GAUS ELIM:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 5 \\ 2 & 3 & 5 & 11 \end{array}\right] \leftarrow -2R_2+3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 5 \\ 0 & -1 & -1 & 1 \end{array}\right]$$

⋮

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$x_3 = 4$
$x_2 = -5$
$x_1 = 3$

LU DECOMP

$$A\vec{x} = \vec{b}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

CREATE EQUATIONS, STARTING FROM THE EASYEST ($u_{11}, u_{..}$) SOLVE AND PUT BACK IN L AND U

$$L\underbrace{U\vec{x}}_{\vec{y}} = \vec{b}$$

$$L\vec{y} = \vec{b}$$

SOLVE FOR $\vec{y}$ USE IN $\vec{y} = U\vec{x}$

CHOLESKY

$$A = \begin{bmatrix} 6 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix}$$

(POSITIVE DEFINITE: FIND EIGENVALUES OF A IN MATLAB $\lambda > 0$)

$$b = \begin{bmatrix} 76 \\ 295 \\ 1259 \end{bmatrix}$$

$$\boxed{\begin{aligned} A\vec{x} &= \vec{b} \\ A &= LL^T \\ L\underbrace{L^T\vec{x}}_{\vec{y}} &= \vec{b} \\ L\vec{y} &= \vec{b} \end{aligned}}$$

$$\overset{L}{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}} \overset{L^T}{\begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}}$$

$$LL^T = \begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31} \\ l_{21}l_{11} & l_{21}^2+l_{22}^2 & l_{21}l_{31}+l_{22}l_{32} \\ l_{31}l_{11} & l_{31}l_{21}+l_{32}l_{22} & l_{31}^2+l_{32}^2+l_{33}^2 \end{bmatrix}$$

EIGENVALUE 3X3:

$$\det(A - \lambda I) = 0$$

$$\det\begin{pmatrix} A-\lambda & A & A \\ A & A-\lambda & A \\ A & A & A-\lambda \end{pmatrix} = 0$$

SOLVE FOR λ

SOLVE FOR ROOTS = EIG

GAUS JACOBI ITT. METH

$$\left.\begin{aligned} 4x_1 - x_2 - x_3 &= 3 \\ -2x_1 + 6x_2 + 3x_3 &= 9 \\ -x_1 + x_2 + 7x_3 &= -6 \end{aligned}\right\}$$ DIAGNALLY DOMINANT!

$x_1 = \dfrac{3 + x_2 + x_3}{4}$ OLD x_2, x_3

$x_2 = \dfrac{9 - 3x_3 + 2x_1}{6}$ OLD x_1, x_3

$x_3 = \dfrac{-6 - x_2 + x_1}{7}$ OLD $x_2 x_1$

INITIAL GUESS:
$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

K	x_1	x_2	x_3
1	0,75	1,5	-0,857
2	0,9107	2,1786	-0,9643
3	1,0536	2,2857	-1,0383
4	1,0619	2,3703	-1,0332
5	1,0843	2,3703	-1.0441

CONVERGING BUT NOT COULD BE BETTER

GAUS SEIDEL

$$\left.\begin{aligned} 12x_1 + 3x_2 - 5x_3 &= 1 \\ x_1 + 5x_2 + 3x_3 &= 28 \\ 3x_1 + 7x_2 + 13x_3 &= 76 \end{aligned}\right\}$$ DIAGONALLY DOMINANT! :)

$x_1 = \dfrac{1 - 3x_2 + 5x_3}{12}$ ← OLD x_2, x_3

$x_2 = \dfrac{28 - x_1 - 3x_3}{5}$ ← NEW x_1, OLD x_3

$x_3 = \dfrac{76 - 3x_1 - 7x_2}{13}$ ← NEW x_1, x_2

K	x_1	x_2	x_3
1	$\frac{1}{2}$	$\frac{49}{10}$	$\frac{201}{65}$
2	0,14679	3,7152	3,81175
3	0,7427	3,1643	3.9708
4	0,94675	3,0281	3.997
5	0.9917	3.003	4.000
6	0.999	3.001	4.000

