FRICTION IN THE TRADING PROCESS AND THE ESTIMATION OF SYSTEMATIC RISK

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ABSTRACT

This paper considers how estimates of the market model beta parameter can be biased by friction in the trading process (information, decision, and transaction costs) that (a) leads to a distinction between observed and "true" returns, (b) causes observed returns to be generated asynchronously for a set of interdependent securities, and (c) thereby introduces serial cross-correlation into security returns. Several propositions are derived from which: consistent estimators of beta are obtained, and the impacts of differencing interval length and of a security's market value on beta estimates are specified. The formulation is contrasted with the related analyses of Scholes-Williams (1977), Dimson (1979), and Theobald (1980).

FRICTION IN THE TRADING PROCESS AND THE ESTIMATION OF SYSTEMATIC RISK

1. INTRODUCTION

Several empirical studies have reported that market model beta estimates vary systematically with the length of the measurement (or "differencing") interval over which security returns are calculated.[1] This intervalling-effect bias has been thought to arise solely from the "Fisher (1966) effect," i.e., the fact that transaction price adjustments lag quotation price adjustments. Scholes-Williams (1977) show that when some securities have their last transactions earlier in the measurement interval than others, an errors-in-variables-type bias occurs in observed beta.[2] Cohen-Maier-Schwartz-Whitcomb [CMSW] (1979) also model transaction prices lagging quotation prices resulting from a Poisson order generating process, and show that it causes observed market index returns to be autocorrelated even if the underlying "true" returns generating process yields serially independent returns.

If transaction prices lagging quotation prices were the only cause of the intervalling-effect bias in beta and of index autocorrelation, these effects could be eliminated by substituting quotation prices observed at a particular time during the measurement interval for "closing" (i.e., last) transaction prices. However, in an active market like the NYSE or AMEX, it is doubtful that the sporadic occurrence of transactions could by itself account for the observed magnitudes of the associated phenomena of index autocorrelation, and the intervalling effect on beta and market model R^2. The present paper shows that the intervalling-effect bias in beta can be caused by friction in the trading process even when "true" (but unobserved) returns are generated by a frictionless process, and presents alternative methods of obtaining a consistent estimator of true beta.

In CHMSW (1980), we show how friction in the trading process causes price-adjustment delays. Three sources of price-adjustment delay are cited: (1) quotation prices can be updated without an accompanying transaction, causing transaction prices to lag quotation prices; (2) specialists/dealers impede quotation price adjustments in the act of satisfying exchange stabilization obligations or redressing inventory imbalances; (3) individual traders update limit orders sporadically due to information, decision, and transaction costs. Item (1) is the source of nonsynchronous trading considered by Fisher (1966), Scholes-Williams (1977), and CMSW (1979). Considerably lengthier price-adjustment delays may be caused by the inventory rebalancing of specialists/dealers and by individuals not revising orders until expected benefits exceed information, decision, and transaction costs.

When the prices for a set of interdependent securities adjust with varying delays, their observed returns are nonsynchronous, and thus serially cross-correlated. In CHMSW (1980), we present a "frictions equation" [equation (2) in the present paper] that comprehends this delay structure. This equation provides the basis for our present analysis of the intervalling-effect bias in beta.

In this paper we show that the friction-induced delays in price adjustments will bias observed beta. We also show how this bias changes as the length of a security's measurement interval is varied. We demonstrate that the beta bias monotonically approaches zero as the interval length is increased. In CHMSW (1980), we suggest that price-adjustment delays will on expectation be shorter for securities with larger market values, and we show here that the beta bias is related to price-adjustment delays, with the result that beta will be underestimated for thinner securities and overestimated for larger issues. These results provide an analytical framework for empirical tests of our model and suggest implementable procedures for eliminating the intervalling-effect bias in beta. Our empirical analysis is contained in CHMSW (1982).

Two alternative approaches to correcting the bias in beta have appeared in the literature. Schwert (1977), Franks-Broyles-Hecht (1977), and Marsh (1979) have directly matched measurement times with trading times to avoid the distortion introduced by transaction prices lagging quotation prices. The difficulty with this approach is that the exact time of each trade must be known and an index must be available that is updated nearly continuously. Moreover, this correction does not compensate for the other (lengthier) price-adjustment delays which cause an intervalling effect. The second approach, of Pogue-Solnik (1974), Ibbotson (1975), Scholes-Williams (1977), Schwert (1977), Dimson (1979), and Theobald (1980) involves regressing individual security returns on synchronous and nonsynchronous market index returns. A difficulty with this approach is that a loss of efficiency due to measurement error restricts the number of lead/lag periods that can be used [see Dimson (1979)], while the restricted number may be insufficient to capture the full price-adjustment-delay structure.

Our analysis suggests two additional approaches to adjusting for the intervalling-effect bias that have not previously been introduced in the literature: (1) an asymptotic beta estimate and (2) an inferred asymptotic estimator based upon a security's market value and its observed beta. These approaches yield consistent estimators of beta even in the presence of long-lived price-adjustment delays. The inferred asymptotic estimator has the additional feature that it requires only a limited number of short-period return observations, making it desirable for use where beta is nonstationary.

In Section 2, we use the "frictions equation" of CHMSW (1980) to prove various propositions regarding the effect of differencing interval length and a security's expected price-adjustment-delay magnitude on its observed beta. From these propositions, consistent estimators of beta are obtained. Section 3 discusses the relationship of our work to that of Scholes-Williams (1977), Dimson (1979), and Theobald (1980). Section 4 contains our concluding remarks, including a brief summary of the empirical tests reported in CHMSW (1982).

2. THE MODEL

2.1. Relationship Between Observed Returns and True Returns

Defining returns to be the log of price relatives adjusted for dividends and splits, we assume:

A1. the true return for security j in period t is generated by the market model[3]

$$r_{j,t} = \alpha_j + \beta_j r_{M,t} + e_{j,t} \tag{1}$$

We then use the following equation [see CHMSW (1980)] to distinguish between true returns and observed returns ($r^o_{j,t}$):

$$r^o_{j,t} = \sum_{n=0}^{N} (\gamma_{j,t-n,n} r_{j,t-n} + \theta_{j,t-n,n}) \tag{2}$$

where we further assume:

A2. $\gamma_{j,t,m}$ and $\gamma_{k,\tau,n}$ are independent for all $j \neq k$, t, τ, m, and n;

A3. $\theta_{j,t,m}$ and $\theta_{k,\tau,n}$ are independent for all $j \neq k$, t, τ, m, and n;

A4. $\gamma_{j,t,n}$ are independent of $r_{M,\tau}$ and $e_{k,\tau}$ for all j, k, t, τ, and n;

A5. $\theta_{j,t,n}$ are independent of $r_{M,\tau}$ and $e_{k,\tau}$ for all j, k, t, τ, and n;

A6. $\gamma_{j,t,m}$ and $\Theta_{k,\tau,n}$ are independent for all $j \neq k$, t, τ, m, and n

A7. $E(\gamma_{j,t,m}) = E(\gamma_{j,\tau,m})$ for all j, t, τ, and m;

A8. $E(\sum_{\ell=0}^{N} \gamma_{j,t,\ell}) = 1$ for all j and t.

