Ravi Dayabhai

$\{\ldots -6, -3, 0, 3, 6, \ldots\}$

$\{\ldots -5, -2, 1, 4, \ldots\}$

$\{\ldots -4, -1, 2, 5 \ldots\}$

Problems for Recitation 11

1. Give a description of the equivalence classes associated with each of the following equivalence relations.

   (a) Integers x and y are equivalent if $x \equiv y \pmod 3$.

   (mod 3) partitions the integers into 3 blocks/equivalence classes, namely the sets consisting of elements whose remainders are the same when divided by 3

   (b) Real numbers x and y are equivalent if $\lceil x \rceil = \lceil y \rceil$, where $\lceil z \rceil$ denotes the smallest integer greater than or equal to z.

   the equivalence classes are given by $\forall n \in \mathbb{Z}\ \{x \in \mathbb{R} \mid n < x \leq n+1\}$; all reals in the interval between two successive integers (upper-bound inclusive) are one such equivalence class — and there is exactly one class for all such intervals

2. Show that neither of the following relations is an equivalence relation by identifying a missing property (reflexivity, symmetry, or transitivity).

   (a) The "divides" relation on the positive integers.

   $\boxed{\text{symmetry}}$ is the missing property

   (b) The "implies" relation on propositional formulas.

   $\boxed{\text{symmetry}}$ is the missing property; note that "iff" relation (the biconditional) is an equivalence relation.

3. Here is prerequistite information for some MIT courses:

$$\begin{array}{rcl} 18.01 \to 6.042 & \quad & 18.01 \to 18.02 \\ 18.01 \to 18.03 & & 6.046 \to 6.840 \\ 8.01 \to 8.02 & & 6.01 \to 6.034 \\ 6.042 \to 6.046 & & 18.03, 8.02 \to 6.02 \\ 6.01, 6.02 \to 6.003 & & 6.01, 6.02 \to 6.004 \\ 6.004 \to 6.033 & & 6.033 \to 6.857 \end{array}$$

   (a) Draw a Hasse diagram for the corresponding partially-ordered set. (A ***Hasse diagram*** is a way of representing a poset $(A, \preceq)$ as a directed acyclic graph. The vertices are the element of A, and there is generally an edge $u \to v$ if $u \preceq v$. However, self-loops and edges implied by transitivity are omitted.) You'll need this diagram for all the subsequent problem parts, so be neat!

   (SEE NEXT PAGE)

   (b) Identify a largest chain. (A ***chain*** in a poset $(S, \preceq)$ is a subset $C \subseteq S$ such that for all $x, y \in C$, either $x \preceq y$ or $y \preceq x$.)

