Supplementary Information Two: Sequential Monte Carlo methods for data assimilation problems in ecology.

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Simulation study One : Linear Gaussian State Space Models

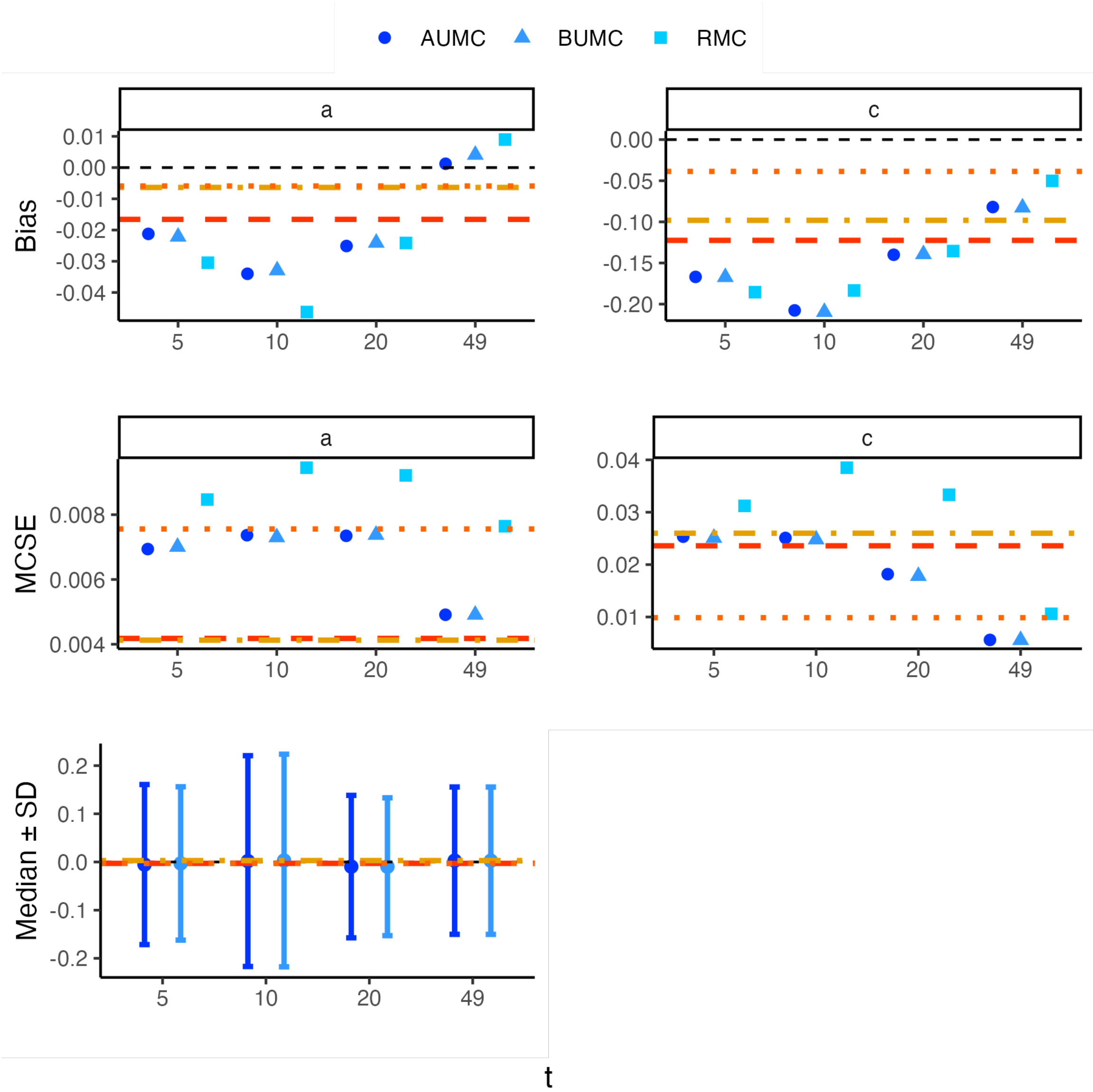

Figure 1: First row: Bias in model parameters (a, c); second row: Monte Carlo standard error (MCSE) of the model parameters (a, c); last row: root mean square error (RMSE) of the latent state distribution $x_{1:50}$ (the length of the error bars represents the standard deviation of the estimated RMSE for each simulation). The horizontal dashed line (— — —) represents the corresponding estimates from the full model fitted with particle Markov Chain Monte Carlo (pMCMC) using auxiliary particle filter algorithm; the horizontal dot-dashed line (—·—) represents the corresponding estimates from the full model fitted with pMCMC using bootstrap particle filter; the horizontal dotted line (· · · · ·) represents the corresponding estimates from the full model fitted with Markov Chain Monte Carlo algorithm.

