

Separasjon av variabler

$$\frac{dy}{dx} = f(x)g(y)$$

$$\frac{1}{g(y)} dy = f(x) dx$$

$$\int \frac{1}{g(y)} dy = \int f(x) dx$$

Generelle løsninger av lineære differensialligninger

En lineær differensialligning på formen:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

har den generelle løsningen: $y(x) = e^{-\int P(x) dx} \left(\int Q(x) e^{\int P(x) dx} dx + C \right)$

Ekspontiell vekst og logistisk vekst

Ekspontiell omskriving av logaritmer

$$\ln \left| \frac{a+u}{a-u} \right| = t + C \implies \frac{a+u}{a-u} = K e^t$$

Ekspontiell vekst eller forfall

Differensialligningen: $\frac{dy}{dt} = ky$
 har løsningen: $y(t) = C e^{kt}$

Logistisk vekst

Differensialligningen: $\frac{dy}{dt} = ky \left(1 - \frac{y}{M} \right)$
 har løsningen: $y(t) = \frac{M}{1 + C e^{-kt}}$

Trigonometriske substitusjoner

Ved integrasjon av komplekse uttrykk kan trigonometriske substitusjoner forenkle prosessen:

- For $\sqrt{a^2 - x^2}$, bruk substitusjonen $x = a \sin \theta$.
- For $\sqrt{x^2 - a^2}$, bruk substitusjonen $x = a \sec \theta$.
- For $\sqrt{a^2 + x^2}$, bruk substitusjonen $x = a \tan \theta$.

Important derivations and rules

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x).$$

$$\frac{d}{dx} (C \cdot f(x)) = C \cdot \frac{d}{dx} f(x),$$

Trigonometric functions,

$$\int \cos(kx) dx = \frac{1}{k} \sin(kx) + C$$

$$\int \sin(kx) dx = -\frac{1}{k} \cos(kx) + C$$

$$\int \tan(x) dx = -\ln |\cos(x)| + C$$

$$\int (1 + \tan^2(x)) dx = \tan(x) + C$$

$$\int \frac{1}{\cos^2(x)} dx = \tan(x) + C$$

$$\int \sin^2(x) dx = -\frac{1}{4} \sin(2x) - \frac{x}{2} + C.$$

$$\int \cos^2(x) dx = \frac{1}{4} \sin(2x) + \frac{x}{2} + C$$

$$\int \tan^2(x) dx = \tan(x) - x + C$$

Sine and Cosine Powers and Products:

$$\sin^3(x) = \frac{3}{4} \sin(x) - \frac{1}{4} \sin(3x)$$

$$\sin(3x) = 3 \sin(x) - 4 \sin^3(x)$$

$$\cos(3x) = 4 \cos^3(x) - 3 \cos(x)$$

$$\sin^2(x) = \frac{1}{2} (1 - \cos(2x))$$

$$\cos^2(x) = \frac{1}{2} (1 + \cos(2x))$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

Cotangent Definition: $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Addition formulas for cosine:

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

Euler's Formulas for Cosine and Sine:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

Euler's Formula

$$e^{\pm ix} = \cos(x) \pm i \sin(x)$$

$$e^{-ix} = \cos(x) - i \sin(x)$$

$$e^{ix} = \cos(x) + i \sin(x)$$

Calculation rules compressed

Rules for complex konjugates

$$\begin{aligned} z = a + bi &\Rightarrow \bar{z} = a - bi \\ \overline{z_1 + z_2} &\Rightarrow \bar{z}_1 + \bar{z}_2 \\ \overline{z_1 - z_2} &\Rightarrow \bar{z}_1 - \bar{z}_2 \\ \overline{z_1 z_2} &\Rightarrow \bar{z}_1 \cdot \bar{z}_2 \\ \frac{\bar{z}_1}{z_2} &\Rightarrow \frac{\bar{z}_1}{\bar{z}_2}, \quad z_2 \neq 0 \\ z \in \mathbb{R} &\Rightarrow \bar{z} = z \\ \bar{i} &\Rightarrow -i \\ \overline{e^z} &\Rightarrow e^{\bar{z}} \\ |\bar{z}| &\Rightarrow |z| \\ \overline{\bar{z}} &\Rightarrow z \\ \langle \psi, \phi \rangle &\Rightarrow \langle \phi, \psi \rangle \end{aligned}$$