   $\{18.01, 18.03, 6.02, 6.004, 6.033, 6.857\}$

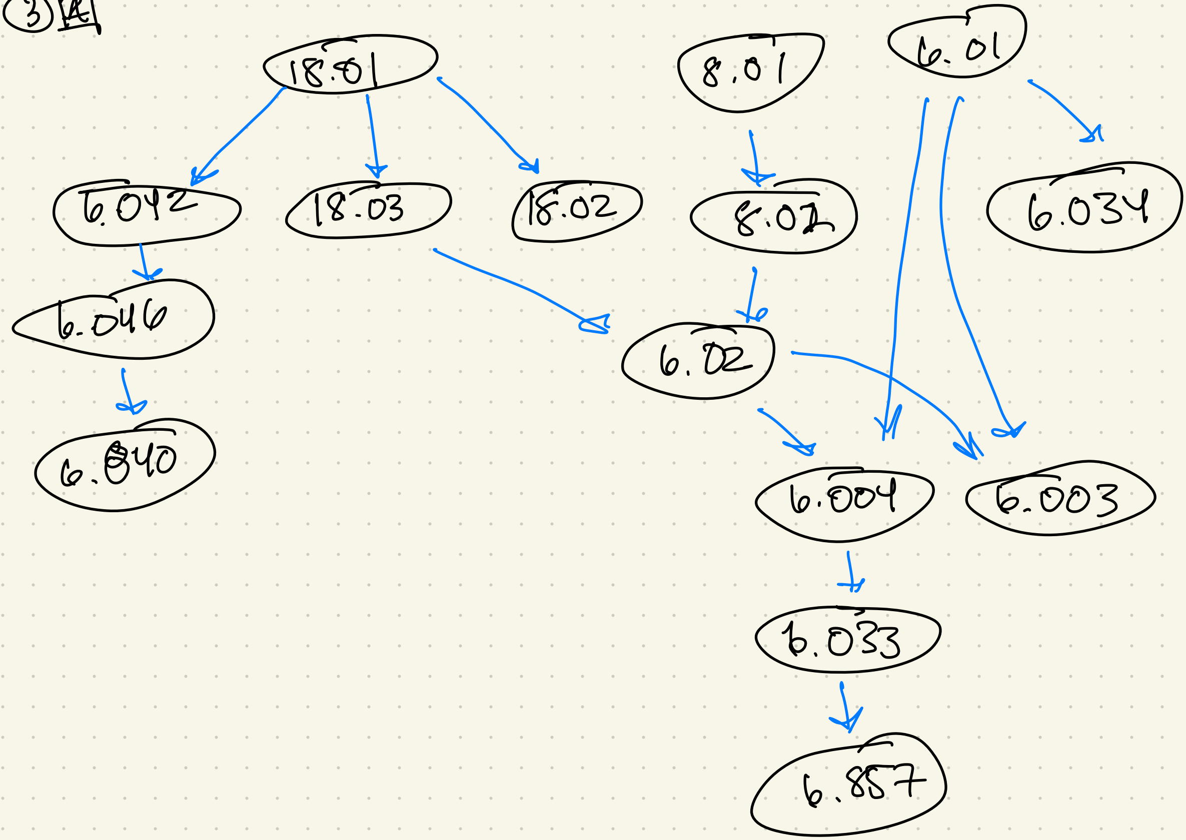
   (c) Suppose that you want to take all the courses. What is the minimum number of terms required to graduate, if you can take as many courses as you want per term?

   6 (corresponds to vertices in longest chain)

   (d) Identify a largest **antichain**. (An ***antichain*** in a poset $(S, \preceq)$ is a subset $A \subseteq S$ such that for all $x, y \in A$ with $x \neq y$, neither $x \preceq y$ nor $y \preceq x$.)

   $A := \{6.042, 18.03, 18.02, 6.02, 6.034\}$

③ $\boxed{A}$



$\boxed{F}$ { 18.01, 8.01, 18.03, 8.02, 6.02, 6.01, 6.003, 6.004, 6.042, 18.02, 6.046, 6.033, 6.840, 6.034, 6.857 }

$\boxed{B}$ See green underlines ⇒ 8 semesters.

(e) What is the maximum number of classes that you could possibly take at once?

8 (corresponds to $|A_1|$)

(f) Identify a topological sort of the classes. (A ***topological sort*** of a poset $(A, \preceq)$ is a total order of all the elements such that if $a_i \preceq a_j$ in the partial order, then a_i precedes a_j in the total order.)

(SEE PRIOR PAGE)

(g) Suppose that you want to take all of the courses, but can handle only two per term. How many terms are required to graduate?

(SEE PRIOR PAGE)

(h) What if you could take three courses per term?

6 (from $\boxed{G}$)

(i) Stanford's computer science department offers n courses, limits students to at most k classes per term, and has its own complicated prerequisite structure. Describe two different lower bounds on the number of terms required to complete all the courses. One should be based on your answers to parts (b) and (c) and a second should be based on your answer to part (g).

$\boxed{B+C}$ $\Rightarrow$ terms required to complete is lower-bounded by the longest chain in the poset on the courses

$\boxed{G}$ $\Rightarrow$ if k is sufficiently small, then another lower bound would be $\lceil \frac{n}{k} \rceil$ as the bottleneck is k rather than the prerequisite structure.

6.042J / 18.062J Mathematics for Computer Science
Fall 2010