Matte alle Formel ark

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Separasjon av variabler

$\frac{dy}{dx} = f(x)g(y)$
$\frac{1}{g(y)}\,dy = f(x)\,dx$
$\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$

Generelle løsninger av lineære differensialligninger

En lineær differensialligning på formen: $\frac{dy}{dx} + P(x)y = Q(x)$
har den generelle løsningen: $y(x) = e^{-\int P(x)\,dx}\left(\int Q(x)e^{\int P(x)\,dx}\,dx + C\right)$

Eksponentiell vekst og logistisk vekst

Eksponentiell omskriving av logaritmer

$\ln\left|\frac{a+u}{a-u}\right| = t + C \implies \frac{a+u}{a-u} = Ke^t$

Eksponentiell vekst eller forfall

Differensialligningen: $\frac{dy}{dt} = ky$
har løsningen: $y(t) = Ce^{kt}$

Logistisk vekst

Differensialligningen: $\frac{dy}{dt} = ky\left(1 - \frac{y}{M}\right)$
har løsningen: $y(t) = \frac{M}{1+Ce^{-kt}}$

Trigonometriske substitusjoner

Ved integrasjon av komplekse uttrykk kan trigonometriske substitusjoner forenkle prosessen:

- For $\sqrt{a^2 - x^2}$, bruk substitusjonen $x = a\sin\theta$.
- For $\sqrt{x^2 - a^2}$, bruk substitusjonen $x = a\sec\theta$.
- For $\sqrt{a^2 + x^2}$, bruk substitusjonen $x = a\tan\theta$.

Important derivations and rules

$\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)\cdot g'(x).$
$\frac{d}{dx}\big(C\cdot f(x)\big) = C\cdot\frac{d}{dx}f(x),$

Trigonometric functions

$\int\cos(kx)dx = \frac{1}{k}\sin(kx) + C$
$\int\sin(kx)dx = -\frac{1}{k}\cos(kx) + C$
$\int\tan(x)dx = -ln\,|\cos(x)| + C$
$\int(1+\tan^2(x))dx = \tan(x) + C$
$\int\frac{1}{\cos^2(x)}dx = \tan(x) + C$
$\int\sin^2(x)\,dx = -\frac{1}{4}\sin(2x) - \frac{x}{2} + C.$
$\int\cos^2(x)dx = \frac{1}{4}\sin(2x) + \frac{x}{2} + C$
$\int\tan^2(x)dx = \tan(x) - x + C$

Sine and Cosine Powers and Products: $\sin^3(x) = \frac{3}{4}\sin(x) - \frac{1}{4}\sin(3x)$
$\sin(3x) = 3\sin(x) - 4\sin^3(x)$
$\cos(3x) = 4\cos^3(x) - 3\cos(x)$
$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$
$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$
$\sin(2x) = 2\sin(x)\cos(x)$
$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Cotangent Definition: $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Addition formulas for cosine:
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

Euler's Formulas for Cosine and Sine:
$\cos(x) = \frac{e^{ix}+e^{-ix}}{2}$
$\sin(x) = \frac{e^{ix}-e^{-ix}}{2i}$

Euler's Formula
$e^{\pm ix} = \cos(x) \pm i\sin(x)$
$e^{-ix} = \cos(x) - i\sin(x)$
$e^{ix} = \cos(x) + i\sin(x)$

Calculation rules compressed

Rules for complex konjugates

$z = a + bi \quad \Rightarrow \quad \overline{z} = a - bi$
$\overline{z_1 + z_2} \quad \Rightarrow \quad \overline{z_1} + \overline{z_2}$
$\overline{z_1 - z_2} \quad \Rightarrow \quad \overline{z_1} - \overline{z_2}$
$\overline{z_1 z_2} \quad \Rightarrow \quad \overline{z_1} \cdot \overline{z_2}$
$\overline{\frac{z_1}{z_2}} \quad \Rightarrow \quad \frac{\overline{z_1}}{\overline{z_2}}, \quad z_2 \neq 0$
$z \in \mathbb{R} \quad \Rightarrow \quad \overline{z} = z$
$\overline{i} \quad \Rightarrow \quad -i$
$\overline{e^z} \quad \Rightarrow \quad e^{\overline{z}}$
$|\overline{z}| \quad \Rightarrow \quad |z|$
$\overline{\overline{z}} \quad \Rightarrow \quad z$
$\overline{\langle \psi, \phi \rangle} \quad \Rightarrow \quad \langle \phi, \psi \rangle$

