-VNA is calibrated (2-port calibration) with center of 20 MHz and span of 1 kHz, aptures live data from Nano-VNA.
2. Calculate the 3dB bandwidth, Quality factors (Q_0 and Q_L), and coupling factor.

1.1 Analysis

[1]:
```
import os
import sys
import numpy as np
from matplotlib import pyplot as plt
import skrf as rf
from math import pi, exp
from scipy.optimize import least_squares
plt.rcParams['figure.figsize'] = (8.0, 8.0)
plt.rcParams['font.size'] = 10
import matplotlib as mpl
mpl.rcParams['axes.formatter.useoffset'] = False
from nanovna import NanoVNA
```

[2]:
```
class CrystalAnalysis:
    def __init__(self, co_fname=None):
        nv = NanoVNA()
        # make sure the calibrated stop and start frequencies are same␣
↪for capturing data
        start_f, stop_f = 19.997600e6, 19.998600e6
        nv.set_sweep(start_f, stop_f)
        f = nv.fetch_frequencies()
        self. freq_data = (np.array(f))/1e6  # frequency in MHz
        self.s21 = nv.data(1)
        self.s11 = nv.data(0)

        self.s21_data = 20 * np.log10(np.abs(self.s21))  # magnitude in dB
        self.s21_deg = np.angle(self.s21)  # phase in radians
        self.s11_data = np.abs(self.s11)  # absolute values
        self.s11_deg = np.angle(self.s11)  # phase in radians
        # Results
        self.fs   = 0.0
        self.df   = 0.0
        self.Q_L = 0.0
        self.Q_0 = 0.0
        self.beta = 0.0
        self.Cm   = 0.0
```

```
        self.Lm  = 0.0
        # smith plot here, can clearly see under/critical/over coupling␣
↪case
        self.s11_ant(nv)
        self.__call__()

    def beta_cal(self):
        s11_min = np.min(self.s11_data)
        ix = np.where(self.s11_data == s11_min)[0][0]
        if self.s11[ix].real < 0:
            s11_min = -s11_min
            print("############ Over-coupled #############")
        else:
            print("############ Under-coupled #############")
        self.beta = (1 - s11_min) / (1 + s11_min)

    def s11_ant(self, nv):
        self.beta_cal()
        fig, ax = plt.subplots()
        n = nv.skrf_network(self.s11)
        n.plot_s_smith()
        plt.title("Smith Chart")
        # text = "beta:{:.5f}".format(self.beta)
        # ax.text(self.beta, 0, text, va='top')

    def Rs_calc(self, s21, R_L):
        """ ESR calculation """
        Rs = 2 * R_L * (pow(10, (-s21 / 20)) - 1)
        Reff = 2 * R_L + Rs
        return Rs, Reff

    def df_calc(self, fl, fh):
        """ bandwidth calculation """
        return fh - fl

    def Cm_Lm_calc(self, fs, df, Rs, Reff):
        """ Calculate motional capacitance and inductance """
        Cm = df / (2.0 * pi * (pow(fs, 2)) * Reff)
        Lm = Reff / (2.0 * pi * df)
        Q_L = fs / df
        Q_0 = (1 + self.beta) * Q_L
        return Cm, Lm, Q_0, Q_L

    def find_max_freq_mag(self, data):
```

