Mathematics for Robotics Assignment 2

Polynomial Approximation

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1 Task 1

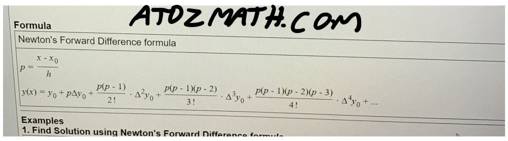
1) The following table gives four values of $f(x) = x^4 - 3x^3 + 6x^2 - 2x + 5$ at $x_i = 1, 2, 3$ and 4. Obtain a 3[rd] degree polynomial, $p(x)$ using Newton forward difference formula. Further, compare the plots of $f(x)$ *vs* x, and $p(x)$ *vs* x in the interval [0 5] in MATLAB.

x:	1	2	3	4
f(x):	7	17	53	157

NEWTON FORWARD DIFFERENCE TABLE:

X	$f(x)$	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$
1	7	10	26	42
2	17	36	68	
3	53	104		
4	157			

ATOZMATH.COM


Formula

Newton's Forward Difference formula

$$p = \frac{x - x_0}{h}$$

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\cdot\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\cdot\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\cdot\Delta^4 y_0 + \dots$$

Examples

1. Find Solution using Newton's Forward Difference formula

$$h = X_1 - X_0 = 2 - 1 = 1$$

$$P = \frac{x-1}{1}$$

$$P_3(x) = 7 + 10P + \frac{P(P-1)}{2!}\cdot 26 + \frac{P(P-1)(P-2)}{3!}\cdot 42$$

$$P_3(x) = 7 + \left(\frac{x-1}{1}\right)\cdot 10 + \frac{\left(\frac{x-1}{1}\right)\left(\frac{x-1}{1} - 1\right)\cdot 26}{2!} + \frac{\left(\frac{x-1}{1}\right)\left(\frac{x-1}{1} - 1\right)\left(\frac{x-1}{1} - 2\right)}{3!}\cdot 42$$

SE MATLAB PLOT:

Figure 1: Task 1

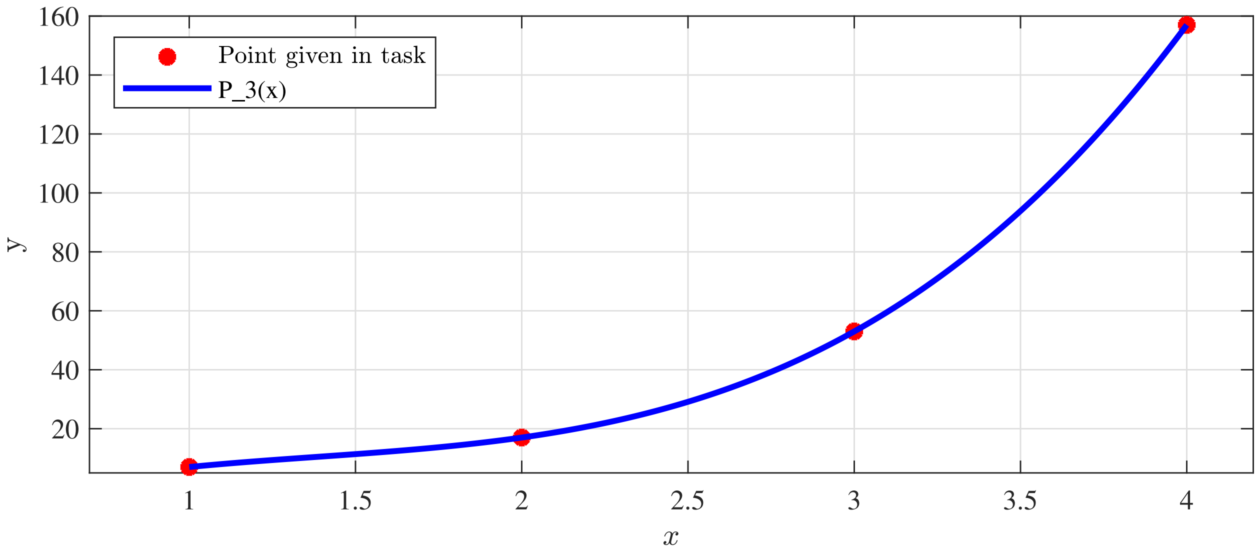


Figure 2: Plot 1

```
clear; close all; clc;

%task 1

pX = [1, 2,3,4];
pY = [7,17, 53, 157];

% plot function
x = 1 : 0.001 : 4;
p = (x-1)/1;
P3_x = 7+10.*p + p.*(p-1).*26/(2*1) + p.*(p-1).*(p-2)*42/(3*2*1);

plot(pX,pY, '*r');
hold on
xlim([0.7 4.2]);
ylim([5 160]);
plot(x,P3_x, 'b')
legend('Point given in task', 'P_3(x)', Location='northwest')

%NB: generated .eps figure in other script
```

2 Task 2

2) Using Newton's backward difference formula, construct an interpolating polynomial of degree 3 for the data: f (– 0.75) = – 0.0718125, f (– 0.5) = – 0.02475, f (– 0.25) = 0.3349375, f (0) = 1.10100. Hence find f (– 1/3).

NEWTON BACKWARDS DIFFERENCE TABLE:

(SAME WAY OF CALCULATING Δ AS FORWARD)

X	$f(x)$	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$
-0,75	-0,0718125			
		0,0471		
-0,5	-0,02475		0,3126	
		0,3597		0,0938
-0,25	0,3349375		0,4064	
		0,7661		
0	1,10100			

Formula

Newton's Backward Difference formula ATOZMATH.COM

$$p = \frac{x - x_n}{h}$$

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\cdot\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\cdot\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\cdot\nabla^4 y_n + \ldots$$

$$p = \frac{(x-0)}{h} = \frac{x}{0{,}25} \qquad h = x_1 - x_0 = -0{,}5-(-0{,}75) = 0{,}25$$

$$P_3(x) = 1{,}10100 + 0{,}7661\cdot p + 0{,}4064\cdot\frac{p(p+1)}{2!} + 0{,}0938\cdot\frac{p(p+1)(p+2)}{3!}$$

$$P_3\left(-\tfrac{1}{3}\right) = 0{,}1745$$

SEE MATLAB PLOT:

Figure 3: Task 2

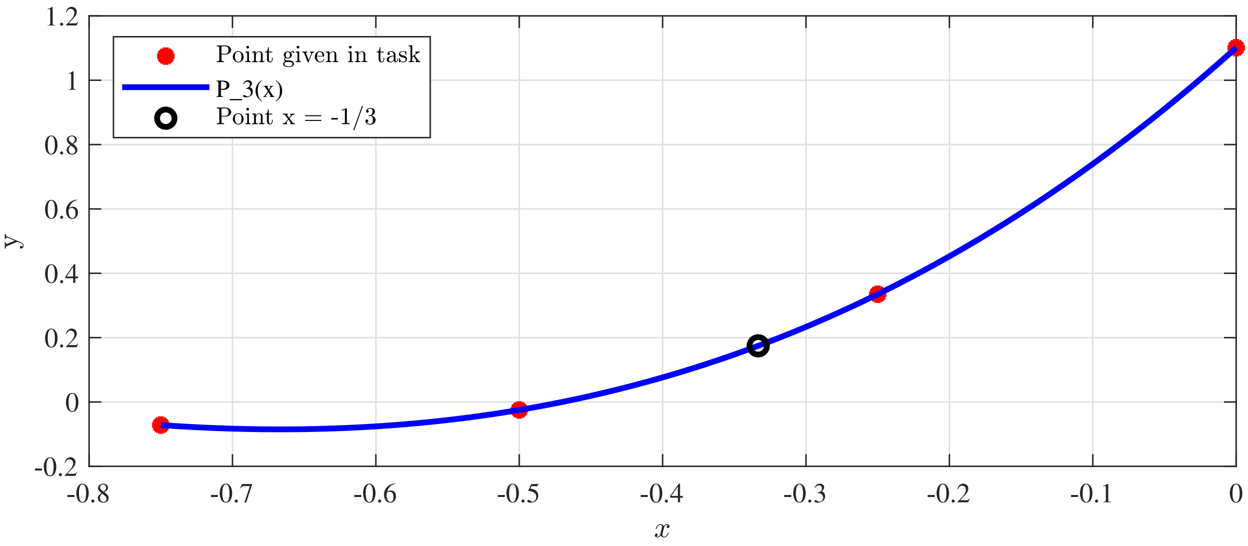


Figure 4: Plot 2

```
%% NewtonBackwards task 2
clear; close all; clc;
pX = [-0.75, -0.5, -0.25, 0]; %Given points along function
pY = [-0.0718125,-0.02475, 0.3349375, 1.10100]; %Given points along ...
    function

%Calculate the Delta1Y
% From self made function
Delta1Y = delta(pY);
%calculate the delta^2Y
Delta2Y = delta(Delta1Y);
%calculate the delta^3Y
Delta3Y = delta(Delta2Y);
xList = -0.75 : 0.01 : 0;
x = xList;
p = x./0.25;
P_3_x = pY(4) + Delta1Y(3).*p + Delta2Y(2).*p.*(p+1)/(2*1) + ...
    Delta3Y(1).*p.*(p+1).*(p+2)/(3*2);
%Specific point
xGiven = -1/3;
x = xGiven;
p = x./0.25;
P_3_givenX = pY(4) + Delta1Y(3)*p + Delta2Y(2).*p.*(p+1)/(2*1) + ...
    Delta3Y(1)*p.*(p+1).*(p+2)/(3*2)

plot(pX,pY, '*r');
hold on
plot(xList,P_3_x, 'b')
plot(xGiven, P_3_givenX, 'ok')
legend('Point given in task', 'P_3(x)','Point x = -1/3', ...
    Location='northwest')
%NB: generated .eps figure in other script
```

```
function [delta] = delta(y)
% y = list of poins to find deltas from
for i=1 : length(y)-1
    delta(i) = y(1+i)-y(i);
end
end
```

3 Task 3

3) Find the distance moved by a particle and its acceleration at the end of 4 seconds, if the time verses velocity data is as follows: (Use Lagrange interpolation to generate the *v(t)* curve).

	X_0	X_1	X_2	X_3
t:	0	1	3	4
v:	21	15	12	10
	Y_0	Y_1	Y_2	Y_3

Formula

Lagrange's Interpolation formula

$$y(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} \times y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\ldots(x_2-x_n)} \times y_2 + \ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})} \times y_n$$

Examples

1. Find Solution using Lagrange's Interpolation formula

x	f(x)

$$V(t) = \frac{(t-1)(t-3)(t-4)}{(0-1)(0-3)(0-4)} \cdot 21 + \frac{(t-0)(t-3)(t-4)}{(1-0)(1-3)(1-4)} \cdot 15 + \frac{(t-0)(t-1)(t-4)}{(3-0)(3-1)(3-4)} \cdot 12 + \frac{(t-0)(t-1)(t-3)}{(4-0)(4-1)(4-3)} \cdot 10$$