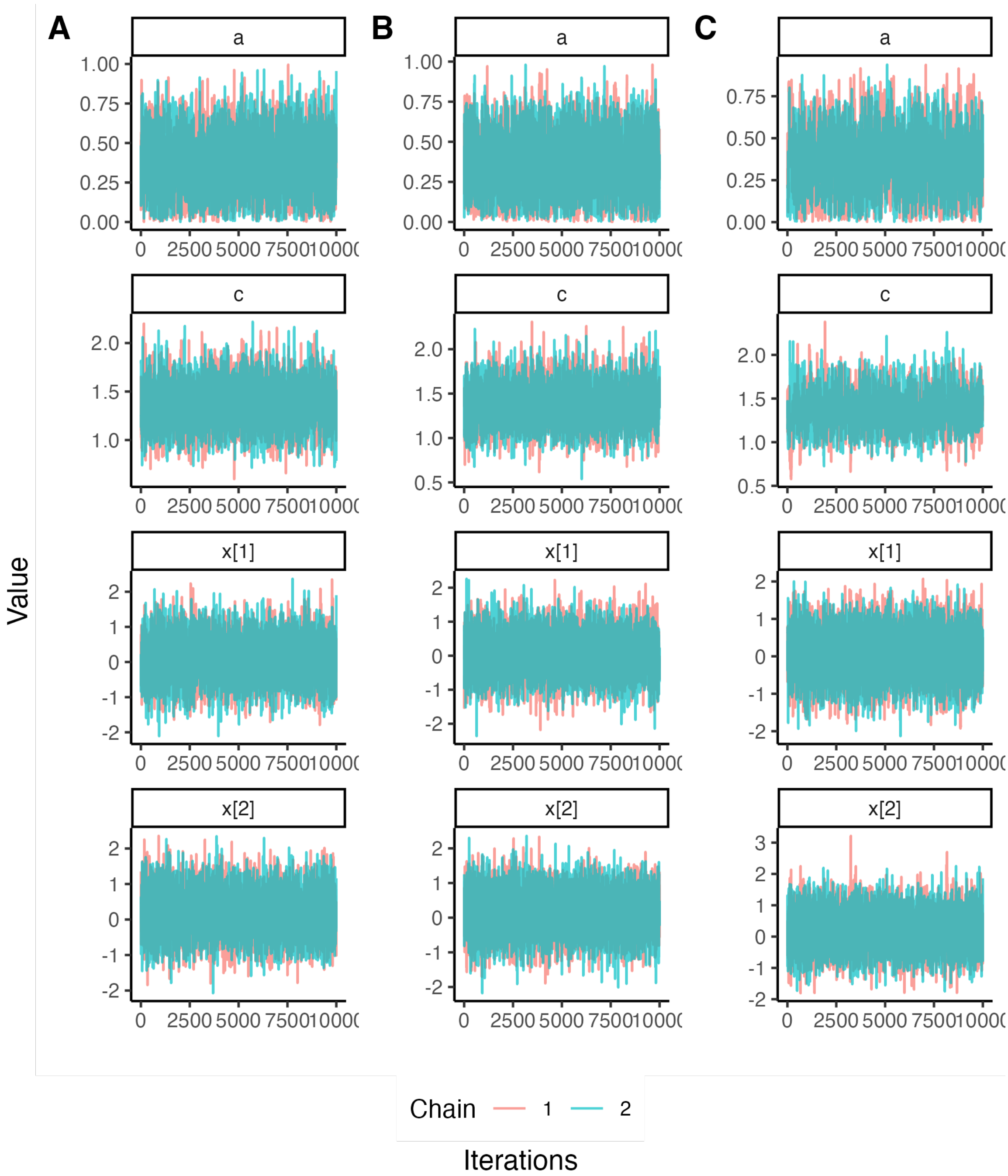

Figure 2: Convergence traceplots for model parameters (a and c) and latent state (x_1 and x_2) estimated from the full models. Each column corresponds to a particular method used to fit the full model: A) particle Markov Chain Monte Carlo (pMCMC) with bootstrap particle filter B) particle Markov Chain Monte Carlo (pMCMC) with bootstrap particle filter C) Markov Chain Monte Carlo algorithm.