Imaginary numbers

$$\begin{aligned} i^2 &= -1 \\ z = a + bi, \quad \text{where } a, b \in \mathbb{R} \\ |z| &= \sqrt{a^2 + b^2} \\ \bar{z} &= a - bi \\ z \cdot \bar{z} &= |z|^2 = a^2 + b^2 \\ e^{i\theta} &= \cos(\theta) + i \sin(\theta) \\ |e^{i\theta}| &= 1 \\ (e^{i\theta})^2 &= e^{i(2\theta)} \\ z^n = 1 &\implies z_k = e^{i \frac{2\pi k}{n}}, \quad k = 0, 1, 2, \dots, n-1 \\ z_1 &= r_1 e^{i\theta_1}, \quad z_2 = r_2 e^{i\theta_2} \implies \\ z_1 \cdot z_2 &= r_1 r_2 e^{i(\theta_1 + \theta_2)} \quad \frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)} \\ (e^{i\theta})^n &= e^{in\theta} = (\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta) \end{aligned}$$

Logarithmic Rules

$$\begin{aligned} \log_b(xy) &= \log_b(x) + \log_b(y) \\ \log_b\left(\frac{x}{y}\right) &= \log_b(x) - \log_b(y) \\ \log_b(x^k) &= k \cdot \log_b(x) \\ \log_b(1) &= 0 \\ \log_b(b) &= 1 \\ \log_b(x) &= \frac{\ln(x)}{\ln(b)} \\ \ln(e^x) &= x \\ \ln(1) &= 0 \\ \ln(ab) &= \ln(a) + \ln(b) \\ \ln\left(\frac{a}{b}\right) &= \ln(a) - \ln(b) \\ \ln(a^k) &= k \cdot \ln(a) \end{aligned}$$

Rules for absolute value

$$\begin{aligned} |ab| &= |a| \cdot |b| \\ \left|\frac{a}{b}\right| &= \frac{|a|}{|b|}, \quad b \neq 0 \\ |a + b| &\leq |a| + |b| \quad (\text{Trekantulikheten}) \\ ||a| - |b|| &\leq |a - b| \\ ||a| &= |a| \\ |a| = 0 &\Leftrightarrow a = 0 \\ |a|^2 &= a^2 \\ a(x) \in \mathbb{R}_{\geq 0} &\implies |a(x)| = a(x) \end{aligned}$$

Rules for expectation E

$$\begin{aligned} E(c) &= c, \quad \text{if } c \text{ is constant.} \\ E(a + b) &= E(a) + E(b) \\ E(c \cdot a) &= c \cdot E(a), \quad \text{if } c \text{ is constant.} \\ E(a \cdot b) &= E(a) \cdot E(b), \quad \text{if } a \text{ and } b \text{ are independent.} \\ E\left(\sum_{i=1}^n a_i\right) &= \sum_{i=1}^n E(a_i) \\ E(a^2) &= \text{Var}(a) + E(a)^2 \\ E\left(\frac{a}{b}\right) &\neq \frac{E(a)}{E(b)}, \quad (\text{not always equal}). \\ E(g(a)) &= \int_{-\infty}^{\infty} g(x) f_a(x) dx, \\ &(\text{for the continuous case with density function } f_a) \\ E(g(a)) &= \sum_x g(x) P(a = x), \quad (\text{for the discrete case}). \end{aligned}$$

Regneregler for potenser

$$\begin{aligned} a^p a^q &= a^{p+q} \\ (a * b)^p &= a^p b^p \\ \frac{a^p}{a^q} &= a^{p-q} \\ a^{-q} &= \frac{1}{a^q} \\ (a^p)^q &= a^{p \cdot q} \\ a^{\frac{1}{p}} &= \sqrt[p]{a} \\ a^p b^p &= (ab)^p \\ \frac{a^p}{b^p} &= \left(\frac{a}{b}\right)^p \\ a^0 &= 1, \quad \text{for } a \neq 0 \\ a^1 &= a \\ 0^p &= 0, \quad \text{for } p > 0 \\ 1^p &= 1 \\ (-a)^p &= \begin{cases} a^p, & \text{hvis } p \text{ er partall} \\ -a^p, & \text{hvis } p \text{ er oddetall} \end{cases} \\ a^{1/2} &= \sqrt{a} \end{aligned}$$

Regneregler for kvadratrøtter

$$\begin{aligned} \sqrt{a \cdot b} &= \sqrt{a} \cdot \sqrt{b}, \quad \text{for } a \geq 0, b \geq 0 \\ \sqrt{\frac{a}{b}} &= \frac{\sqrt{a}}{\sqrt{b}}, \quad \text{for } a \geq 0, b > 0 \\ (\sqrt{a})^2 &= a, \quad \text{for } a \geq 0 \\ \sqrt{a^2} &= |a| \\ \sqrt[3]{a} &= a^{1/3} \\ \sqrt[n]{a^n} &= a^{n/n}, \quad \text{for } a \geq 0 \\ \sqrt{k \cdot a} &= \sqrt{k} \cdot \sqrt{a} \\ \sqrt{a}, &\quad \text{for positiv konstant } k \\ \sqrt{a} &\neq \sqrt{b} \quad \text{med mindre } a = b \\ \sqrt{a^2 + b^2} &\neq \sqrt{a^2} + \sqrt{b^2} \quad (\text{ikke likhet}) \end{aligned}$$

