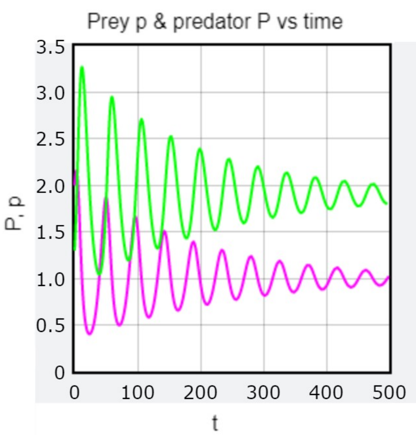
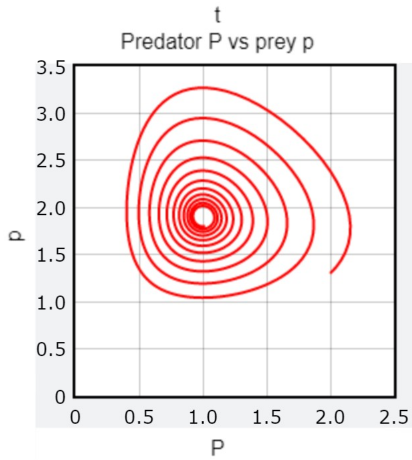
Exercise 3.3: Chaos in population dynamics models

Output of the PredatorPrey.py:





For this question I adapt the "Predator-prey equations", also known as "Lotka-Volterra equations", which defined in such a way:

$$\frac{dp}{dt} = Ap - BpP, \tag{1}$$

$$\frac{dP}{dt} = CpP - DP, \tag{2}$$

where $p(t)$ is the population of prey, $P(t)$ is the population of predator, with constants A, B, C, D.

Part(a):

Explain in words why their is a correlation between extremums in prey and predator populations?

Ans: **The equation (1) denotes that the prey population growth rate is increasing exponentially with the constant A and under the control of term "BpP". When the p gets to zero, the growth rate of prey becomes zero which makes the predator population growth rate becomes negative forever.**

The equation (2)'s mechanism is similar to (1), which denotes that the predator population growth rate is decreasing exponentially with the constant D limited by the term "CpP".

In conclusion, there is a correlation between extremums in prey and predator populations.

Part(b):

Because predators eat prey, one might expect the existence of a large number of predators to lead to the eating of a large number of prey. Explain why the maxima in predator population and the minima in prey population do not occur at the same time.

Ans: **From the diagram "Prey p predator P vs time" indicates that the prey population reaches maximum, as the predator got the highest growth rate (determined by slope of the curve).**

Limited by equation (1) and (2), it is impossible for both of prey and predator reaches the maximum at the same time. Because the fastest population growth rate and the population can't be maximum at the same time.

Part(c):

Why do the extreme values of the population just repeat with no change in their values?

Ans: The population of prey and predator will reach a balance after certain amount of time since A, B, C, D is constant, which means the other condition(e.g. food, space) is fixed in this model.

Part(d):

Explain the meaning of the spirals in the predator?prey phase space diagram.

Ans: The "Predator P vs prey p" graph converges to approximately point (1,2), denotes $P(t) : p(t) = 1 : 2$. Which also means there is 1 predator for every 2 prey.

Part(e):

Explain why the phase-space orbits closed?

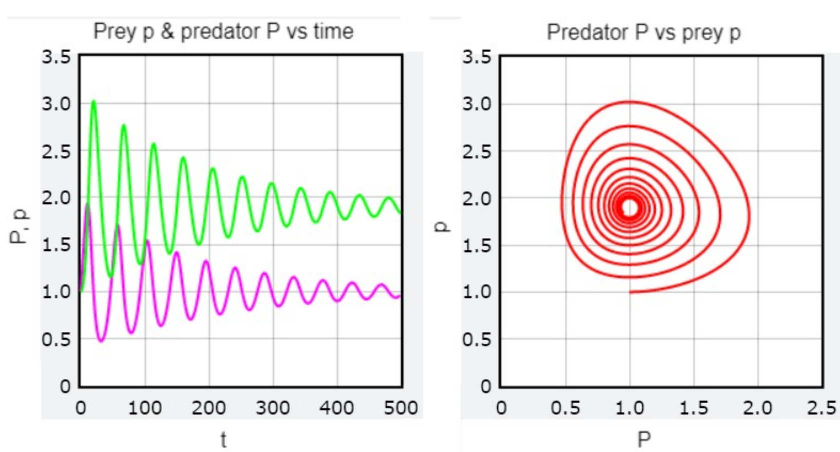
Ans: The given model with fixed conditions (For example, 10 tiger and 50 buffalo in an area of $10km^2$ with infinite food.) will reach a balance, because of the limitation applied to each other by equations (1) and (2).