$|K_6 - K_5| = \varepsilon$
LOOKS LIKE CONVERGING TO $[1, 3, 4]$

DERIVATIVE AND INTEGRATION:

$$(fg)' = f'g + g'f$$

$$\left(\frac{f}{g}\right)' = \frac{f'g + g'f}{g^2}$$

$$\int uv\,dx = u\int v\,dx - \int\left(u'\int v\,dx\right)dx$$

POLYNOMIAL APPROXIMATION

NEWTON FORWARD:

x	$f(x)$	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
1	7			
		10		
2	17		26	
		36		42
3	53		68	
		104		
4	157			

$h = x_1 - x_0$

$p = \frac{x-1}{1}$

$$P_3(x) = 7 + 10p + \frac{p(p-1)}{2!}\cdot 26 + \frac{p(p-1)(p-2)}{3!}\cdot 42$$

$$P_3(x) = 7 + \left(\frac{x-1}{1}\right)\cdot 10 + \frac{\left(\frac{x-1}{1}\right)\left(\frac{x-1}{1}-1\right)\cdot 26}{2!} + \frac{\left(\frac{x-1}{1}\right)\left(\frac{x-1}{1}-1\right)\left(\frac{x-1}{1}-2\right)}{3!}\cdot 42$$

FORMULA:

$p = \frac{x-x_0}{h}$ $\quad h = x_1 - x_0$

$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i$

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 \ldots\ldots$$

NEWTON BACKWARD:

x	$f(x)$	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
1	7			
		10		
2	17		26	
		36		42
3	53		68	
		104		
4	157			

$h = x_1 - x_0$

$p = \frac{x-1}{1}$

FORMULA: $h = x_1 - x_0$

$p = \frac{x-x_n}{h}$

$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i$

$$y(x) = y_n + p\Delta y_n + \frac{p(p+1)}{2!}\Delta^2 y_n + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_n \ldots\ldots$$

LAGRANGES INT. FORM

x	x_0	x_1	.....	x_n
y	y_0	x_0		y_n

$$y(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}\cdot y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}\cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)\ldots(x_2-x_n)}\cdot y_2$$

LAGRANGES INVERSE INT

$$X(y) = \frac{(y-y_1)(y-y_2)\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots(y_0-y_n)}\cdot x_0 + \frac{(y-y_0)(y-y_2)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots(y_1-y_n)}\cdot x_1 + \frac{(y-y_0)(y-y_1)\ldots(y-y_n)}{(y_2-y_0)(y_2-y_1)\ldots(y_2-y_n)}\cdot x_2$$

$\ldots\ldots\ldots\ldots$

NEWTON DIVIDED DIFF

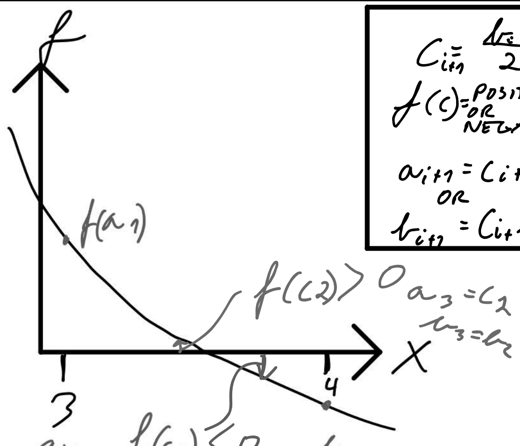
x	$f(x)$	$f[x_0,x_1]$	$f[x_0,x_1,x_2]$	$f[x_0,x_1,x_2,x_3]$	$f[x_0,x_1,x_2,x_3,x_4]$
5	150				4TH ORDER
		$\frac{392-150}{7-5} = \boxed{121}$			
7	392		$\frac{265-121}{11-5} = 24$		
		$\frac{1452-392}{11-7} = \boxed{265}$		$\frac{32-24}{13-5} = 1$	
11	1452		$\frac{457-265}{13-7} = \boxed{32}$		$\frac{1-1}{17-5} = 0$
		$\frac{2366-1452}{13-11} = 457$		$\frac{42-32}{17-7} = 1$	
13	2366		$\frac{709-457}{17-11} = \boxed{42}$		
		$\frac{5202-2366}{17-13} = 709$			
17	5202				