The random variables $\gamma_{j,t-n,n}$ relate the observed return generated in one period to the true returns generated in that period and in N prior periods. Alternatively stated, the $\gamma_{j,t,0}$, $\gamma_{j,t,1}$, ..., $\gamma_{j,t,N}$ comprise a delay distribution that shows how the true return generated in period t impacts on the returns actually observed during period t and the next N periods (we assume that the true return generated at period t-N will have no remaining impact on returns observed after period t). Moreover, assumption A8 above indicates that on average each real return $r_{j,t}$ will be fully reflected in the future observed returns.

The random variables $\Theta_{j,t-n,n}$ reflect the "bouncing" of transaction prices between the bid and the ask quotations. We arbitrarily let the bid define the base level price, view each transaction at the ask as a momentary jump in the process, and specify the process as:

$$\Theta_{j,t,0} = \begin{cases} \text{market bid-ask spread (in the returns dimension) at the time of the last transaction} & \text{if there was a transaction in period } t \text{ and the last transaction prior to the end of measurement period } t \text{ executed at the ask} \\ 0 & \text{otherwise} \end{cases}$$

$$\Theta_{j,t,n} = -\Theta_{j,t,0} \qquad \text{in the first period } (t+n) \text{ following } t \text{ in which a transaction occurred}$$

$$\Theta_{j,t,n} = 0 \qquad \text{for all other } n$$

The existence of the bid-ask spread can be explained by transaction costs causing trading to be noncontinuous [see CMSW (1981)]. In a compatible manner, $\gamma_{j,t,n} \neq 0$ for $n > 0$ results from friction in the trading process that causes price-adjustment delays. The structure of the $\gamma_{j,t,n}$ is of particular importance here because if the $\gamma_{j,t,n}$ are nonzero for $n > 0$ and differ across securities, the observed returns for a set of securities will adjust asynchronously to common movements in their true returns. We shall show that with such asynchronous adjustments across securities, cross-serial correlation is introduced into observed returns, and observed beta estimates are biased. Further, with greater (smaller) price-adjustment delays for thinner (thicker) securities, for any given differencing interval the bias in beta will be shown to vary systematically across securities, with its signed value being positively related to the size of the issue.

Our model of the observed returns generation process can be utilized to establish the relationship between the systematic risk calculated from observed returns and the true systematic risk. This in turn allows us to examine the properties of the estimated beta coefficient and to derive a consistent estimator of a security's systematic risk. We proceed as follows. In Proposition 1, we establish the relationship between the covariances of observed returns and the covariances of true returns for pairs of securities. From Proposition 1, we derive the relationship between the covariance of the observed returns of a security and the observed returns of a market index, stated in Corollary 1, and the relationship between the variance of observed returns of a market index and the variance of its true returns, stated in Corollary 2. Using Corollaries 1 and 2, we establish the relationship between a security's beta coefficient calculated from observed returns and its true beta coefficient. This result is stated in Proposition 2. We then derive two alternative consistent estimators of true beta, one in Corollary 3 and the other in Proposition 3. Propositions 4, 5, and 6 provide yet another method by which a consistent estimator of true beta can be calculated.

2.2. Relationship between Observed Beta and True Beta

Proposition 1

The contemporaneous covariance between the observed returns of securities j and k plus the sum of the serial cross-covariances of their observed returns for all leads and lags up to N periods equals the contemporaneous covariance between their true returns. For all $j \neq k$ we have:

$$\mathrm{COV}(r^o_{j,t}, r^o_{k,t}) + \sum_{n=1}^{N} \mathrm{COV}(r^o_{j,t}, r^o_{k,t-n}) + \sum_{n=1}^{N} \mathrm{COV}(r^o_{j,t-n}, r^o_{k,t})$$

$$= \mathrm{COV}(r_{j,t}, r_{k,t}) \qquad (3)$$

The proof of Proposition 1 is given in the Appendix. Note that the proof does not require that the price-adjustment delay variables $\gamma_{j,t,\ell}$ and $\gamma_{j,t,n}$ be independent. As we shall discuss in Section 3, recognition of this is the key to obtaining the appropriate form for Dimson's model, and to relaxing the stationarity assumption of Scholes and Williams (which requires that a transaction be observed in each measurement period used in the estimation process).

We now examine the covariance between the returns of securities and those of a market index (Corollary 1) and the variance of the returns of a market index (Corollary 2). Define an observed market index $r^o_{M,t}$ and the true market index $r_{M,t}$ used in equation (1) by:

$$r^o_{M,t} = \sum_k x_k \, r^o_{k,t} \qquad \text{and} \qquad r_{M,t} = \sum_k x_k \, r_{k,t}$$

where the x_k are appropriate weights for each security.

Corollary 1

The contemporaneous covariance between the observed returns of any security j and a market index M, plus the sum of the serial cross-covariances of their observed returns for all leads and lags up to N periods, equals the contemporaneous covariance between the true returns of j and M. That is:

$$COV(r^o_{j,t}, r^o_{M,t}) + \sum_{n=1}^{N} COV(r^o_{j,t}, r^o_{M,t-n}) + \sum_{n=1}^{N} COV(r^o_{j,t-n}, r^o_{M,t})$$
$$= COV(r_{j,t}, r_{M,t})$$

The proof of Corollary 1 follows directly from Proposition 1. Summing equation (3) over all securities with the appropriate weights yields Corollary 1.[4]

Corollary 2

The variance of the observed returns of a market index M, plus twice the sum of the serial covariances of the observed returns of M for all lags up to N periods, equals the variance of the true market index returns. That is:

$$VAR(r^o_{M,t}) + 2 \sum_{n=1}^{N} COV(r^o_{M,t}, r^o_{M,t-n}) = VAR(r_{M,t})$$

where we define:

$$VAR(r^o_{M,t}) = \sum_{j}\sum_{k} x_j x_k COV(r^o_{j,t}, r^o_{k,t})$$

$$COV(r^o_{M,t}, r^o_{M,t-n}) = \sum_{j}\sum_{k} x_j x_k COV(r^o_{j,t}, r^o_{k,t-n})$$

$$VAR(r_{M,t}) = \sum_{j}\sum_{k} x_j x_k COV(r_{j,t}, r_{k,t})$$

The proof of Corollary 2 follows directly from Corollary 1. Summing equation (3) over the index j yields the desired result.[5]