FROM SIMPLIFICATION IN MATLAB:

$$v(t) = -\frac{5}{12}t^3 + \frac{19 \cdot t^2}{6} - \frac{35t}{4} + 21$$

$$a(t) = -\frac{5}{12} \cdot 3t^2 + \frac{19 \cdot 2t}{6} - \frac{35}{4}$$

$$X(t) = \int v(t) = -\frac{5}{12 \cdot 4}t^4 + \frac{19}{6 \cdot 3}t^3 - \frac{35}{4 \cdot 2}t^2 + 21t + X_0$$

$$X(4) = 54{,}8889$$

$$a(4) = -3{,}4167$$

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Figure 5: Task 3

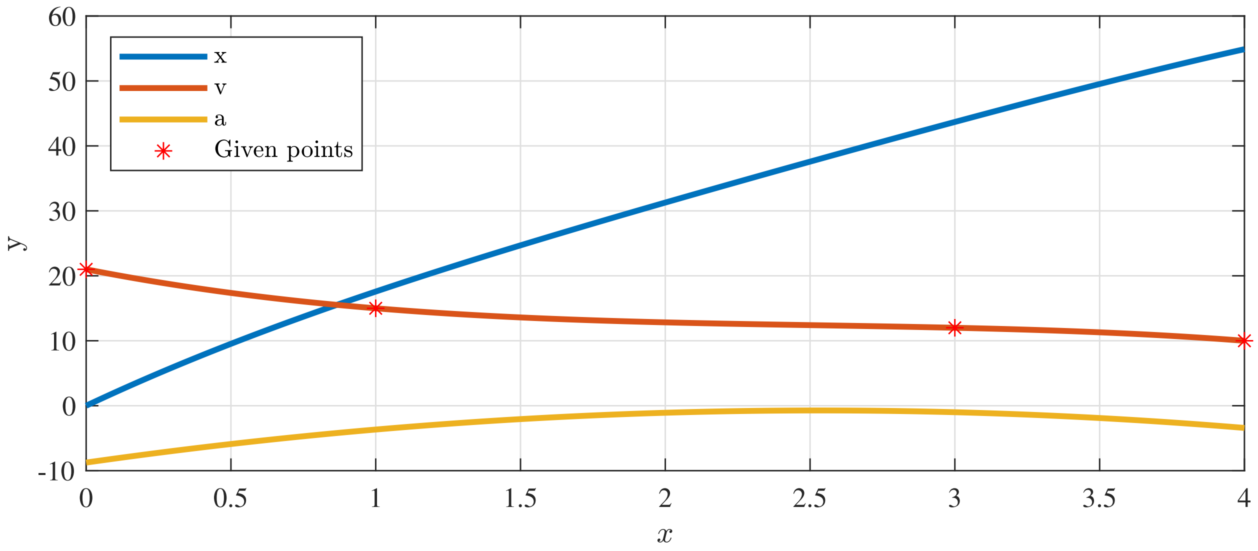

Figure 6: Plot 3

```
%% lagrange task 3
clear; close all; clc;

time = [0, 1, 3, 4]; % given time
v = [21, 15, 12, 10]; %given velocities at corresponding time

syms t
p1 = (t-1)*(t-3)*(t-4)*21/((-1*-3*-4));
p1 = expand(p1);

p2 = (t-0)*(t-3)*(t-4)*15/((1*(1-3)*(1-4)));
p2 = expand(p2);

p3 = (t-0)*(t-1)*(t-4)*12/(((3-0)*(3-1)*(3-4)));
p3 = expand(p3);

p4 = (t-0)*(t-1)*(t-3)*10/(((4-0)*(4-1)*(4-3)));
p4 = expand(p4);

v_t = p1+p2+p3+p4
x_t = int(v_t,t)
a_t = diff(v_t,t)

tlist = 0 : 0.01 : 4;
%from symbolic function to plotable values
vPlot = subs(v_t, t, tlist);
xPlot = subs(x_t, t, tlist);
aPlot = subs(a_t, t, tlist);

plot(tlist,xPlot)
hold on
plot(tlist,vPlot)
plot(tlist,aPlot)
plot(time, v, '*r')
legend('x','v','a','Given points', Location='northwest')

x_4_s = double(subs(x_t, t, 4))
a_4_s = double(subs(a_t, t, 4))
%NB: generated .eps figure in other script
```

