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Unsmoothing Returns of Illiquid Funds

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Abstract

Funds that invest in illiquid assets report returns with spurious autocorrelation. Consequently, investors need to unsmooth these funds' returns when evaluating their risk exposures. We show that funds with similar investments have a common source of spurious autocorrelation not fully resolved by traditional unsmoothing methods, leading to underestimation of systematic risk. As such, we propose a generalized unsmoothing technique and apply it to hedge funds and private commercial real estate funds. Our method significantly improves the measurement of funds' risk exposures and risk-adjusted performance, especially for highly illiquid funds. Overall, the average illiquid fund alpha is lower than previously thought.

JEL Classification: G11, G12, G23.

Keywords: Illiquidity, Return Unsmoothing, Performance Evaluation, Hedge Funds, Commercial Real Estate.

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Introduction

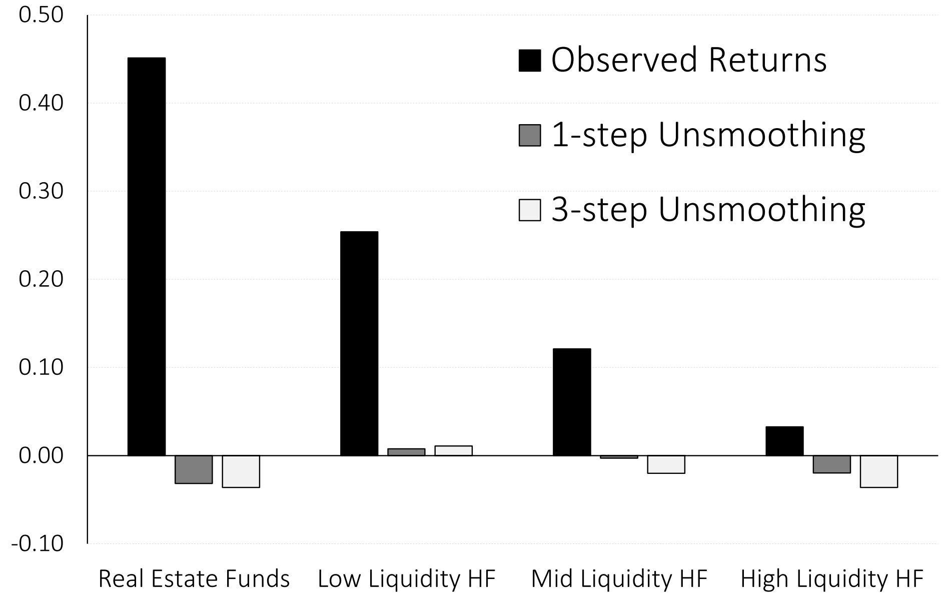
The market size of intermediaries investing in illiquid assets has grown dramatically over the last two decades.[1] However, there is a lot we do not know about their risks and performance due to the difficulty in measuring these quantities with standard techniques. Specifically, the reported returns of a fund reflect valuation changes with a partial lag when the fund's assets trade infrequently or are sporadically valued. This smoothing effect creates spurious return autocorrelation and invalidates traditional risk and performance measures. The crux of the problem is we only observe reported (or smoothed) returns, while we need economic (or unsmoothed) returns to estimate risk and performance metrics such as betas and alphas.

In some influential papers, Geltner (1991, 1993) and Getmansky, Lo, and Makarov (2004) provide different ways to recover economic return estimates by unsmoothing observed returns. In this paper, we argue that while these previous techniques represent an important first step in measuring the risks of illiquid funds, they do not fully unsmooth the common component of returns, and thus understate the importance of risk factors in explaining illiquid fund returns. We then provide a novel return unsmoothing technique to address this issue and apply our methodology to hedge funds and private commercial real estate (CRE) funds, demonstrating its usefulness in measuring the risk exposures and risk-adjusted performance of illiquid funds.[2] Our unsmoothing method builds on Geltner (1991, 1993) and Getmansky, Lo, and Makarov (2004), and thus can be seen as an extension of their methodologies. Our main finding is that systematic risk and risk-adjusted performance are better measured when returns are unsmoothed using our new method, with the average alpha of illiquid funds being lower than previously thought.

[1]For instance, a report by the PWC Asset and Wealth Management Research Center shows a growth of roughly 400% (from \$2.5 trillion to \$10.1 trillion) in assets under management for alternative investments (which are typically illiquid) from 2004 to 2016 (see pwc (2018)). The same report indicates that the hedge fund industry (the focus of a substantial part of our empirical analysis) has grown from \$1.0 trillion to \$3.3 trillion over the same period.

[2]We focus on hedge funds and private CRE funds because smoothed returns is a common problem with these asset classes. For instance, Getmansky, Lo, and Makarov (2004) and Geltner (1991, 1993) introduced their influential return unsmoothing methods in the context of hedge funds and real estate indices, respectively.

(a) **Average Autocorrelation of Fund-level Returns**



(b) **Autocorrelation of Average Fund Returns**

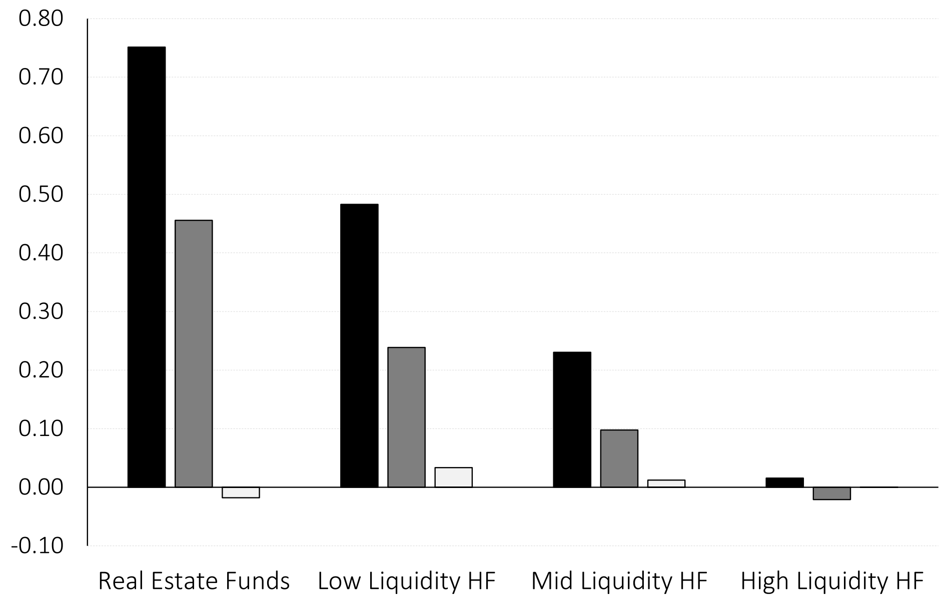


Figure 1

Real Estate and Hedge Fund Return Autocorrelation: Fund-level vs Aggregate

Panel (a) plots the average 1st order autocorrelation coefficient for returns of private commercial real estate (CRE) funds (quarterly returns from 1994 to 2017) and Hedge Funds (monthly returns from 1995 to 2017), with the latter sorted on strategy liquidity (see Subsection 2.2 for the details on the strategy liquidity sort). Panel (b) plots the analogous measure, but for average returns (i.e., first taking the equal-weighted average of fund-level returns and then calculating the autocorrelations). We consider three definitions of returns: observed returns, 1-step unsmoothed returns (Geltner (1991, 1993) for private CRE funds and Getmansky, Lo, and Makarov (2004) for Hedge funds), and 3-step unsmoothed returns. See Subsections 2.1 and 3.2 for further empirical details.

The basic idea behind return unsmoothing techniques is simple. They assume observed returns are weighted averages of current and past economic returns, and then estimate these weights to recover economic return estimates, which are otherwise unobservable.

Traditional return unsmoothing techniques (Geltner (1991, 1993) for private CRE funds and Getmansky, Lo, and Makarov (2004) for Hedge funds), which we refer to as 1-step unsmoothing, seem to perform well when we only analyze unsmoothing at the fund-level. Figure 1(a) shows that fund-level autocorrelations are high in private CRE and hedge funds, but effectively disappear after 1-step unsmoothing. These results suggest 1-step unsmoothed returns better reflect price movements in the funds' underlying assets than reported returns.

Despite this apparent success, we find that averaging 1-step unsmoothed returns produces aggregate returns that display significant autocorrelation, as demonstrated in Figure 1(b). This result indicates that the common component of fund-level returns is not fully unsmoothed based on 1-step unsmoothing, potentially biasing fund risk exposure estimates.

To deal with this issue, we propose a generalization of traditional return unsmoothing methods that directly accounts for aggregate smoothing effects. We assume the observed returns of each fund are weighted averages of current and past economic returns on both the fund and the aggregate of similar funds (i.e., an equal-weighted portfolio of funds in the same asset class). We then show how to use a 3-step procedure to estimate the weights and obtain more accurate economic return estimates. As Figure 1 shows, our 3-step unsmoothing method is better able to unsmooth the common component of fund returns than 1-step unsmoothing. In particular, both fund and aggregate autocorrelations effectively disappear after 3-step unsmoothing.

The 3-step unsmoothing method we propose reduces to 1-step unsmoothing if observed fund returns do not directly reflect lagged aggregate returns of similar funds. However, our empirical evidence suggests this condition is not met because, otherwise, 1-step and 3-step unsmoothing would lead to similar unsmoothed returns. To explore this issue further, we perform a Monte Carlo simulation exercise. The simulation results indicate the autocorrelation patterns in observed returns are highly consistent with the underlying smoothing process of our 3-step unsmoothing method. In contrast, the observed return patterns are inconsistent with the underlying smoothing process of the 1-step unsmoothing method.

To understand why the smoothing process of our 3-step unsmoothing method is more consistent with the autocorrelation in observed returns, we provide a simple economic model. In our model, funds do not observe the current value of the assets they hold. Instead, funds receive different signals about the shocks common to all assets in their asset class and the shocks affecting the relative value of their assets. If funds report the Bayesian estimate of their asset value each period, then observed returns are based on current and past economic returns, with the smoothing process reflecting past fund and aggregate returns with different intensities, in line with the underlying smoothing process of our 3-step unsmoothing method. The intuition is that the signal-to-noise ratios of the two different signals drive the smoothing parameters and they differ across signals that are either fund-specific or common to an entire asset class.

The aforementioned results point to a misspecification in unsmoothed returns obtained using 1-step unsmoothing methods. Motivated by this finding, we explore the implications of our 3-step unsmoothing method to the measurement of risk exposures and risk-adjusted performance of hedge funds and private CRE funds.

In the case of hedge funds, we perform two main exercises. In the first exercise, we sort funds into three groups based on the liquidity of their underlying strategy, and apply 1-step and 3-step unsmoothing to funds in each of these groups. We then measure, for each group, average Sharpe ratio as well as risk and risk-adjusted performance based on a standard factor model used in the hedge fund literature (the FH 8-Factor model that builds on Fung and Hsieh (2001)). We find that Sharpe ratios substantially decrease after unsmoothing returns, but the decrease is roughly the same whether we use 1-step or 3-step unsmoothing. In contrast, 3-step unsmoothing produces economic returns that comove more strongly with FH risk factors (relative to 1-step unsmoothing) and display lower alphas as a consequence. Moreover, we also show in an out-of-sample exercise that past alphas estimated using 3-step unsmoothed returns provide a better signal for future alphas than past alphas obtained from observed returns or 1-step unsmoothed returns. All the aforementioned results hold only in the low and mid liquidity groups. The performance of funds in the high liquidity group is largely unaffected by unsmoothing. This last finding indicates unsmoothing techniques do not produce unintended distortions in the estimates of economic returns for liquid funds.