We can now derive the relationship between a security's systematic risk calculated from observed returns (the observed beta β_j^o) and the security's true systematic risk (β_j). Define the following variables:

$$\beta_j^o = \frac{COV(r_{j,t}^o, r_{M,t}^o)}{VAR(r_{M,t}^o)}$$

$$\beta_{M_{+n}}^o = \frac{COV(r_{M,t+n}^o, r_{M,t}^o)}{VAR(r_{M,t}^o)}$$

$$\beta_{j_{+n}}^o = \frac{COV(r_{j,t+n}^o, r_{M,t}^o)}{VAR(r_{M,t}^o)}$$

$$\beta_{j_{-n}}^o = \frac{COV(r_{j,t-n}^o, r_{M,t}^o)}{VAR(r_{M,t}^o)}$$

$$\beta_j = \frac{COV(r_{j,t}, r_{M,t})}{VAR(r_{M,t})}$$

where we call β_j^o the observed security beta, $\beta_{M_{+n}}^o$ the observed intertemporal market beta, $\beta_{j_{+n}}^o$ the observed lead security beta, $\beta_{j_{-n}}^o$ the observed lag security beta, and β_j the true security beta. The relationship between these variables is expressed in Proposition 2.

<u>Proposition 2</u>

The observed beta of security j is a function of the true beta of j, the observed intertemporal market beta, and the observed lead and lag betas of security j up to N periods; in particular:

$$\beta_j^o = \beta_j(1+2\sum_{n=1}^{N}\beta_{M_{+n}}^o) - \sum_{n=1}^{N}(\beta_{j_{+n}}^o + \beta_{j_{-n}}^o) \qquad (4)$$

To prove Proposition 2, note that by definition the right-hand side of equation (4) is equal to:

$$\frac{COV(r_{j,t},r_{M,t})}{VAR(r_{M,t})}\left[1+2\sum_{n=1}^{N}\frac{COV(r^o_{M,t+n},r^o_{M,t})}{VAR(r^o_{M,t})}\right]-\sum_{n=1}^{N}\frac{COV(r^o_{j,t+n},r^o_{M,t})+COV(r^o_{j,t-n},r^o_{M,t})}{VAR(r^o_{M,t})}$$

Now by recognizing that our stationarity assumptions A1-A8 imply that $COV(r^o_{j,t},r^o_{k,t-n}) = COV(r^o_{j,t+n},r^o_{k,t})$ and substituting Corollaries 1 and 2, we obtain:

$$\frac{COV(r_{j,t},r_{M,t})}{VAR(r_{M,t})}\,\frac{VAR(r_{M,t})}{VAR(r^o_{M,t})}-\frac{COV(r_{j,t},r_{M,t})-COV(r^o_{j,t},r^o_{M,t})}{VAR(r^o_{M,t})}$$

$$=\frac{COV(r^o_{j,t},r^o_{M,t})}{VAR(r^o_{M,t})}=\beta^o_j \qquad \text{Q.E.D.}$$

Using Proposition 2, an estimator for true beta can easily be determined as shown in the following corollary:

<u>Corollary 3</u>

A consistent estimator of true beta can be found by the formula:

$$\hat{\beta}_j=\frac{b^o_j+\sum_{n=1}^{N}b^o_{j_{+n}}+\sum_{n=1}^{N}b^o_{j_{-n}}}{1+2\sum_{n=1}^{N}b^o_{M_{+n}}} \tag{5}$$

where b^o_j, $b^o_{j_{+n}}$, $b^o_{j_{-n}}$ and $b^o_{M_{+n}}$ are OLS regression estimators of

β^o_j, $\beta^o_{j_{+n}}$, $\beta^o_{j_{-n}}$ and $\beta^o_{M_{+n}}$, respectively.

The proof of Corollary 3 follows immediately from equation (4). It is clear from Corollary 3 that b^o_j, the OLS regression estimate of systematic risk, will be a biased estimator of true β_j.

2.3. <u>Intervalling Effect and Asymptotic Estimator</u>

We now address the issue of what happens when we increase the differencing interval over which returns are measured. Let $\hat{\beta}_j(L)$ be the estimator defined by equation (5) where the OLS estimators $b_j^o(L)$, $b_{j+n}^o(L)$, $b_{j-n}^o(L)$, and $b_{M_{+n}}^o(L)$ are computed using nonoverlapping periods consisting of L consecutive smaller measurement periods. Since $\hat{\beta}_j(L)$ is a consistent estimator of β_j, we can also obtain the following result:

Proposition 3

The asymptotic estimator defined by:

$$\beta_j^* = \lim_{L\to\infty} b_j^o(L)$$

is a consistent estimator of the true beta β_j where $b_j^o(L)$ is the observed OLS regression beta based on nonoverlapping periods of length L.

The proof of Proposition 3 is given in the Appendix.

Corollary 3 and Proposition 3 provide alternative methods by which consistent estimators of true beta (β_j) can be calculated. The Corollary 3 estimator is similar to the methods used by Scholes-Williams (1977) and Dimson (1979) and is discussed in Section 3. To implement the asymptotic estimator of Proposition 3 (since taking L to the limit is not practical) one can regress observations of $b_j^o(L)$ for different L on an appropriate function of L, obtaining β_j^* as the asymptote. We turn next to the use of a single $b_j^o(L)$ plus a security's market value in estimating β_j.

Proposition 4

The observed systematic risk, β_j^o satisfies:

$$\sum_j x_j \ \beta_j^o = 1$$

To prove Proposition 4, take the sum of equation (4) over all securities weighted by their participation in the market index. This yields:

$$\sum_j x_j \ \beta_j^o = (1 + 2 \sum_{n=1}^{N} \beta_{M_{+n}}^o) \ \sum_j x_j \ \beta_j - \sum_j x_j \sum_{n=1}^{N} (\beta_{j_{+n}}^o + \beta_{j_{-n}}^o)$$

$$\text{but} \quad \sum_j x_j \, \beta^o_{j_{+n}} = \sum_j x_j \frac{COV(r^o_{j,t+n}, r^o_{M,t})}{VAR(r^o_{M,t})} = \beta^o_{M_{+n}}$$

$$\text{and} \quad \sum_j x_j \, \beta^o_{j_{-n}} = \sum_j x_j \frac{COV(r^o_{j,t-n}, r^o_{M,t})}{VAR(r^o_{M,t})} = \beta^o_{M_{-n}}$$

Also recognize that our assumptions on the generation process given by equation (2) implies that

$$\beta^o_{M_{-n}} = \beta^o_{M_{+n}}$$

Therefore

$$\sum_j x_j \, \beta^o_j = (1+2 \sum_{n=1}^{N} \beta^o_{M_{+n}}) \sum_j x_j \beta_j - 2 \sum_{n=1}^{N} \beta^o_{M_{+n}}$$

The Proposition then follows from the property $\sum_j x_j \beta_j = 1$ [see Fama (1976, pp. 60-61) for a proof of this property]. Q.E.D.

