Dyanmics Equations for Smartmouse 2018

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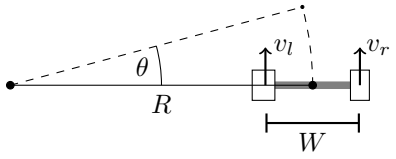
Deriving the kinematic equations of a differential drive robot.



At an instant in time we say that the robot is turning around some point ICC, the instanteous center. The radius R about that point is what we want to solve for. We start with the knowedge that since the robot doesn't tear itself apart while driving, the rate w at which both wheels (and the robot) move around that center is the same.

$$\omega_l = \omega_r = w \tag{1}$$

Let R_l and R_l be the radius to the right and left wheels.

$$\omega_l = \frac{v_l}{R_l}, \omega_r = \frac{v_r}{R_r} \tag{2}$$

We can then combine 1 and 2

$$\frac{v_l}{R_l} = \frac{v_r}{R_r} \tag{3}$$

We can then substitute $R_l = R - \frac{w}{2}$ and $R_r = R + \frac{w}{2}$

$$\frac{v_l}{R - \frac{w}{2}} = \frac{v_r}{R + \frac{w}{2}} \tag{4}$$

No do some algebra...

$$\frac{v_l}{R - \frac{w}{2}} = \frac{v_r}{R + \frac{w}{2}}$$

$$\frac{R - \frac{w}{2}}{v_l} = \frac{R + \frac{w}{2}}{v_r}$$

$$\frac{R}{v_l} - \frac{w}{2v_l} = \frac{R}{v_r} + \frac{w}{2v_r}$$

$$\frac{R}{v_l} - \frac{R}{v_r} = \frac{w}{2v_r} + \frac{w}{2v_l}$$

$$\frac{R(v_r - v_l)}{v_r v_l} = \frac{w(v_l + v_r)}{2v_r v_l}$$

$$R(v_r - v_l) = w(v_l + v_r)$$

$$R = \frac{w(v_l + v_r)}{2(v_r - v_l)}$$

mic drop.