In the second exercise, we separately study funds in each major hedge fund strategy category and find results consistent with the previous paragraph. However, grouping funds based on their underlying strategies allows us to study funds exposed to similar risks, and thus to explore how our 3-step unsmoothing technique improves the measurement of systematic risk exposures. We find that our 3-step unsmoothing method tends to change risk exposure estimates in ways consistent with economic intuition despite no risk factor information being used during the unsmoothing process. For example, after unsmoothing returns using our 3-step method, the exposure of emerging market funds to the emerging market risk factor strongly increases, while other risk exposures of emerging market funds display little change.

Turning to private CRE funds, the overall results are similar to those we obtain with hedge funds. However, the degree to which the 3-step unsmoothing process improves upon 1-step unsmoothing is much higher given the extreme illiquidity of real estate assets. For instance, the average beta of private CRE funds to the public CRE market increases from 0.07 to 0.34, driving the 4.3% annual alpha of private CRE funds (measured with observed returns) to 1.6% after 3-step unsmoothing.

In summary, we develop a 3-step process which improves upon traditional return unsmoothing techniques for illiquid assets in order to better estimate their systematic risk exposures. We then apply our new return unsmoothing method to hedge funds, finding that the measurement of risk exposures and risk-adjusted performance substantially improves relative to what is obtained from returns unsmoothed using traditional methods. Finally, we perform a similar analysis based on private CRE funds and find that results are even more pronounced for these funds given the high degree of illiquidity in their underlying assets.

Our paper provides a general contribution to the literature on illiquid funds as it develops a simple way to recover economic return estimates from observed returns to measure their risk exposures and risk-adjusted performance. Several papers in this body of literature attempt to measure the illiquidity premium (e.g., Aragon (2007), Khandani and Lo (2011), and Barth and Monin (2020)). Our contribution is particularly important in this area because unsmoothing returns is an essential part of measuring the illiquidity premium. Without properly unsmoothing the common component of returns, any attempt to measure this premium would not correctly control for exposures to other sources of risk and, as a consequence, could attribute the premium associated with other risk factors to illiquidity.

Our 3-step method builds on and improves upon Getmansky, Lo, and Makarov (2004), thereby contributing to the hedge fund literature as it is standard practice to apply 1-step unsmoothing before studying hedge fund returns.[3,4] Our empirical analysis further adds to these previous papers by demonstrating that hedge fund alphas are lower than previously recognized once systematic risk is properly measured using our 3-step unsmoothing method.

Some papers in the hedge fund literature explore what drives illiquidity by studying the determinants of the unsmoothing parameters in Getmansky, Lo, and Makarov (2004) (e.g., Cao et al. (2017) and Cassar and Gerakos (2011)). We further add to this subset of the literature by demonstrating that the Getmansky, Lo, and Makarov (2004) unsmoothing parameters do not distinguish between fund-specific and systematic components of observed autocorrelation, which can directly affect the inferences associated with illiquidity drivers.

We also contribute to the real estate literature by showing that our unsmoothing technique can be used to improve upon the autoregressive unsmoothing method introduced in Geltner (1991, 1993), which is the basis for many papers unsmoothing returns in the real estate

[3]See, for example, Kosowski, Naik, and Teo (2007), Fung et al. (2008), Patton (2009), Agarwal, Daniel, and Naik (2009), Teo (2009), Jagannathan, Malakhov, and Novikov (2010), Kang et al. (2010), Aragon and Nanda (2011), Avramov et al. (2011), Teo (2011), Titman and Tiu (2011), Billio et al. (2012), Berzins, Liu, and Trzcinka (2013), Bollen (2013), Patton and Ramadorai (2013), Li, Xu, and Zhang (2016), Agarwal, Ruenzi, and Weigert (2017), Agarwal, Green, and Ren (2018), Gao, Gao, and Song (2018), and Bollen, Joenväärä, and Kauppila (2021).

[4]While we interpret our results through the lens of illiquidity, misreporting can also induce smoothed returns (e.g., Bollen and Pool (2008, 2009), Aragon and Nanda (2017)). We do not attempt to disentangle the two sources of return smoothness because, when considering the perspective of an investor or econometrician attempting to estimate economic returns, the degree of return smoothness is the relevant variable rather than the mechanism through which smoothness arises. Moreover, Cassar and Gerakos (2011) and Cao et al. (2017) find that asset illiquidity is the major driver of spurious autocorrelation in hedge fund returns.

literature.[5]

Our addition to the set of potential unsmoothing techniques also reaches beyond the hedge fund and real estate literatures. For instance, unsmoothing methods have been applied to other types of illiquid funds such as private equity, venture capital, and bond mutual funds (Chen, Ferson, and Peters (2010) and Ang et al. (2018)), to highly illiquid assets such as collectible stamps and art investments (Campbell (2008) and Dimson and Spaenjers (2011)), and even to unsmooth other economic series such as aggregate consumption (Kroencke (2017)).

Some papers in the literature rely on the Dimson (1979) method instead of return unsmoothing techniques when estimating alphas of illiquid funds (e.g., Chen (2011), Bali, Brown, and Caglayan (2012), and Cao et al. (2013)). Alas, the Dimson (1979) method does not provide a way to recover economic returns and it dramatically increases the number of parameters to be estimated since it requires adding lags for each risk factor in the factor regressions needed to estimate alphas. This is an important limitation as investors often face the problem of measuring multiple risk exposures based on a relatively short time-series. For instance, with a 2-month autocorrelation (common in hedge funds), the Dimson (1979) method requires 25 regression parameters to estimate the 8-factor model of Fung and Hsieh (2001). As a consequence of this low efficiency of the Dimson (1979) method, we find that in our setting the 3-step unsmoothing technique performs better than the Dimson (1979) method in reflecting fund-level alphas both in simulations and in an out-of-sample empirical analysis.

The rest of this paper is organized as follows. Section 1 introduces the traditional, 1-step, return unsmoothing framework and develops our 3-step unsmoothing process to improve upon it; Section 2 applies 1-step and 3-step unsmoothing to hedge fund returns and demonstrates that the latter improves upon the former in measuring risk exposures and risk-adjusted performance; Section 3 extends the analysis to private CRE funds; and Section 4 concludes. The Internet Appendix provides supplementary results.

[5]See, for example, Fisher, Geltner, and Webb (1994), Barkham and Geltner (1995), Corgel et al. (1999), Fisher and Geltner (2000), Fisher et al. (2003), Pagliari Jr, Scherer, and Monopoli (2005), and Rehring (2012).

1 A New Return Unsmoothing Method

Academics and practitioners primarily rely on two methods to estimate economic (or unsmoothed) returns, R_t, from observed (or smoothed) returns, R_t^o. Both unsmoothing methods assume R_t^o is a weighted average of current and past R_t, but the two techniques differ in how the weights are specified. The first method, developed by Getmansky, Lo, and Makarov (2004), leaves weights unconstrained, but requires a finite number of smoothing lags. We refer to this framework as MA unsmoothing as it implies a moving average time-series process for R_t^o. The second method, developed by Geltner (1991, 1993), imposes an infinite number of smoothing lags, but constrains weights to decay exponentially. We refer to this framework as AR unsmoothing as it implies an autoregressive time-series process for R_t^o. The former method is heavily used in the hedge fund literature while the latter is most commonly applied in the real estate literature.

We refer to both the MA and AR methods generally as "1-step unsmoothing" and develop a 3-step generalization for both methods, which better unsmoothes the common component of returns. As this section makes clear, our 3-step unsmoothing method requires aggregate indexes of funds to be used in the unsmoothing of each individual fund. In the main text, we take these aggregate indexes as given by grouping funds based on the strategy classification provided by data vendors. This approach is consistent with the underlying economic framework for 3-step unsmoothing (see Subsection 1.5) and makes the 3-step method easy to understand and implement. However, Internet Appendix B provides an extended 3-step unsmoothing method that does not require any prior knowledge of the aggregate indexes (with its overall empirical results being similar to the ones we report in the main text). The idea is to estimate unsmoothing parameters simultaneously with the aggregate index for each fund, which we show is a fund-specific linear combination of common latent factors.

Subsection 1.1 details the 1-step MA unsmoothing method; Subsection 1.2 demonstrates that it does not fully unsmooth the systematic component of returns; Subsection 1.3 develops our 3-step MA unsmoothing technique to address this issue; Subsection 1.4 explains why the

1-step method is unable to unsmooth the systematic component of returns, and Subsection 1.5 provides an economic framework that justifies our 3-step unsmoothing method. The description of the AR unsmoothing framework is provided in Section 3, where we also apply AR unsmoothing to study the risk and performance of private CRE funds.

1.1 The 1-step MA Unsmoothing Method

Table 1 provides the basic characteristics of the hedge funds we study (the data sources and sample construction are detailed in the data appendix). There are 10 different hedge fund strategies (sorted by the average fund-level 1st order return autocorrelation coefficient) with a total of 5,069 funds and an average of 92 monthly returns per fund. Hedge funds display (annualized) average excess returns varying from 1.5% to 5.1% and (annualized) Sharpe ratios varying from 0.15 to 0.51, with more illiquid funds displaying higher Sharpe ratios.

It is well known that some hedge fund strategies rely on illiquid assets, and thus the observed returns of many hedge funds are smoothed. To deal with this issue, Getmansky, Lo, and Makarov (2004) (henceforth GLM) propose a method to unsmooth hedge fund returns. GLM assume the observed return of fund j at time t is given by (see GLM for the economic motivation):[6]

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} \ + \ \theta_j^{(1)} \cdot R_{j,t-1} \ + ... + \ \theta_j^{(H)} \cdot R_{j,t-H_j} \tag{1}$$

$$= \mu_j \ + \Sigma_{h=0}^{H} \, \theta_j^{(h)} \cdot \eta_{j,t-h} \tag{2}$$

where θs represent the smoothing weights with $\Sigma_{h=0}^{H} \theta_j^{(h)} = 1$ and the second equality follows from GLM's assumption that $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \sim IID$.

The first equality represents the economic assumption that the observed fund return, $R^o_{j,t}$, is a weighted average of the fund's economic returns, $R_{j,t}$, over the most recent $H+1$ periods, including the current period. The second equality is the econometric implication that observed fund returns follow a Moving Average process of order H, MA(H), so long as

[6]The fact that H does not depend on j simplifies the notation but does not imply that the number of MA lags is not fund dependent. Specifically, letting H_j represent the number of MA lags with a non-zero weight for fund j, we define $H = max(H_j)$ and set $\theta_j^{(h)} = 0$ for any $h > H_j$.

economic returns are not autocorrelated.[7]

Given Equation 2, we can estimate economic returns by estimating an MA(H) process for observed returns, extracting the estimated residuals, $\eta_{j,t}$, and adding the estimated expected return, μ_j, such that $R_{j,t} = \mu_j + \eta_{j,t}$. GLM also provide the basic steps to estimate θs by maximum likelihood under the added parametric assumption that $\eta_{j,t} \overset{iid}{\sim} N(0, \sigma^2_{\eta,j})$.[8] This procedure is used by several papers in the hedge fund literature to unsmooth returns (see the citations in Foonote 3).