Proposition 4 states that whatever bias is introduced into the measurement of β_j will be positive for some securities and negative for others, with the weighted average over all securities equal to zero. Proposition 5 stated below will allow us to determine which securities will have positive biases and which negative.

Proposition 5

Whether observed beta overestimates or underestimates true beta depends upon the variance-covariance structure and the expected price-adjustment delay structure; specifically, for any security j whose true beta is positive, $\beta^o_j \gtreqless \beta_j$ as

$$\frac{1}{\beta_j} \sum_k x_k COV(r_{j,t}, r_{k,t}) \left\{ \sum_{n=1}^{N} E(\gamma_{k,t,n}) + \sum_{\ell=1}^{N} E(\gamma_{j,t,\ell}) [E(\gamma_{k,t,0}) - E(\gamma_{k,t,\ell})] \right\}$$

$$\gtreqless 2 \, VAR(r^o_{M,t}) \sum_{n=1}^{N} \beta^o_{M_{+n}}$$

The proof of Proposition 5 can be found in the Appendix.

Proposition 5 suggests a second approach for estimating β_j without the use of equation (5). Note that the size of the bias in β_j^o is related to the $E(\gamma_{j,t,\ell})$ where the random variables $\gamma_{j,t,\ell}$ are a direct measure of the price adjustment delay. A comparative measure of price adjustment delay can be defined as follows:

Definition

A security j has <u>strictly greater expected price-adjustment delay</u> than security i if:

$$\sum_{\ell=p}^{N} E(\gamma_{j,t,\ell}) > \sum_{\ell=p}^{N} E(\gamma_{i,t,\ell})$$

for all delay periods $p = 1,\ldots,N$.[6]

Alternatively stated: security j has strictly greater expected price-adjustment delay than security i if for all $p < N$ a smaller proportion of the true return generated for j in period t is expected to be reflected in j's observed returns for periods t to t+p than is the case for i.

This definition can now be used to prove Proposition 6.

Proposition 6

If the price-adjustment delay variables are such that $E(\gamma_{k,t,0}) \geq E(\gamma_{k,t,1}) \geq \cdots \geq E(\gamma_{k,t,N})$ for all securities k with at least one inequality holding strictly for each security, and if security j has strictly greater expected price-adjustment delay than security i and is otherwise identical ($x_j = x_i$, $\beta_j = \beta_i$, and securities i and j have the same covariances with all other securities), then:

$$\beta_j^o \lesseqgtr \beta_i^o \quad \text{as} \quad \beta_j \gtreqless 0$$

The proof of Proposition 6 is given in the Appendix. Put differently, Proposition 6 states that under the given reasonable conditions for the price-adjustment delay variables, the observed beta will underestimate a positive true beta to a greater extent, cet. par., the greater the expected price-adjustment delay of the security.

It should be noted that over all securities in the market, not all observed betas will be underestimates. Proposition 4 guarantees that on average (where average is taken to mean the weighted average as represented in the market index) the observed betas will be correct. That is, some observed betas are likely to be above their true beta values, while others are likely to be below. The importance of Proposition 6 is that it identifies securities with positive betas and relatively lengthy price-adjustment delays as being those securities for which beta will be underestimated by the usual OLS estimators. Conversely, securities with positive betas and relatively short price-adjustment delays are likely to have their betas overestimated by the usual OLS technique. The opposite is true for those few securities having negative betas.

Proposition 6 is the key to our second new approach to estimating β_j. The approach is to estimate the difference between β_j^* and $\beta_j(L)$ as a function of a measurable surrogate for the price-adjustment delay characteristics of the security. Possible proxies for price-adjustment delay are the security's value of shares traded or its value of shares outstanding. A bias correction factor can then be computed which, together with an OLS regression determined $b_j^o(L)$, can give an unbiased estimate of β_j^* and, therefore, of β_j. An empirical implementation of this approach is presented in CHMSW (1982).

3. RELATIONSHIP TO THE SCHOLES-WILLIAMS (1977), DIMSON (1979), and THEOBALD (1980) MODELS

The original work on the impact of nonsynchronous trading for the measurement of beta was by Scholes and Williams (1977). They developed a consistent estimator of β_j that is a special case of our own. Their estimator is:

$$\hat{\beta}_j = \frac{b_j^o + b_{j_{+1}}^o + b_{j_{-1}}^o}{1 + 2\, b_{M_{+1}}^o} \tag{6}$$

which is identical to our estimator given in equation (5) when N = 1. The reason that their estimator does not contain the higher order lead and lag betas is related to two of their fundamental assumptions. First, they assumed that only nonsynchronous trading leads to the observed bias in beta. Our frictions equation (2) encompasses many other possible reasons for observed prices lagging the underlying true returns process, and some of these reasons imply substantial lags, as discussed in Section 1. Second, Scholes and Williams assumed that a transaction occurs in each measurement period, so any information lost due to an interval of non-trading will be observed in the immediately following period when the next trade is assumed to occur. Thus, their assumptions lead to a special form of our frictions equation (2):

$$r_{j,t}^o = \gamma_{j,t,0}\, r_{j,t} + \gamma_{j,t-1,1}\, r_{j,t-1}$$

which in turn could only hold if N = 1 in our formulation, which by our equation (5) then yields equation (6) above.

Of special interest in the Scholes and Williams formulation is their assumption that the process generating the delay structure in observed betas was both independent and identically distributed over time. This assumption made it impossible to extend their basic result shown in equation (6) to the case where securities did not trade at least once in each period. Even with securities that trade relatively frequently, a day can arise where no trade occurs; Scholes and Williams had to discard such observations in their empirical work. As pointed out by Dimson (1979), this in turn leads to a loss of information which can increase the size of the confidence interval around the Scholes and Williams beta estimator. Even more important, however, is the observation that the higher order lead and lag betas could be substantial, as we would expect for thinly traded securities; hence ignoring them can lead to an inconsistent beta estimator. Our assumption for frictions equation (2) is only that the delay variables <u>across</u> securities are stationary and independent. There can, however, be complex dependency patterns over time for a given security.