Imaginary numbers

$i^2 = -1$
$z = a + bi, \quad \text{where } a, b \in \mathbb{R}$
$|z| = \sqrt{a^2 + b^2}$
$\overline{z} = a - bi$
$z \cdot \overline{z} = |z|^2 = a^2 + b^2$
$e^{i\theta} = \cos(\theta) + i\sin(\theta)$
$|e^{i\theta}| = 1$
$\left(e^{i\theta}\right)^2 = e^{i(2\theta)}$
$z^n = 1 \implies z_k = e^{i\frac{2\pi k}{n}}, \quad k = 0, 1, 2, \ldots, n-1$
$z_1 = r_1 e^{i\theta_1}, \quad z_2 = r_2 e^{i\theta_2} \implies z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$ $\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$
$\left(e^{i\theta}\right)^n = e^{in\theta} = (\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

Logarithmic Rules

$\log_b(xy) = \log_b(x) + \log_b(y)$
$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
$\log_b(x^k) = k \cdot \log_b(x)$
$\log_b(1) = 0$
$\log_b(b) = 1$
$\log_b(x) = \frac{\ln(x)}{\ln(b)}$
$\ln(e^x) = x$
$\ln(1) = 0$
$\ln(ab) = \ln(a) + \ln(b)$
$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
$\ln(a^k) = k \cdot \ln(a)$

Rules for absolute value

$|ab| = |a| \cdot |b|$
$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$
$|a + b| \leq |a| + |b|$ (Trekantulikheten)
$||a| - |b|| \leq |a - b|$
$||a|| = |a|$
$|a| = 0 \quad \Leftrightarrow \quad a = 0$
$|a|^2 = a^2$
$a(x) \in \mathbb{R}_{\geq 0} \implies |a(x)| = a(x)$

Rules for expectation E

$E(c) = c, \quad \text{if } c \text{ is constant.}$
$E(a + b) = E(a) + E(b)$
$E(c \cdot a) = c \cdot E(a), \quad \text{if } c \text{ is constant.}$
$E(a \cdot b) = E(a) \cdot E(b), \quad \text{if } a \text{ and } b \text{ are independent.}$
$E\left(\sum_{i=1}^n a_i\right) = \sum_{i=1}^n E(a_i)$
$E(a^2) = \mathrm{Var}(a) + E(a)^2$
$E\left(\frac{a}{b}\right) \neq \frac{E(a)}{E(b)}, \quad$ (not always equal).
$E(g(a)) = \int_{-\infty}^{\infty} g(x) f_a(x)\, dx,$
(for the continuous case with density function $f_a(x)$).
$E(g(a)) = \sum_x g(x) P(a = x), \quad$ (for the discrete case).

Regneregler for potenser

$a^p a^q = a^{p+q}$
$(a * b)^p = a^p b^p$
$\frac{a^p}{a^q} = a^{p-q}$
$a^{-q} = \frac{1}{a^q}$
$(a^p)^q = a^{p \cdot q}$
$a^{\frac{1}{p}} = \sqrt[p]{a}$
$a^p b^p = (ab)^p$
$\frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$
$a^0 = 1, \quad \text{for } a \neq 0$
$a^1 = a$
$0^p = 0, \quad \text{for } p > 0$
$1^p = 1$
$(-a)^p = \begin{cases} a^p, & \text{hvis } p \text{ er partall} \\ -a^p, & \text{hvis } p \text{ er oddetall} \end{cases}$
$a^{1/2} = \sqrt{a}$

Regneregler for kvadratrøtter

$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}, \quad \text{for } a \geq 0, b \geq 0$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \quad \text{for } a \geq 0, b > 0$
$(\sqrt{a})^2 = a, \quad \text{for } a \geq 0$
$\sqrt{a^2} = |a|$
$\sqrt[2]{a} = a^{1/2}$
$\sqrt{a^n} = a^{n/2}, \quad \text{for } a \geq 0$
$\sqrt{k \cdot a} = \sqrt{k} \cdot \sqrt{a}, \quad \text{for positiv konstant } k$
$\sqrt{a} \neq \sqrt{b} \quad \text{med mindre } a = b$
$\sqrt{a^2 + b^2} \neq \sqrt{a^2} + \sqrt{b^2} \quad$ (ikke likhet)

Rules for summation

$\sum_{i=1}^n (a + b) = \sum_{i=1}^n a + \sum_{i=1}^n b$
$\sum_{i=1}^n c \cdot a_i = c \cdot \sum_{i=1}^n a_i, \quad \text{if } c \text{ is constant.}$
$\sum_{i=1}^n c = n \cdot c, \quad \text{if } c \text{ is constant.}$
$\sum_{i=1}^n (a_i \cdot b_i) \neq \left(\sum_{i=1}^n a_i\right) \cdot \left(\sum_{i=1}^n b_i\right), \quad$ (not always equal!)
$\sum_{i=1}^n i = \frac{n(n+1)}{2}$
$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^n (X_i - \bar{X}) X_i = \sum_{i=1}^n (X_i - \bar{X})^2$
$\sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n X_i^2 - n\bar{X}^2$
$\sum_{i=1}^n (aX_i + b)^2 = a^2 \sum_{i=1}^n X_i^2 + 2ab \sum_{i=1}^n X_i + nb^2$
$\sum_{i=1}^n (x_i - \bar{x}) = 0$ $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$
$\mathrm{Cov}(X, Y) = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$
$\mathrm{Var}(X) = E(X^2) - E(X)^2$
$\mathrm{Var}(aX + b) = a^2 \mathrm{Var}(X)$