We apply this 1-step MA unsmoothing to each hedge fund in our dataset (empirical details are provided in Subsection 2.1). Table 2 contains the (average) autocorrelations (at 1, 2, and 3 monthly lags) for hedge fund returns (observed and unsmoothed) as well as the percentage of funds with a significant autocorrelation at the 10% level. Observed returns display relatively high autocorrelations. For instance, relative value funds have an average 1st order autocorrelation of 0.29, with 61% of these funds displaying statistically significant autocorrelations. After 1-step MA unsmoothing, average autocorrelations are basically zero at all lags and the percentage of funds displaying statistically significant autocorrelations is in line with the statistical error of the test.

These results indicate that 1-step MA unsmoothing produces economic return estimates that are largely unsmoothed at the fund level. This correction is important to properly analyse hedge funds because smoothed returns understate volatilities and betas, and thus overstate Sharpe ratios and alphas, as demonstrated by GLM.

[7] While the assumption that economic returns are not autocorrelated is standard in the return unsmoothing literature, there is evidence of momentum in liquid assets (e.g., Jegadeesh and Titman (1993) and Ehsani and Linnainmaa (2022)). In Internet Appendix C, we show how to control for momentum during the return unsmoothing process and find that, empirically, controlling for momentum yields results that are very similar to the ones we report in the main text. We also generally show that ignoring reasonable levels of autocorrelation in economic returns leads to a relatively small bias in alphas, which is present regardless of whether one estimates alphas based on unsmoothed returns or the Dimson (1979) method.

[8] The method is almost identical to the one used in most econometric packages. The only difference is econometric packages tend to impose the normalization $\theta_j^{(0)} = 1$ as opposed to $\Sigma_{h=0}^{H}\theta_j^{(h)} = 1$. As such, econometric packages yield $\eta^*_{j,t}$ and $\theta_j^{(h)*}$ with $\theta_j^{(h)} \cdot \eta_{j,t-h} = \theta_j^{(h)*} \cdot \eta^*_{j,t}$ and $\theta_j^{(h)*}/\theta_j^{(h)} = 1 + \theta_j^{(1)*} + ... + \theta_j^{(H)*}$. We can then recover $\eta_{j,t}$ and $\theta_j^{(h)}$ by dividing $\theta_j^{(h)*}$ (and multiplying $\eta^*_{j,t}$) by $1 + \theta_j^{(1)*} + ... + \theta_j^{(H)*}$.

1.2 Implications to Aggregate Fund Returns

If the smoothing process postulated by GLM is correct, strategy-level 1-step unsmoothed returns should not be autocorrelated. Specifically, for any set of time-invariant weights, $\{w_j\}_{j=1}^{J}$, the assumption that $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$ implies:

$$\begin{aligned}\overline{R}_t &\equiv \Sigma_{j=1}^{J}\, w_j \cdot R_{j,t} \\ &= \Sigma_{j=1}^{J}\, w_j \cdot \mu_j \;+\; \Sigma_{j=1}^{J}\, w_{j,t} \cdot \eta_{j,t} \\ &= \overline{\mu} \;+\; \overline{\eta}_t \end{aligned} \tag{3}$$

where $\overline{\eta}_t \sim IID$.

Table 3 shows monthly autocorrelations for each (equal-weighted) strategy return, with returns unsmoothed at the fund-level before aggregation. Observed returns on these strategies display quite high autocorrelations (in fact, higher than the average autocorrelations of their respective funds). Moreover, the autocorrelation coefficients remain high after aggregating 1-step unsmoothed returns. For instance, the relative value strategy has a 1st order autocorrelation of 0.51 (statistically significant at 1%) and the autocorrelation is still 0.28 (statistically significant at 1%) after 1-step MA unsmoothing.

The results suggest that 1-step unsmoothing delivers strategy indexes with substantial autocorrelation, which indicates that the approach used by the previous literature to unsmooth returns does not fully unsmooth the systematic component of returns. This result is important because risk exposure estimates are understated (and alpha estimates are overstated) even after return unsmoothing if the systematic return component is not properly unsmoothed.

1.3 The 3-step MA Unsmoothing Method

We develop a 3-step unsmoothing procedure to address the issue raised in the previous subsection. The basic idea is to allow the aggregate and fund-specific components of returns to be smoothed with different weights. This subsection focuses on the econometrics of the

3-step MA unsmoothing method, with Subsection 1.5 providing an economic framework that justifies the procedure.

We refer to the "aggregate" of a generic variable, $y_{j,t}$, as $\overline{y}_t = \Sigma_{j=1}^{J} w_j \cdot y_{j,t}$ and (in the case of returns) its "relative return" as $y_{j,t} - \overline{y}_t$, where w_j are arbitrary (but time-invariant) weights with $\Sigma_{j=1}^{J} w_j = 1$. Moreover, we keep the total number of funds, J, constant over time while developing our aggregation results.

This subsection relies on the fact that, for arbitrary variables x_j and $y_{j,t}$, we have:[9]

$$\begin{aligned} \Sigma_{j=1}^{J} w_j \cdot x_j \cdot y_{j,t} &= \overline{x} \cdot \overline{y}_t \ + \ \Sigma_{j=1}^{J} w_j \cdot (x_j - \overline{x}) \cdot (y_{j,t} - \overline{y}_t) \\ &= \overline{x} \cdot \overline{y}_t \ + \ \widehat{Cov}(x_j, y_{j,t}) \end{aligned} \tag{4}$$

We generalize the underlying assumption in Equation 1 so that the aggregate and relative economic returns can be smoothed with different weights:[10]

$$R^o_{j,t} = \Sigma_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} \ + \ \Sigma_{h=0}^{L} \pi_j^{(h)} \cdot \overline{R}_{t-h} \tag{5}$$

$$= \mu_j \ + \ \Sigma_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^{L} \pi_j^{(h)} \cdot \overline{\eta}_{t-h} \tag{6}$$

where $\overline{R}_t = \Sigma_{j=1}^{J} w_j \cdot R_{j,t}$ are aggregate economic returns, $\widetilde{R}_j = R_{j,t} - \overline{R}_t$ are relative economic returns, and $\overline{\eta}$ and $\widetilde{\eta}$ are the respective shocks.

In GLM, the weights on past economic returns, θs, add to one to assure that information is eventually incorporated into observed prices. Similarly, we impose $\Sigma_{h=0}^{H} \phi_j^{(h)} = \Sigma_{h=0}^{L} \pi_j^{(h)} = 1$ so that both relative and aggregate information eventually gets incorporated into prices.

[9] This result is a generalization of the typical covariance decomposition for the case of non-equal weights:

$$\begin{aligned} \Sigma_{j=1}^{J} w_j \cdot (x_j - \overline{x}) \cdot (y_{j,t} - \overline{y}_t) &= \Sigma_{j=1}^{J} w_j \cdot x_j \cdot y_{t,j} - \overline{y}_t \cdot \Sigma_{j=1}^{J} w_j \cdot x_j - \overline{x} \cdot \Sigma_{j=1}^{J} w_j \cdot y_{t,j} + \overline{x} \cdot \overline{y}_t \cdot \Sigma_{j=1}^{J} w_j \\ &= \Sigma_{j=1}^{J} w_j \cdot x_j \cdot y_{t,j} - \overline{x} \cdot \overline{y}_t \end{aligned}$$

[10] This smoothing process reduces to Equation 1 (the 1-step unsmoothing process in GLM) if we set $\pi_j^{(h)} = \phi_j^{(h)} = \theta_j^{(h)}$. Moreover, as in the 1-step method, the fact that H and L do not depend on j simplifies the notation but does not imply the number of MA lags is not fund dependent. Specifically, letting H_j and L_j represent the number of MA lags with non-zero weight for fund j, we have $H = max(H_j)$ and $L = max(L_j)$ with $\phi_j^{(h)} = 0$ for any $h > H_j$ and $\pi_j^{(h)} = 0$ for any $h > L_j$.

Aggregating Equation 6 yields:

$$\begin{aligned}\overline{R}_t^o &= \overline{\mu} \ + \ \overline{\pi}^{(0)} \cdot \overline{\eta}_t \ + \ \overline{\pi}^{(1)} \cdot \overline{\eta}_{t-1} \ + ... + \ \overline{\pi}^{(L)} \cdot \overline{\eta}_{t-\overline{H}} \\ &\quad + \widehat{Cov}(\phi_j^{(0)}, \widetilde{\eta}_{j,t}) \ + \ \widehat{Cov}(\phi_j^{(1)}, \widetilde{\eta}_{j,t-1}) \ + ... + \ \widehat{Cov}(\phi_j^{(H)}, \widetilde{\eta}_{j,t-H}) \\ &\approx \overline{\mu} \ + \ \Sigma_{h=0}^{L} \, \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h} \end{aligned} \tag{7}$$

where the first equality relies on Equation 4 and the second equality is based on a large sample approximation that uses $\underset{J\to\infty}{Plim} \, \widehat{Cov}(\phi_j^{(h)}, \widetilde{\eta}_{j,t-h}) = Cov(\phi_j^{(h)}, \widetilde{\eta}_{j,t-h}) = 0$. The restriction $\Sigma_{h=0}^{L}\pi_j^{(h)} = 1$ assures the aggregate moving average parameters satisfy $\Sigma_{h=0}^{L}\overline{\pi}^{(h)} = 1$ so that aggregate information is eventually incorporated into aggregate prices.[11]

Subtracting Equation 7 from Equation 6, we have observed relative returns:

$$\begin{aligned}\widetilde{R}_{j,t}^o &= \Sigma_{h=0}^{H} \, \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} \ + \ \Sigma_{h=0}^{L} \, \psi_j^{(h)} \cdot \overline{R}_{t-h} \\ &= \widetilde{\mu}_j \ + \ \Sigma_{h=0}^{H} \, \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^{L} \, \psi_j^{(h)} \cdot \overline{\eta}_{t-h}\end{aligned} \tag{8}$$

where $\psi_j^{(h)} = \pi_j^{(h)} - \overline{\pi}_j^{(h)}$.

Equations 7 and 8 provide a simple way to recover aggregate and fund-level economic returns in an internally consistent way. First, we estimate aggregate economic returns from $\overline{R}_t = \overline{\mu} + \overline{\eta}_t$, where $\overline{\eta}_t$ are residuals of a MA(L) fit to $\overline{R}_t^o$. Second, we obtain fund-level economic relative returns from $\widetilde{R}_{j,t} = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$, where $\widetilde{\eta}_{j,t}$ are residuals from a MA(H) fit (with $\overline{\eta}_t$, $\overline{\eta}_{t-1}$, ..., $\overline{\eta}_{t-L}$ as covariates) to $\widetilde{R}_{j,t}^o$.[12] Third, we recover fund-level economic returns from $R_{t,j} = \overline{R}_t + \widetilde{R}_{j,t} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$. This procedure summarizes our 3-step unsmoothing process.