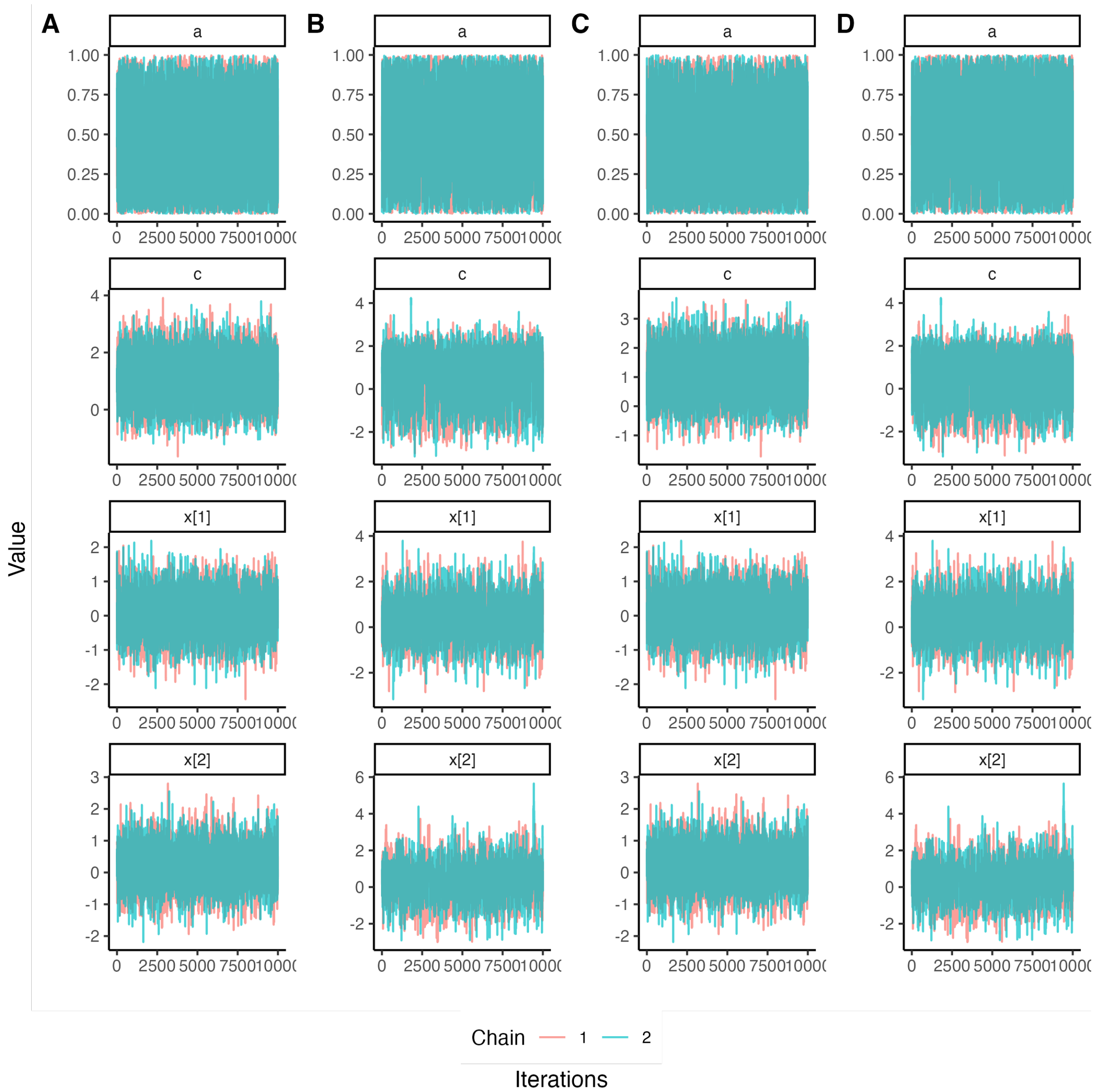

Figure 3: MCMC convergence traceplots for model parameters (a and c) and latent state (x_1 and x_2) estimated from the updated models. Each column corresponds to a particular method used to fit the full model: A) particle Markov Chain Monte Carlo (pMCMC) with bootstrap particle filters using $t = 5$, B) pMCMC with bootstrap particle filters using $t = 45$, C) pMCMC with auxiliary particle filters using $t = 5$ D) pMCMC with auxiliary particle filters using $t = 45$.

Example 2 : Dynamic Occupancy Models

Table 1: Summary of dynamic occupancy model parameters (with standard error in paranthesis). The columns labelled truth refers to the true parameter values, BMC refers to the parameter estimates from the full model fitted with Markov Chain Monte Carlo (MCMC), RMC refers to the parameter estimates from the reduced model fitted with MCMC and BUMC and AUMC refer to the estimates from the updated model fitted with bootstrap and auxiliary particle filters respectively. The values are rounded to two decimal places, so the standard error values of 0 means the estimate was less than 0.005.