Rules for summation

$$\begin{aligned} \sum_{i=1}^n (a + b) &= \sum_{i=1}^n a + \sum_{i=1}^n b \\ \sum_{i=1}^n c \cdot a_i &= c \cdot \sum_{i=1}^n a_i \\ \sum_{i=1}^n a_i, &\quad \text{if } c \text{ is constant.} \\ \sum_{i=1}^n c &= n \cdot c, \quad \text{if } c \text{ is constant.} \\ \sum_{i=1}^n (a_i \cdot b_i) &\neq \left(\sum_{i=1}^n a_i\right) \cdot \left(\sum_{i=1}^n b_i\right), \quad (\text{not always equal!}) \\ \sum_{i=1}^n i &= \frac{n(n+1)}{2} \\ \sum_{i=1}^n i^2 &= \frac{n(n+1)(2n+1)}{6} \\ \sum_{i=1}^n (X_i - \bar{X}) X_i &= \sum_{i=1}^n (X_i - \bar{X})^2 \\ \sum_{i=1}^n (X_i - \bar{X})^2 &= \sum_{i=1}^n X_i^2 - n \bar{X}^2 \\ \sum_{i=1}^n (a X_i + b)^2 &= a^2 \sum_{i=1}^n X_i^2 + 2ab \sum_{i=1}^n X_i + nb^2 \\ \sum_{i=1}^n (x_i - \bar{x}) &= 0 \quad \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \\ \text{Cov}(X, Y) &= \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) \\ \text{Var}(X) &= E(X^2) - E(X)^2 \\ \text{Var}(aX + b) &= a^2 \text{Var}(X) \end{aligned}$$

Rules for e and ln

$$\begin{aligned} e^0 &= 1 \\ e^1 &= e \\ e^x \cdot e^y &= e^{x+y} \\ \frac{e^x}{e^y} &= e^{x-y} \\ e^{-y} &= \frac{1}{e^y} \\ (e^x)^y &= e^{xy} \\ e^{\frac{1}{x}} &= \sqrt[x]{e} \\ e^{\ln(x)} &= x \\ \ln(e^x) &= x \\ \ln(1) &= 0 \\ \ln(ab) &= \ln(a) + \ln(b) \\ \ln\left(\frac{a}{b}\right) &= \ln(a) - \ln(b) \\ \ln(a^k) &= k \cdot \ln(a) \\ \ln(e) &= 1 \\ \ln(x) &= \frac{\log(x)}{\log(e)} \end{aligned}$$

sectionIntegration

subsectionBestemte integraler

$$\begin{aligned}\int_a^b F'(x) dx &= F(b) - F(a) \\ \int_a^b cf(x) dx &= c \int_a^b f(x) dx \\ \int_a^b (f+g) dx &= \int_a^b f dx + \int_a^b g dx \\ \int_a^b uv' dx &= [uv]_a^b - \int_a^b u'v dx \\ \int_a^b f(u)u' dx &= \int_{u(a)}^{u(b)} f(u) du\end{aligned}$$

subsectionImportant integrals

$$\begin{aligned}\int_{-\infty}^{\infty} e^{-x^2} dx &= \sqrt{\pi} \\ \int_{-\infty}^{\infty} e^{-ax^2} dx &= \sqrt{\frac{\pi}{a}}, \quad a > 0 \\ \int_{-\infty}^{\infty} x^2 e^{-ax^2} dx &= \sqrt{\frac{\pi}{a}} \cdot \frac{1}{2a}, \quad a > 0 \\ \int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx &= \sqrt{\frac{\pi}{a}} \cdot \frac{(2n-1)!!}{(2a)^n}, \quad n \in \mathbb{N}_0, a > 0 \\ \int_a^b uv' dx &= [uv]_a^b - \int_a^b u'v dx \\ \int_a^b f(u)u' dx &= \int_{u(a)}^{u(b)} f(u) du \\ u = ax + b &\Rightarrow \frac{du}{dx} = a \Rightarrow du = a dx \Rightarrow dx = \frac{du}{a}\end{aligned}$$

Egenskaper til ubestemte integral

$$\begin{aligned}\int uv' dx &= uv - \int u'v dx \quad \int f(u)u' dx = \int f(u) du \quad \int f(ax + b) dx = \frac{1}{a}F(ax + b) + C\end{aligned}$$