4 Task 4

4) Given the values

x:	5	7	11	13	17
$f(x)$:	150	392	1452	2366	5202

Evaluate $f(9)$, using Newton's divided difference formula

Formula

Newton's Divided Difference Interpolation formula

$$y(x) = y_0 + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \ldots$$

X	f(x)	$f[X_0,X_1]$ 1ST ORDER	$f[X_0,X_1,X_2]$ 2 ORDER	$f[X_0,X_1,X_2,X_3]$ 3. ORDER	$f[X_0,X_1,X_2,X_3,X_4]$ 4. ORDER
5	150	$\frac{392-150}{7-5}=121$	$\frac{265-121}{11-5}=24$	$\frac{32-24}{13-5}=1$	$\frac{1-1}{17-5}=0$
7	392	$\frac{1452-392}{11-7}=265$	$\frac{457-265}{13-7}=32$	$\frac{42-32}{17-7}=1$	
11	1452	$\frac{2366-1452}{13-11}=457$	$\frac{709-457}{17-11}=42$		
13	2366	$\frac{5202-2366}{17-13}=709$			
17	5202				

$$Y(x) = 150 + (X-5)\cdot 121 + (X-5)(X-7)\cdot 24 + (X-5)(X-7)(X-11)\cdot 1$$

$$Y = X^3 + X^2$$

$$\underline{\underline{Y(9) = 9^3 + 9^2 = 810}}$$

SE MATLAB PLOT:

Figure 7: Task 4

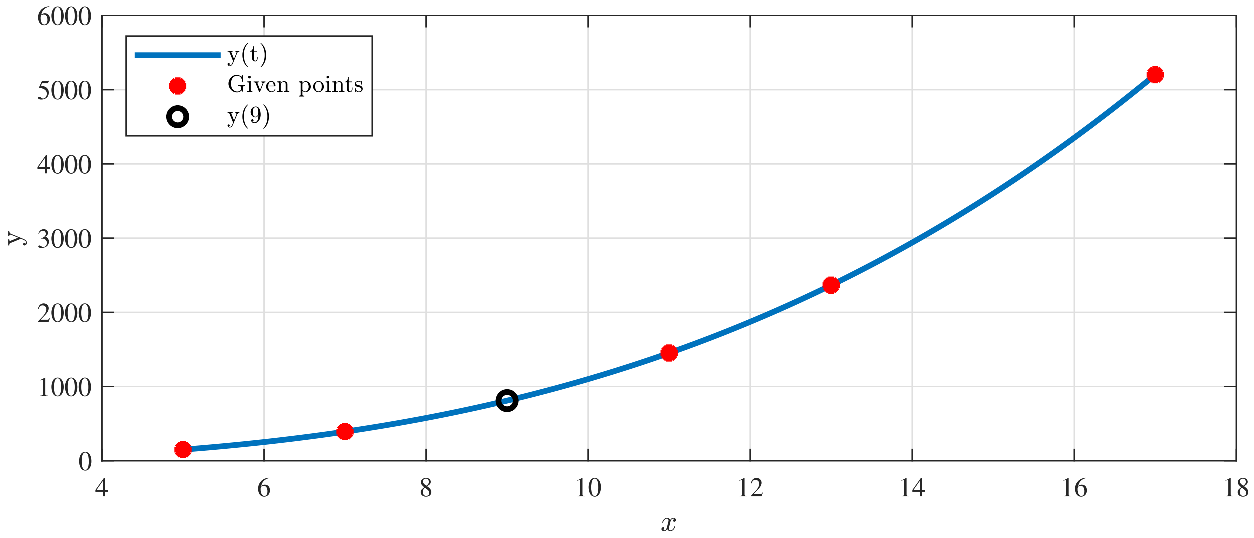

Figure 8: Plot 4

```
pX = [5 7 11 13 17]; %Given points along function
pY = [150 392 1452 2366 5202]; %Given points along function

% 1st order diveded difference
for i=1 : length(pY)-1
    delta_ND(i) = (pY(1+i)-pY(i))/(pX(1+i)-pX(i));
    i = i+1;
end

% 2nd order diveded difference
delta_ND2 = [(265-121)/(11-5),(457-265)/(13-7),(709-457)/(17-11)]

% 3rd order
delta_ND3 = [(32-24)/(13-5),(42-32)/(17-7)]
% 3th order = 0
syms x
y = 150 + (x-5)*121 + (x-5)*(x-7)*24+ (x-5)*(x-7)*(x-11)*1;
y = expand(y);
xlist = 5 :0.01 : 17;
yPlot = subs(y, x, xlist);

y_9 = subs(y, x, 9);
plot(xlist,yPlot)
hold on
plot(pX,pY, '*r')
plot(9,y_9, 'ok')
legend('y(t)','Given points','y(9)', Location='northwest')
%NB: generated .eps figure in other script
```

```
function [delta_ND] = delta_ND_1stOrder(x,y)
% y = list of poins to find deltas from Newton divided differece
for i=1 : length(y)-1
    delta_ND(i) = (y(1+i)-y(i))/(x(1+i)-x(i));
    i = i+1;
end
end
```

5 Task 5

5) Apply Lagrange's formula inversely to obtain a root of the equation $f(x) = 0$, given that $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, and $f(42) = 18$.

Formula

Lagrange's Inverse Interpolation formula

$$x(y) = \frac{(y-y_1)(y-y_2)\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots(y_0-y_n)} \times x_0 + \frac{(y-y_0)(y-y_2)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots(y_1-y_n)} \times x_1$$
$$+ \frac{(y-y_0)(y-y_1)(y-y_3)\ldots(y-y_n)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)\ldots(y_2-y_n)} \times x_2 + \ldots + \frac{(y-y_0)(y-y_1)\ldots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\ldots(y_n-y_{n-1})} \times x_n$$

X	f(x)
30	-30
34	-13
38	3
42	18

COULD GO STRAIGHT TO X(0), BUT FIND THE FUNCTION AS USUAL, TO GIVE PLOT

$$X(Y) = \frac{(Y-(-13))(Y-3)(Y-18)}{(-30-13)(-30-3)(-30-18)} \cdot 30 + \frac{(Y-(-30))(Y-3)(Y-18)}{(-13-(-30))(-13-3)(-13-18)} \cdot 34$$
$$+ \frac{(Y-(-30))(Y-(-13))(Y-18)}{(3-(-30))(3-(-13))(3-18)} \cdot 38 + \frac{(Y-(-30))(Y-(-13))(Y-3)}{(18-(-30))(18-(-13))(18-3)} \cdot 42$$

$$X(y) = \frac{y^3}{521730} + \frac{109}{208692} \cdot y^2 + \frac{88679}{347820} y + \frac{431649}{11594}$$

