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Math 21a: Multivariable calculus Fall 2019

# Homework 1: Geometry and Distance

This homework is due on Wednesday, 9/11 at the beginning of class.

1 a) Find its center and radius of the sphere $S$ given by $x^2 - 4x + y^2 + 2y + z^2 - 8z = 30$. (SEE BACK)

b) Find the distance from the center of $S$ defined in a) to the sphere $T : x^2 + y^2 + z^2 = 900$. (SEE BACK)

c) Find the minimal distance between the spheres $S$ and $T$. This is the minimal distance between two points $x, y$ where $x$ is in $S$ and $y$ is in $T$. (SEE BACK)

2 a) Find the distance from $P = (-21, -7, -6)$ to $y = 0$. $\boxed{7}$

b) Find the distance from $P$ to the $x$-axes. $\sqrt{49+36} = \boxed{\sqrt{85}}$

c) Find a point which has distance 5 from the $x$ axes and distance 2 from the $yz$-plane. $\boxed{\langle 2, 5, 0 \rangle}$

3 a) Find an equation of the largest sphere with center $(4, 11, 9)$ that is contained in the first octant $\{x \geq 0, y \geq 0, z \geq 0\}$. (SEE BACK)

b) Find the equation for the sphere centered at $(6, 10, 8)$ which passes through the center $(4, 11, 9)$ of the sphere in a).

$x - 2y + z = \pm 2 \Rightarrow$ $\boxed{\text{2 parallel planes}}$

4 a) Describe the surface $S$ given by $(x - 2y + z)^2 = 4$ in $\mathbb{R}^3$.

If you like to see a bit of a story behind this, on the website, under "data", you find something about prime numbers related to this surface $S$.

b) What surface is $x^2 + y^2 - 3 = 0$ in $\mathbb{R}^3$? infinitely long "tube" along z dir. (radius $\sqrt{3}$)

c) What is the set $x^2 + y^2 - 3 = 0$ in $\mathbb{R}^2$? $\boxed{\text{circle}}$ (centered at $\langle 0, 0 \rangle$ with radius $\sqrt{3}$)

5 An ant moves on the unit cube bound by the walls $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ from the point $A = (0.4, 0.8, 1)$ to the point $B = (1, 0.8, 0.5)$. Compute the length of the two obvious paths, where one passes over three faces, the other only over two. ~~Which one is shorter? See the figures~~ on the third page.

$\boxed{\text{The path over 3 faces is shorter}}$ (SEE THIRD PAGE).

① A. $x^2 - 4x + y^2 + 2y + z^2 - 8z = 30$ gives a sphere S.

$$[(x-2)^2 - 4] + [(y+1)^2 - 1] + [(z-4)^2 - 16] = 30$$

$$(x-2)^2 + (y+1)^2 + (z-4)^2 = 51 \Longrightarrow \boxed{r_S = \sqrt{51},\ \text{center of } S: \begin{bmatrix}2\\-1\\4\end{bmatrix}}$$

B. We need to minimize $\sqrt{(2-a)^2 + (-1-b)^2 + (4-c)^2}$ for $a, b, c$ that satisfy $a^2 + b^2 + c^2 = 30^2$ to find the distance between <2,-1,4> and sphere T. But notice that we have S "contained" in T. The centers are offset by $\begin{bmatrix}2\\-1\\4\end{bmatrix}$, the vector from $\begin{bmatrix}0\\0\\0\end{bmatrix}$ pointing to center of S. If the spheres were both centered at $0 = \begin{bmatrix}0\\0\\0\end{bmatrix}$, the distance between center of S (and T) and T = radius of T. This offset shifts the center of S by the same amount it decreases the distance to T $\Longleftrightarrow$ $30 - \sqrt{2^2 + (-1)^2 + 4^2} = \boxed{30 - \sqrt{21}}$ is distance from center of S to T because radius is shortest distance to T along $\begin{bmatrix}2\\-1\\4\end{bmatrix}$.

C. Using the same logic as above, we just need to subtract the radius of S: $\boxed{30 - \sqrt{21} - \sqrt{51}}$ is shortest distance from S to T.

③ A. The largest sphere centered at <4,11,9> contained in octant given by $\{x \geq 0, y \geq 0, z \geq 0\}$ has radius 4 because any larger would cross the x-axis. This sphere is given by $\boxed{(x-4)^2 + (y-11)^2 + (z-9)^2 = 4^2}$.

B. The radius of the desired sphere is the distance between $(4,11,9)$ and $(6,10,8)$ $\Longrightarrow$ the desired sphere is given by