Ekspontentialfunksjoner

$$\begin{aligned}\int e^{kx} dx &= \frac{1}{k}e^{kx} + C, \quad k \neq 0 \\ \int a^x dx &= \frac{a^x}{\ln(a)} + C, \quad a > 0, a \neq 1\end{aligned}$$

Trigonometriske funksjoner

$$\begin{aligned}\int \sin(kx) dx &= -\frac{1}{k} \cos(kx) + C \\ \int \cos(kx) dx &= \frac{1}{k} \sin(kx) + C \\ \int \tan(x) dx &= -\ln |\cos(x)| + C \\ \int (1 + \tan^2(x)) dx &= \tan(x) + C \\ \int \frac{1}{\cos^2(x)} dx &= \tan(x) + C \\ \int \sin^2(x) dx &= -\frac{1}{4} \sin(2x) - \frac{x}{2} + C \\ \int \cos^2(x) dx &= \frac{1}{4} \sin(2x) + \frac{x}{2} + C \\ \int \tan^2(x) dx &= \tan(x) - x + C \\ \int \sec(kx) \tan(kx) dx &= \frac{1}{k} \sec(kx) + C \\ \int \sec^2(kx) dx &= \frac{1}{k} \tan(kx) + C \\ \int \csc(kx) \cot(kx) dx &= -\frac{1}{k} \csc(kx) + C \\ \int \csc^2(kx) dx &= -\frac{1}{k} \cot(kx) + C\end{aligned}$$

Hyperbolske funksjoner

$$\begin{aligned}\int \sinh(kx) dx &= \frac{1}{k} \cosh(kx) + C \\ \int \cosh(kx) dx &= \frac{1}{k} \sinh(kx) + C\end{aligned}$$

Inverse trigonometriske funksjoner

$$\begin{aligned}\int \frac{1}{\sqrt{a^2-x^2}} dx &= \frac{1}{|a|} \arcsin\left(\frac{x}{a}\right) + C \\ \int \frac{1}{1+x^2} dx &= \arctan(x) + C\end{aligned}$$

Logaritmiske funksjoner

$$\begin{aligned}\int \frac{1}{x} dx &= \ln |x| + C \\ \int \ln(x) dx &= x \ln(x) - x + C\end{aligned}$$

Potensfunksjoner

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Spesielle integraler

$$\begin{aligned}\int \frac{1}{\sqrt{x^2+a^2}} dx &= \ln |x + \sqrt{x^2+a^2}| + C \\ \int \frac{1}{\sqrt{x^2-a^2}} dx &= \ln |x + \sqrt{x^2-a^2}| + C, \quad x > a \\ \int x^r dx &= \frac{x^{r+1}}{r+1} + C, \quad r \neq -1 \\ \int \frac{1}{x} dx &= \ln |x| + C \\ \int e^x dx &= e^x + C \\ \int e^{kx} dx &= \frac{1}{k} e^{kx} + C, \quad k \neq 0 \\ \int \sin x dx &= -\cos x + C \\ \int \cos x dx &= \sin x + C \\ \int \frac{1}{\cos^2 x} dx &= \tan x + C \\ \int \frac{1}{\sqrt{1-x^2}} dx &= \sin^{-1} x + C \\ \int \frac{1}{1+x^2} dx &= \tan^{-1} x + C\end{aligned}$$

Rotasjon om x-aksen

$$\begin{aligned}A_x &= 2\pi \int_a^b |f(x)| \sqrt{1 + (f'(x))^2} dx \\ V_x &= \pi \int_a^b f(x)^2 dx\end{aligned}$$

Rotasjon om y-aksen

$$V_y = 2\pi \int_a^b x |f(x)| dx$$

Gjennomsnitt

$$\bar{y} = \frac{1}{b-a} \int_a^b y(x) dx$$

Trigonometriske funksjoner

Eksakte verdier til sin og cos

u	u	$\sin u$	$\cos u$	$\tan u$
0	0°	0	1	0
$\pi/6$	30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	90°	1	0	—

Trigonometriske formler

$$1 = \sin^2 u + \cos^2 u$$

$$\tan u = \frac{\sin u}{\cos u}$$

$$\sin u = \sin(u + 2\pi n), \quad n \in \mathbb{Z}$$

$$\cos u = \cos(u + 2\pi n), \quad n \in \mathbb{Z}$$

$$\tan u = \tan(u + \pi n), \quad n \in \mathbb{Z}$$

$$\sin(\pi - u) = \sin u$$

$$\cos(-u) = \cos u$$

$$-\sin(u) = \sin(-u)$$

$$\cos^2 u = \frac{1 + \cos(2u)}{2}$$

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\sin(2u) = 2 \sin u \cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