The consistent estimator of beta derived by Dimson (1979) and given in his equation (12), p. 204 is (in our notation):

$$\hat{\beta}_j = b^o_j + \sum_{n=1}^{N} b^o_{j_{+n}} + \sum_{n=1}^{N} b^o_{j_{-n}}$$

Unfortunately, Dimson's estimator is incorrect.[7] Dimson also suggests that the coefficients $b^o_{j_{-n}}$, $b^o_{j_{+n}}$ all be simultaneously estimated using multiple regression as opposed to independently estimated as suggested by Scholes and Williams. Such an approach would not yield estimators consistent with our definitions of β^o_j, $\beta^o_{j_{+n}}$, and $\beta^o_{j_{-n}}$.

Dimson brings up the important issue of the correct selection of the value for the parameter N. The appropriate selection of this parameter represents a direct conflict between model and statistical accuracy. On the one hand, the greater the number of lead and lag terms estimated, the better potentially is the model's representation of reality. On the other hand, the more such terms included in equation (5), the greater the potential noise introduced in the estimation process. In other words, as N increases, the model becomes both more realistic and less biased, but the statistical efficiency of the estimator declines. Dimson also proposes a further adjustment in the estimator b_j^o, $b_{j_{+n}}^o$, and $b_{j_{-n}}^o$ using the method of Vasicek (1973). A further discussion of the work of Scholes-Williams and Dimson can be found in Fung (1981).

In a later paper, Theobald (1980) approaches the problem from a somewhat different analytical framework, but produces an estimator identical to that of Scholes and Williams (1977). One slight difference is that Theobald makes a correction to the Scholes and Williams estimator for the fact that the number of observations would be less for lead and lag betas than for simultaneous betas. For most applictions this would be of no practical significance. The remainder of his paper discusses the implication of biased beta estimation on tests of capital market efficiency, and in particular, estimation of market model residuals.

4. CONCLUSION

In this paper we have developed an analytical model that shows how friction in the trading process can cause estimates of beta to be biased even when "true" returns are generated by an underlying process that is unbiased. In several important respects our analysis might be viewed as an extension of the works of Scholes-Williams (1977) and Dimson (1979), which we have accordingly discussed.

Trading frictions result from information, decision, and transaction costs that result in nonsynchronous trading, delayed portfolio rebalancing, "stale" limit orders remaining on the book, the breaking-up and sequential execution of large orders, and the delayed redressing of inventory imbalances by dealers/specialists. As a consequence of these delays, the ultimate adjustment of security prices to change in the information set lags the advent of news. Hence, observed returns for an interdependent set of securities are expected to be generated asynchronously, and a serial cross-correlation structure is introduced into the data which causes bias in the estimation of beta by OLS regression.

We have modeled the bias in beta by postulating a delay distribution $(\gamma_{j,t,0}, \gamma_{j,t,1}, \ldots, \gamma_{j,t,N})$ that relates true returns generated in period t to observed returns over the periods t,...,t+N. With this formulation, we show that, for a security, the bias in beta will go to zero as the differencing interval is lengthened without bound; in essence, the returns for a set of securities are less asynchronous for longer differencing intervals. We also suggest that, for a given differencing interval, the bias in beta will be arithmetically smaller for issues of larger total market value; the reason is that the beta bias for a security is itself related to the relative magnitude of the security's expected price-adjustment delays, and price-adjustment delays are expected to be less for the larger issues.

As betas have increasingly been estimated from short-period data, evidence of a protracted delay structure in stock price movements has emerged. Viewed only on a stock-by-stock basis, the evidence can be confusing since the intervalling effect does not have the same sign for all securities, and for a large number of securities, the effect appears to be insignificant. The pattern clarifies, however, when stocks are arrayed according to the value of their shares outstanding.

In CHMSW (1982) we report the results of an empirical study that measures the intervalling effect and implements the two new approaches to adjusting for the intervalling-effect bias developed in the present paper. Our test design consists of three regression passes. In the first pass, OLS betas are estimated for 14 different values of the differencing interval, L, for each of 50 NYSE common stocks selected by value-stratified random sampling. The second pass regressions then relate beta to L for each security, yielding estimates of the asymptotic value of beta that is approached as L is increased without bound. The third pass is a cross-sectional regression of the slope coefficient from the second pass regressions on the logarithm of the value of shares outstanding. The results of the second and third passes conform to expectations: thin stocks' first pass betas rise as L is increased; the strength of the intervalling effect diminishes as value of shares outstanding rises, and the effect's sign reverses for stocks with very high market values; and market value explains two-thirds of the intervalling effect.

The estimation of asymptotic betas in the second pass regressions results in a nontrivial adjustment for the intervalling-effect bias. The third pass regressions provide another method of adjusting for the intervalling-effect bias that is well-suited for nonstationary betas where only a short calendar span of recent daily data is considered representative. The third pass regression coefficients, unadjusted (first pass) one-day betas, and value of shares outstanding are used to infer the asymptotic betas. The mean square deviation of the inferred asymptotic betas from the estimated asymptotic betas is about one-quarter that of the unadjusted one-day betas. Thus, it would appear that the empirical results conform to our theoretical predictions and that the proposed adjustment procedures are meaningful.

Parallel tests of our beta estimators and the Scholes-Williams estimator on Paris Bourse data by Fung-Schwartz-Whitcomb (1981) show that the Scholes-Williams estimator still contains a significant intervalling effect, while our inferred asymptotic beta estimator does not. The intervalling-effect bias in the Scholes-Williams estimator is shown to be explained just as well by the size proxy (log of value of shares outstanding) as is the intervalling-effect bias in unadjusted OLSE betas.

Recent empirical work by Reinganum (1981) and Banz (1981) shows that firms with small market value have significantly larger risk-adjusted ("abnormal") returns than large market value firms, and that the excess returns persist for lengthy periods. Reinganum, using daily data to estimate OLS betas, observes that excess returns are positive for portfolios of small firms, decrease almost monotonically with firm size, and are negative for portfolios of the larger firms.[8] Banz, using monthly returns to estimate OLS betas, finds positive abnormal returns for the smallest twenty per cent of the firms in his sample, but no significant abnormal returns for the rest. Both Banz and Reinganum conclude that their findings demonstrate that the equilibrium asset pricing model is misspecified in that it omits a factor related to the market value of firms (or to whatever more fundamental economic variable market value might proxy).