Parameter	t	Truth	BMC	RMC	BUMC M10	AUMC M10	BUMC M25	AUMC M25	BUMC M50	AUMC M50	BUMC M100	AUMC M100
α^p	25	5.44	5.45 (0.16)	5.33 (0.17)	5.34 (0.25)	5.34 (0.26)	5.33 (0.23)	5.34 (0.25)	5.33 (0.21)	5.33 (0.19)	5.33 (0.18)	5.34 (0.22)
α^p	29	5.44	5.45 (0.16)	5.41 (0.16)	5.42 (0.17)	5.42 (0.17)	5.42 (0.17)	5.42 (0.18)	5.42 (0.17)	5.42 (0.17)	5.42 (0.17)	5.42 (0.17)
α^ϕ	25	-2	-1.75 (0.18)	-1.75 (0.17)	-1.75 (0.34)	-1.74 (0.34)	-1.74 (0.3)	-1.75 (0.31)	-1.74 (0.25)	-1.75 (0.26)	-1.75 (0.22)	-1.75 (0.29)
α^ϕ	29	-2	-1.75 (0.18)	-1.74 (0.17)	-1.74 (0.22)	-1.74 (0.23)	-1.74 (0.22)	-1.74 (0.22)	-1.74 (0.2)	-1.74 (0.21)	-1.74 (0.21)	-1.74 (0.2)
α^ψ	25	7.4	7.69 (0.65)	7.44 (0.66)	7.45 (0.72)	7.46 (0.73)	7.45 (0.71)	7.45 (0.71)	7.45 (0.68)	7.45 (0.69)	7.45 (0.68)	7.45 (0.7)
α^ψ	29	7.4	7.69 (0.65)	7.66 (0.67)	7.66 (0.68)	7.66 (0.68)	7.66 (0.68)	7.66 (0.68)	7.66 (0.68)	7.66 (0.68)	7.66 (0.67)	7.66 (0.68)
σ_{α^p}	25	2	2.58 (1.54)	2.43 (1.47)	2.43 (1.47)	2.43 (1.47)	2.43 (1.47)	2.43 (1.47)	2.43 (1.46)	2.43 (1.47)	2.43 (1.47)	2.43 (1.46)
σ_{α^p}	29	2	2.58 (1.54)	2.52 (1.51)	2.52 (1.51)	2.52 (1.51)	2.52 (1.51)	2.52 (1.51)	2.52 (1.51)	2.52 (1.51)	2.52 (1.51)	2.52 (1.51)
σ_{α^ψ}	25	2	4.11 (1.56)	3.99 (1.56)	3.99 (1.56)	3.99 (1.55)	4 (1.56)	3.99 (1.55)	3.99 (1.55)	3.99 (1.56)	3.99 (1.56)	3.99 (1.56)
σ_{α^ψ}	29	2	4.11 (1.56)	4.1 (1.54)	4.09 (1.54)	4.1 (1.54)	4.09 (1.54)	4.1 (1.54)	4.1 (1.54)	4.1 (1.54)	4.09 (1.54)	4.1 (1.54)
β^p	25	3.74	3.7 (0.21)	3.62 (0.23)	3.62 (0.32)	3.62 (0.32)	3.61 (0.3)	3.62 (0.31)	3.62 (0.27)	3.62 (0.25)	3.62 (0.25)	3.62 (0.29)
β^p	29	3.74	3.7 (0.21)	3.67 (0.21)	3.67 (0.22)	3.67 (0.23)	3.67 (0.22)	3.67 (0.23)	3.67 (0.22)	3.67 (0.22)	3.67 (0.22)	3.67 (0.22)
β^ϕ	25	1.5	1.03 (0.3)	1.03 (0.3)	1.04 (0.41)	1.03 (0.42)	1.04 (0.4)	1.04 (0.4)	1.03 (0.35)	1.04 (0.35)	1.03 (0.32)	1.04 (0.37)
β^ϕ	29	1.5	1.03 (0.3)	1.03 (0.3)	1.03 (0.34)	1.03 (0.34)	1.03 (0.33)	1.03 (0.34)	1.03 (0.33)	1.03 (0.32)	1.03 (0.33)	1.03 (0.32)
β^ψ	25	-1.07	-0.81 (1.04)	-0.81 (1.07)	-0.81 (1.1)	-0.81 (1.1)	-0.81 (1.1)	-0.82 (1.1)	-0.82 (1.08)	-0.81 (1.09)	-0.81 (1.08)	-0.81 (1.09)
β^ψ	29	-1.07	-0.81 (1.04)	-0.81 (1.05)	-0.81 (1.06)	-0.81 (1.06)	-0.81 (1.06)	-0.81 (1.06)	-0.81 (1.05)	-0.81 (1.05)	-0.81 (1.05)	-0.81 (1.05)
σ^{β_p}	25	3	4.51 (1.53)	4.48 (1.56)	4.47 (1.56)	4.47 (1.56)	4.47 (1.56)	4.47 (1.56)	4.47 (1.56)	4.47 (1.56)	4.47 (1.56)	4.47 (1.56)
σ^{β_p}	29	3	4.51 (1.53)	4.5 (1.53)	4.49 (1.53)	4.49 (1.53)	4.49 (1.53)	4.49 (1.53)	4.49 (1.53)	4.49 (1.53)	4.49 (1.53)	4.49 (1.53)
σ^{β_ψ}	25	3	2.61 (1.64)	2.49 (1.6)	2.49 (1.6)	2.49 (1.6)	2.49 (1.6)	2.49 (1.6)	2.49 (1.6)	2.49 (1.6)	2.49 (1.6)	2.49 (1.6)
σ^{β_ψ}	29	3	2.61 (1.64)	2.56 (1.61)	2.57 (1.61)	2.57 (1.61)	2.57 (1.61)	2.57 (1.61)	2.57 (1.61)	2.57 (1.61)	2.57 (1.61)	2.57 (1.61)
γ	25	0.05	0.05 (0)	0.05 (0)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)
γ	29	0.05	0.05 (0)	0.05 (0)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)