Rules for e and ln

$e^0 = 1$
$e^1 = e$
$e^x \cdot e^y = e^{x+y}$
$\frac{e^x}{e^y} = e^{x-y}$
$e^{-y} = \frac{1}{e^y}$
$(e^x)^y = e^{xy}$
$e^{\frac{1}{x}} = \sqrt[x]{e}$
$e^{\ln(x)} = x$
$\ln(e^x) = x$
$\ln(1) = 0$
$\ln(ab) = \ln(a) + \ln(b)$
$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
$\ln(a^k) = k \cdot \ln(a)$
$\ln(e) = 1$
$\ln(x) = \frac{\log(x)}{\log(e)}$

sectionIntegration
subsectionBestemte integraler
$\int_a^b F'(x)\,dx = F(b) - F(a)$
$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
$\int_a^b (f+g)\,dx = \int_a^b f\,dx + \int_a^b g\,dx$
$\int_a^b uv'\,dx = [uv]_a^b - \int_a^b u'v\,dx$
$\int_a^b f(u)u'\,dx = \int_{u(a)}^{u(b)} f(u)\,du$

subsectionImportant integrals
$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$
$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}, \quad a > 0$
$\int_{-\infty}^{\infty} x^2 e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}} \cdot \frac{1}{2a}, \quad a > 0$
$\int_{-\infty}^{\infty} x^{2n} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}} \cdot \frac{(2n-1)!!}{(2a)^n}, \quad n \in \mathbb{N}_0,\, a > 0$
$\int_a^b uv'\,dx = \big[uv\big]_a^b - \int_a^b u'v\,dx$
$\int_a^b f(u)u'\,dx = \int_{u(a)}^{u(b)} f(u)\,du$
$u = ax + b \Rightarrow \frac{du}{dx} = a \Rightarrow du = a\,dx \Rightarrow dx = \frac{du}{a}$

Egenskaper til ubestemte integral
$\int uv'\,dx = uv - \int u'v\,dx$ $\int f(u)u'\,dx = \int f(u)\,du$ $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$

Eksponentialfunksjoner
$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C, \quad k \neq 0$
$\int a^x\,dx = \frac{a^x}{\ln(a)} + C, \quad a > 0, a \neq 1$

Trigonometriske funksjoner
$\int \sin(kx)\,dx = -\frac{1}{k}\cos(kx) + C$
$\int \cos(kx)\,dx = \frac{1}{k}\sin(kx) + C$
$\int \tan(x)dx = -\ln|\cos(x)| + C$
$\int (1+\tan^2(x))dx = \tan(x) + C$
$\int \frac{1}{\cos^2(x)}dx = \tan(x) + C$
$\int \sin^2(x)\,dx = -\frac{1}{4}\sin(2x) - \frac{x}{2} + C$
$\int \cos^2(x)dx = \frac{1}{4}\sin(2x) + \frac{x}{2} + C$
$\int \tan^2(x)dx = \tan(x) - x + C$
$\int \sec(kx)\tan(kx)\,dx = \frac{1}{k}\sec(kx) + C$
$\int \sec^2(kx)\,dx = \frac{1}{k}\tan(kx) + C$
$\int \csc(kx)\cot(kx)\,dx = -\frac{1}{k}\csc(kx) + C$
$\int \csc^2(kx)\,dx = -\frac{1}{k}\cot(kx) + C$

Hyperbolske funksjoner
$\int \sinh(kx)\,dx = \frac{1}{k}\cosh(kx) + C$
$\int \cosh(kx)\,dx = \frac{1}{k}\sinh(kx) + C$

Inverse trigonometriske funksjoner
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \frac{1}{|a|}\arcsin\left(\frac{x}{a}\right) + C$
$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$

Logaritmiske funksjoner
$\int \frac{1}{x}\,dx = \ln|x| + C$
$\int \ln(x)\,dx = x\ln(x) - x + C$

Potensfunksjoner
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

Spesielle integraler
$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left|x + \sqrt{x^2+a^2}\right| + C$
$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x + \sqrt{x^2-a^2}\right| + C, \quad x > a$
$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C,\, r \neq -1$
$\int \frac{1}{x}\,dx = \ln|x| + C$
$\int e^x\,dx = e^x + C$
$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C,\, k \neq 0$
$\int \sin x\,dx = -\cos x + C$
$\int \cos x\,dx = \sin x + C$
$\int \frac{1}{\cos^2 x}\,dx = \tan x + C$
$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$
$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$

