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Separasjon av variabler

$$\frac{dy}{dx} = f(x)g(y)$$

$$\frac{1}{g(y)}\,dy = f(x)\,dx$$

$$\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$$

Generelle løsninger av lineære differensialligninger

En lineær differensialligning på formen:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

har den generelle løsningen:

$$y(x) = e^{-\int P(x)\,dx}\left(\int Q(x)e^{\int P(x)\,dx}\,dx + C\right)$$

Eksponentiell vekst og logistisk vekst

Eksponentiell omskriving av logaritmer

$$\ln\left|\frac{a+u}{a-u}\right| = t + C \implies \frac{a+u}{a-u} = Ke^t$$

Eksponentiell vekst eller forfall

Differensialligningen:

$$\frac{dy}{dt} = ky$$

har løsningen:

$$y(t) = Ce^{kt}$$

Logistisk vekst

Differensialligningen:

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{M}\right)$$

har løsningen:

$$y(t) = \frac{M}{1 + Ce^{-kt}}$$

Trigonometriske substitusjoner

Ved integrasjon av komplekse uttrykk kan trigonometriske substitusjoner forenkle prosessen:

- For $\sqrt{a^2 - x^2}$, bruk substitusjonen $x = a\sin\theta$.
- For $\sqrt{x^2 - a^2}$, bruk substitusjonen $x = a\sec\theta$.
- For $\sqrt{a^2 + x^2}$, bruk substitusjonen $x = a\tan\theta$.

Important derivations and rules

$$\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)\cdot g'(x).$$

$$\frac{d}{dx}\big(C\cdot f(x)\big) = C\cdot\frac{d}{dx}f(x),$$

Trigonometric functions

$$\int \cos(kx)dx = \frac{1}{k}\sin(kx) + C$$

$$\int \sin(kx)dx = -\frac{1}{k}\cos(kx) + C$$

$$\int \tan(x)dx = -ln\left|\cos(x)\right| + C$$

$$\int (1 + \tan^2(x))dx = \tan(x) + C$$

$$\int \frac{1}{\cos^2(x)}dx = \tan(x) + C$$

$$\int \sin^2(x)\,dx = -\frac{1}{4}\sin(2x) - \frac{x}{2} + C.$$

$$\int \cos^2(x)dx = \frac{1}{4}\sin(2x) + \frac{x}{2} + C$$

$$\int \tan^2(x)dx = \tan(x) - x + C$$

Sine and Cosine Powers and Products:

$$\sin^3(x) = \frac{3}{4}\sin(x) - \frac{1}{4}\sin(3x)$$

$$\sin(3x) = 3\sin(x) - 4\sin^3(x)$$

$$\cos(3x) = 4\cos^3(x) - 3\cos(x)$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

Cotangent Definition:

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Addition formulas for cosine:

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

Partial Derivatives of a Function $f(x,y)$:

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_0,y=y_0} = \lim_{h\to 0}\frac{f(x_0+h,y_0) - f(x_0,y_0)}{h}$$

$$\left.\frac{\partial f}{\partial y}\right|_{x=x_0,y=y_0} = \lim_{h\to 0}\frac{f(x_0,y_0+h) - f(x_0,y_0)}{h}$$

Euler's Formulas for Cosine and Sine:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}, \quad \sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

Euler's Formula

$$e^{\pm ix} = \cos(x) \pm i\sin(x)$$

Specific Forms of Euler's Formula (see +- syntax)

$$e^{-ix} = \cos(x) - i\sin(x)$$

$$e^{ix} = \cos(x) + i\sin(x)$$

Calculation rules compressed

Rules for complex konjugates

$z = a + bi \quad \Rightarrow \quad \overline{z} = a - bi$
$\overline{z_1 + z_2} \quad \Rightarrow \quad \overline{z_1} + \overline{z_2}$
$\overline{z_1 - z_2} \quad \Rightarrow \quad \overline{z_1} - \overline{z_2}$
$\overline{z_1 z_2} \quad \Rightarrow \quad \overline{z_1} \cdot \overline{z_2}$
$\overline{\frac{z_1}{z_2}} \quad \Rightarrow \quad \frac{\overline{z_1}}{\overline{z_2}}, \quad z_2 \neq 0$
$z \in \mathbb{R} \quad \Rightarrow \quad \overline{z} = z$
$\overline{i} \quad \Rightarrow \quad -i$
$\overline{e^z} \quad \Rightarrow \quad e^{\overline{z}}$
$|\overline{z}| \quad \Rightarrow \quad |z|$
$\overline{\overline{z}} \quad \Rightarrow \quad z$
$\overline{\langle \psi, \phi \rangle} \quad \Rightarrow \quad \langle \phi, \psi \rangle$

