DAVE3705-1 25V FORMELARK

Function	**Laplace Transform**
$f(t)$	$F(s) = \int_0^\infty e^{-st} f(t) dt$
$af(t) + bg(t)$	$aF(s) + bG(s)$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$
$f^{(n)}(t)$	$s^nF(s) - \sum_{k=0}^{n-1} s^{n-1-k} f^{(k)}(0)$
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
$e^{at}f(t)$	$F(s-a)$
$u(t-a)f(t-a)$	$e^{-as}F(s)$
$\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$
$tf(t)$	$-F'(s)$
$t^nf(t)$	$(-1)^nF^{(n)}(s)$
$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma)d\sigma$
$f(t+p) = f(p)$ (periodic function)	$\frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t) dt$

Laplace Transforms of Common Functions

Function	Laplace Transform
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
t^a	$\frac{\Gamma(a+1)}{s^{a+1}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$
$e^{at}\cos(kt)$	$\frac{s-a}{(s-a)^2+k^2}$
$e^{at}\sin(kt)$	$\frac{k}{(s-a)^2+k^2}$
$\frac{t}{2k}\sin(kt)$	$\frac{s}{(s^2+k^2)^2}$
$u(t-a)$ (step function)	e^{-as}

Derivatives

Function	Derivative
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
x^a	ax^{a-1}
$(f(x)\cdot g(x))'$	$f'(x)\cdot g(x) + f(x)\cdot g'(x)$
$f(x) = h(g(x))$	$h'(g(x))\cdot g'(x)$

Partial Fraction Expansion

$$\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$$

$$\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$$

Trigonometric Functions

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

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Integrals

$$\int a\,dx = ax + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{c}{ax+b} dx = \frac{c}{a}\ln|ax+b| + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin^2 x\,dx = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C$$

$$\int \cos^2 x\,dx = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C$$

$$\int x\cos(ax) dx = \frac{\cos(ax)}{a^2} + \frac{x\sin(ax)}{a} + C$$

$$\int x\sin(ax) dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a} + C$$

Fourier Series

For $f(x+T) = f(x)$, $T = 2L$:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)\right)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right) dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi}{L}x\right) dx$$

If $f(x)$ is Antisymmetric (Odd):

If $f(-x) = -f(x)$, then:

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$$a_0 = a_n = 0$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx$$

If $f(x)$ is Symmetric (Even):

If $f(-x) = f(x)$, then:

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$$b_n = 0$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi}{L} x\right) dx$$

Characteristic Equation Method

The general form of a second-order differential equation:

$$ay''(x) + by'(x) + cy(x) = 0$$

Using the ansatz $y(x) = e^{rx}$, we obtain the characteristic equation:

$$ar^2 + br + c = 0 \Rightarrow r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Distinct Roots: $r_1 \neq r_2$, both real

$$y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

Single Root: $r_1 = r_2 = r$

$$y(x) = C_1 e^{rx} + C_2 x e^{rx}$$

Complex Roots: $r_{1,2} = \alpha \pm i\beta$

$$y(x) = e^{\alpha x} \left(C_1 \cos \beta x + C_2 \sin \beta x \right)$$

Heat Equation

In one dimension:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Boundary Conditions:

- $u(0,t) = 0$, $u(L,t) = 0$ (Zero temperature at the ends of the rod)
- $u_x(0,t) = 0$, $u_x(L,t) = 0$ (Insulated ends)

Initial Condition:

$$u(x,0) = f(x)$$

Method to Solve: Separation of Variables

Assume:

$$u(x,t) = X(x)T(t)$$

Substituting into the equation:

$$\frac{\dot{T}(t)}{kT(t)} = \frac{X''(x)}{X(x)} = \text{constant} = -\lambda$$

Solution for Zero-Temperature End Boundary Condition

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-\frac{n^2\pi^2}{L^2}kt} \sin\left(\frac{n\pi}{L}x\right)$$

where

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx$$

Solution for Insulated Ends

$$u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \exp\left(-\frac{n^2\pi^2}{L^2}kt\right)\cos\left(\frac{n\pi}{L}x\right)$$

where

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

Wave Equation

In one dimension:

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

$$\begin{cases} y(0,t) = 0 \\ y(L,t) = 0 \end{cases}$$

$$\begin{cases} y(x,0) = f(x) & \text{Initial displacement} \\ y_t(x,0) = g(x) & \text{Initial velocity} \end{cases}$$

Method to Solve: Separation of Variables

Assume:

$$u(x,t) = X(x)T(t)$$