Rotasjon om x-aksen
$A_x = 2\pi\int_a^b |f(x)|\sqrt{1+(f'(x))^2}\,dx$
$V_x = \pi\int_a^b f(x)^2\,dx$

Rotasjon om y-aksen
$V_y = 2\pi\int_a^b x|f(x)|\,dx$

Gjennomsnitt
$\bar{y} = \frac{1}{b-a}\int_a^b y(x)\,dx$

Trigonemetriske funksjoner

Eksakte verdier til sin og cos

u	u	$\sin u$	$\cos u$	$\tan u$
0	0°	0	1	0
$\pi/6$	30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	90°	1	0	$-$

Trigonometriske formler

$1 = \sin^2 u + \cos^2 u$

$\tan u = \frac{\sin u}{\cos u}$

$\sin u = \sin(u + 2\pi n), \quad n \in \mathbb{Z}$

$\cos u = \cos(u + 2\pi n), \quad n \in \mathbb{Z}$

$\tan u = \tan(u + \pi n), \quad n \in \mathbb{Z}$

$\sin(\pi - u) = \sin u$

$\cos(-u) = \cos u$

$-\sin(u) = \sin(-u)$

$\cos^2 u = \frac{1+\cos(2u)}{2}$

$\sin^2 u = \frac{1-\cos(2u)}{2}$

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$\cos(u + v) = \cos u \cos v - \sin u \sin v$

$\sin(2u) = 2 \sin u \cos u$

$\cos(2u) = \cos^2 u - \sin^2 u$

Derivatives

Partial Derivatives of a Function $f(x,y)$:

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_0,y=y_0} = \lim_{h\to 0}\frac{f(x_0+h,y_0)-f(x_0,y_0)}{h}$$

$$\left.\frac{\partial f}{\partial y}\right|_{x=x_0,y=y_0} = \lim_{h\to 0}\frac{f(x_0,y_0+h)-f(x_0,y_0)}{h}$$

Derivasjons regler

$(u+v)' = u'+v'$
$(u-v)' = u'-v'$
$(cu(x))' = cu'(x)$
$(uv)' = u'v+uv'$
$(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$
$(u\circ v)' = u'(v)\cdot v'$
$(u^{-1})' = -\frac{u'}{u^2}$
$(u^n)' = nu^{n-1}u'$
$(e^u)' = u'e^u$
$(a^u)' = u'a^u\ln(a)$
$(\ln(u))' = \frac{u'}{u}$
$(\log_a(u))' = \frac{u'}{u\ln(a)}$
$(\sin(u))' = u'\cos(u)$
$(\cos(u))' = -u'\sin(u)$
$(\tan(u))' = u'\sec^2(u)$
$(\cot(u))' = -u'\csc^2(u)$
$(\sec(u))' = u'\sec(u)\tan(u)$
$(\csc(u))' = -u'\csc(u)\cot(u)$
$(\arcsin(u))' = \frac{u'}{\sqrt{1-u^2}}$
$(\arccos(u))' = -\frac{u'}{\sqrt{1-u^2}}$
$(\arctan(u))' = \frac{u'}{1+u^2}$

Important Derivatives

$\frac{d}{dx}(c) = 0$
$\frac{d}{dx}(x) = 1$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(a^x) = a^x\ln(a)$
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$
$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$
$\frac{d}{dx}(\sin(x)) = \cos(x)$
$\frac{d}{dx}(\cos(x)) = -\sin(x)$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$
$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\arccos(x)) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