The columns under "3-step Unsmoothing" in Tables 2 and 3 show return autocorrelations after our 3-step unsmoothing process. From Table 2, unsmoothed fund-level returns display autocorrelations comparable to the ones obtained from 1-step unsmoothing (effectively no

[11]Since $\Sigma_{h=0}^{L}\overline{\pi}^{(h)} = \Sigma_{h=0}^{L}\Sigma_{j=1}^{J} w_j \cdot \pi_j^{(h)} = \Sigma_{j=1}^{J} w_j \cdot (\Sigma_{h=0}^{L}\pi_j^{(h)}) = 1$

[12]As in the 1-step method, if the aggregate and fund-level MA processes are estimated by standard statistical packages (which would normalize $\overline{\pi}^{(0)} = 1$ in step 1 and $\theta_j^{(0)} = 1$ in step 2), then we need to divide the coefficients estimated by the package (and multiple the estimated residuals) by $1 + \overline{\pi}^{(1)} + ... + \overline{\pi}^{(L)}$ in Step 1 and by $1 + \phi_j^{(1)} + ... + \phi_j^{(H)}$ in Step 2. The $\psi_j^{(h)}$ coefficients do not need to be adjusted in step 2.

autocorrelation). Moreover, Table 3 shows that, in contrast to 1-step unsmoothing, our 3-step unsmoothing method drives strategy-level autocorrelations to virtually zero.

One may worry that the autocorrelations reported in Table 3 are close to zero after 3-step unsmoothing simply because we are exploring aggregate returns on the same strategies used in our unsmoothing process. To address this concern, we perform two other exercises.

First, within each strategy, we divide funds into two groups based on the alphabetic order of their names at each inception year. Then, we use the aggregate index from the second (first) group during the 3-step unsmoothing process for the first (second) group. The last three columns of Tables 2 and 3 report results based on the first group of each strategy (with similar untabulated results for the second group). Overall, the results are very similar to what we find for the baseline 3-step unsmoothing method. Consequently, the good unsmoothing performance of the 3-step method is not driven by some mechanical relation between the unsmoothing process and the reported aggregate returns (since the aggregate indexes reported in this analysis do not overlap with the indexes used during the unsmoothing process).[13]

Second, we take the fund-level unsmoothed returns from our baseline 3-step unsmoothing method and aggregate them into β-sorted portfolios instead of aggregating them into hedge fund strategy portfolios (see Table 4). We start by estimating the Fung and Hsieh (2001) factor model betas for each fund using observed returns (see Subsection 2.1 for details on the factor model). We then sort all funds into three groups based on their exposures to each of the eight factors in the model, with Table 4 reporting the first order return autocorrelations of each of the resulting ($3 \times 8 = 24$) portfolios. As is clear from the table, autocorrelations obtained from aggregating observed returns are relatively high for many portfolios and autocorrelations based on 1-step unsmoothed returns are still roughly half of the autocorre-

[13]Note that strategy-level returns from the baseline 3-step unsmoothing method are obtained by aggregating $R_{t,j}$, not by directly using $\overline{\mu}+\overline{\eta}_t$. As such, the baseline autocorrelations obtained from 3-step unsmoothed returns are not mechanical and instead reflect the fact that the 3-step unsmoothing method is able to unsmooth the systematic portion of fund-level returns. Nevertheless, it could still be the case that the Table 3 results under "3-step Unsmoothing" hold only for the same aggregate index used during the unsmoothing process. The results in the last three columns of Tables 3 show that this is not the case.

lations of observed returns. In contrast, autocorrelations are typically close to zero for the portfolios constructed by aggregating 3-step unsmoothed returns. Consequently, the 3-step unsmoothing method goes beyond unsmoothing the aggregate returns of each hedge fund strategy.

The overall evidence suggests that our 3-step MA unsmoothing method properly unsmoothes the systematic component of fund-level returns. This finding has important implications for the measurement of risk exposures and risk-adjusted performance, as we demonstrate in Section 2.

1.4 Understanding Autocorrelation in 1-Step Unsmoothed Returns

The previous subsection shows that the 3-step MA unsmoothing method effectively eliminates the autocorrelation in aggregate returns while the 1-step MA unsmoothing method does not. In this subsection, we explain why this happens. Specifically, we consider the case in which the econometrician assumes Equation 2 is valid (i.e., believes the smoothing process is consistent with the 1-step method), but the true return smoothing process is given by Equation 6 (i.e., it is consistent with the 3-step method).

1.4.1 Analytical Analysis

In this case, the econometrician's 1-step unsmoothed returns are given by:[14]

$$R^{1s}_{j,t} = \mu_j + \eta_{j,t} + \Sigma_{h=0}^{max(H,L)} \lambda_j^{(h)} \cdot \bar{\epsilon}_{j,t-h} \tag{9}$$

[14]To derive Equations 9 and 10, substitute $\overline{\eta}_t = b_j \cdot \eta_{j,t} + \bar{\epsilon}_{j,t}$ into the true smoothing process (Equation 6) to get:

$$R^o_{j,t} = \mu_j + \Sigma_{h=1}^{max(H,L)} \theta_j^{(h)} \cdot \eta_{j,t-h} + u_{j,t}$$

where $\theta_j^{(h)} = \phi_j^{(h)} + (\pi_j^{(h)} - \phi_j^{(h)}) \cdot b_j$ and $u_{j,t} = \theta_j^{(0)} \cdot \eta_{j,t} + \Sigma_{h=0}^{max(H,L)} (\pi_j^{(h)} - \phi_j^{(h)}) \cdot \bar{\epsilon}_{j,t-h}$. Since $\theta_j^{(0)} = 1 - \Sigma_{h=1}^{max(H,L)} \theta_j^{(h)}$ and the econometrician believes $u_{j,t} = \theta_j^{(0)} \cdot \eta_{j,t}$, s/he recovers economic returns as $R^{1s}_{j,t} = \mu_j + u^{1s}_{j,t}/\theta_j^{(0)}$, which yields Equation 9. Then, since $\Sigma_j \, w_j \cdot \bar{\epsilon}_{j,t} \approx (1-\bar{b}) \cdot \overline{\eta}_t$, we have that aggregating Equation 9 yields Equation 10.

and

$$\overline{R}_t^{1s} \approx \overline{\mu} + \overline{\eta}_t + \Sigma_{h=0}^{max(H,L)} \overline{\lambda}^{(h)} \cdot (1-\overline{b}) \cdot \overline{\eta}_{t-h} \tag{10}$$

where $\lambda_j^{(h)} = (\pi_j^{(h)} - \phi_j^{(h)})/(\phi_j^{(0)} + (\pi_j^{(0)} - \phi_j^{(0)}) \cdot b_j)$ and $\overline{\epsilon}_{j,t}$ represents the error process of the projection $\overline{\eta}_t = b_j \cdot \eta_{j,t} + \overline{\epsilon}_{j,t}$.

If $\pi_j^{(h)} = \phi_j^{(h)}$ or $\overline{\epsilon}_{j,t} = 0$, the 1-step method properly recovers the true economic returns.[15] As such, the 3-step method can be seen as a generalization of the 1-step method that allows aggregate returns and returns relative to the aggregate to have different effects on the return smoothing process ($\pi_j^{(h)} \neq \phi_j^{(h)}$). Both methods produce the same economic return estimates (as $T \to \infty$) if the underlying assumption in the 1-step method ($\pi_j^{(h)} = \phi_j^{(h)}$) is valid or if the economic returns are perfectly correlated across funds (in which case $\overline{\epsilon}_{j,t} = 0$ and the aggregate provides no extra information).

Empirically, $\pi_j^{(h)} = \phi_j^{(h)}$ and $\overline{\epsilon}_{j,t} = 0$ are not valid conditions because, otherwise, the 1-step and 3-step unsmoothed returns would generate a similar systematic autocorrelation structure (and identical as T grows). Consequently, Equation 9 shows that $R_{j,t}^{1s}$ reflects true economic returns, $R_{j,t} = \mu_j + \eta_{j,t}$, but also a moving average component related to the portion of aggregate returns "unexplained" by fund returns, $\Sigma_{h=0}^{max(H,L)} \lambda_j^{(h)} \cdot \overline{\epsilon}_{j,t-h}$. Since fund-returns tend to be much more volatile than aggregate returns, the first term tends to dominate the autocorrelation structure so that fund-level 1-step unsmoothed returns have almost no autocorrelation. In contrast, Equation 10 shows that aggregate 1-step unsmoothed returns follow a MA(H) process so that autocorrelation is easy to detect, which explains why the autocorrelations remain high at the aggregate after unsmoothing returns with the 1-step method.[16]

Intuitively, the fund-level autocorrelation in 1-step unsmoothed returns is small because

[15] Note that $b_j = Cor(\overline{\eta}_t, \eta_{j,t}) \cdot \sigma[\overline{\eta}_t]/\sigma[\eta_{j,t}]$. Since $Cor(\overline{\eta}_t, \eta_{j,t}) = 1$ implies $\sigma[\overline{\eta}_t] = \sigma[\eta_{j,t}]$, we have $b_j = \overline{b} = 1$ if $\epsilon_{j,t} = 0$ for all funds. As such, the condition $\epsilon_{j,t} = 0$ leads the 1-step method to properly recover the true aggregate economic returns even though $\epsilon_{j,t} = 0$ is not explicitly present in Equation 10.

[16] Since $b_j = Cor(\overline{\eta}_t, \eta_{j,t}) \cdot \sigma[\overline{\eta}_t]/\sigma[\eta_{j,t}]$ we have that $Cor(\overline{\eta}_t, \eta_{j,t}) < 1$ and $\sigma[\overline{\eta}_t] < \sigma[\eta_{j,t}]$ (conditions that are empirically valid) imply $\overline{b} < 1$. Moreover, we have $\overline{\pi}^{(h)} > \overline{\phi}^{(h)}$ in the data, which predicts a positive autocorrelation for aggregate 1-step unsmoothed returns, exactly what we observe empirically.

the 1-step method misspecification is related to the systematic component of returns, which is small relative to the idiosyncratic component of returns.[17] Yet, this misspecification has important implications for the measurement of fund-level risk exposures (and risk-adjusted performance) because these quantities heavily depend on the systematic component of returns.

1.4.2 Simulations Analysis

To better understand the autocorrelation in 1-step unsmoothed returns from a quantitative perspective, we simulate returns on a panel of 670 funds over a 85 month period. The monthly economic returns of each fund j satisfy:

$$R_{j,t} = \alpha_j + \beta_j \cdot f_t + \varepsilon_{j,t} \tag{11}$$

where $\alpha_j \overset{iid}{\sim} N(\mu_\alpha, \sigma_\alpha^2)$, $\beta_j \overset{iid}{\sim} N(1, \sigma_\beta^2)$, $f_t \overset{iid}{\sim} N(\mu_f, \sigma_f^2)$, and $\varepsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$.[18]

We then smooth these returns according to the smoothing process outlined in our 3-step method (i.e., Equation 5). Specifically, we consider $H = L = 1$ (i.e., MA(1) smoothing) and set $\phi_j^{(1)} = \phi^{(1)}$ and $\pi_j^{(1)} = \pi^{(1)}$ for simplicity. Finally, we estimate economic returns for each fund in the panel using the 1- and 3-step unsmoothing methods and study the properties of observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. The average results obtained from 1,000 simulations of this panel of funds are provided in Table 5 (Panel A).

The first column shows results for a specification in which the aggregate and fund-specific components of returns are smoothed with the same intensity ($\phi^{(1)} = \pi^{(1)} = 0.3$). In this case, fund- and aggregate-level R_t^o display autocorrelation, but R_t^{1s} and R_t^{3s} do not, indicating that the 1- and 3-step methods both work well in unsmoothing returns when $\phi^{(1)} = \pi^{(1)}$. However, we also observe that $\overline{R}_t^{1s}$ display no autocorrelation and the amount of autocorrelation in R_t^o

[17] This result is analogous to the result in Granger (1987) that macrovariables are driven by the common component of microunits. We thank an anonymous referee for pointing out this connection.