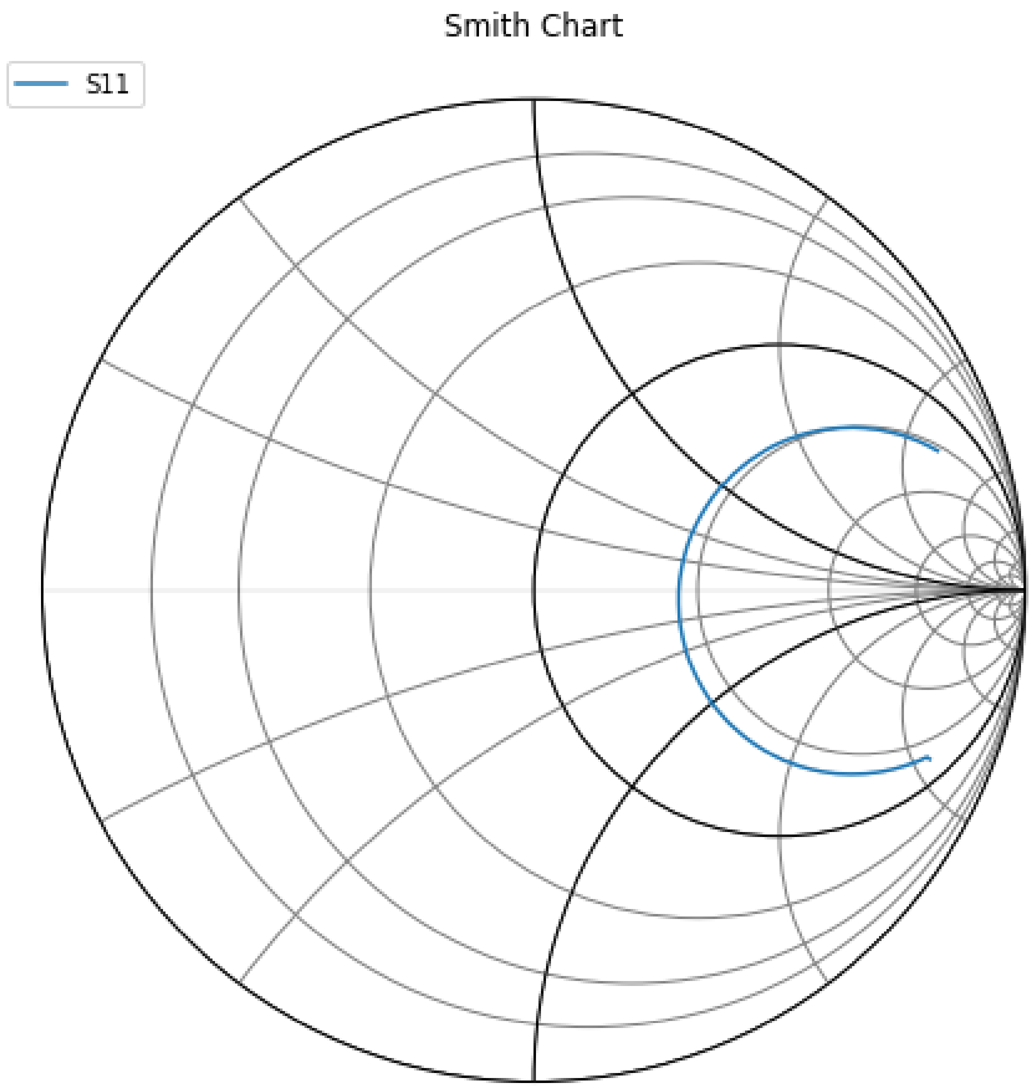
```
        """ find the maximum S21 magnitude and it's corresponding␣
↪frequency """
        max_idx = np.where(data == np.amax(data))
        mag = np.max(data)
        freq = self.freq_data[max_idx][0]
        return (freq, mag)

    def find_nearest(self, array, value):
        """ find the value closest to the given data """
        idx = (np.abs(array - value)).argmin()
        return array[idx]

    def annot(self, ax, freq, mag, y, offset=0.1):
        """ used for annotating max, -3 dB points on S21 plot   """
        text= "{:.3f} dBm\n{:.5f} MHz".format(mag, freq)
        plt.axvline(x=freq, color = 'b', label = 'axvline - full height')
        ax.text(freq+offset, y,text, rotation=0, va='top')

    def ann_stats(self, ax, xloc, yloc):
        """ used for annotating crystal parameters on the S21 plot """
        txt  = "Cm: {:8.3f} pF\n".format(self.Cm/1e-15)
        txt += "Rs:  {:8.3f} Ohm\n".format(self.Rs)
        txt += "Lm: {:8.3f} mH\n".format(self.Lm/1e-3)
        txt += "bw: {:8.3f} Hz\n".format(self.df)
        txt += "Q_0:  {:8.3f}\n".format(self.Q_0)
        txt += "Q_L:  {:8.3f}\n".format(self.Q_L)
        txt += "Beta:  {:.5f}".format(self.beta)
        ax.text(xloc, yloc, txt, va='top')

    def find_3dB_points(self):
        """ find the -3 dB points """
        fs, mag   = self.find_max_freq_mag(self.s21_data)
        center_idx = np.where(self.s21_data == mag)[0][0]

        left_mag = self.find_nearest(self.s21_data[:center_idx], mag-3)
        left_idx = np.where(self.s21_data == left_mag)[0][0]
        left_freq = self.freq_data[left_idx]

        right_mag = self.find_nearest(self.s21_data[center_idx:], mag-3)
        right_idx = np.where(self.s21_data == right_mag)[0][0]
        right_freq = self.freq_data[right_idx]
        return ((left_freq, left_mag), (right_freq, right_mag))

    def plot_3db(self, fname):
```