nr	Formel	Derivert	$\frac{d}{dx}$ Formel	$\frac{d}{dx}$ Derivert	Alt Formel	Alt Derivert
1	c'	0	$\frac{d}{dx}(c)$	0	C'	0
2	x'	1	$\frac{d}{dx}(x)$	1	X'	1
2	$(x+v)'$	$x'+v'$	$\frac{d}{dx}(x+v)$	$\frac{d}{dx}+\frac{d}{dv}$	$(X(x)+V(x))'$	$X'(x)+V'(x)$
3	cx'	cx'	$\frac{d}{dx}(cx)$	$c\frac{d}{dx}$	$(cX(x))'$	$cX'(x)$
4	$(UV)'$	$U'V+V'U$	$\frac{d}{dx}(UV)$	$\frac{d}{dx}(U)\cdot V+\frac{d}{dx}(V)\cdot U$	$X(x)\cdot V(x)$	$X'(x)V(x)+X(x)V'(x)$
5	$\left(\frac{x}{v}\right)'$	$\frac{x'v-xv'}{v^2}$	$\frac{d}{dx}\left(\frac{x}{v}\right)$	$\frac{\frac{d}{dx}(x)v-x\frac{d}{dx}(v)}{v^2}$	$\frac{X(x)}{V(x)}$	$\frac{X'(x)V(x)-X(x)V'(x)}{V(x)^2}$
6	$(x^n)'$	$nx^{n-1}x'$	$\frac{d}{dx}(x^n)$	nx^{n-1}	$X(x)^n$	$nX(x)^{n-1}X'(x)$
7	$(e^x)'$	$x'e^x$	$\frac{d}{dx}(e^x)$	e^x	$e^{X(x)}$	$X'(x)e^{X(x)}$
8	$(a^x)'$	$x'a^x\ln(a)$	$\frac{d}{dx}(a^x)$	$a^x\ln(a)$	$a^{X(x)}$	$X'(x)a^{X(x)}\ln(a)$
9	$(\ln(x))'$	$\frac{x'}{x}$	$\frac{d}{dx}(\ln(x))$	$\frac{1}{x}$	$\ln(X(x))$	$\frac{X'(x)}{X(x)}$
10	$(\log_a(x))'$	$\frac{x'}{x\ln(a)}$	$\frac{d}{dx}(\log_a(x))$	$\frac{1}{x\ln(a)}$	$\log_a(X(x))$	$\frac{X'(x)}{X(x)\ln(a)}$
11	$(\sin(x))'$	$x'\cos(x)$	$\frac{d}{dx}(\sin(x))$	$\cos(x)$	$\sin(X(x))$	$X'(x)\cos(X(x))$
12	$(\cos(x))'$	$-x'\sin(x)$	$\frac{d}{dx}(\cos(x))$	$-\sin(x)$	$\cos(X(x))$	$-X'(x)\sin(X(x))$
13	$(\tan(x))'$	$x'\sec^2(x)$	$\frac{d}{dx}(\tan(x))$	$\sec^2(x)$	$\tan(X(x))$	$X'(x)\sec^2(X(x))$
14	$(\cot(x))'$	$-x'\csc^2(x)$	$\frac{d}{dx}(\cot(x))$	$-\csc^2(x)$	$\cot(X(x))$	$-X'(x)\csc^2(X(x))$
15	$(\sec(x))'$	$x'\sec(x)\tan(x)$	$\frac{d}{dx}(\sec(x))$	$\sec(x)\tan(x)$	$\sec(X(x))$	$X'(x)\sec(X(x))\tan(X(x))$
16	$(\csc(x))'$	$-x'\csc(x)\cot(x)$	$\frac{d}{dx}(\csc(x))$	$-\csc(x)\cot(x)$	$\csc(X(x))$	$-X'(x)\csc(X(x))\cot(X(x))$
17	$(\arcsin(x))'$	$\frac{x'}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arcsin(x))$	$\frac{1}{\sqrt{1-x^2}}$	$\arcsin(X(x))$	$\frac{X'(x)}{\sqrt{1-X(x)^2}}$
18	$(\arccos(x))'$	$-\frac{x'}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x))$	$-\frac{1}{\sqrt{1-x^2}}$	$\arccos(X(x))$	$-\frac{X'(x)}{\sqrt{1-X(x)^2}}$
19	$(\arctan(x))'$	$\frac{x'}{1+x^2}$	$\frac{d}{dx}(\arctan(x))$	$\frac{1}{1+x^2}$	$\arctan(X(x))$	$\frac{X'(x)}{1+X(x)^2}$

Differensialligning ansatser

Homogene ligninger

Ansatser for $y'' + by' + cy = 0$: $y = e^{\lambda x}$
$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$
$y = e^{\alpha x}(C_1 \cos(\beta x) + C_2 \sin(\beta x))$
$y = (C_1 + C_2 x)e^{\lambda x}$

Inhomogene ligninger

Ansatser for $g(x)$: $y_p = B_0 + B_1 x + \cdots + B_n x^n$
$y_p = Ce^{ax}$
$y_p = Cxe^{ax}$
$y_p = C_1 \cos(ax) + C_2 \sin(ax)$
$y_p = x(C_1 \cos(ax) + C_2 \sin(ax))$
$y_p = e^{ax}(R_n(x)\cos(bx) + S_n(x)\sin(bx))$

Ikke-lineære ligninger

$\frac{dy}{dx} = g(x)h(y)$ $z = y^{1-n}$ $y = vx, \quad v = \frac{y}{x}$ $y = \frac{1}{z}$ $y = e^z$