[18] We rely on simple (but realistic) parameters given by $\mu_\alpha = 0\%$, $\sigma_\alpha = 2\%$, $\sigma_\beta = 0.25$, $\mu_f = 0.6\%$, $\sigma_f = 4\%$, $\sigma_\varepsilon = 4\%$. However, the general insights from our simulations exercise are not sensitive to these baseline parameters.

is roughly the same at the fund and aggregate levels (0.35 with R_t^o and 0.36 with $\overline{R}_t^o$), with both of these results being inconsistent with what we observe in our hedge fund analysis.

To explore this issue further, the second column considers a specification in which the fund-level component of returns is smoothed less than the aggregate-level component ($\phi^{(1)} = 0.2$ and $\pi^{(1)} = 0.4$). In this case, observed returns have an autocorrelation that is lower at the fund-level in comparison to the aggregate-level (0.31 with R_t^o and 0.46 with $\overline{R}_t^o$), which is in line with what we observed in our hedge fund analysis. Moreover, while both the 1- and 3-step methods reduce R_t^o autocorrelation to virtually zero, roughly half of the autocorrelation in $\overline{R}_t^o$ persists in $\overline{R}_t^{1s}$ (also in line with the results we observe empirically).

Finally, the third column considers an alternative scenario in which the fund-level component of returns is smoothed more than the aggregate-level component ($\phi^{(1)} = 0.4$ and $\pi^{(1)} = 0.2$). This case results in a counterfactual autocorrelation structure (relative to our empirical results) since $\overline{R}_t^o$ is more autocorrelated at the fund-level than at the aggregate level and $\overline{R}_t^{1s}$ is negatively autocorrelated.

Overall, the results suggest our hedge fund analysis is in line with a smoothing process in which the fund-level component of returns is smoothed less than the aggregate-level component. This finding explains why the 1-step method (which implicitly assumes the two components are smoothed with the same intensity) does not fully unsmooth the systematic component of returns.

Given that the second column is the empirically relevant scenario, the last six rows of Table 5 (Panel A) suggest that a performance evaluation of hedge funds that relies on the 1-step method to unsmooth returns is likely to overstate alphas whereas relying on the 3-step method does not ($\mu_\alpha = 0$ in our simulations). Note also that the 3-step method is more efficient than the 1-step method in terms of Mean Absolute Errors (MAEs) and Mean Squared Errors (MSEs). Specifically, rows labeled $|\widehat{\alpha}|$ report MAE = Average($|\widehat{\alpha}|$) and rows labeled $\widehat{\alpha}^2$ report MSE = Average($\widehat{\alpha}^2$), with the 3-step method leading to lower MAE and MSE values than the 1-step method. As such, the 3-step method dominates the 1-step method both in terms of bias and efficiency.

Dimson (1979) provides an alternative alpha estimation method that does not create unsmoothed returns, but instead obtains alpha directly from smoothed returns by adding lags of the risk factors to the regression. The results from Table 5 (Panel A) suggest that the Dimson (1979) method is unbiased and has a similar efficiency as the 3-step unsmoothing method in estimating alphas when there is one factor and one lag. Specifically, the Dimson MAE and MSE are only slightly larger than our 3-step MAE and MSE. Applying Dimson on unsmoothed returns yields similar qualitative results (with Dimson becoming slightly less efficient in this case).

Table 5 (Panel B) provides a more realistic simulation at the cost of complexity. In particular, we consider an 8-factor model and calibrate the properties of risk factors, economic returns, and smoothing parameters to match our empirical analysis of relative value hedge funds (the details are described in the table header). The results provided in Panel B are qualitatively similar to those in Panel A. Moreover, in this more realistic simulation, the quantitative results are stronger because the efficiency loss of using Dimson (1979) is much higher. Specifically, the increase from the 3-step MAE and MSE to the Dimson MAE and MSE is much larger.[19] The reason is that estimating Dimson (1979) with multiple risk factors results in "too many" parameters in the factor regressions. For instance, with our 8-factor model and our 3 lags, we have a total of 33 parameters in the factor regression of each fund. To further explore the effect of parameter proliferation, we also consider whether a constrained version of Dimson (1979), that reduces this proliferation, achieves better results (we refer to it as "CDimson" for the rest of the paper).[20] We find that the CDimson method has only a small bias (average alpha is between 0.1% and 0.2%) and is much more efficient than

[19]In these simulations and in our empirical analysis, we put Dimson (1979) and unsmoothing methods on the same playing field by using three lags of risk factors when estimating Dimson regressions (just as we use three MA lags when unsmoothing returns). The only exception is when a fund has less than four years of data in our empirical analysis, in which case we estimate Dimson (1979) with only two lags (because adding a third lag would result in 33 parameters and only 48 observations to estimate them).

[20]The CDimson method relies on the non-linear regression (estimated by Nonlinear Least Squares) $R^o_{j,t} = \alpha + \beta_0^{'} f_t + \omega_1 \cdot \beta_0^{'} f_{t-1} + \omega_2 \cdot \beta_0^{'} f_{t-2} + \omega_3 \cdot \beta_0^{'} f_{t-3} + \epsilon_{j,t}$ so that only three common beta decay parameters (ω_1, ω_2, and ω_3) are added to the β_0 vector. We find similar results (untabulated) when exploring a CDimson version that imposes $\omega_1 = \omega$, $\omega_2 = \omega^2$, and $\omega_3 = \omega^3$ (so that only parameter ω is added to the β_0 vector). We thank two anonymous referees for the suggestion to explore these two CDimson methods.

the Dimson (1979) method. However, we also find that the CDimson method is dominated by 3-step unsmoothing in terms of both bias and efficiency.

1.5 The Economics of 3-Step MA Unsmoothing

The previous subsections demonstrate two main stylized facts. First, 1-step MA unsmoothing does not fully unsmooth the systematic component of returns. Second, allowing the aggregate and fund-specific components of returns to be smoothed with different weights (i.e., $\pi_j^{(h)} \neq \phi_j^{(h)}$) through our proposed 3-step MA unsmoothing method solves the problem. This subsection provides an economic framework that clarifies why allowing for $\pi_j^{(h)} \neq \phi_j^{(h)}$ is economically more sensible than restricting the smoothing process to satisfy $\pi_j^{(h)} = \phi_j^{(h)} \equiv \theta_j^{(h)}$ as in the 1-step MA unsmoothing method. In a nutshell, $\pi_j^{(h)} \neq \phi_j^{(h)}$ arises naturally if funds do not observe the current value of the assets they hold, but instead receive different signals about the shocks common to all assets in their asset class and the shocks affecting the relative value of their assets.

There are J funds, indexed by j, operating in a common asset class. The log value of each fund in this asset class evolves as:

$$V_{j,t} = \mu_j + V_{j,t-1} + \overline{\eta}_t + \widetilde{\eta}_{j,t} \tag{12}$$

where $\overline{\eta}_t \overset{iid}{\sim} N(0, \overline{\sigma}^2)$ is an asset class shock, $\widetilde{\eta}_{j,t} \overset{iid}{\sim} N(0, \widetilde{\sigma}_j^2)$ is a shock specific to the assets of fund j, and the parameter μ_j is common knowledge.

The specification above implies log economic returns are given by (assuming no cash flows are paid from $t-1$ to t):

$$\begin{aligned} r_{j,t} &= V_{j,t} - V_{j,t-1} \\ &= \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t} \end{aligned} \tag{13}$$

so that aggregate log economic returns are $\overline{r}_t = \sum_{j=1}^{J} w_j \cdot r_{j,t} = \overline{\mu} + \overline{\eta}_t$ and relative log economic returns are $\widetilde{r}_{j,t} = r_{j,t} - \overline{r}_t = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$.[21]

[21]While $\overline{r}_t = \sum_{j=1}^{J} w_j \cdot r_{j,t} = \overline{\mu} + \overline{\eta}_t$ holds in general, the interpretation of $\overline{r}_t$ as an aggregate return relies

However, funds do not observe $V_{j,t}$ at time t, and thus the returns calculated from reported value (i.e., observed returns) differ from economic returns. Specifically, at each time t, each fund learns its $V_{j,t-1}$ but not its $V_{j,t}$. Instead, each fund receives a (different) signal about $\overline{\eta}_t$:

$$\widehat{\overline{\eta}}_{j,t} = \overline{\eta}_t + \overline{u}_{j,t} \tag{14}$$

as well as a separate signal about $\widetilde{\eta}_{j,t}$:

$$\widehat{\widetilde{\eta}}_{j,t} = \widetilde{\eta}_{j,t} + \widetilde{u}_{j,t} \tag{15}$$

where $\overline{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\overline{\sigma}}_j^2)$ and $\widetilde{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\widetilde{\sigma}}_j^2)$.

After receiving the signals, each fund reports the posterior mean of its log value, $V^o_{j,t} = \mathbb{E}[V_{j,t}|V_{j,t-1}, \widehat{\overline{\eta}}_t, \widehat{\widetilde{\eta}}_t]$, in its books, which is given by:

$$\begin{aligned} V^o_{j,t} &= \mu_j + V_{j,t-1} + \mathbb{E}[\overline{\eta}_t|\widehat{\overline{\eta}}_t] + \mathbb{E}[\widetilde{\eta}_{j,t}|\widehat{\widetilde{\eta}}_{j,t}] \\ &= \mu_j + V_{j,t-1} + \pi_j \cdot \widehat{\overline{\eta}}_t + \phi_j \cdot \widehat{\widetilde{\eta}}_{j,t} \end{aligned} \tag{16}$$

and implies observed log returns are given by:

$$\begin{aligned} r^o_{j,t} &= V^o_{j,t} - V^o_{j,t-1} \\ &= (V_{j,t-1} - V_{j,t-2}) + \pi_j \cdot (\widehat{\overline{\eta}}_t - \widehat{\overline{\eta}}_{t-1}) + \phi_j \cdot (\widehat{\widetilde{\eta}}_{j,t} - \widehat{\widetilde{\eta}}_{j,t-1}) \\ &= r_{j,t-1} + \pi_j \cdot (\overline{\eta}_t - \overline{\eta}_{t-1}) + \phi_j \cdot (\widetilde{\eta}_{j,t} - \widetilde{\eta}_{j,t-1}) + \xi_{j,t} \\ &= \pi_j \cdot \overline{r}_t + (1-\pi_j) \cdot \overline{r}_{t-1} + \phi_j \cdot \widetilde{r}_{j,t} + (1-\phi_j) \cdot \widetilde{r}_{j,t-1} + \xi_{j,t} \end{aligned} \tag{17}$$

where
$\pi_j = (1/\widehat{\overline{\sigma}}_j^2)/(1/\widehat{\overline{\sigma}}_j^2 + 1/\overline{\sigma}^2)$
$\phi_j = (1/\widehat{\widetilde{\sigma}}_j^2)/(1/\widehat{\widetilde{\sigma}}_j^2 + 1/\widetilde{\sigma}_j^2)$
$\xi_{j,t} = \pi_j \cdot (\overline{u}_{j,t} - \overline{u}_{j,t-1}) + \phi_j \cdot (\widetilde{u}_{j,t} - \widetilde{u}_{j,t-1})$

Equation 17 is the same as the smoothing process we assume for the 3-step MA unsmoothing method (in Equation 5), except that Equation 17 applies to log returns (instead

on the approximation in Campbell, Chan, and Viceira (2003) because the aggregate of log returns is not generally equal to the log of aggregate returns.

of regular returns) and the smoothing process in Equation 17 has an extra latent component, $\xi_{j,t}$, which we abstract from in our econometric framework to maintain the tractability of our unsmoothing method.[22] Moreover, Equation 17 can be generalized to an MA(H) process by assuming that, at time t, funds only learn $V_{j,t-H}$ and have to rely on signals about the shocks from $t-H$ to t in order to obtain their posterior distribution for $V^o_{j,t}$.