Imaginary numbers

$i^2 = -1$
$z = a + bi, \quad \text{where } a, b \in \mathbb{R}$
$|z| = \sqrt{a^2 + b^2}$
$\overline{z} = a - bi$
$z \cdot \overline{z} = |z|^2 = a^2 + b^2$
$e^{i\theta} = \cos(\theta) + i\sin(\theta)$
$|e^{i\theta}| = 1$
$\left(e^{i\theta}\right)^2 = e^{i(2\theta)}$
$z^n = 1 \implies z_k = e^{i\frac{2\pi k}{n}}, \quad k = 0, 1, 2, \ldots, n-1$
$z_1 = r_1 e^{i\theta_1}, \quad z_2 = r_2 e^{i\theta_2} \implies z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)} \ \frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$
$\left(e^{i\theta}\right)^n = e^{in\theta} = (\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

Logarithmic Rules

$\log_b(xy) = \log_b(x) + \log_b(y)$
$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
$\log_b(x^k) = k \cdot \log_b(x)$
$\log_b(1) = 0$
$\log_b(b) = 1$
$\log_b(x) = \frac{\ln(x)}{\ln(b)}$
$\ln(e^x) = x$
$\ln(1) = 0$
$\ln(ab) = \ln(a) + \ln(b)$
$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
$\ln(a^k) = k \cdot \ln(a)$

Rules for absolute value

$|ab| = |a| \cdot |b|$
$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$
$|a + b| \leq |a| + |b|$ (Trekantulikheten)
$||a| - |b|| \leq |a - b|$
$||a|| = |a|$
$|a| = 0 \quad \Leftrightarrow \quad a = 0$
$|a|^2 = a^2$
$a(x) \in \mathbb{R}_{\geq 0} \implies |a(x)| = a(x)$

Rules for summation

$\sum_{i=1}^n (a + b) = \sum_{i=1}^n a + \sum_{i=1}^n b$
$\sum_{i=1}^n c \cdot a_i = c \cdot \sum_{i=1}^n a_i, \quad \text{if } c \text{ is constant.}$
$\sum_{i=1}^n c = n \cdot c, \quad \text{if } c \text{ is constant.}$
$\sum_{i=1}^n (a_i \cdot b_i) \neq \left(\sum_{i=1}^n a_i\right) \cdot \left(\sum_{i=1}^n b_i\right), \quad \text{(not always equal!)}$
$\sum_{i=1}^n i = \frac{n(n+1)}{2}$
$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^n (X_i - \bar{X}) X_i = \sum_{i=1}^n (X_i - \bar{X})^2$
$\sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n X_i^2 - n\bar{X}^2$
$\sum_{i=1}^n (aX_i + b)^2 = a^2 \sum_{i=1}^n X_i^2 + 2ab \sum_{i=1}^n X_i + nb^2$
$\sum_{i=1}^n (x_i - \bar{x}) = 0 \ \ \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$
$\text{Cov}(X, Y) = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$
$\text{Var}(X) = E(X^2) - E(X)^2$
$\text{Var}(aX + b) = a^2 \text{Var}(X)$

Rules for expectation E

$E(c) = c, \quad \text{if } c \text{ is constant.}$
$E(a + b) = E(a) + E(b)$
$E(c \cdot a) = c \cdot E(a), \quad \text{if } c \text{ is constant.}$
$E(a \cdot b) = E(a) \cdot E(b), \quad \text{if } a \text{ and } b \text{ are independent.}$
$E\left(\sum_{i=1}^n a_i\right) = \sum_{i=1}^n E(a_i)$
$E(a^2) = \text{Var}(a) + E(a)^2$
$E\left(\frac{a}{b}\right) \neq \frac{E(a)}{E(b)}, \quad$ (not always equal, except in special case
$E(g(a)) = \int_{-\infty}^{\infty} g(x) f_a(x)\, dx, \quad$ (for the continuous case wi
$E(g(a)) = \sum_x g(x) P(a = x), \quad$ (for the discrete case).