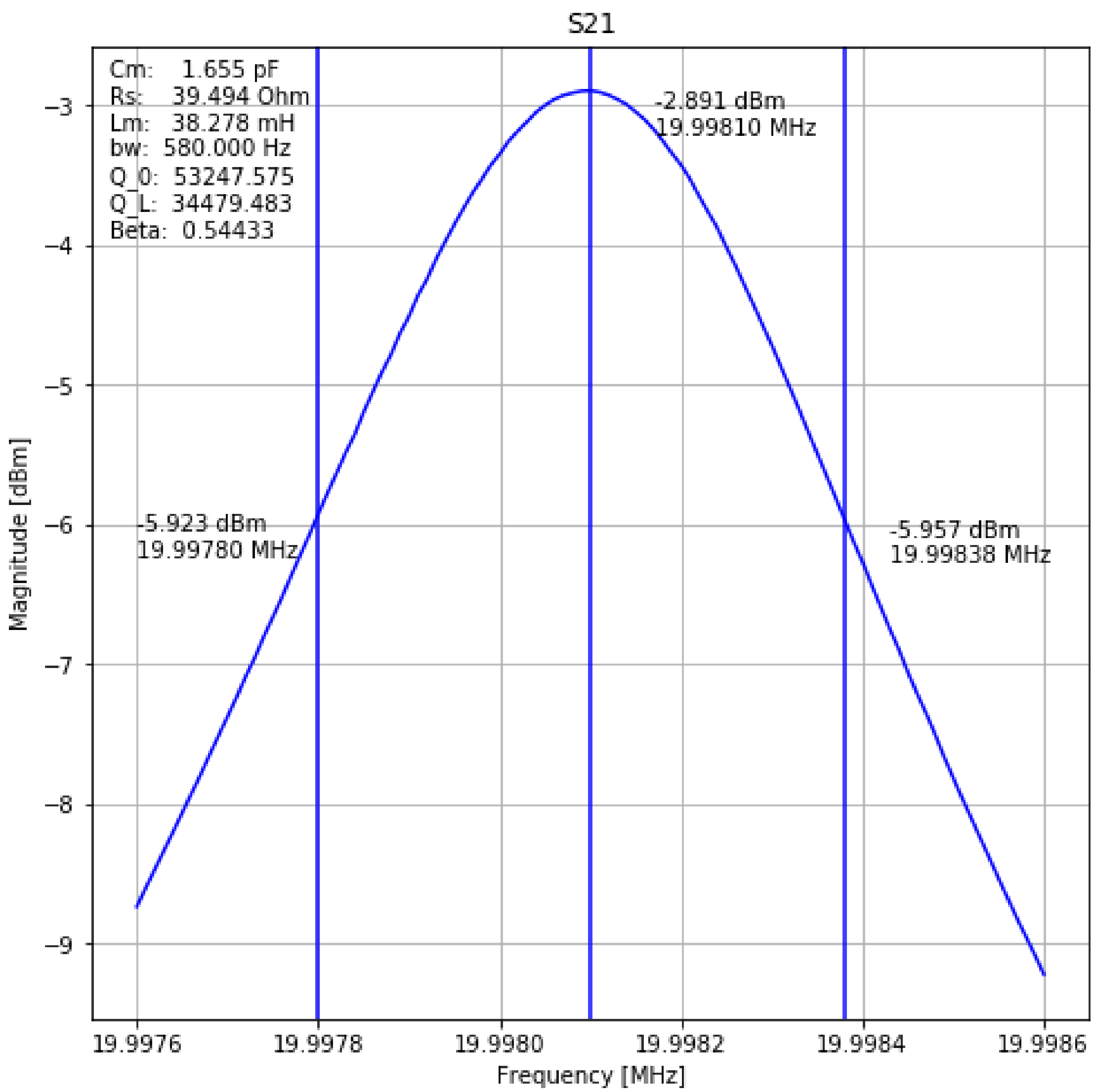
```
        """ S21 magnitude plot """
        fig, ax = plt.subplots()
        plt.plot(self.freq_data, self.s21_data, "-b")
        self.annot(ax, self.freq_0deg, self.S21_0deg, np.max(self.
↪s21_data), offset=0.00007)
        #self.annot(ax, self.freq_460MHz, self.S21_460MHz, np.max(self.
↪s21_data)+0.7)
        idx = np.where(self.s21_data == self.left[1])
        self.annot(ax, self.left[0], self.left[1], self.s21_data[idx][0],␣
↪offset=-0.0002)
        idx = np.where(self.s21_data == self.right[1])
        self.annot(ax, self.right[0], self.right[1], self.
↪s21_data[idx][0], offset=0.00005)
        plt.title("S21")
        self.ann_stats(ax, plt.xlim()[0]+0.00002, plt.ylim()[1]-0.1)
        plt.grid(True)
        plt.xlabel("Frequency [MHz]")
        plt.ylabel("Magnitude [dBm]")
        #plt.show()
        plt.savefig(fname)

    def __call__(self):
        self.freq_0deg, self.S21_0deg = self.find_max_freq_mag(self.
↪s21_data)
        self.fs = self.freq_0deg

        self.left, self.right = self.find_3dB_points()
        self.Rs, self.Reff = self.Rs_calc(self.S21_0deg, 50)
        self.df = self.df_calc(self.left[0]*1e6, self.right[0]*1e6)
        self.Cm, self.Lm, self.Q_0, self.Q_L = self.Cm_Lm_calc(
            self.freq_0deg * 1e6, self.df, self.Rs, self.Reff)
        print("Resonance Freq: %3.4f MHz at %3.4f dBm" %(self.freq_0deg,␣
↪self.S21_0deg))
        print("Full bandwidth: %3.4f Hz" %self.df)
        print("Q_L: %3.4f" %self.Q_L)
        print("Q_0: %3.4f" %self.Q_0)
        print("Coupling factor (beta): %.4f" %self.beta)
```