Derivatives

Derivasjons regler

$$(u + v)' = u' + v'$$

$$(u - v)' = u' - v'$$

$$(cu(x))' = cu'(x)$$

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$(u \circ v)' = u'(v) \cdot v'$$

$$(u^{-1})' = -\frac{u'}{u^2}$$

$$(u^n)' = nu^{n-1}u'$$

$$(e^u)' = u'e^u$$

$$(a^u)' = u'a^u \ln(a)$$

$$(\ln(u))' = \frac{u'}{u}$$

$$(\log_a(u))' = \frac{u'}{u \ln(a)}$$

$$(\sin(u))' = u' \cos(u)$$

$$(\cos(u))' = -u' \sin(u)$$

$$(\tan(u))' = u' \sec^2(u)$$

$$(\cot(u))' = -u' \csc^2(u)$$

$$(\sec(u))' = u' \sec(u) \tan(u)$$

$$(\csc(u))' = -u' \csc(u) \cot(u)$$

$$(\arcsin(u))' = \frac{u'}{\sqrt{1-u^2}}$$

$$(\arccos(u))' = -\frac{u'}{\sqrt{1-u^2}}$$

$$(\arctan(u))' = \frac{u'}{1+u^2}$$

Important Derivatives

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln(a)}$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x)$$

$$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x) \tan(x)$$

$$\frac{d}{dx}(\csc(x)) = -\csc(x) \cot(x)$$

$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arccos(x)) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$$

Partial Derivatives of a Function $f(x, y)$:

$$\left. \frac{\partial f}{\partial x} \right|_{x=x_0, y=y_0} = \lim_{h \rightarrow 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h}$$

$$\left. \frac{\partial f}{\partial y} \right|_{x=x_0, y=y_0} = \lim_{h \rightarrow 0} \frac{f(x_0, y_0 + h) - f(x_0, y_0)}{h}$$

nr	Formel	Derivert	$\frac{d}{dx}$ Formel	$\frac{d}{dx}$ Derivert	Alt Formel	Alt Derivert
1	c'	0	$\frac{d}{dx}(c)$	0	C'	0
2	x'	1	$\frac{d}{dx}(x)$	1	X'	1
2	$(x+v)'$	$x' + v'$	$\frac{d}{dx}(x+v)$	$\frac{d}{dx} + \frac{d}{dv}$	$(X(x) + V(x))'$	$X'(x) + V'(x)$
3	cx'	cx'	$\frac{d}{dx}(cx)$	$c \frac{d}{dx}$	$(cX(x))'$	$cX'(x)$
4	$(UV)'$	$U'V + V'U$	$\frac{d}{dx}(UV)$	$\frac{d}{dx}(U) \cdot V + \frac{d}{dx}(V) \cdot U$	$X(x) \cdot V(x)$	$X'(x)V(x) + X(x)V'(x)$
5	$\left(\frac{x}{v}\right)'$	$\frac{x'v - xv'}{v^2}$	$\frac{d}{dx}\left(\frac{x}{v}\right)$	$\frac{\frac{d}{dx}(x)v - x\frac{d}{dx}(v)}{v^2}$	$\frac{X(x)}{V(x)}$	$\frac{X'(x)V(x) - X(x)V'(x)}{V(x)^2}$
6	$(x^n)'$	$nx^{n-1}x'$	$\frac{d}{dx}(x^n)$	nx^{n-1}	$X(x)^n$	$nX(x)^{n-1}X'(x)$
7	$(e^x)'$	$x'e^x$	$\frac{d}{dx}(e^x)$	e^x	$e^{X(x)}$	$X'(x)e^{X(x)}$
8	$(a^x)'$	$x'a^x \ln(a)$	$\frac{d}{dx}(a^x)$	$a^x \ln(a)$	$a^{X(x)}$	$X'(x)a^{X(x)} \ln(a)$
9	$(\ln(x))'$	$\frac{x'}{x}$	$\frac{d}{dx}(\ln(x))$	$\frac{1}{x}$	$\ln(X(x))$	$\frac{X'(x)}{X(x)}$
10	$(\log_a(x))'$	$\frac{x'}{x \ln(a)}$	$\frac{d}{dx}(\log_a(x))$	$\frac{1}{x \ln(a)}$	$\log_a(X(x))$	$\frac{X'(x)}{X(x) \ln(a)}$
11	$(\sin(x))'$	$x' \cos(x)$	$\frac{d}{dx}(\sin(x))$	$\cos(x)$	$\sin(X(x))$	$X'(x) \cos(X(x))$
12	$(\cos(x))'$	$-x' \sin(x)$	$\frac{d}{dx}(\cos(x))$	$-\sin(x)$	$\cos(X(x))$	$-X'(x) \sin(X(x))$
13	$(\tan(x))'$	$x' \sec^2(x)$	$\frac{d}{dx}(\tan(x))$	$\sec^2(x)$	$\tan(X(x))$	$X'(x) \sec^2(X(x))$
14	$(\cot(x))'$	$-x' \csc^2(x)$	$\frac{d}{dx}(\cot(x))$	$-\csc^2(x)$	$\cot(X(x))$	$-X'(x) \csc^2(X(x))$
15	$(\sec(x))'$	$x' \sec(x) \tan(x)$	$\frac{d}{dx}(\sec(x))$	$\sec(x) \tan(x)$	$\sec(X(x))$	$X'(x) \sec(X(x)) \tan(X(x))$
16	$(\csc(x))'$	$-x' \csc(x) \cot(x)$	$\frac{d}{dx}(\csc(x))$	$-\csc(x) \cot(x)$	$\csc(X(x))$	$-X'(x) \csc(X(x)) \cot(X(x))$
17	$(\arcsin(x))'$	$\frac{x'}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arcsin(x))$	$\frac{1}{\sqrt{1-x^2}}$	$\arcsin(X(x))$	$\frac{X'(x)}{\sqrt{1-X(x)^2}}$
18	$(\arccos(x))'$	$-\frac{x'}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x))$	$-\frac{1}{\sqrt{1-x^2}}$	$\arccos(X(x))$	$-\frac{X'(x)}{\sqrt{1-X(x)^2}}$
19	$(\arctan(x))'$	$\frac{x'}{1+x^2}$	$\frac{d}{dx}(\arctan(x))$	$\frac{1}{1+x^2}$	$\arctan(X(x))$	$\frac{X'(x)}{1+X(x)^2}$