Given the above paragraph, it is natural to ask under which economic conditions the 1-step MA unsmoothing restriction ($\pi_j = \phi_j = \theta_j$) would hold. Inspecting Equation 17, in order for the economic condition $\pi_j = \phi_j$ to hold, the variance of the aggregate signal relative to the variance of aggregate returns, $\widehat{\overline{\sigma}}_j^2/\overline{\sigma}^2$, must be the same as the analogous quantity for relative returns, $\widehat{\widetilde{\sigma}}_j^2/\widetilde{\sigma}_j^2$, for all funds. As such, it seems implausible to expect the $\pi_j = \phi_j$ condition to hold for hedge funds (or any set of funds). The implausibility of this condition explains why the systematic component is not fully unsmoothed when we apply 1-step MA unsmoothing to hedge fund returns, but it is when we use our proposed 3-step MA unsmoothing method.

2 Unsmoothing Hedge Fund Returns

In this section, we explore hedge fund risk exposures and risk-adjusted performance after unsmoothing returns using the 1-step and 3-step MA unsmoothing methods. Subsection 2.1 explains the empirical details; Subsection 2.2 presents the main results after separating funds into liquidity groups; and Subsection 2.3 reports results by hedge fund strategy to explore the improvement in the measurement of risk exposures.

[22] In Internet Appendix D.2, we find that our main results are very similar if we rely on log returns instead of raw returns. Moreover, Internet Appendix A demonstrates that our 3-step unsmoothing method yields unbiased estimates of betas and alpha despite ignoring the $\xi_{j,t}$ component when unsmoothing returns. Internet Appendix A also shows how to recover economic returns while accounting for the $\xi_{j,t}$ term (at the cost of complexity). Specifically, it provides a state space representation of the model with $\xi_{j,t}$, which we estimate by conditional maximum likelihood and a Kalman filter algorithm (with the resulting α estimates being similar to the ones obtained from our 3-step unsmoothing process).

2.1 Empirical Details

Our final hedge fund dataset is based on a merge of the Lipper Trading Advisor Selection System database with the BarclayHedge database. It covers a total of 5,069 funds with at least 36 uninterrupted monthly observations over the period from January 1995 to December 2017. Further details are provided in the data appendix.

Our analysis of risk exposures and risk-adjusted performance is based on the FH 8-Factor model, which augments the 7-Factor model in Fung and Hsieh (2001) with an emerging market factor. The risk-free rate and trend-following factors are obtained respectively from Kenneth French's and David A. Hsieh's online data libraries.[23] The 3 equity-oriented risk factors are calculated using equity index data from Datastream, and the 2 bond-oriented factors are calculated using data from the Federal Reserve Bank of St. Louis (both equity and bond factors follow the instructions given on David A. Hsieh's webpage).

We perform 1-step MA unsmoothing following a procedure similar to Getmansky, Lo, and Makarov (2004). Specifically, we use $H = 3$ as the number of smoothing lags in the MA process for observed returns ($R^o_{j,t}$), extract estimated residuals ($\eta_{j,t}$), and add the average return (μ_j) back to obtain economic returns, $R_{j,t} = \mu_j + \eta_{j,t}$.[24] The MA process is estimated using maximum likelihood under $\eta_{j,t} \overset{iid}{\sim} N(0, \sigma^2_{\eta,j})$, as described in GLM.

We follow an analogous procedure for our 3-step MA unsmoothing with $H = L = 3$. First, we take the average return of all funds in a given strategy each month to obtain strategy indexes and perform GLM unsmoothing (as described in the previous paragraph) for each strategy index separately to recover unsmoothed strategy-level returns.[25] Second, we obtain

[23] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
https://faculty.fuqua.duke.edu/~dah7/HFRFData.htm

[24] It is common in the literature to fix $H = 2$, but we use $H = 3$ to be conservative (since a process with $H = 2$ is consistently estimated when we use $H > 2$). To address potential efficiency issues, Internet Appendix D.4 shows that choosing H using the AIC criterion for each fund separately yields results that are similar to the ones we report in the main text.

[25] We rely on equal-weights to construct the strategy indexes because this approach is consistent with our derivations of the 3-step unsmoothing method (which relies on time-invariant weights). However, Internet Appendix D provides results (similar to our baseline analysis) using value-weights based on lagged assets under management to construct strategy-level returns.

unsmoothed relative returns from Equation 8 (an MA process for observed relative returns with aggregate unsmoothed returns as covariates). And third, we sum the unsmoothed strategy returns with each fund unsmoothed excess return to obtain fund-level economic returns.

For our baseline empirical analysis of hedge funds, we follow the hedge fund literature and rely on regular returns (as opposed to log returns as our economic framework in Subsection 1.5 suggests).[26] However, Internet Appendix D shows that our results are consistent if we rely on log returns instead.

2.2 Results by Liquidity Group

This subsection demonstrates that our 3-step unsmoothing method improves the measurement of risk-adjusted performance relative to traditional (or 1-step) unsmoothing.

Since unsmoothing methods are designed to affect only the returns of illiquid funds (i.e., funds with significant return autocorrelation), our analysis classifies funds in groups based on liquidity. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies).[27] We then measure fund-level information within each

[26] When estimating smoothing parameters, some funds have extreme unsmoothed returns. To deal with this issue, we set any fund with extreme smoothing parameter values to the default of "no smoothing" (i.e., θ_0=1 for the 1-step method and π_0=ϕ_0=1 for the 3-step method). For simplicity, we define a fund to have "extreme parameter values" if any of its smoothing parameters is above 1.25 or below -0.45. We calibrate these two values based on a simulation. In a nutshell, we simulate economic returns and smooth them based on empirically reasonable parameters. Then, we asked what bounds need to be imposed for estimated smoothing methods to recover, on average, the true smoothing parameters in the simulations (i.e., we prevent outliers in unsmoothed returns from having a major effect on average parameter values). That said, using different reasonable bounds on smoothing parameters (e.g., 1.50 and -0.50) leads to very similar empirical results. Moreover, setting a subset of funds to "no smoothing" as we do bias our results towards finding no effect of unsmoothing methods, which is the opposite of what we find empirically.

[27] The liquidity ranking obtained from this approach (which is based on the first column of Table 3) is consistent with economic logic. The most illiquid strategy is the relative value strategy, which contains funds that attempt to profit from mispricing across securities. If capital markets work well, hedge funds are unlikely to find mispricing opportunities among the pool of liquid securities, and thus tend to invest in relatively illiquid assets. At the other extreme, CTAs represent the most liquid hedge funds as their underlying strategies tend to be based on trend-following and are usually executed using futures contracts, which are marked to market daily.

group and report averages.

2.2.1 Hedge Fund Ex-Post Performance Before and After Unsmoothing

A fundamental question in the literature is whether hedge funds are able to generate positive risk-adjusted performance. This question naturally leads to an ex-post analysis of hedge fund performance, so we start by exploring how unsmoothing returns affects the ex-post measurement of risk-adjusted performance.

The basic problem with smoothed returns is that they understate risk (Getmansky, Lo, and Makarov (2004)), even though they do not affect average (i.e., risk unadjusted) performance. As such, unsmoothing methods are designed to increase return volatility without affecting average returns, which decreases Sharpe ratios.

Figures 2(a) and 2(b) show, for the three liquidity groups, the average (annualized) volatility and Sharpe ratio based on (i) reported returns; (ii) 1-step unsmoothed returns; and (iii) 3-step unsmoothed returns.[28] For the low and mid liquidity strategies, the average Sharpe ratio (volatility) strongly decreases (increases) after unsmoothing. For instance, the average fund Sharpe ratio in the low liquidity strategies decreases by 28.5% (from 0.43 to 0.31) as we unsmooth returns. In contrast, there is almost no change in average the Sharpe ratio as we unsmooth returns of funds in the high liquidity strategies. This result shows that unsmoothing methods work well as they should not strongly affect the returns of funds that invest in liquid assets. Comparing the 3-step method with 1-step method, we see little change in average Sharpe ratios. For instance, after the 3-step unsmoothing, the average Sharpe ratio of funds in the low liquidity strategies only changes by 3.5% (from 0.31 to 0.32). It is not surprising that the 3-step unsmoothing method has a very small effect on fund Sharpe ratios beyond 1-step unsmoothing. The 3-step approach is designed to better unsmooth the sys-

[28]We "annualize" volatilities and Sharpe ratios by multiplying them by $\sqrt{12}$ (which, strictly speaking, is the correct annualization factor only for returns that are not autocorrelated). However, multiplying by this fixed constant does not affect any of the relative comparisons between smoothing methods. Moreover, in the case of Sharpe ratios, we report cross-fund medians as oppose to averages since the Sharpe ratios of funds with negative average excess returns increase as volatility increases. Nevertheless, results based on cross-fund average Sharpe ratios are similar and so we still refer to the reported values as "average Sharpe ratios".

tematic portion of returns, not to increase the degree of unsmoothing. As such, the improved risk measurement provided by the 3-step method (detailed below) is not a mechanical result of increased return volatility.

Figure 2(c) explores systematic risk by focusing on average R^2s based on the FH 8-Factor model, which effectively captures how much of fund-level return variability is explained by the risk factors most commonly used in the hedge fund literature. 1-step MA unsmoothing has basically no effect on R^2s (if anything, R^2s decrease), which indicates that even though 1-step unsmoothing increases volatility relative to reported returns, it does not increase the fraction of volatility explained by standard risk factors. In stark contrast, 3-step MA unsmoothing substantially increases R^2s for funds in the low and mid liquidity strategies. For instance, after 3-step unsmoothing, the average R^2 of funds in the low liquidity strategies increases by 14.9% (from 34.3% to 39.3%) relative to reported returns and by 21.3% (from 32.4% to 39.3%) relative to 1-step unsmoothing.

These R^2 patterns suggest that the 3-step MA unsmoothing method allows us to better uncover the true systematic risk exposure of hedge funds, which is usually partially concealed because observed returns are smoothed. Given this R^2 result, we should be able to better measure risk-adjusted performance using our 3-step MA unsmoothing method. Figure 2(d) explores this issue by reporting average (annualized) αs for the three liquidity groups. The 3-step unsmoothing strongly decreases αs for the low and mid liquidity strategies relative to the αs obtained with observed returns or after 1-step unsmoothing. For both groups, average αs decrease by more than 1 percentage point relative to observed returns (from 3.1% to 1.9% for the low liquidity group and from 0.9% to -0.4% for the mid liquidity group), which is substantially larger than the improvement provided by the 1-step unsmoothing method.

Figures 2(e) and 2(f) gauge how unsmoothing affects the statistical significance of fund-level αs. Overall, there is a strong decline in average t^{α}_{stat} as well as on the percentage of significant αs for the low liquidity group. The mid liquidity group still displays an effect, but much weaker since αs are not (on average) very significant in the first place.