$$\underline{\underline{X(0) = 37{,}2304}}$$

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Figure 9: Task 5

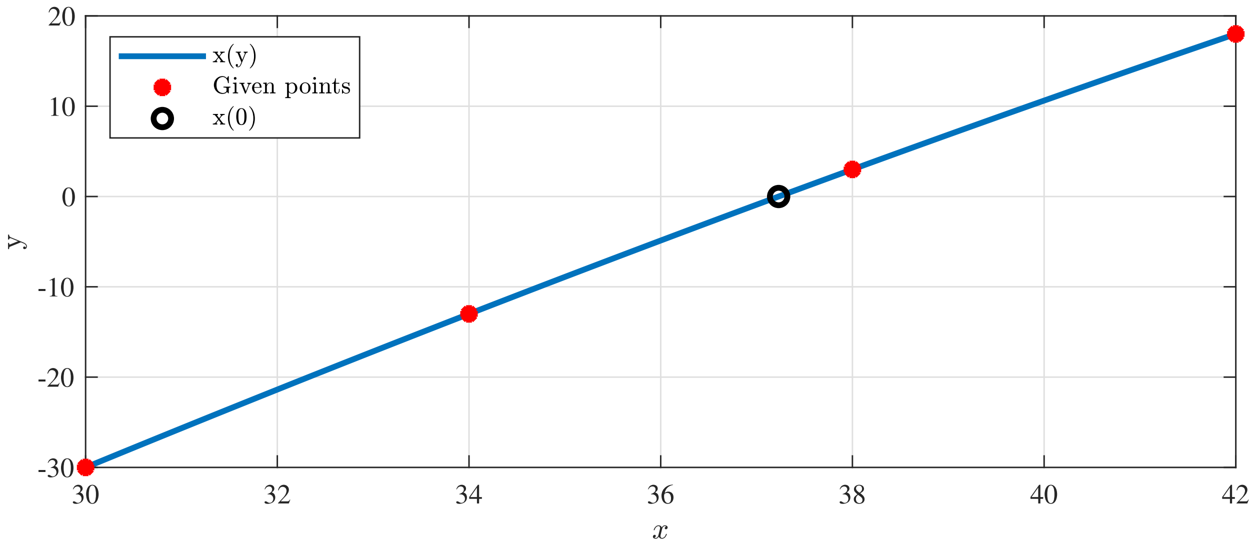

Figure 10: Plot 5

```
%% Lagranges task 5
clear; close all; clc;
pX = [30 34 38 42]; %Given points along function
pY = [-30 -13 3 18]; %Given points along function

syms y
x1 = (y+13)*(y-3)*(y-18)*30/((-30+13)*(-30-3)*(-30-18));
x2 = (y+30)*(y-3)*(y-18)*34/((-13+30)*(-13-3)*(-13-18));
x3 = (y+30)*(y+13)*(y-18)*38/((3+30)*(3+13)*(3-18));
x4 = (y+30)*(y+13)*(y-3)*42/((18+30)*(18+13)*(18-3));
x = x1+x2+x3+x4;

x = expand(x);

x_0 = double(subs(x, y, 0));
ylist = -30 : 0.01 :18;
xPlot= (subs(x, y, ylist));

plot(xPlot,ylist)
hold on
plot(pX,pY, '*r')
plot(x_0,0, 'ok')
legend('x(y)','Given points','x(0)', Location='northwest')
%NB: generated .eps figure in other script
```

6 Task 6

6) Using Newton's divided differences formula, evaluate *f(8)*

x:	4	5	7	10	11
f(x):	48	100	294	900	1210

Later, add a new data point f(13) = 2028 to the interpolated polynomial and evaluate f(12).

Formula

Newton's Divided Difference Interpolation formula

$$y(x) = y_0 + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \ldots$$

x	$f(x)$	$f[x_0,x_1]$ 1ST ORDER	$f[x_0,x_1,x_2]$ 2 ORDER	$f[x_0,x_1,x_2,x_3]$ 3. ORDER	$f[x_0,x_1,x_2,x_3,x_4]$ 4. ORDER
4	48	52	$\frac{97-52}{7-4}=15$	$\frac{21-15}{10-4}=1$	$\frac{1-1}{m}=0$
5	100	97	$\frac{202-97}{10-5}=21$	$\frac{27-21}{11-5}=1$	$\frac{1-1}{m}=0$
7	294	202	$\frac{310-202}{11-7}=27$	$\frac{33-27}{13-7}=1$	
10	900	310	$\frac{409-310}{13-10}=33$		
11	1210	$\frac{2028-1210}{13-11}=409$			
13	2028				

$$y(x) = 48 + (x-4)\cdot 52 + (x-4)(x-5)\cdot 15 + (x-4)(x-5)(x-7)\cdot 1$$

$$y(8) = 448$$

FUNCTION $y(x)$ IS UNCHANGED DUE TO THE ADDED POINT.
THIS IS PROBABLY NOT A COINCIDENCE.
HOWEVER THE POINT IS PLOTTED IN MATLAB TO DOUBBLE CHECK!