Differensialligning ansatser

Homogene ligninger

Ansatser for $y'' + by' + cy = 0$: $y = e^{\lambda x}$

$$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$$

$$y = e^{\alpha x} (C_1 \cos(\beta x) + C_2 \sin(\beta x))$$

$$y = (C_1 + C_2 x) e^{\lambda x}$$

Inhomogene ligninger

Ansatser for $g(x)$: $y_p = B_0 + B_1 x + \dots + B_n x^n$

$$y_p = C e^{ax}$$

$$y_p = C x e^{ax}$$

$$y_p = C_1 \cos(ax) + C_2 \sin(ax)$$

$$y_p = x(C_1 \cos(ax) + C_2 \sin(ax))$$

$$y_p = e^{ax} (R_n(x) \cos(bx) + S_n(x) \sin(bx))$$

Ikke-lineære ligninger

$$\frac{dy}{dx} = g(x)h(y) \quad z = y^{1-n} \quad y = vx, \quad v = \frac{y}{x} \quad y = \frac{1}{z} \quad y = e^z$$

Spesialtilfeller

$$y = x^r \quad y = A \cos(kx) + B \sin(kx) \quad y = e^{kx}$$

Vanlige tegn i Statistikk

\sum : Summasjon, brukes for å summere verdier.
 \prod : Produkt, brukes for å multiplisere verdier.
 \bar{x} : Gjennomsnitt, representerer gjennomsnittet av et datasett.
 $\hat{\beta}$: Estimat, en estimert parameterverdi, ofte brukt i regresjon.
 ε : Feilledd, tilfeldige feil eller støy i en modell.
 n : Antall, antall observasjoner eller datapunkter.
 σ^2 : Varians, måler spredningen i et datasett.
 σ : Standardavvik, kvadratroten av variansen.
 $P(A)$: Sannsynlighet, sannsynligheten for hendelsen A .
 $f(x)$: Tetthetsfunksjon, sannsynlighetstetthet for en kontinuerlig variabel.
 $F(x)$: Fordelingsfunksjon, kumulativ sannsynlighet for en variabel.
 μ : Forventningsverdi, populasjonsgjennomsnittet.
 x_i : Observasjon, den i -te observasjonen i et datasett.
 X : Tilfeldig variabel, en stokastisk variabel.
 z : Z-score, standardisert verdi for en observasjon.
 $|x|$: Absoluttverdi, avstanden fra 0 på tallinjen.
 a^\top : Transponert. a^\top transponert av a .
 Derfor orthogonal $y = \hat{y} + \hat{\varepsilon}$, med $\hat{\varepsilon}^\top \hat{y} = 0$, $X^\top \hat{\varepsilon} = 0$.