Spesialtilfeller

$y = x^r$ $y = A\cos(kx) + B\sin(kx)$ $y = e^{kx}$

Statistikk

Mamo3100

Per Erik Grønvik 2025

Vanlige tegn i Statistikk

$\sum$: Summasjon, brukes for å summere verdier.
$\prod$: Produkt, brukes for å multiplisere verdier.
$\bar{x}$: Gjennomsnitt, representerer gjennomsnittet av et datasett.
$\hat{\beta}$: Estimat, en estimert parameterverdi, ofte brukt i regresjon.
ε: Feilledd, tilfeldige feil eller støy i en modell.
n: Antall, antall observasjoner eller datapunkter.
σ^2: Varians, måler spredningen i et datasett.
σ: Standardavvik, kvadratroten av variansen.
$P(A)$: Sannsynlighet, sannsynligheten for hendelsen A.
$f(x)$: Tetthetsfunksjon, sannsynlighetstetthet for en kontinuerlig variabel.
$F(x)$: Fordelingsfunksjon, kumulativ sannsynlighet for en variabel.
μ: Forventningsverdi, populasjonsgjennomsnittet.
x_i: Observasjon, den i-te observasjonen i et datasett.
X: Tilfeldig variabel, en stokastisk variabel.
z: Z-score, standardisert verdi for en observasjon.
$|x|$: Absoluttverdi, avstanden fra 0 på tallinjen.
$a^\top$: Transponert. $a^\top$ transponert av a.
Derfor orthogonal $y = \hat{y} + \hat{\varepsilon}, \quad \text{med} \quad \hat{\varepsilon}^\top \hat{y} = 0, \quad X^\top \hat{\varepsilon} = 0.$

Regressjon

Sum of Squared Errors (SSE): Linear regression formula
$(x_1, y_1), \ldots, (x_n, y_n)$
$y = \alpha + \beta x + \varepsilon_i$
$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$
$\hat{\beta} = \frac{\sum_{i=1}^n (x_i - \bar{x}) y_i}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n X_i Y_i - n\bar{X}\bar{Y}}{\sum_{i=1}^n X_i^2 - n\bar{X}^2}$
$SSE(\alpha, \beta) = \sum (y_i - \alpha - \beta x_i)^2$
$\mathrm{Var}(\hat{\beta}) = \frac{\sigma^2}{\sum_{i=1}^n (X_i - \bar{X})^2}$
$\mathrm{Var}(\hat{\beta}) = \frac{\mathrm{Var}\left(\sum_{i=1}^n (X_i - \bar{X})\varepsilon_i\right)}{\left(\sum_{i=1}^n (X_i - \bar{X})^2\right)^2} = \frac{\sigma^2 \sum_{i=1}^n (X_i - \bar{X})^2}{\left(\sum_{i=1}^n (X_i - \bar{X})^2\right)^2} = \frac{\sigma^2}{\sum_{i=1}^n (X_i - \bar{X})^2}.$

Konfidansintervall for T-interverall

$$\beta \pm t_{(\alpha, n-2)} \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^n (X_i - \overline{X})^2}}$$

Least Squares (LS):

$$\mathrm{LS}(\boldsymbol{\beta}) = \sum_{i=1}^n (y_i - \mathbf{x}_i' \boldsymbol{\beta})^2 = \sum_{i=1}^n \varepsilon_i^2 = \boldsymbol{\varepsilon}' \boldsymbol{\varepsilon},$$

Least Absolute Deviations (LA):

$$\mathrm{LA}(\boldsymbol{\beta}) = \sum_{i=1}^n |y_i - \mathbf{x}_i' \boldsymbol{\beta}| = \sum_{i=1}^n |\varepsilon_i|.$$

Linearity of Differentiation:

$$\frac{\partial}{\partial x} \sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{\partial f_n(x)}{\partial x},$$

provided the following conditions are satisfied:

- The series $\sum_{n=1}^{\infty} f_n(x)$ **converges uniformly** in the region where differentiation is applied.
- Each term $f_n(x)$ is **differentiable** in the region.

Rules for expectation E
$E(c) = c, \quad \text{if } c \text{ is constant.}$
$E(a + b) = E(a) + E(b)$
$E(c \cdot a) = c \cdot E(a), \quad \text{if } c \text{ is constant.}$
$E(a \cdot b) = E(a) \cdot E(b), \quad \text{if } a \text{ and } b \text{ are independent.}$
$E\left(\sum_{i=1}^n a_i\right) = \sum_{i=1}^n E(a_i)$
$E(a^2) = \mathrm{Var}(a) + E(a)^2$
$E\left(\frac{a}{b}\right) \neq \frac{E(a)}{E(b)}, \quad \text{(not always equal, except in special cases).}$
$E(g(a)) = \int_{-\infty}^{\infty} g(x) f_a(x)\, dx, \quad$ (for the continuous case with density fu
$E(g(a)) = \sum_x g(x) P(a = x), \quad \text{(for the discrete case).}$

Regneregler for varians

$$\mathrm{Var}\left(\sum_{i=1}^n a_i \varepsilon_i\right) = \sum_{i=1}^n a_i^2 \mathrm{Var}(\varepsilon_i),$$