Our analysis suggests an alternative explanation that is consistent with both the efficient markets hypothesis and the single-factor CAPM. We have shown that OLS regression systematically underestimates (overestimates) true beta for small (large) firms. Thus, with properly adjusted beta, abnormal returns would evidence a weaker (or no) relationship with firm size.[9] When attention is focused on the bias in OLSE beta, we see that delays in the price-adjustment process provide a fundamental explanation for the empirical findings, but such delays do not necessarily lead to abnormal returns.

There is no reason to believe that the existence of price-adjustment delays implies a violation of the efficient markets hypothesis. However, a focus on such delays in the trading process calls attention to a different sense of the term "efficiency"; in an operational sense, efficiency is related to the amount of friction in a system. It is intriguing that empirical examination of the bias in unadjusted beta provides an indirect test of the degree of operational efficiency of a trading system. Stock price movements can tell an incredibly rich story.

FOOTNOTES

1. The studies include Pogue-Solnik (1974), Altman-Jacquillat-Levasseur (1974), Scholes-Williams (1977), Hawawini (1977, 1980), Levhari-Levy (1977), Smith (1978), Dimson (1979), Cohen-Hawawini-Maier-Schwartz-Whitcomb [CHMSW] (1982), and Fung-Schwartz-Whitcomb (1981); several of these papers contain theoretical analyses as well. It is important (theoretically, at least) to distinguish between (arithmetic) returns measured as periodic rates and (log) returns measured as continuously compounded rates. Levhari-Levy (1977) have explored an intervalling effect which arises purely as an artifact of measuring returns as periodic rates, rather than arising from trading frictions. The Levhari-Levy model assumes cross-serial independence of returns, whereas in our model cross-serial dependence of observed returns is of the essence. Under the Levhari-Levy assumptions, the intervalling effect could be eliminated by expressing returns as continuously compounded rates (since then long-interval returns would be the sum of short-interval returns), whereas with cross-serial dependence this is not so, nor is it so empirically; see Hawawini-Vora (1981).
2. As a convenient shorthand, we often use "observed beta" or "unadjusted beta" to refer to the estimate of beta obtained by OLS regression of observed security returns on a single independent variable, contemporaneous observed market index returns.

3. We make the usual assumptions about the process generating $r_{j,t}$, that is: (i) $r_{M,t}$ and $r_{M,\tau}$ are independent and identically distributed for all $t \neq \tau$; (ii) $e_{j,t}$ and $e_{j,\tau}$ are independent and identically distributed for all $t \neq \tau$ and $E(e_{j,t})=0$ for all j and t; (iii) $e_{j,t}$ and $e_{k,\tau}$ are independent for all $j \neq k$ and all t and τ; and (iv) $r_{M,t}$ and $e_{j,\tau}$ are independent for all j, t, and τ.

4. This argument is somewhat inaccurate. The summation over all values of k in equation (3) would include the term for which k = j. However, we have not shown that equation (3) holds in this case. In fact, this is not true. The variables γ_k and θ_k are usually not independent. Moreover, $\gamma_{j,t,n}$ and $\gamma_{j,\tau,m}$ are also not usually independent. For example, the existence of a sample period in which no trade takes place can cause complex interrelationships amongst the γ_j and θ_j variables. Without these independence assumptions, equation (3) cannot be shown to hold for the case k = j. However, this is not of any real significance, since for market indices with large numbers of securities the proportion x_j is small for any one security. Moreover, the most actively traded securities, those with the largest x_j fractions in a value-weighted market index, are also the ones for which the frictions discussed in this paper are likely to have the least effect. Thus, we believe that one can safely ignore the distortion of summing over a value of k = j, and do so to facilitate the proofs of subsequent propositions.

5. Again the argument is somewhat inaccurate in that the summation includes variance terms for the securities as well as all possible covariance terms. However, as explained in footnote 4 above, we believe this distortion can be safely ignored.

6. Note that this definition necessarily implies that $E(\gamma_{j,t,0}) < E(\gamma_{i,t,0})$ in order that A8 be satisfied.

7. The error in Dimson's analysis appears to be in the derivation of equation (11) on page 203 of Dimson (1979). Dimson attempts to equate the appropriate terms in his equation (9) with those in his equation (10) to derive equation (11). The basic problem is that all terms that are dependent upon M_{t+k} are not equivalenced. This is because $\hat{\beta}_{i+k}$ is itself dependent upon M_{t+k}, therefore, equation (11) omitted at least one term. In addition, the equivalence used to derive equation (11) requires only that all terms associated with M_t in equation (9) must equal all terms associated with M_t in equation (10), not that each subterm be equal. Thus the corrected form of Dimson's estimator is our equation (5).

8. Note that Reinganum calculates excess returns rather than abnormal returns, but because his OLSE betas are close to unity for most of his portfolios, excess and abnormal returns behave similarly. However, this need not be the case if OLSE betas are systematically biased with respect to firm size.

9. The contrast between the Banz and Reinganum findings with respect to the range of firm size over which abnormal returns are significant is also consistent with our analysis. That is, with price-adjustment delays having an impact that decays as the differencing interval is lengthened, the abnormal returns - firm size relationship should decay as the differencing interval used for OLS beta estimates is increased. We would predict that when yet-longer intervals are used, the relationship would disappear for even the smallest firms.

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APPENDIX

Proof of Proposition 1

By the definition of $r^o_{j,t}$ from equation (2) we have:

$$\mathrm{COV}(r^o_{j,t}, r^o_{k,t}) = \mathrm{COV}\left[\sum_{\ell=o}^{N} (\gamma_{j,t-\ell,\ell}\, r_{j,t-\ell} + \Theta_{j,t-\ell,\ell}),\ \sum_{m=o}^{N} (\gamma_{k,t-m,m}\, r_{k,t-m} + \Theta_{k,t-m,m})\right]$$

By the assumption of the independence of Θ_j with γ_k, r_k and Θ_k for all $j \neq k$ we have:

$$= \mathrm{COV}\left(\sum_{\ell=o}^{N} \gamma_{j,t-\ell,\ell}\, r_{j,t-\ell},\ \sum_{m=o}^{N} \gamma_{k,t-m,m}\, r_{k,t-m}\right)$$

Now applying the identify $\mathrm{COV}(X,Y) = E[\mathrm{COV}(X,Y|\Phi)] + \mathrm{COV}[E(X|\Phi), E(Y|\Phi)]$

$$= \sum_{\ell=o}^{N} \sum_{m=o}^{N} E(\gamma_{j,t-\ell,\ell}\, \gamma_{k,t-m,m}) \mathrm{COV}(r_{j,t-\ell}, r_{k,t-m})$$

$$+ \sum_{\ell=o}^{N} \sum_{m=o}^{N} E(r_{j,t-\ell}) E(r_{k,t-m}) \mathrm{COV}(\gamma_{j,t-\ell,\ell}, \gamma_{k,t-m,m})$$