Regressjon

Sum of Squared Errors (SSE): Linear regression formula

$$\begin{aligned}
 (x_1, y_1), \dots, (x_n, y_n) \\
 y &= \alpha + \beta x + \varepsilon_i \\
 \hat{\alpha} &= \bar{y} - \hat{\beta} \bar{x} \\
 \hat{\beta} &= \frac{\sum_{i=1}^n (x_i - \bar{x}) y_i}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n X_i Y_i - n \bar{X} \bar{Y}}{\sum_{i=1}^n X_i^2 - n \bar{X}^2} \\
 SSE(\alpha, \beta) &= \sum (y_i - \alpha - \beta x_i)^2 \\
 \text{Var}(\hat{\beta}) &= \frac{\sigma^2}{\sum_{i=1}^n (X_i - \bar{X})^2} \\
 \text{Var}(\hat{\beta}) &= \frac{\text{Var}(\sum_{i=1}^n (X_i - \bar{X}) \varepsilon_i)}{(\sum_{i=1}^n (X_i - \bar{X})^2)^2} = \\
 &= \frac{\sigma^2 \sum_{i=1}^n (X_i - \bar{X})^2}{(\sum_{i=1}^n (X_i - \bar{X})^2)^2} = \frac{\sigma^2}{\sum_{i=1}^n (X_i - \bar{X})^2}.
 \end{aligned}$$

Konfidansintervall for T-intervall

$$\beta \pm t_{(\alpha, n-2)} \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2}}$$

Least Squares (LS):

$$\text{LS}(\beta) = \sum_{i=1}^n (y_i - \mathbf{x}_i' \beta)^2 = \sum_{i=1}^n \varepsilon_i^2 = \varepsilon' \varepsilon,$$

Least Absolute Deviations (LA):

$$\text{LA}(\beta) = \sum_{i=1}^n |y_i - \mathbf{x}_i' \beta| = \sum_{i=1}^n |\varepsilon_i|.$$

Linearity of Differentiation:

$$\frac{\partial}{\partial x} \sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{\partial f_n(x)}{\partial x},$$

provided the following conditions are satisfied:

- The series $\sum_{n=1}^{\infty} f_n(x)$ **converges uniformly** in the region where differentiation is applied.
- Each term $f_n(x)$ is **differentiable** in the region.

Rules for expectation E

$$\begin{aligned}
 E(c) &= c, \quad \text{if } c \text{ is constant.} \\
 E(a + b) &= E(a) + E(b) \\
 E(c \cdot a) &= c \cdot E(a), \quad \text{if } c \text{ is constant.} \\
 E(a \cdot b) &= E(a) \cdot E(b), \quad \text{if } a \text{ and } b \text{ are independent.} \\
 E\left(\sum_{i=1}^n a_i\right) &= \sum_{i=1}^n E(a_i) \\
 E(a^2) &= \text{Var}(a) + E(a)^2 \\
 E\left(\frac{a}{b}\right) &\neq \frac{E(a)}{E(b)}, \quad (\text{not always equal, except in special cases}). \\
 E(g(a)) &= \int_{-\infty}^{\infty} g(x) f_a(x) dx, \quad (\text{for the continuous case with density } f) \\
 E(g(a)) &= \sum_x g(x) P(a = x), \quad (\text{for the discrete case}).
 \end{aligned}$$

Regneregler for varians

$$\text{Var}\left(\sum_{i=1}^n a_i \varepsilon_i\right) = \sum_{i=1}^n a_i^2 \text{Var}(\varepsilon_i),$$

Formler og regneregler for lineær regresjon

1. Modell og grunnleggende formler

Lineær regresjonsmodell: $Y_i = \alpha + \beta X_i + \varepsilon_i$ Forventning og varians:

$$\begin{aligned}
 E(Y_i) &= \alpha + \beta X_i, \quad \text{Var}(Y_i) = \sigma^2 \\
 \mathbf{2. Minste kvadraters metode (OLS)} \\
 \text{Estimat for } \beta:
 \end{aligned}$$

$$\hat{\beta} = \frac{\sum_{i=1}^n (X_i - \bar{X}) Y_i}{\sum_{i=1}^n (X_i - \bar{X})^2}$$

Estimat for α :

$$\hat{\alpha} = \bar{Y} - \hat{\beta} \bar{X}$$

Sum of Squared Errors (SSE):

$$SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n (Y_i - \hat{\alpha} - \hat{\beta} X_i)^2$$

3. Varians og standardfeil

Estimert varians til residualene (s^2):

$$\hat{\sigma}^2 = s^2 = \frac{1}{n-2} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