Overall, the results indicate that volatility strongly increases after unsmoothing and the fraction of volatility due to systematic risk only increases when the 3-step MA unsmoothing process is used. Moreover, unsmoothing returns decreases α, and this effect is stronger when the 3-step method is used. Finally, all of these results are only present when evaluating relatively illiquid funds.

2.2.2 Do Unsmoothed Alphas Better Identify Funds with Superior Alphas Ex-Ante?

In some applications, researchers and investors are interested in identifying funds with superior ex-ante performance. To explore this issue, we follow the literature by sorting hedge funds into portfolios using past estimated αs and study the αs of such portfolios in the subsequent months.

Specifically, following Bollen, Joenväärä, and Kauppila (2021), we form quintile portfolios within each liquidity group according to the t-stat of their FH 8-Factor αs calculated using a 24-month rolling window.[29] The portfolios are formed anew at the beginning of each year and are initially equal-weighted, but not rebalanced within the year so that the portfolio weights evolve according to the realized returns of the underlying funds. We then concatenate the returns of each quintile portfolio across the sample to construct a full time-series of portfolio returns, and estimate their αs.[30] We perform this sorting procedure using the t-stats of αs estimated using three different versions of past returns: observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. Importantly, we unsmooth returns using data available only up to the month of portfolio formation to avoid look-ahead bias. In the first few years of the sample, some fund strategies include only a few funds and therefore the estimated MA parameters would be unstable. For this reason, we require at least 6 years

[29] Specifically, we require all funds to have at least 48 past monthly returns, compute betas using all past monthly returns available, and estimate α t-stats using only the most recent 24 monthly returns as in Bollen, Joenväärä, and Kauppila (2021). Since α t-stats are normalized αs that adjust for cross-sectional differences in estimation noise, we still refer to our quintile portfolios as sorted by α to simplify exposition. Internet Appendix D.6 provides an analysis in which funds are directly sorted on alphas, with the noise adjustment following Vasicek (1973). The results from this analysis are similar to the ones we report in the main text.

[30] To estimate the αs of the final portfolios, we use 1-step unsmoothing (note that 3-step unsmoothing is not needed since these are not fund-level alphas). However, as we show in Internet Appendix D.6, further applying Dimson or constrained Dimson when estimating portfolio αs yields similar qualitative results.

of data to unsmooth returns before forming the portfolios. That is, the first set of portfolios is formed at the end of December 2000, using data from January 1995 to December 2000 to unsmooth returns.

Figure 3 summarizes the results from our portfolio exercise. Figure 3(a) shows under the "Baseline" category that sorting on αs obtained based on 3-step unsmoothed returns produces quintile portfolios that have a larger spread in ex-ante αs than portfolios sorted on αs estimated from observed returns or 1-step unsmoothed returns. Interestingly, the "+Dimson" and "+CDimson" categories show that applying Dimson or constrained Dimson to observed returns or unsmoothed returns yields portfolio spreads with much lower αs. These results are consistent with our simulations in Table 5 in the sense that Dimson and constrained Dimson applied to observed or unsmoothed returns tend to increase the noise associated with α estimates, which deteriorates the efficacy of the ex-ante identification of high and low α funds in the context of our out-of-sample analysis.

Since hedge fund portfolios cannot easily be shorted, the α spreads analysed in Figure 3(a) do not reflect αs that can be achieved in the market. While our point is simply that αs estimated using 3-step unsmoothed returns better predict future αs, and thus the tradability of the strategy is not relevant, Figure 3(b) also provides the αs of the highest past α quintiles, which reflect only the long positions on the strategies analysed in Figure 3(a). The results are largely similar, with αs estimated using 3-step unsmoothed returns providing a better signal for future α, and thus allowing researchers and investors to identify hedge funds that, ex-ante, have higher αs.

To ensure our analysis is out-of-sample, Figures 3(a) and 3(b) create portfolios with all funds, without using the liquidity classification we rely on in the prior subsection. However, our framework predicts that the superior performance of the 3-step method is due to illiquid funds. To test this prediction, Figures 3(c) to 3(f) replicate Figures 3(a) to 3(b) separately for low and high liquidity funds. As it is clear from the figures, the superior performance of the 3-step method is only present among low liquidity funds. When focusing on high liquidity funds, there is little difference between the performance of portfolios formed based on αs estimated from observed returns or 3-step unsmoothed returns.

Overall, the results indicate that 3-step unsmoothing helps not only to measure risk-adjusted performance ex-post, but also ex-ante. Moreover, this finding helps to validate our ex-post α estimates. Specifically, our 3-step unsmoothing method performs better than 1-step unsmoothing and the Dimson method in the identification of true α, with such results holding only for illiquid funds.

2.3 Results by Hedge Fund Strategy

The previous results suggest that our 3-step MA unsmoothing method improves the risk-adjusted performance measurement relative to the 1-step method. To better understand which risks (betas) are better measured, it is useful to analyze groups of funds that engage in similar activity, and thus are exposed to similar risks. As such, this subsection reports results by hedge fund strategy.

2.3.1 Volatilities, Sharpe Ratios, R^2s, and Alphas

Figure 4 shows several average statistics by hedge fund strategy. In our description, we refer to "illiquid strategies" as the strategies with autocorrelation coefficient higher than 0.20 (these include all strategies in high and mid liquidity strategies of the previous section, except for market neutral, which tends to display results more consistent with the high liquidity group).

Figures 4(a) and 4(b) demonstrate that, for each of the illiquid strategies, volatilities increase and Sharpe ratios decrease after unsmoothing, but it also shows that 3-step unsmoothing has little effect beyond 1-step unsmoothing. Figure 4(c) plots R^2s relative to the FH 8-Factor model and shows that the results observed in the low and mid liquidity groups are present for each of the illiquid strategies separately (with basically no effect on liquid strategies). That is, R^2s do not increase as we 1-step unsmooth returns (if anything, they decrease), but they strongly increase after our 3-step unsmoothing. Figure 4(d) shows that the average α declines in every illiquid strategy while Figures 4(e) and 4(f) make it clear that the statistical decline in alphas (i.e., decline in average t^{α}_{stat} and in the percentage of funds with significant α) is much larger for the more illiquid strategies.

Figure 5 reports the main statistics analyzed in Figure 4, but focuses on the economic and statistical changes to these metrics as analyzed returns move from (i) observed returns to 1-step unsmoothed returns and (ii) 1-step unsmoothed returns to 3-step unsmoothed returns. The t_{stat} for each average change is provided on the top of the respective bar. Figures 5(a) to 5(c) reinforce the inferences from Figure 4 and add that the changes tend to be statistically significant as well. Figure 5(c) also emphasizes that 1-step unsmoothing significantly changes the measurement of risk-adjusted performance (i.e., decreases αs) relative to reported returns, with the change obtained from moving from 1-step unsmoothing to 3-step unsmoothing being comparable to (sometimes even larger than) the change obtained by unsmoothing through the 1-step process. This result suggests that moving from the 1-step to 3-step unsmoothing is at least as economically important as unsmoothing returns in the first place.

Overall, the results when separating funds into three liquidity groups are largely present for individual strategies as well.

2.3.2 Risk Exposures

The previous results indicate that the FH 8-Factor model explains a significantly higher fraction of the volatility of hedge fund returns than suggested by looking at observed returns (or at 1-step unsmoothed returns). Below, we ask how much each risk factor contributes to the improvement.

In a factor model with two risk factors, $R_t = \alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t} + \epsilon_t$, R^2 can be decomposed as (the decomposition is analogous for an arbitrary number of risk factors):

$$\begin{aligned} R^2 &= Var(\alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t})/Var(R_t) \\ &= Cov(\alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t}, R_t)/Var(R_t) \\ &= \underbrace{\beta_1 \cdot \frac{Cov(f_{1,t}, R_t)}{Var(R_t)}}_{R_1^2} + \underbrace{\beta_2 \cdot \frac{Cov(f_{2,t}, R_t)}{Var(R_t)}}_{R_2^2} \end{aligned} \tag{18}$$

where the second equality follows from the projection orthogonality condition, $Cov(f_{1,t}, \epsilon_t) = Cov(f_{2,t}, \epsilon_t) = 0$, and R_i^2 represents the R^2 portion due to risk factor i.

Figure 6 reports the average R^2 due to each of the risk factors in the FH 8-Factor model for each hedge fund strategy. For all illiquid strategies except the emerging market strategy, the results indicate market risk and emerging market risk are significantly more important in explaining returns after 3-step unsmoothing. For instance, market risk accounts for about 15.5% of the volatility of Event Driven funds after 3-step unsmoothing (an increase of 54.0% relative to the importance of market risk when we look at observed returns). For Emerging Market funds, the only risk factor that displays a substantial increase after 3-step unsmoothing is the emerging market risk factor itself, which is consistent with economic intuition. Similarly, the exposure of Event Driven funds to the size factor increases substantially after 3-step unsmoothing, which is also in line with economic intuition as these funds are often focused on relatively small and illiquid firms. For liquid funds, there is no change in the importance of different risk factors after unsmoothing returns.

Overall, the results indicate that most of the improvement coming from the 3-step unsmoothing method stems from better measuring exposures to market risk and emerging market risk in the underlying illiquid assets held by hedge funds.

3 Unsmoothing Returns of Commercial Real Estate Funds

The previous two sections introduce our 3-step MA unsmoothing method and demonstrate that it provides a substantial improvement relative to 1-step MA unsmoothing in the context of hedge funds. While unsmoothing hedge fund returns is a natural application of our 3-step unsmoothing process, unsmoothing is even more important for private CRE funds, which are highly illiquid given the appraisal nature of real estate valuation. As such, this section demonstrates how we can extract economic returns for private CRE funds using a 3-step version of the AR unsmoothing framework proposed in Geltner (1991, 1993), which is more commonly applied in the real estate literature. Subsection 3.1 outlines the 1-step AR unsmoothing method and extends our 3-step process to improve upon it; and Subsection 3.2

applies our 3-step AR unsmoothing to private CRE fund returns.

3.1 Autoregressive Return Unsmoothing Framework

The baseline unsmoothing framework for real estate assets comes from Geltner (1991, 1993) and is often referred to in the literature as AR unsmoothing since it implies observed returns follow an autoregressive process.

Geltner (1991, 1993) assume the observed return of fund j at time t is given by (see original paper for the economic motivation):

$$R^o_{j,t} = \theta^{(0)}_j \cdot R_{j,t} \ + \ \Sigma^H_{h=1}\, \theta^{(h)}_j \cdot R^o_{j,t-h} \tag{19}$$

$$= \mu_j \ + \ \Sigma^H_{h=1}\, \theta^{(h)}_j \cdot (R^o_{t-h} - \mu_j) \ + \ \theta^{(0)}_j \cdot \eta_{j,t} \tag{20}$$

where θs capture the level of "staleness" in observed returns with $\Sigma^H_{h=0}\theta^{(h)}_j = 1$, and the second equality follows from $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$.[31].

The first equality represents the economic assumption that prices are only partially updated so that observed returns partially reflect the economic returns of the reporting period as well as observed returns of the H previous periods. The second equality is the econometric implication that, under the given assumption, the observed fund returns follow an AR(H) process.