Regneregler for potenser

$a^p a^q = a^{p+q}$
$\frac{a^p}{a^q} = a^{p-q}$
$a^{-q} = \frac{1}{a^q}$
$(a^p)^q = a^{p \cdot q}$
$a^{\frac{1}{p}} = \sqrt[p]{a}$
$a^p b^p = (ab)^p$
$\frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$
$a^0 = 1, \quad \text{for } a \neq 0$
$a^1 = a$
$0^p = 0, \quad \text{for } p > 0$
$1^p = 1$
$(-a)^p = \begin{cases} a^p, & \text{hvis } p \text{ er partall} \\ -a^p, & \text{hvis } p \text{ er oddetall} \end{cases}$
$a^{1/2} = \sqrt{a}$

Regneregler for kvadratrøtter

$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}, \quad \text{for } a \geq 0, b \geq 0$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \quad \text{for } a \geq 0, b > 0$
$(\sqrt{a})^2 = a, \quad \text{for } a \geq 0$
$\sqrt{a^2} = |a|$
$\sqrt[2]{a} = a^{1/2}$
$\sqrt{a^n} = a^{n/2}, \quad \text{for } a \geq 0$
$\sqrt{k \cdot a} = \sqrt{k} \cdot \sqrt{a}, \quad \text{for positiv konstant } k$
$\sqrt{a} \neq \sqrt{b} \quad \text{med mindre } a = b$
$\sqrt{a^2 + b^2} \neq \sqrt{a^2} + \sqrt{b^2} \quad \text{(ikke likhet)}$

sectionIntegration
subsectionImportant integrals

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}, \quad a > 0$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \cdot \frac{1}{2a}, \quad a > 0$$

$$\int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \cdot \frac{(2n-1)!!}{(2a)^n}, \quad n \in \mathbb{N}_0,\, a > 0$$

$$\int_a^b uv' \, dx = \left[uv\right]_a^b - \int_a^b u'v \, dx$$

$$\int_a^b f(u)u' \, dx = \int_{u(a)}^{u(b)} f(u) \, du$$

$$u = ax + b \Rightarrow \frac{du}{dx} = a \Rightarrow du = a\,dx \Rightarrow dx = \frac{du}{a}$$

Eksponentialfunksjoner

$\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C, \quad k \neq 0$
$\int a^x \, dx = \frac{a^x}{\ln(a)} + C, \quad a > 0, a \neq 1$

Trigonometriske funksjoner

$\int \sin(kx) \, dx = -\frac{1}{k} \cos(kx) + C$
$\int \cos(kx) \, dx = \frac{1}{k} \sin(kx) + C$
$\int \sec^2(kx) \, dx = \frac{1}{k} \tan(kx) + C$
$\int \csc^2(kx) \, dx = -\frac{1}{k} \cot(kx) + C$
$\int \sec(kx) \tan(kx) \, dx = \frac{1}{k} \sec(kx) + C$
$\int \csc(kx) \cot(kx) \, dx = -\frac{1}{k} \csc(kx) + C$

Hyperbolske funksjoner

$\int \sinh(kx) \, dx = \frac{1}{k} \cosh(kx) + C$
$\int \cosh(kx) \, dx = \frac{1}{k} \sinh(kx) + C$

Inverse trigonometriske funksjoner

$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \arcsin\left(\frac{x}{a}\right) + C$
$\int \frac{1}{1+x^2} \, dx = \arctan(x) + C$

Logaritmiske funksjoner

$\int \frac{1}{x} \, dx = \ln|x| + C$
$\int \ln(x) \, dx = x \ln(x) - x + C$

Potensfunksjoner

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

Spesielle integraler

$\int \frac{1}{\sqrt{x^2+a^2}} \, dx = \ln\left|x + \sqrt{x^2+a^2}\right| + C$
$\int \frac{1}{\sqrt{x^2-a^2}} \, dx = \ln\left|x + \sqrt{x^2-a^2}\right| + C, \quad x > a$
$\int x^r \, dx = \frac{x^{r+1}}{r+1} + C,\ r \neq -1$
$\int \frac{1}{x} \, dx = \ln|x| + C$
$\int e^x \, dx = e^x + C$
$\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C,\ k \neq 0$
$\int \sin x \, dx = -\cos x + C$
$\int \cos x \, dx = \sin x + C$
$\int \frac{1}{\cos^2 x} \, dx = \tan x + C$
$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$
$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$