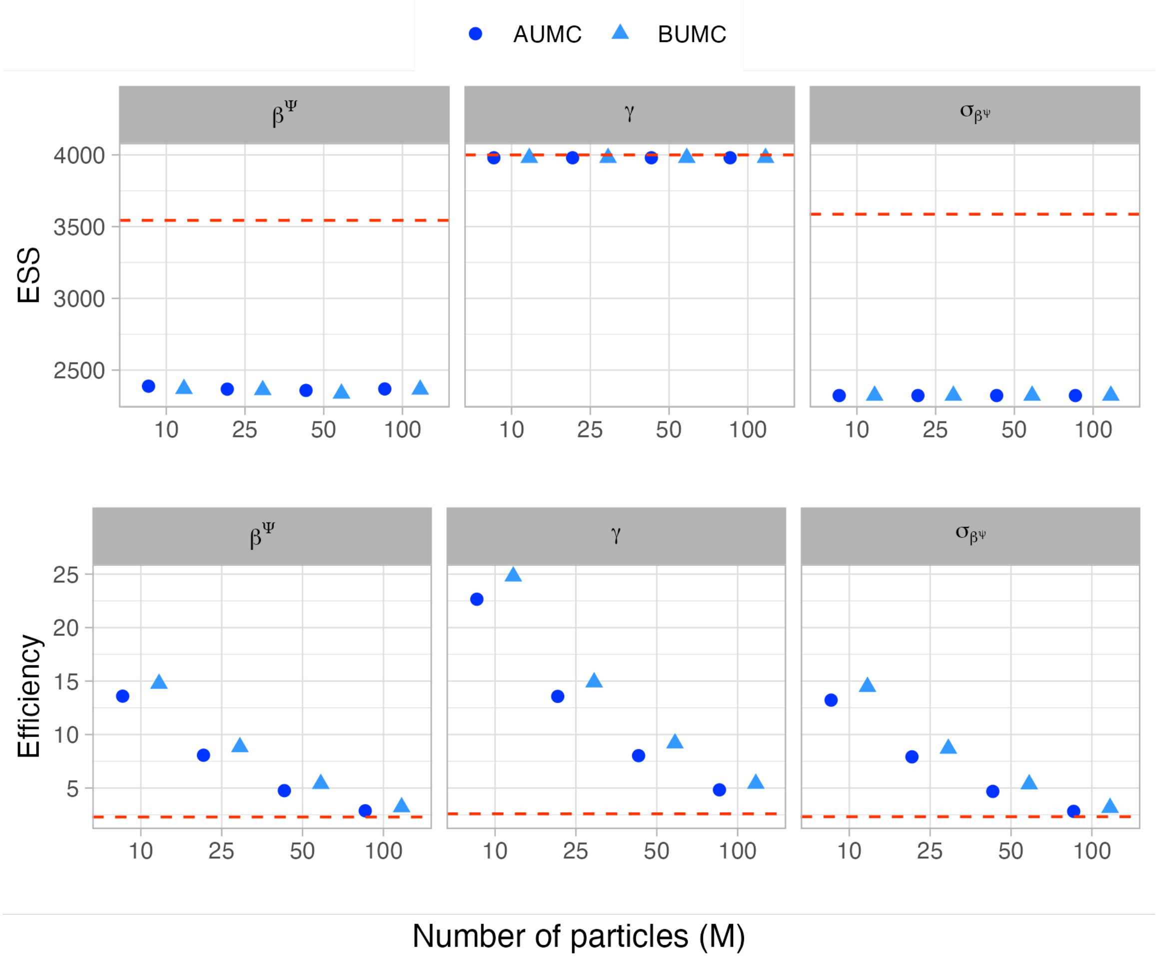

Figure 4: First row: Effective sample size (ESS) of some selected model parameters estimated from dynamic occupancy model fitted with the proposed Monte Carlo algorithms: BUMC and AUMC. Second column: Efficiency of model parameters estimated from dynamic occupancy model fitted with the proposed Monte Carlo algorithms: BUMC and AUMC. The models were fitted using $M = 50$ particles. Out of the 30 years data we simulated, $t = 29$ years were used to fit the reduced model.