The assumption that r_j and r_k are stationary with $\mathrm{COV}(r_{j,t}, r_{k,\tau})=0$ for all t and τ nonoverlapping yields

$$= \mathrm{COV}(r_{j,t}, r_{k,t}) \sum_{\ell=o}^{N} E(\gamma_{j,t-\ell,\ell}\, \gamma_{k,t-\ell,\ell})$$

$$+ E(r_{j,t}) E(r_{k,t}) \sum_{\ell=o}^{N} \sum_{m=o}^{N} \mathrm{COV}(\gamma_{j,t-\ell,\ell}, \gamma_{k,t-m,m})$$

A similar analysis for the second and third terms on the left-hand side of equation (3) yields:

$$\mathrm{COV}(r^o_{j,t}, r^o_{k,t}) + \sum_{n=1}^{N} \mathrm{COV}(r^o_{j,t}, r^o_{k,t-n}) + \sum_{n=1}^{N} \mathrm{COV}(r^o_{j,t-n}, r^o_{k,t})$$

$$= \mathrm{COV}(r_{j,t}, r_{k,t}) \Big[\sum_{\ell=o}^{N} E(\gamma_{j,t-\ell,\ell}\, \gamma_{k,t-\ell,\ell})$$

$$+ \sum_{n=1}^{N} \sum_{\ell=o}^{N-n} E(\gamma_{j,t-\ell,\ell}\, \gamma_{k,t-\ell,\ell+n})$$

$$+ \sum_{n=1}^{N} \sum_{\ell=o}^{N-n} E(\gamma_{j,t-\ell,\ell+n}\, \gamma_{k,t-\ell,\ell}) \Big]$$

$$+ E(r_{j,t})E(r_{k,t}) \; [\sum_{\ell=0}^{N} \sum_{m=0}^{N} COV(\gamma_{j,t-\ell,\ell}\ \gamma_{k,t-m,m})$$

$$+ \sum_{n=1}^{N} \sum_{\ell=0}^{N} \sum_{m=0}^{N} COV(\gamma_{j,t-\ell,\ell}\gamma_{k,t-m,m+n})$$

$$+ \sum_{n=1}^{N} \sum_{\ell=0}^{N} \sum_{m=0}^{N} COV(\gamma_{j,t-\ell,\ell+p}\ \gamma_{k,t-m,m})]$$

Now since $\gamma_{j,t,\ell}$ and $\gamma_{k,\tau,m}$ are assumed independent for all $j \neq k, t, \tau, \ell$ and m and $E\ (\gamma_{j,t,m}) = E\ (\gamma_{j,\tau,m})$ for all j,t,τ and m, the second half of the above expression is zero and the first half simplifies to:

$$= COV(r_{j,t}, r_{k,t}) \; [\sum_{\ell=o}^{N} E(\gamma_{j,t,\ell})\ E(\gamma_{k,t,\ell})$$

$$+ \sum_{n=1}^{N} \sum_{\ell=o}^{N-n} E(\gamma_{j,t,\ell})\ E(\gamma_{k,t,\ell+n})$$

$$+ \sum_{n=1}^{N} \sum_{\ell=o}^{N-n} E(\gamma_{j,t,\ell+n})\ E(\gamma_{k,t,\ell})]$$

$$= COV(r_{j,t}, r_{k,t})E(\sum_{\ell=o}^{N} \gamma_{j,t,\ell})\ E(\sum_{m=o}^{N} \gamma_{k,t,m})$$

By the assumption that $E(\sum_{\ell=o}^{N} \gamma_{j,t,\ell}) = 1$ for all j and t, we obtain:

$$= COV(r_{j,t}, r_{k,t}) \qquad\qquad \text{Q.E.D.}$$

Proof of Proposition 3

By definition we have:

$$\beta^{o}_{j_{-n}} = \frac{COV(r^{o}_{j,t-n}, r^{o}_{M,t})}{VAR(r^{o}_{M,t})}$$

$$= \frac{\sum_{k} x_k\ COV(r^{o}_{j,t-n}, r^{o}_{k,t})}{\sum_{j}\sum_{k} x_j x_k\ COV(r^{o}_{j,t}, r^{o}_{k,t})}$$

From the proof of Proposition 1 we have:

$$\mathrm{COV}(r^o_{j,t-n}, r^o_{k,t}) = \mathrm{COV}(r_{j,t}, r_{k,t}) \sum_{\ell=o}^{N-n} E(\gamma_{j,t,\ell+n})\, E(\gamma_{k,t,\ell})$$

$$\mathrm{COV}(r^o_{j,t}, r^o_{k,t}) = \mathrm{COV}(r_{j,t}, r_{k,t}) \sum_{\ell=o}^{N} E(\gamma_{j,t,\ell})\, E(\gamma_{k,t,\ell})$$

Now the effect of lengthening the measurement interval is to create $\gamma(L)$ as follows:

$$\gamma_{j,t,0}\,(L) = \sum_{\ell=o}^{L-1} \gamma_{j,t,\ell} \qquad \text{and} \qquad \gamma_{j,t,n}\,(L) = \sum_{\ell=Ln}^{L(n+1)-1} \gamma_{j,t,\ell}$$

For $L > N$, we have $\gamma_{j,t,n}(L) = 0$ for all $n \geq 1$.

Notice that since $\gamma_{j,t,n}(L) = 0$ for sufficiently large L, $\beta^o_{j_{-n}}(L)$ would also be zero for $n \geq 1$ and such a value of L. (The expression for $\beta^o_{j_{-n}}(L)$ is the same as $\beta^o_{j_{-n}}$ except that $\gamma_{j,t,n}(L)$ is substituted for $\gamma_{j,t,n}$.)

A similar argument shows that $\beta^o_{j_{+n}}(L)$ and $\beta^o_{M_{+n}}(L)$ would also be zero for $L > N$ and $n \geq 1$. Thus equation (4) reduces to

$$\beta^o_j(L) = \beta_j \text{ for } L > N$$

where since $b^o_j(L)$ is just the OLSE of $\beta^o_j(L)$ the result follows.

It should be noted that an assumption that the price adjustment begun in period t could be observed indefinitely into the future could be substituted for our assumption of a finite N as long as the price adjustment for lengthy periods was small. For example, if for any $\varepsilon > 0$, no matter small, there exists an k such that

$$\left| \sum_{\ell=k}^{\infty} \gamma_{j,t,\ell} \right| < \varepsilon$$

then a similar proof could be constructed. Q.E.D.