Part(f):

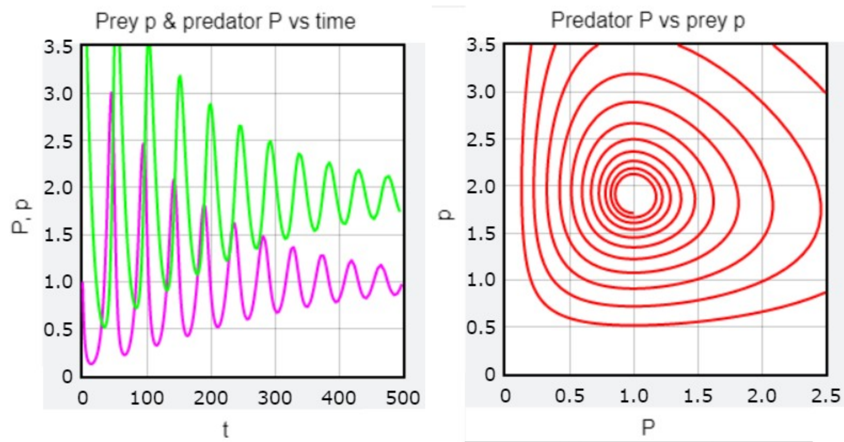
What different initial conditions would lead to different phase-space orbits?

Ans: By manipulating the initial value of y[0] and y[1] in the PredatorPrey.py program, I found out the system reaches the balance faster when the initial number of prey is more than the number of predator with a reasonable range.

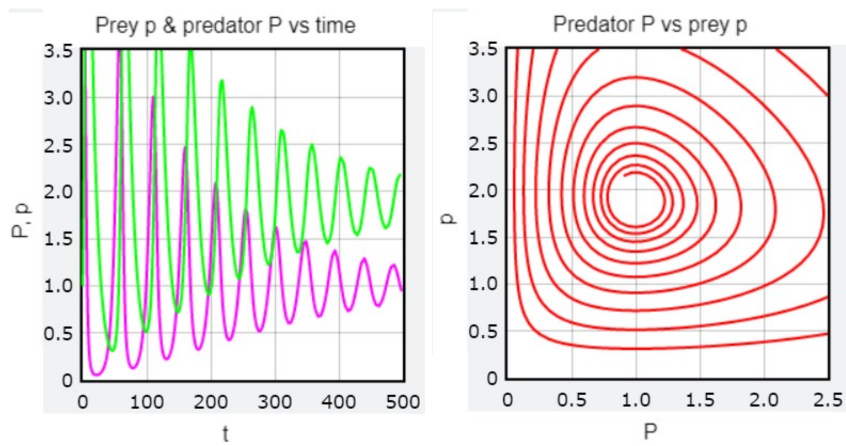
Here are some sample outputs with different p(o) and P(o) value:



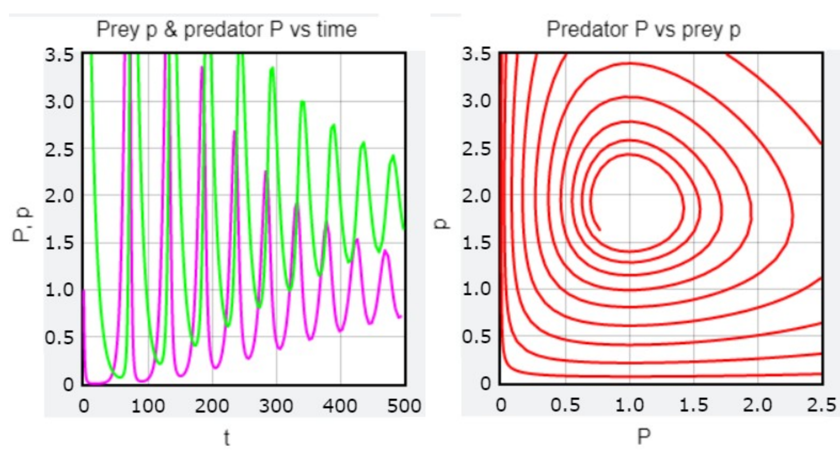
Plots for P(0) = 1 p(0) = 1:



Plots for $\mathbf{P(0) = 1 \ \ p(0) = 5}$:



Plots for $\mathbf{P(0) = 5 \ \ p(0) = 1}$:



Plots for $\mathbf{P(0) = 1 \ \ p(0) = 10}$:

Part(g):

Discuss the symmetry and lack of symmetry in the phase-space orbits.

Ans: **The graph is not symmetry when the different between 2 populations is big, which will take the system more time to reach balance.**