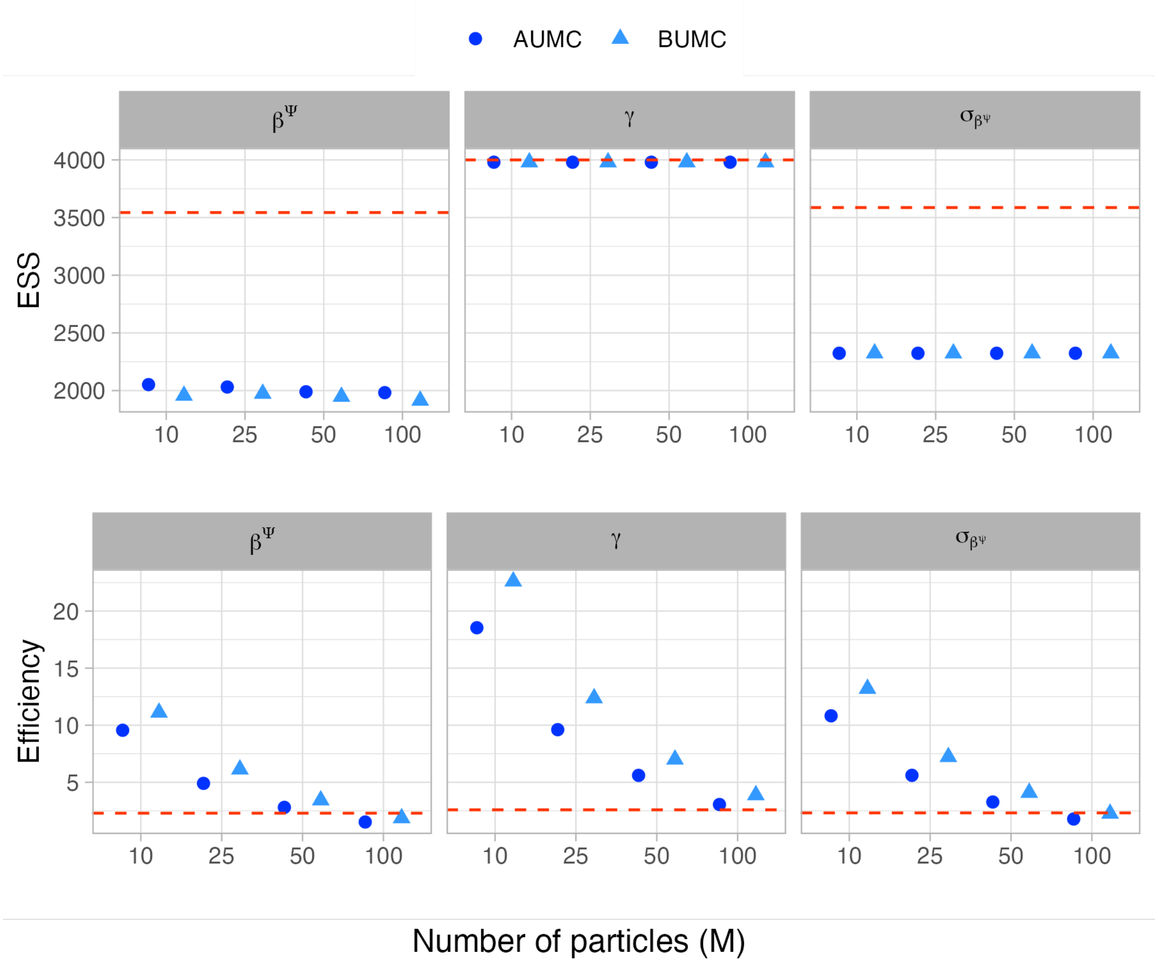

Figure 5: First row: Effective sample size (ESS) of some selected model parameters estimated from dynamic occupancy model fitted with the proposed Monte Carlo algorithms: BUMC and AUMC. Second column: Efficiency of model parameters estimated from dynamic occupancy model fitted with the proposed Monte Carlo algorithms: BUMC and AUMC. The models were fitted using $M = 50$ particles. Out of the 30 years data we simulated, $t = 25$ years were used to fit the reduced model.