$$\boxed{y(x) = y_0 + (x-x_0)f[x_0,x_1] + (x-x_0)(x-x_1)f[x_0,x_1,x_2] \ldots}$$

$$Y(x) = 150 + (x-5)\cdot 121 + (x-5)(x-7)\cdot 24 + (x-5)(x-7)(x-11)\cdot 1$$

ROOT FINDING:

BISECTIONAL



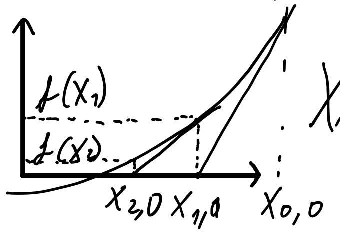
$a_2 = a_1 \quad b_2 = c_1$

$c_{i+1} = \frac{b_i + a_i}{2}$

$f(c)$ = POSITIVE OR NEGATIVE

$a_{i+1} = c_{i+1}$
OR
$b_{i+1} = c_{i+1}$

NEWTON RAPHSON:



$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

LIMITATION:

$f'(x_n) = 0$

$f'(x_n) = \infty$

COULD ALSO OSSICATE

SET OF EQUATIONS:

$$X^{(i)} = X^{(i-1)} - J^{-1} f_j\left(X_j^{(i)}\right)$$

$$\begin{bmatrix} X_0^{(i)} \\ \vdots \\ X_j^{(i)} \end{bmatrix} = \begin{bmatrix} x_0^{(i-1)} \\ \vdots \\ x_j^{(i-1)} \end{bmatrix} - \begin{bmatrix} J^{-1} \end{bmatrix} \begin{bmatrix} f_0(X_0^{(i)} \dots X_j^{(i)}) \\ f_j(X_0^{(i)} \dots X_j^{(i)}) \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_1} \end{bmatrix}$$

FIXED POINT ITT METHOD:

$$f(x) = x - e^{-x}$$

SOLVE FOR $x = g(x)$

$$x = e^{-x}$$

CHOSE POINT UNTILL ERROR IS <u>OK</u>

$$x_i = e^{-x(i-1)}$$

SECANT METHOD:

$$X_{n+1} = X_n - f(X_n)\left(\frac{X_i - X_{i-1}}{f(X_i) - f(X_{i-1})}\right)$$

$\boxed{\text{TWO INITIAL CONDITION}}$

NUMERICAL DIFFERENCIATION

DIVIDED DIFF:

THRE POINTS:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

ERROR: $\boxed{\frac{h^2}{3!} f'''(x)\ldots}$

FIVE POINTS DIFF:

$$f'(x) = \frac{1}{12h}\left(f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)\right) \quad \boxed{+h^4}$$

ERROR

FINITE MUST BE MORE POINTS?

NUMERICAL INTEGRATION:

EQUAL SPACED:

→ NEWTON-COTES METHOD
- → TRAPEZOIDAL
- → SIMPSONS $\frac{1}{3}$
- → SIMPSONS $\frac{3}{8}$ RULE

UNEQUAL SPACED:

→ GAUS-LEGENDRE METHOD

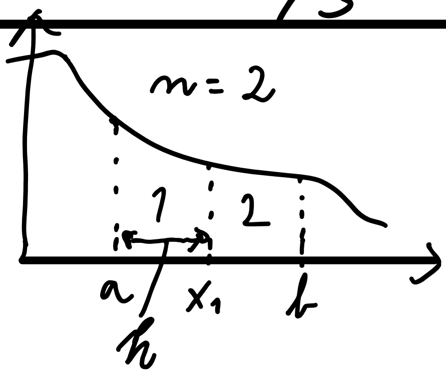
NOT MIDTERM

GENERAL EQUATION:

$$\int_a^b f(x)\,dx = h\int_0^1 \left(\overset{T_1}{y_n} + \overset{T_2}{P\Delta y_0} + \overset{T_3}{\frac{P(P-1)}{2!}\Delta^2 y_0} \cdots\right) dp$$

$$P = \frac{X - X_0}{h} \qquad h\,dp = dx$$

SIMPSONS $\frac{1}{3}$ RULE



$$I = \frac{h}{3}\left[f(a) + 4f(x_1) + f(b)\right]$$

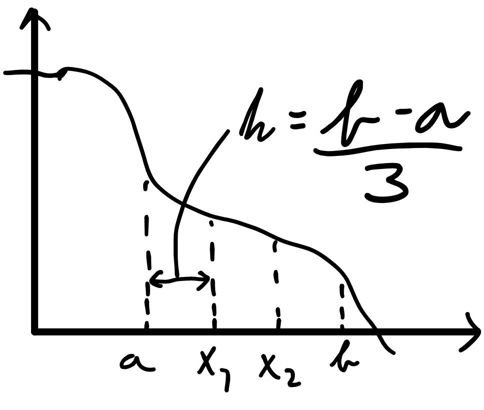
$$\text{ERROR} = \frac{-h^5}{90} f''''(x)$$

TRAPEZOIDAL RULE:

$$I = \frac{h}{2}\left(y_0 + 2(y_1 + \cdots + y_{n-1}) + y_n\right)$$

ERROR: $\int_a^b T_2\,dx = -\frac{h^3}{12} f''(x)$

SIMPSONS $\frac{3}{8}$ RULE



$$I = \frac{3h}{8}\left[f(a) + 3f(x_1) + 3f(x_2) + f(b)\right]$$

$$\text{ERROR: } -\frac{3h^5}{80} f''''(x)$$

SOLVING ODE

TAILOR S. METHOD

$y(x) = y(x_0) + (x - x_0) y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) \ldots$

GIVEN $y' = y^2 + 3x$ $\quad y(0) = \boxed{1}$

DERIVE $y'' = 2y \cdot y' + 3$

$y(x) = \boxed{1} + (x - 0)(1^2 + 3) + \ldots$ ... MORE ACCURACY WITH MORE TERMS

$\frac{(x-x_0)^3}{3!} y'''(x)$ = ERROR FOR EACH TIMESTEP. SMALL TIMESTEP REQUIRED

MAJOR PROBLEM:

DERIVATIVES NEEDS TO BE FOUND ANALYTICALLY OR DERIVED NUMMER. BUT WILL CREATE EVEN MORE ERROR

EULER'S METHOD

ONLY TWO FIRST TERMS OF TAILOR

$y = y(x_0) + (x - x_0) y'(x_0)$

ERROR $= \frac{y''(x_1) h^2}{2}$

$y_{i+1}(x) = y_i(x) + (x_{i+1} - x_i) y'(x_i)$

HEUN'S METHOD

$k_2 = f(x_i + h, y_i + k_1 h) \quad k_1 = f(x_i, y_i)$

$y_{i+1} = y_i + (k_1 + k_2) \frac{h}{2} \quad h = x_1 - x_0 \quad$ ERROR $= h^3$

RUNGE-KUTTA 4TH ORDER

$K1 = hf(x_n, y_n)$

$K2 = hf(x_n + \frac{h}{2}, y_n + \frac{K1}{2})$

$K3 = hf(x_n + \frac{h}{2}, y_n + \frac{K2}{2})$

$K3 = hf(x_n + h, y_n + K3)$

$y_{i+1} = y_i + \frac{K1}{6} + \frac{K2}{3} + \frac{K3}{3} + \frac{K4}{6}$

ERROR (h^5)