Formler og regneregler for lineær regresjon

1. Modell og grunnleggende formler
Lineær regresjonsmodell: $Y_i = \alpha + \beta X_i + \varepsilon_i$ Forventning og varians:

$E(Y_i) = \alpha + \beta X_i, \quad \mathrm{Var}(Y_i) = \sigma^2$
2. Minste kvadraters metode (OLS)
Estimat for β:

$$\hat{\beta} = \frac{\sum_{i=1}^n (X_i - \bar{X}) Y_i}{\sum_{i=1}^n (X_i - \bar{X})^2}$$

Estimat for α:

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$$

Sum of Squared Errors (SSE):

$$SSE = \sum_{i=1}^n \left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^n \left(Y_i - \hat{\alpha} - \hat{\beta} X_i\right)^2$$

3. Varians og standardfeil
Estimert varians til residualene (s^2):

$$\hat{\sigma}^2 = s^2 = \frac{1}{n-2} \sum_{i=1}^n \left(Y_i - \hat{Y}_i\right)^2$$

Varians til $\hat{\beta}$:

$$\mathrm{Var}(\hat{\beta}) = \frac{\sigma^2}{\sum_{i=1}^n (X_i - \bar{X})^2}$$

Standardfeil for $\hat{\beta}$:

$$SE(\hat{\beta}) = \sqrt{\frac{s^2}{\sum_{i=1}^n (X_i - \bar{X})^2}}$$

4. Hypotesetesting for β
Null- og alternativhypotese:

$$H_0 : \beta = 0 \quad \text{(ingen sammenheng)},$$

$$H_1 : \beta \neq 0 \quad \text{(sammenheng eksisterer)}$$

T-test-statistikk:

$$t_{\text{obs}} = \frac{\hat{\beta}}{SE(\hat{\beta})} = \frac{\hat{\beta}\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}}{s}$$

Forkastningsregel:

Forkast H_0 hvis $|t_{\text{obs}}| > t_{\alpha/2,n-2}$

5. Konfidensintervall for β
$\hat{\beta} - t_{\alpha/2,n-2} \cdot SE(\hat{\beta}) < \beta < \hat{\beta} + t_{\alpha/2,n-2} \cdot SE(\hat{\beta})$

6. Fordeling av estimatene
Fordeling av $\hat{\beta}$:

$$\hat{\beta} \sim N\left(\beta, \text{Var}(\hat{\beta})\right)$$

Fordeling av $\hat{\alpha}$:

$$\hat{\alpha} \sim N\left(\alpha, \text{Var}(\hat{\alpha})\right)$$

7. Bruk av T-fordeling
Når variansen σ^2 ikke er kjent:

$$t = \frac{\hat{\beta} - \beta}{SE(\hat{\beta})} \sim T_{n-2}$$

8. Summasjonsregler
Summasjonsregler for X_i:

$$\sum_{i=1}^{n}(X_i - \bar{X}) = 0, \quad \sum_{i=1}^{n}(X_i - \bar{X})^2 = \text{total varians i } X$$

9. Linearitet av forventning
Lineær kombinasjon av forventninger:

$$E(aX + b) = aE(X) + b$$

10. Varians av lineær kombinasjon
Varians av skalering:

$$\text{Var}(aX) = a^2\text{Var}(X)$$

Varians av summen av uavhengige variabler:

$$\text{Var}(X+Y) = \text{Var}(X)+\text{Var}(Y) \quad \text{(hvis } X \text{ og } Y \text{ er uavhengige)}$$

Dumping av til kanskje brukbert senere
$Y_i = \alpha + \beta X_i + \varepsilon_i$
$E(\alpha) = E(Y_i - \beta X_i - \varepsilon_i)$
$\mathbb{E}\left[\hat{\beta}_0\right] = \mathbb{E}\left[\bar{Y} - \hat{\beta}_1\bar{x}\right]$

Matriser

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Vektorer

Geometri

Kvantemekanikk

Div matte

Rett linje: $y - y_0 = a(x - x_0)$
Sirkel: $(x - x_0)^2 + (y - y_0)^2 = r^2$

Numeriske metoder

Numerisk løsning av startverdiproblem

$y' = F(x, y), \quad y(x_0) = y_0$

$x_n = x_0 + n \cdot h$ og $y(x_n) \approx y_n$ med

Eulers metode:

$y_{n+1} = y_n + F(x_n, y_n)h$

Eulers midtpunktmetode:

$y_{n+1} = y_n + F(\hat{x}_n, \hat{y}_n)h$

der

$\hat{x}_n = x_n + h/2$ og $\hat{y}_n = y_n + F(x_n, y_n)h/2$

Numerisk integrasjon

Trapesmetoden

$\int_a^b f(x)\,dx \approx T_n$ der

$T_n = h\left(\frac{f(x_0)}{2} + f(x_1) + \cdots + f(x_{n-1}) + \frac{f(x_n)}{2}\right)$