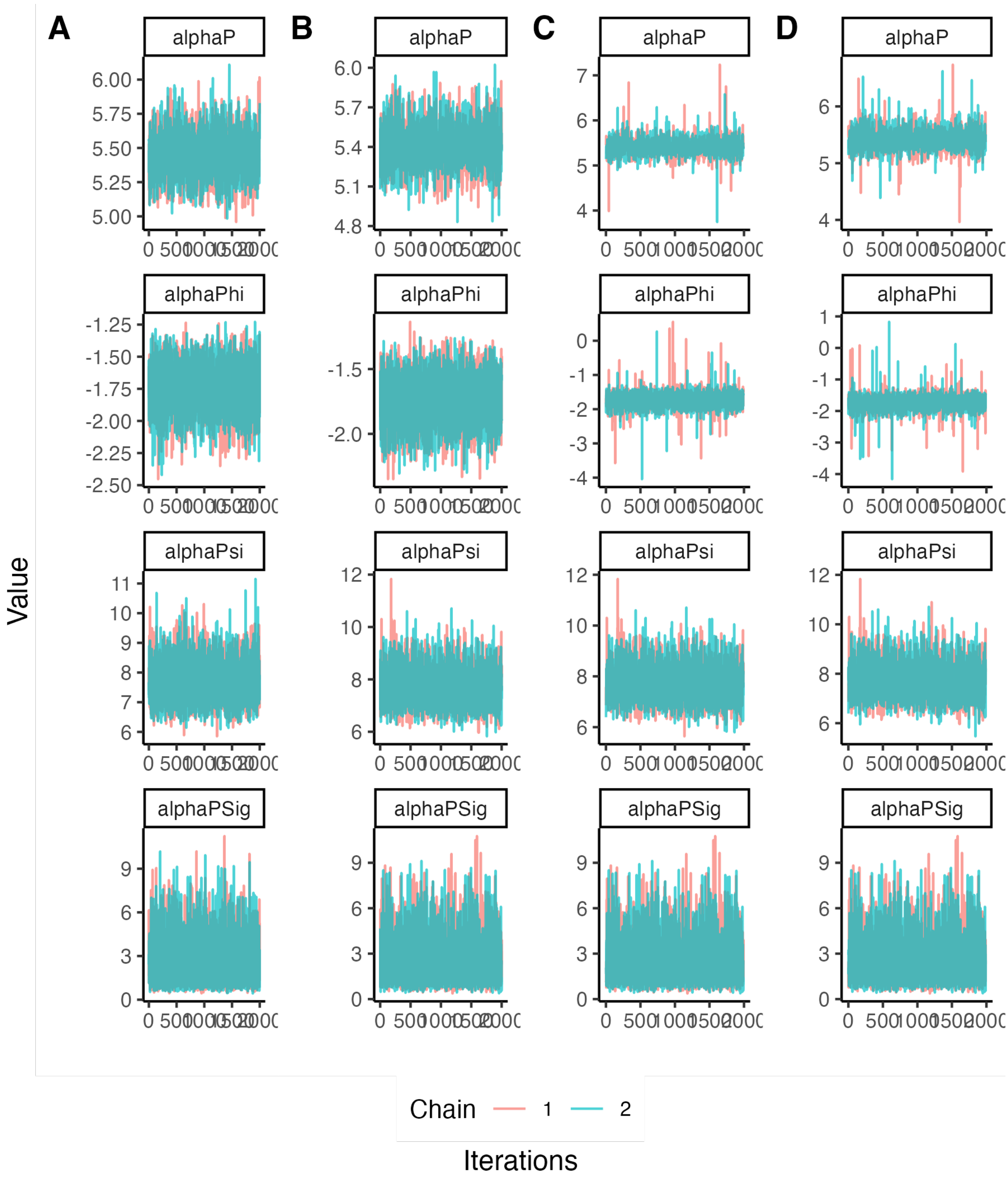

Figure 6: Converge traceplots of model parameters estimated from A) full model fitted using Markov Chain Monte Carlo (MCMC), B) reduced model using MCMC, C) updated model using particle MCMC (pMCMC) with bootstrap particle filter and D) updated model using pMCMC with auxiliary particle filter. The model parameters presented here are: first row - alphaP (α^P), second row - alphaPhi (α^ϕ), third row - alphaPsi (α^ψ), last row - alphaPsig (σ_{α^P})