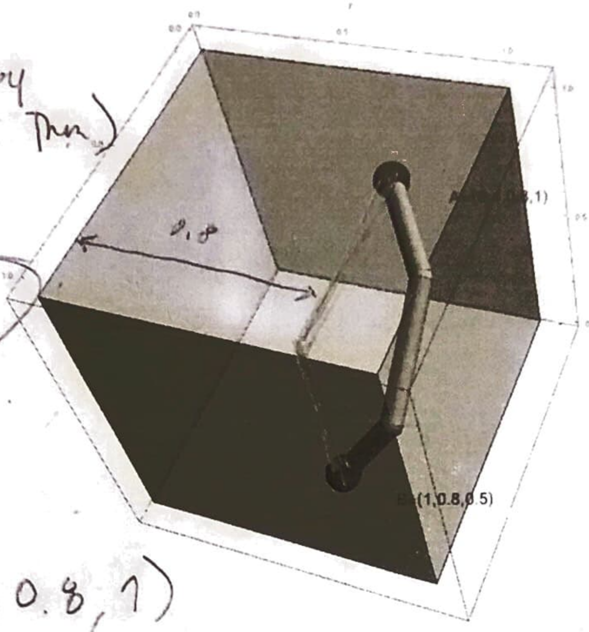
$$\boxed{(x-6)^2 + (y-10)^2 + (z-8)^2 = 2^2 + 1^2 + 1^2 = 6}.$$

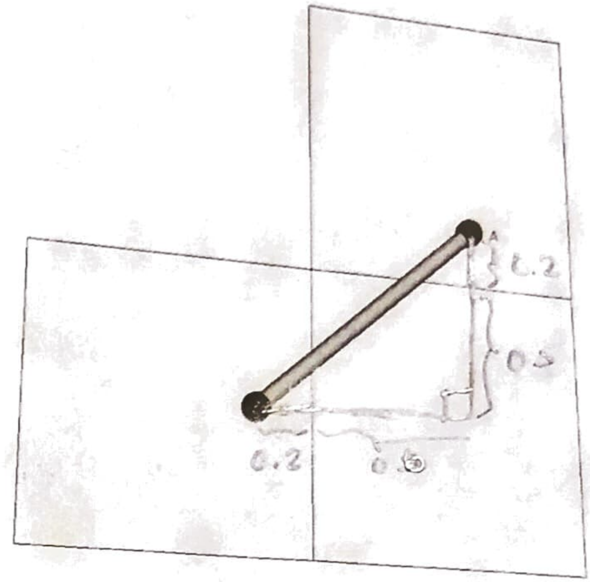
-B

distance (by Pythagorean Thm.) is given by:

$\sqrt{0.8^2 + 0.7^2}$

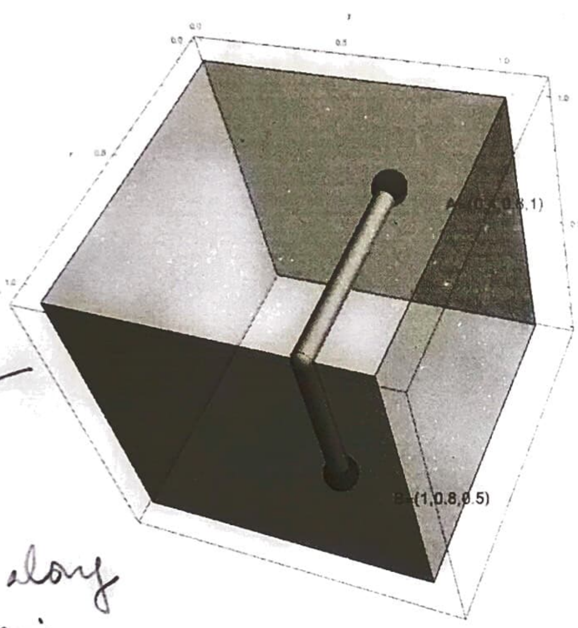
$\approx 1.063$

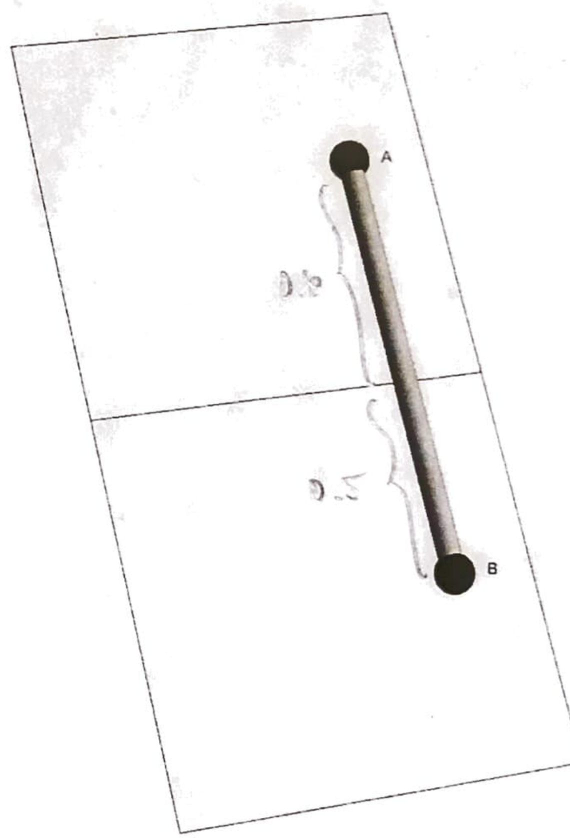



$A = (0.4, 0.8, 1)$

$B = (1, 0.8, 0.5)$




"Manhattan" distance along unit cube:

$= \underbrace{|0.4-1|}_{a_x - b_x} + \underbrace{|\cancel{0.8-0.8}|}_{a_y - b_y} + \underbrace{|1-0.5|}_{a_z - b_z}$

$= 0.6 + 0.5 = 1.1.$