Figure 11: Task 6

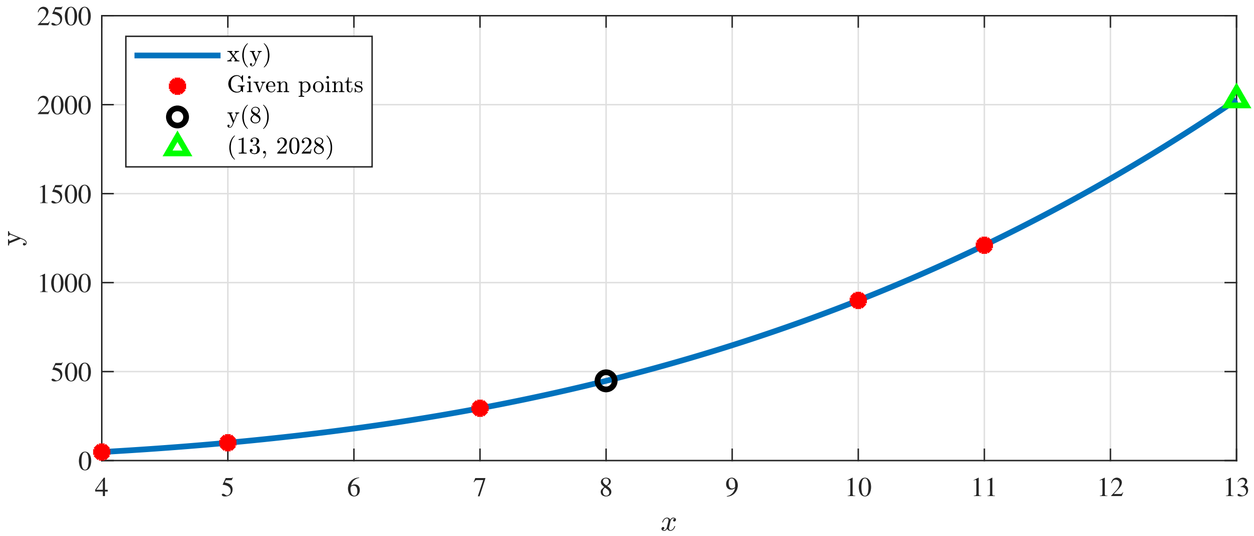

Figure 12: Plot 6

```
%% Newton divided task 6
clear; close all; clc;
pX = [4 5 7 10 11]; %Given points along function
pY = [48 100 294 900 1210]; %Given points along function
% Evaluate f(8)


% 1st order diveded difference
for i=1 : length(pY)-1
    delta_ND(i) = (pY(1+i)-pY(i))/(pX(1+i)-pX(i));
    i = i+1;
end

% 2nd order diveded difference
delta_ND2 = [(97-52)/(7-4),(202-97)/(10-5),(310-202)/(11-7)]
delta_ND3 = [(21-15)/(10-4), (27-21)/(11-5)]

%ADD POINT (13,2028)
delta_ND(5) = (2028-1210)/(13-11);
delta_ND2(4) = (409-310)/(13-10);
delta_ND3(3) = (33-27)/(13-7);

syms x
%Function y is unchanged with added point
y = 48 + (x-4)*52 + (x-4)*(x-5)*15 + (x-4)*(x-5)*(x-7);
y_8 = double(subs(y, x, 8))
%Point lands good on the function as expexted
y_12 = double(subs(y, x, 12))
xlist = 4 : 0.01 : 13;
ylist= (subs(y, x, xlist));

plot(xlist,ylist)
hold on
plot(pX,pY, '*r')
plot(8,y_8, 'ok')
plot(13,2028,'^g')
legend('x(y)','Given points','y(8)','(13, 2028)', Location='northwest')
%NB: generated .eps figure in other script
```

7 Citing

Formulas from website atozmath.com [1].

Bibliography

[1] atoZmath. *atoZmath*. URL: https://atozmath.com/example/CONM/NumeInterPola.aspx?q=LII&q1=E1. (accessed: 21.09.2023).