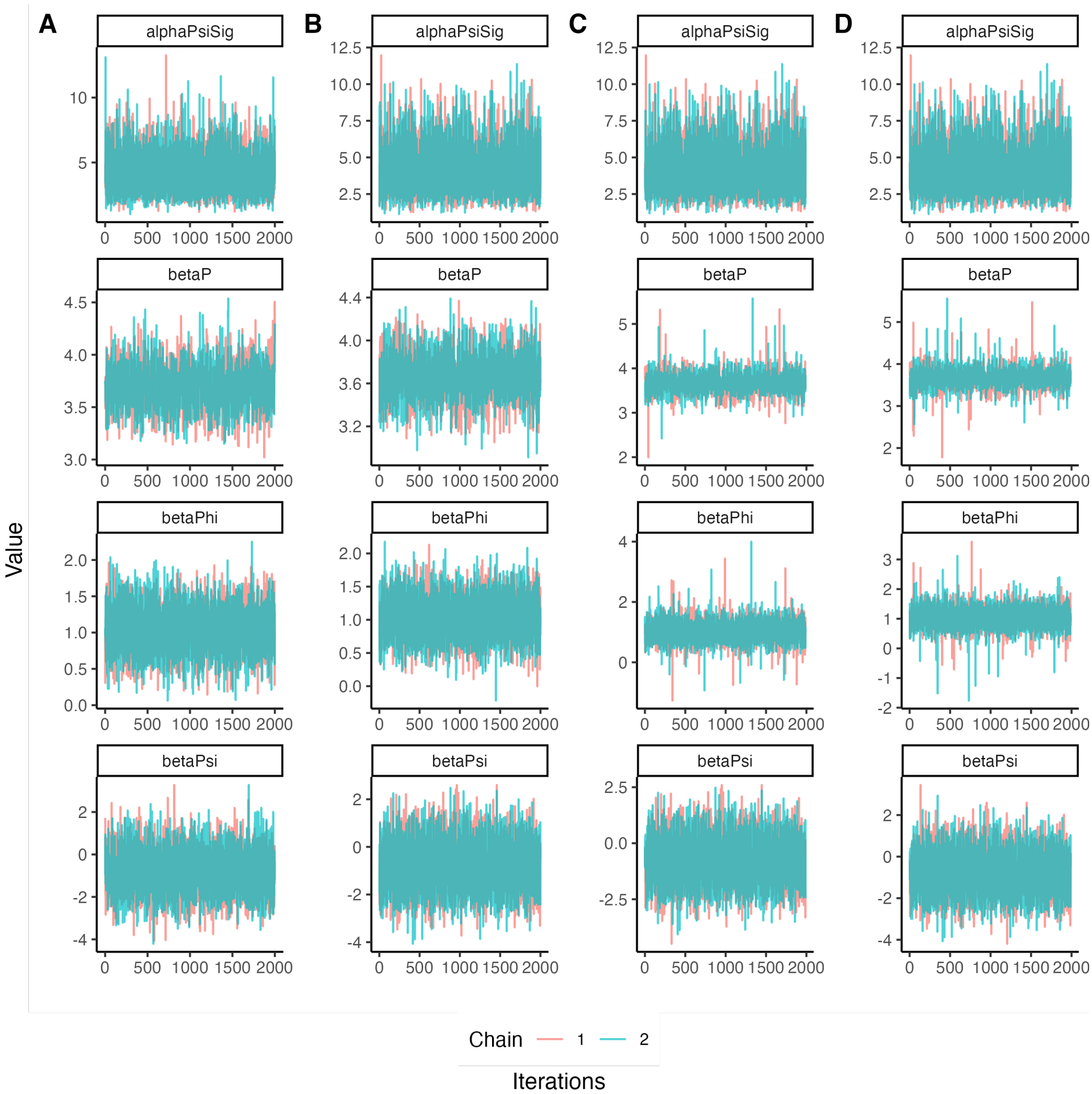

Figure 7: Converge traceplots of model parameters estimated from A) full model fitted using Markov Chain Monte Carlo (MCMC), B) reduced model using MCMC, C) updated model using particle MCMC (pMCMC) with bootstrap particle filter and D) updated model using pMCMC with auxiliary particle filter The model parameters presented here are: first row - alphaPsiSig ($\sigma_{\alpha^{\psi}}$), second row - betaP (β_p), third row - betaPhi (β^{ϕ}), last row - betaPsig (σ_{β^P})