Varians til $\hat{\beta}$:

$$\text{Var}(\hat{\beta}) = \frac{\sigma^2}{\sum_{i=1}^n (X_i - \bar{X})^2}$$

Standardfeil for $\hat{\beta}$:

$$SE(\hat{\beta}) = \sqrt{\frac{s^2}{\sum_{i=1}^n (X_i - \bar{X})^2}}$$

4. Hypotesetesting for β

Null- og alternativhypotese:

$$H_0 : \beta = 0 \quad (\text{ingen sammenheng}),$$

$$H_1 : \beta \neq 0 \quad (\text{sammenheng eksisterer})$$

T-test-statistikk:

$$t_{\text{obs}} = \frac{\hat{\beta}}{SE(\hat{\beta})} = \frac{\hat{\beta} \sqrt{\sum_{i=1}^n (X_i - \bar{X})^2}}{s}$$

Forkastningsregel:

Forkast H_0 hvis $|t_{\text{obs}}| > t_{\alpha/2, n-2}$

5. Konfidensintervall for β

$$\hat{\beta} - t_{\alpha/2, n-2} \cdot SE(\hat{\beta}) < \beta < \hat{\beta} + t_{\alpha/2, n-2} \cdot SE(\hat{\beta})$$

6. Fordeling av estimatene

Fordeling av $\hat{\beta}$:

$$\hat{\beta} \sim N(\beta, \text{Var}(\hat{\beta}))$$

Fordeling av $\hat{\alpha}$:

$$\hat{\alpha} \sim N(\alpha, \text{Var}(\hat{\alpha}))$$

7. Bruk av T-fordeling

Når variansen σ^2 ikke er kjent:

$$t = \frac{\hat{\beta} - \beta}{SE(\hat{\beta})} \sim T_{n-2}$$

8. Summasjonsregler

Summasjonsregler for X_i :

$$\sum_{i=1}^n (X_i - \bar{X}) = 0, \quad \sum_{i=1}^n (X_i - \bar{X})^2 = \text{total varians i } X$$

9. Linearitet av forventning

Lineær kombinasjon av forventninger:

$$E(aX + b) = aE(X) + b$$

10. Varians av lineær kombinasjon

Varians av skalering:

$$\text{Var}(aX) = a^2 \text{Var}(X)$$

Varians av summen av uavhengige variabler:

$$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) \quad (\text{hvis } X \text{ og } Y \text{ er uavhengige})$$

Dumping av til kanskje brukbart senere

$$Y_i = \alpha + \beta X_i + \varepsilon_i$$

$$E(\alpha) = E(Y_i - \beta X_i - \varepsilon_i)$$

$$\mathbb{E}[\hat{\beta}_0] = \mathbb{E}[\bar{Y} - \hat{\beta}_1 \bar{x}]$$

Matriser

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Div matte

Rett linje: $y - y_0 = a(x - x_0)$

Sirkel: $(x - x_0)^2 + (y - y_0)^2 = r^2$

Numeriske metoder

Numerisk løsning av startverdiproblem

$$y' = F(x, y), \quad y(x_0) = y_0$$

$$x_n = x_0 + n \cdot h \text{ og } y(x_n) \approx y_n \text{ med}$$

Eulers metode:

$$y_{n+1} = y_n + F(x_n, y_n)h$$

Eulers midtpunktm metode:

$$y_{n+1} = y_n + F(\hat{x}_n, \hat{y}_n)h$$

der

$$\hat{x}_n = x_n + h/2 \text{ og } \hat{y}_n = y_n + F(x_n, y_n)h/2$$

Numerisk integrasjon

Trapecmetoden

$$\int_a^b f(x) dx \approx T_n \text{ der}$$

$$T_n = h \left(\frac{f(x_0)}{2} + f(x_1) + \cdots + f(x_{n-1}) + \frac{f(x_n)}{2} \right)$$

Simpsons metode

$$\int_a^b f(x) dx \approx S_n \text{ der}$$

$$S_n = \frac{h}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n))$$

Newtons metode

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Trapecmetoden

$$\int_a^b f(x) dx \approx T_n \text{ der}$$

$$T_n = h \left(\frac{f(x_0)}{2} + f(x_1) + \cdots + f(x_{n-1}) + \frac{f(x_n)}{2} \right)$$

Simpsons metode

$$\int_a^b f(x) dx \approx S_n \text{ der}$$

$$S_n = \frac{h}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n))$$

Rotasjon om x-aksen

$$A_x = 2\pi \int_a^b |f(x)| \sqrt{1 + (f'(x))^2} dx$$

$$V_x = \pi \int_a^b f(x)^2 dx$$

Rotasjon om y-aksen

$$V_y = 2\pi \int_a^b x |f(x)| dx$$

Gjennomsnitt

$$\bar{y} = \frac{1}{b-a} \int_a^b y(x) dx$$

Newtons metode

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Finite Difference Approximation:

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1}))}{2\Delta x}$$

$$f''(x_i) \approx \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1}))}{(\Delta x)^2}$$