Simpsons metode

$\int_a^b f(x)\,dx \approx S_n$ der

$S_n = \frac{h}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n)\right)$

Newtons metode

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Trapesmetoden

$\int_a^b f(x)\,dx \approx T_n$ der

$T_n = h\left(\frac{f(x_0)}{2} + f(x_1) + \cdots + f(x_{n-1}) + \frac{f(x_n)}{2}\right)$

Simpsons metode

$\int_a^b f(x)\,dx \approx S_n$ der

$S_n = \frac{h}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n)\right)$

Rotasjon om x-aksen

$A_x = 2\pi \int_a^b |f(x)|\sqrt{1 + (f'(x))^2}\,dx$

$V_x = \pi \int_a^b f(x)^2\,dx$

Rotasjon om y-aksen

$V_y = 2\pi \int_a^b x|f(x)|\,dx$

Gjennomsnitt

$\bar{y} = \frac{1}{b-a}\int_a^b y(x)\,dx$

Newtons metode

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Finite Difference Approximation:

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x}$$

$$f''(x_i) \approx \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{(\Delta x)^2}$$

Løsning av en lineær 1. ordens differensiallikning med integrerende faktor

Vi starter med en differensiallikning på standard form:

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

Vi skal finne en integrerende faktor $\mu(x)$ slik at venstresiden $\frac{dy}{dx} + P(x)\,y$ kan skrives som den deriverte av et produkt $\mu(x) \cdot Q(x)$. Dette gjøres i tre trinn:

1. Finn den integrerende faktoren. Den integrerende faktoren $\mu(x)$ er gitt ved:

$$\mu(x) = e^{\int P(x)\,dx}$$

2. Multipliser hele likningen med $\mu(x)$

$$\mu(x) \cdot \frac{dy}{dx} + \mu(x) \cdot P(x) \cdot y = \mu(x) \cdot Q(x)$$

Venstresiden kan nå skrives som den deriverte av et produkt:

$$\frac{d}{dx}\left(\mu(x) \cdot y\right) = \mu(x) \cdot Q(x)$$

3. Integrer begge sider

$$\int \frac{d}{dx}\left(\mu(x) \cdot y\right)\, dx = \int \mu(x) \cdot Q(x)\,dx$$

$$\mu(x) \cdot y = \int \mu(x) \cdot Q(x)\,dx + C$$

4. Nå kan du løse for $y(x)$

$$y(x) = \frac{1}{\mu(x)}\left(\int \mu(x) \cdot Q(x)\,dx + C\right)$$

Dette er den generelle løsningen.

Eksempel

Vi løser differensiallikningen

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

med

$$P(x) = \frac{1}{2x}, \quad Q(x) = \frac{1}{2}$$

1. Finn den integrerende faktoren, Den integrerende faktoren er

$$\mu(x) = e^{\int P(x)\,dx} = e^{\int \frac{1}{2x}\,dx} = e^{\frac{1}{2}\ln|x|} = |x|^{1/2}$$

Vi antar $x > 0$, så:

$$\mu(x) = x^{1/2}$$

2. Multipliser hele likningen med $\mu(x)$

$$\mu(x) \cdot \left(\frac{dy}{dx} + P(x)\,y\right) = \mu(x) \cdot Q(x)$$

Siden $\mu(x) \cdot \left(\frac{du}{dx} + P(x)\,u\right) = \frac{d}{dx}\left(\mu(x) \cdot u\right)$, kan vi skrive:

$$\frac{d}{dx}\left(\mu(x) \cdot u\right) = \mu(x) \cdot Q(x)$$

3. Integrer begge sider

$$\int \frac{d}{dx}\left(\mu(x) \cdot y\right)\, dx = \int \mu(x) \cdot Q(x)\,dx$$

$$\mu(x) \cdot y = \int \mu(x) \cdot Q(x)\,dx + C$$

4. Løser for $y(x)$

$$y(x) = \frac{1}{\mu(x)} \left(\int \mu(x) \cdot Q(x)\, dx + C \right)$$

5. Sett inn de opprinnelige uttrykkene for $P(x)$ og $Q(x)$
Vi har:

$$\mu(x) = x^{1/2}, \quad Q(x) = \frac{1}{2}$$

Sett dette inn i løsningen:

$$y(x) = \frac{1}{x^{1/2}} \left(\int x^{1/2} \cdot \frac{1}{2}\, dx + C \right)$$

Integralet:

$$\int \frac{1}{2} x^{1/2}\, dx = \frac{1}{3} x^{3/2}$$

Så:

$$y(x) = \frac{1}{x^{1/2}} \left(\frac{1}{3} x^{3/2} + C \right) = \frac{1}{3} x + C x^{-1/2}$$