Trigonemetriske funksjoner

Eksakte verdier til sin og cos

u	u	$\sin u$	$\cos u$	$\tan u$
0	$0°$	0	1	0
$\pi/6$	$30°$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$45°$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$60°$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	$90°$	1	0	$-$

Trigonometriske formler

$1 = \sin^2 u + \cos^2 u$

$\tan u = \frac{\sin u}{\cos u}$

$\sin u = \sin(u + 2\pi n), \quad n \in \mathbb{Z}$

$\cos u = \cos(u + 2\pi n), \quad n \in \mathbb{Z}$

$\tan u = \tan(u + \pi n), \quad n \in \mathbb{Z}$

$\sin(\pi - u) = \sin u$

$\cos(-u) = \cos u$

$-\sin(u) = \sin(-u)$

$\cos^2 u = \frac{1+\cos(2u)}{2}$

$\sin^2 u = \frac{1-\cos(2u)}{2}$

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$\cos(u + v) = \cos u \cos v - \sin u \sin v$

$\sin(2u) = 2 \sin u \cos u$

$\cos(2u) = \cos^2 u - \sin^2 u$

Derivatives

Derivasjons regler

$(u+v)' = u' + v'$
$(u-v)' = u' - v'$
$(cu(x))' = cu'(x)$
$(uv)' = u'v + uv'$
$(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$
$(u \circ v)' = u'(v) \cdot v'$
$(u^{-1})' = -\frac{u'}{u^2}$
$(u^n)' = nu^{n-1}u'$
$(e^u)' = u'e^u$
$(a^u)' = u'a^u \ln(a)$
$(\ln(u))' = \frac{u'}{u}$
$(\log_a(u))' = \frac{u'}{u\ln(a)}$
$(\sin(u))' = u' \cos(u)$
$(\cos(u))' = -u' \sin(u)$
$(\tan(u))' = u' \sec^2(u)$
$(\cot(u))' = -u' \csc^2(u)$
$(\sec(u))' = u' \sec(u) \tan(u)$
$(\csc(u))' = -u' \csc(u) \cot(u)$
$(\arcsin(u))' = \frac{u'}{\sqrt{1-u^2}}$
$(\arccos(u))' = -\frac{u'}{\sqrt{1-u^2}}$
$(\arctan(u))' = \frac{u'}{1+u^2}$

Important Derivatives

$\frac{d}{dx}(c) = 0$
$\frac{d}{dx}(x) = 1$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(a^x) = a^x \ln(a)$
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$
$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$
$\frac{d}{dx}(\sin(x)) = \cos(x)$
$\frac{d}{dx}(\cos(x)) = -\sin(x)$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$
$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\arccos(x)) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

Differensialligning ansatser

Homogene ligninger

Ansatser for $y'' + by' + cy = 0$: $y = e^{\lambda x}$
$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$
$y = e^{\alpha x}(C_1 \cos(\beta x) + C_2 \sin(\beta x))$
$y = (C_1 + C_2 x)e^{\lambda x}$

Inhomogene ligninger

Ansatser for $g(x)$: $y_p = B_0 + B_1 x + \cdots + B_n x^n$
$y_p = Ce^{ax}$
$y_p = Cxe^{ax}$
$y_p = C_1 \cos(ax) + C_2 \sin(ax)$
$y_p = x(C_1 \cos(ax) + C_2 \sin(ax))$
$y_p = e^{ax}(R_n(x) \cos(bx) + S_n(x) \sin(bx))$

Ikke-lineære ligninger

$\frac{dy}{dx} = g(x)h(y)$ $z = y^{1-n}$ $y = vx, \quad v = \frac{y}{x}$ $y = \frac{1}{z}$ $y = e^z$

Spesialtilfeller

$y = x^r$ $y = A\cos(kx) + B\sin(kx)$ $y = e^{kx}$

Statistikk

Matriser

Vektorer

Geometri

Kvantemekanikk

Numeriske metoderbb

numpy

```
np.mean(data)
np.median(data)
np.std(data)
np.var(data)
np.percentile(data, 25)
np.corrcoef(data1, data2)
np.cov(data1, data2)
np.histogram(data)
np.histogram2d(data1, data2)
np.histogram_bin_edges(data)
np.histogramdd(data)
np.sqrt(data) 2rot
np.cbrt 3rot, 4rot potens++
np.exp(data)
np.log(data) 2,10 osv..
np.sin(data) osv..
```