$\aleph_0$ Weekly Problem

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Problem

Show that for any positive integer n, there exists a positive multiple of n that contains only the digits 7 and 0.

Solution

Consider the set of positive integers $\{7, 77, 777, 7777, \ldots\}$. By the Pigeonhole Principle, there must be two elements in this set that have the same remainder when divided by n. Let a and b be two such numbers, with $a > b$. Then $a - b$ is a positive multiple of n that contains only the digits 7 and 0.