Proof of Proposition 5

$$\text{If } \frac{1}{2\beta_j} \sum_{n=1}^{N} (\beta^o_{j_{+n}} + \beta^o_{j_{-n}}) \gtreqless \sum_{n=1}^{N} \beta^o_{M_{+n}} \text{ then } \beta^o_j \lesseqgtr \beta_j. \qquad \text{(A-1)}$$

Expanding $\sum_{n=1}^{N} (\beta^o_{j_{+n}} + \beta^o_{j_{-n}})$

$$= \frac{1}{VAR(r^o_{M,t})} \sum_{n=1}^{N} [COV(r^o_{j,t+n}, r^o_{M,t}) + COV(r^o_{j,t-n}, r^o_{M,t})]$$

$$= \frac{1}{VAR(r^o_{M,t})} \sum_k x_k \sum_{n=1}^{N} [COV(r^o_{j,t+n}, r^o_{k,t}) + COV(r^o_{j,t-n}, r^o_{k,t})]$$

Then using the definition of $r^o_{j,t}$ given by equation (2) and following the same reasoning used in the proof of Proposition 1 we obtain:

$$= \frac{1}{VAR(r^o_{M,t})} \sum_k x_k \, COV(r_{j,t}, r_{k,t}) \sum_{n=1}^{N} \sum_{\ell=0}^{N-n} [E(\gamma_{j,t,\ell+n}) E(\gamma_{k,t,\ell}) + E(\gamma_{j,t,\ell}) E(\gamma_{k,t,\ell+}$$

Now using the assumption $\sum_{n=o}^{N} E(\gamma_{j,t,n}) = 1$ to eliminate $E(\gamma_{j,t,0})$ from the above relationship and rearranging terms we obtain:

$$= \frac{1}{VAR(r^o_{M,t})} \sum_k x_k \, COV(r_{j,t}, r_{k,t}) \left\{ \sum_{n=1}^{N} E(\gamma_{k,t,n}) + \sum_{\ell=1}^{N} E(\gamma_{j,t,\ell}) \, [E(\gamma_{k,t,0}) - E(\gamma_{k,t} \right.$$

which when combined with equation (A-1) yields the desired result. Q.E.D.

Proof of Proposition 6

Rewriting equation (4) we obtain:

$$\beta^o_j - \beta_j = 2\beta_j \sum_{n=1}^{N} \beta^o_{M_{+n}} - \sum_{n=1}^{N} (\beta^o_{j_{+n}} + \beta^o_{j_{-n}})$$

Expanding the last summation as was done in the proof of Proposition 5 we obtain:

$$= 2\beta_j \sum_{n=1}^{N} \beta^o_{M+n} -$$

$$\frac{1}{VAR(r^o_{M,t})} \sum_k x_k \, COV(r_{j,t}, r_{k,t}) \left\{ \sum_{n=1}^{N} E(\gamma_{k,t,n}) + \sum_{\ell=1}^{N} E(\gamma_{j,t,\ell}) [E(\gamma_{k,t,0}) - E(\gamma_{k,t,\ell})] \right.$$

Using the expression, we can now write an expression for the difference between $\beta^o_j - \beta_j$ and $\beta^o_i - \beta_i$:

$$\beta^o_j - \beta^o_i = - \frac{1}{VAR(r^o_{M,t})} \sum_k x_k \, COV(r_{j,t}, r_{k,t}) \tag{A-2}$$

$$\left\{ \sum_{\ell=1}^{N} [E(\gamma_{j,t,\ell}) - E(\gamma_{i,t,\ell})] \; [E(\gamma_{k,t,0}) - E(\gamma_{k,t,\ell})] \right\}$$

where all identical terms have been cancelled. (Note: By assumption $\beta_i - \beta_j$ and $COV(r_{j,t}, r_{k,t}) = COV(r_{i,t}, r_{k,t})$ for all k.)

Since $\sum_k x_k \, COV(r_{j,t}, r_{k,t}) = COV(r_{j,t}, r_{M,t}) = \beta_j \, VAR(r_{M,t})$, the first half of equation (A-2) is obviously negative, positive, or zero for β_j positive, negative, or zero, respectively. If we can show that the term $\{\}$ is always positive, we will have established our result.

Rewriting $\{\}$ from equation (A-2) we have:

$$= E(\gamma_{k,t,0}) \sum_{\ell=1}^{N} [E(\gamma_{j,t,\ell}) - E(\gamma_{i,t,\ell})]$$

$$- \sum_{\ell=1}^{N} E(\gamma_{k,t,\ell}) \; [E(\gamma_{j,t,\ell}) - E(\gamma_{i,t,\ell})]$$

Substituting the identities $\sum_{\ell=1}^{N} E(\gamma_{j,t,\ell}) = 1-E(\gamma_{j,t,0})$ for securities i and j in the above expression yields:

$$= \sum_{\ell=0}^{N} E(\gamma_{k,t,\ell}) \ [E(\gamma_{i,t,\ell}) - E(\gamma_{j,t,\ell})]$$

which by algebraic manipulation is equal to:

$$= E(\gamma_{k,t,0}) \sum_{\ell=0}^{N} [E(\gamma_{i,t,\ell}) - E(\gamma_{j,t,\ell})] \qquad \text{(A-3)}$$

$$+ \sum_{\ell=1}^{N} [E(\gamma_{k,t,\ell-1}) - E(\gamma_{k,t,\ell})] \sum_{n=\ell}^{N} [E(\gamma_{j,t,n}) - E(\gamma_{i,t,n})]$$

The first term of this expression is zero since $\sum_{\ell=0}^{N} E(\gamma_{i,t,\ell}) = \sum_{\ell=0}^{N} E(\gamma_{j,t,\ell}) = 1$.

By the assumption of the proposition, the left half of the second term is nonnegative for all securities and strictly positive for at least one value of ℓ for each security k. The right half of the second term is positive for all values of ℓ, since security j is assumed to have strictly greater expected price adjustment delay than security i. Thus, equation (A-3) is positive for all values of k.

Q.E.D.

LIST OF INSEAD RESEARCH WORKING PAPERS

80/01 "Identifying cognitive style determinants of retail patronage, by Christian PINSON, Arun K. JAIN and Naresh K. MALHOTRA, January 1980.

80/02 "Dimensions culturelles des conceptions de management - une analyse comparative internationale", par André LAURENT Février 1980.

80/03 "Cognitive style and effective communication", by Arun K. JAIN, Naresh K. MALHOTRA and Christian PINSON, Dec. 1979.

80/04 "Accomodative cognitive style differences in consumer reduction of alternatives", by Naresh K. MALHOTRA, Christian PINSON and Arun K. JAIN, October 1979.

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