[3]:
```
c = CrystalAnalysis()
c.plot_3db("Res_bw.png")
guess = (c.fs, c.Q_L)
```

```
firmware version: 1.1.1
############ Under-coupled #############
```

```
Resonance Freq: 19.9981 MHz at -2.8911 dBm
Full bandwidth: 580.0000 Hz
Q_L: 34479.4828
Q_0: 53247.5749
Coupling factor (beta): 0.5443
```





2 Expected crystal measurement results

Crystal	fs (MHz)	max(S_21) (dB)	BW (Hz)	Q_L	Q_0	Beta
1	19.998	-11.87	310	64509.7	185670.7	1.878
2	19.998	-1.29	500	39996.1	68749.8	0.718
3	19.998	-3.42	630	31742.9	47512.4	0.496
4	19.998	-2.89	590	33894.9	52353.3	0.544
5	19.998	-12.07	320	62493.6	173206.2	1.771
6	19.998	-11.77	300	64509.7	190427.6	1.951
7	19.998	-3.05	420	47614.4	93406.3	0.961
8	19.998	-11.54	300	66660.1	204672.0	2.070

Crystal	fs (MHz)	max(S_21) (dB)	BW (Hz)	Q_L	Q_0	Beta
9	19.998	-2.87	580	34479.5	53230.0	0.544