Given Equation 20, we can recover economic returns by estimating an AR(H) process for $R^o_{j,t}$, extracting the estimated residuals, $\epsilon_{j,t} = \theta^{(0)}_j \cdot \eta_{j,t}$, and using them to obtain economic returns, $R_{j,t} = \mu_j + \epsilon_{j,t}/(1 - \Sigma^H_{h=1}\theta^{(h)}_j)$. There are many methods to estimate AR(H) processes, with Ordinary Least Squares (OLS) being the simplest consistent estimator, and thus the method we use. This procedure is used by several papers in the literature to unsmooth the returns of real estate assets and funds (see the citations in Foonote 5).

However, as we empirically demonstrate in the next subsection, this AR unsmoothing method faces the same aggregation issue observed with the MA unsmoothing. As such,

[31] Note also that, under invertibility, Equation 20 implies an MA(∞) representation with coefficients (i.e., weights) summing to one.

analogously to the MA case, we generalize the assumption in Equation 19 so that aggregate economic returns are directly included in the return smoothing process. Specifically, we assume:[32]

$$R^o_{j,t} = \phi^{(0)}_j \cdot \widetilde{R}_{j,t} \; + \; \Sigma^H_{h=1} \, \phi^{(h)}_j \cdot \widetilde{R}^o_{j,t-h} \; + \; \pi^{(0)}_j \cdot \overline{R}_t \; + \; \Sigma^H_{h=1} \, \pi^{(h)}_j \cdot \overline{R}^o_{j,t-h} \tag{21}$$

$$= \mu_j \; + \; \Sigma^H_{h=1} \, \phi^{(h)}_j \cdot (\widetilde{R}^o_{j,t-h} - \widetilde{\mu}_j) \; + \; \Sigma^H_{h=1} \, \pi^{(h)}_j \cdot (\overline{R}^o_{t-h} - \overline{\mu}) \; + \; \phi^{(0)}_j \cdot \widetilde{\eta}_{j,t} \; + \; \pi^{(0)}_j \cdot \overline{\eta}_t$$

$$= \mu_j \; + \; \Sigma^H_{h=1} \, \phi^{(h)}_j \cdot (\widetilde{R}^o_{j,t-h} - \widetilde{\mu}_j) \; + \; \Sigma^H_{h=1} \, \pi^{(h)}_j \cdot (\overline{R}^o_{t-h} - \overline{\mu}) \; + \; \epsilon_{j,t} \tag{22}$$

where $\Sigma^H_{h=0}\phi^{(h)}_j = \Sigma^H_{h=0}\pi^{(h)}_j = 1$, the second equality follows from $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$, and the third equality defines $\epsilon_{j,t} = \phi^{(0)}_j \cdot \widetilde{\eta}_{j,t} \; + \; \pi^{(0)}_j \cdot \overline{\eta}_t$.

Since the covariates in Equation 22 are observable (in contrast to the MA(H) process), we can directly estimate Equation 22 (by OLS) and obtain coefficient estimates, $\phi^{(h)}_j$ and $\pi^{(h)}_j$ (including $\phi^{(0)}_j = 1 - \Sigma^H_{h=1}\phi^{(h)}_j$ and $\pi^{(0)}_j = 1 - \Sigma^H_{h=1}\pi^{(h)}_j$), as well as residual estimates, $\epsilon_{j,t}$. The challenge is that $\epsilon_{j,t}$ reflects both $\widetilde{\eta}_{j,t}$ and $\overline{\eta}_t$. We rely on an aggregation step to separate the two components. Specifically, aggregating $\epsilon_{j,t}$ yields:

$$\begin{aligned} \overline{\epsilon}_t \; &= \overline{\pi}^{(0)} \cdot \overline{\eta}_t \; + \; \widehat{Cov}(\phi^{(0)}_j, \widetilde{\eta}_{j,t}) \\ &\approx \overline{\pi}^{(0)} \cdot \overline{\eta}_t \end{aligned} \tag{23}$$

Similar to the MA(H) case, this framework provides a simple way to recover aggregate and fund-level economic returns in an internally consistent way given the estimates for $\phi^{(h)}_j$, $\pi^{(h)}_j$, and $\epsilon_{j,t}$ obtained from Equation 22. First, we obtain aggregate economic returns from $\overline{R}_t = \overline{\mu} + \overline{\eta}_t$ where $\overline{\eta}_t = \overline{\epsilon}_t / \overline{\pi}^{(0)}$ with $\overline{\epsilon}_t = \Sigma^J_{j=1} w_j \cdot \epsilon_{j,t}$ and $\overline{\pi}^{(0)} = \Sigma^J_{j=1} w_j \cdot \pi^{(0)}_j$. Second, we obtain fund-level economic relative returns from $\widetilde{R}_{j,t} = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$ where $\widetilde{\eta}_{j,t} = (\epsilon_{j,t} - \pi^{(0)}_j \cdot \overline{\eta}_t)/\phi^{(0)}_j$. Third, we recover fund-level economic returns from $R_{t,j} = \overline{R}_t + \widetilde{R}_{j,t} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$. This procedure summarizes our AR 3-step unsmoothing method.

Similarly to the MA(H) case, our 3-step AR unsmoothing procedure can be seen as a

[32] This smoothing process reduces to Equation 20 (the smoothing process in Geltner (1991, 1993)) if we set $\pi^{(h)}_j = \phi^{(h)}_j = \theta^{(h)}_j$.

generalization of Geltner (1991, 1993) that allows aggregate and relative economic returns to have different effects on observed fund-level returns ($\pi_j \neq \phi_j$), but that would be identical (up to sampling variation) to the 1-step AR unsmoothing if the underlying assumption in Geltner (1991, 1993) ($\pi_j = \phi_j$) was empirically valid.

In our empirical analysis of real estate funds, we unsmooth log observed returns instead of regular observed returns and then transform the unsmoothed log return into unsmoothed regular returns to calculate all reported statistics. This approach is consistent with the 1-step unsmoothing framework in Geltner (1991, 1993) (derived from a model of appraisal valuation) and much of the prior real estate literature. However, as demonstrated in Internet Appendix D, the overall results are similar whether we log returns or not.

3.2 Unsmoothing Returns of Commercial Real Estate Funds

3.2.1 Empirical Details

Our final private commercial real estate (CRE) dataset is from the National Council of Real Estate Investment Fiduciaries (NCREIF). It covers a total of 66 funds with at least 36 uninterrupted quarterly observations over the period from Q1 1994 through Q4 2017. Further details are provided in the data appendix.

There is no consensus on the appropriate factor model to measure risk exposures of private CRE funds. As such, our analysis relies on two simple factor models. The first has only one risk factor: returns (in excess of the risk-free rate) on an index capturing the public CRE market. The second factor model includes the same public CRE factor, but adds returns (in excess of the risk-free rate) on an index capturing the public equity market.[33] Despite this simplified approach, we show that these factor models drive the average α of private CRE

[33]For excess returns on the equity market, we use the market risk factor in Kenneth French's data library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). For excess returns on the public real estate market, we use excess returns on the FTSE NAREIT All Equity REIT Index available on the website of the National Association of REITs (https://www.reit.com/). We compound the monthly returns on both indexes to obtain quarterly returns and subtract the one-month Treasury bill rate compounded over each quarter to get excess returns.

funds to (close to) zero after 3-step unsmoothing.[34]

3.2.2 Autocorrelations of Private CRE Fund Returns

Table 6 provides average autocorrelations for private CRE fund returns as well as autocorrelations for the aggregate (equal-weighted average) of all private CRE fund returns. Confirming the intuition that the appraisal nature of real estate valuation induces (highly) smoothed returns, we find that the average 1-quarter autocorrelation of observed returns is 0.45, with 63.6% of the funds displaying a statistically significant autocorrelation. In fact, returns are so persistent that the average autocorrelation is still 0.21 at 4-quarters (and statistically significant for 40.9% of the funds). Returns are even more autocorrelated at the aggregate level, with the aggregate private CRE returns displaying a 1-quarter autocorrelation of 0.75 (with p-value=0.0%) and a 4-quarter autocorrelation of 0.31 (with p-value=0.3%).

The fund-level partial autocorrelations indicate the return autocorrelation structure of most funds is well described by an AR structure with one or two lags. As such, we apply AR unsmoothing to these private CRE funds using an AR(2) model, which nests the AR(1) structure.

After 1-step AR unsmoothing, the return autocorrelations at the fund level mostly disappear. However, the 1-quarter autocorrelation for the aggregate series remains extremely strong (at 0.46 with p-value=0.0%) and even the 2-quarter autocorrelation remains high (at 0.31 with p-value=0.3%). This result indicates the 1-step AR unsmoothing does not fully unsmooth the systematic component of private CRE fund-level returns.

In contrast, 3-step AR unsmoothed returns display little autocorrelation both at the fund-level and aggregate-level, with the highest autocorrelation being the aggregate 2-lags autocorrelation of 0.24 (p-value=2.3%). This result suggests the 3-step AR unsmoothing goes a long way in unsmoothing the systematic component of private CRE fund-level returns.

[34] Some private CRE funds also invest in mortgage-related assets. We also explore a third model that adds to our equity and real estate factors a mortgage factor based on public REITs. The results are very similar to the ones we report, and thus are omitted for brevity.

3.2.3 Performance of Private CRE Funds after 1-step and 3-step AR Unsmoothing

The upper panel of Table 7 reports private CRE fund statistics based on observed return, 1-step unsmoothed returns, and 3-step unsmoothed returns. Annualized expected returns are 5.0% and, by construction, do not change as we unsmooth returns, while (annualized) volatility starts at 13.1% for observed returns, increases to 25.3% after 1-step unsmoothing, and remains stable at 24.1% after 3-step unsmoothing. Interestingly, in the case of private CRE funds, R^2 increases as we 1-step unsmooth returns (from 3.2% to 7.9% in the 1-factor model and from 4.6% to 9.9% in the 2-factor model). R^2 increases even further as we move from 1-step to 3-step unsmoothing (from 7.9% to 13.7% in the 1-factor model and from 9.9% to 16.0% in the 2-factor model). These results indicate that unsmoothing has the potential to largely affect risk measurement and, consequently, estimated risk-adjusted performance.

Analyzing the performance relative to the 2-factor model (results for the 1-factor model are similar), private CRE funds seem to provide a substantial average α of 4.0% per year with roughly half the funds displaying statistically significant α. This result is a consequence of the extremely low average exposure to the real estate ($\beta_{re} = 0.02$) and equity ($\beta_e = 0.10$) public markets. After 1-step unsmoothing returns, the average exposures to the real estate ($\beta_{re} = 0.15$) and equity ($\beta_e = 0.15$) markets increase, driving the average α down to 2.4%, with 13.6% of the funds displaying statistically significant α. Average risk exposures increase even further after 3-step unsmoothing ($\beta_{re} = 0.21$ and $\beta_e = 0.26$) so that the average α becomes 0.8% and statistically insignificant (or significantly negative) for 90.9% of the funds.

The lower panel of Table 7 reports the same results as the upper panel, but focuses on how βs and α change as we unsmooth returns. The key message is that the increase in βs and decline in α obtained by 1-step unsmoothing returns is about the same as the improvement obtained when moving from 1-step to 3-step unsmoothing. The t_{stat} values in brackets also show that the average changes are highly significant from a statistical perspective.

Overall, the results indicate that the 3-step AR unsmoothing method provides a substantial improvement over 1-step AR unsmoothing in terms of measuring risk exposure and risk-adjusted performance of private CRE funds. The economic gains obtained by moving from 1-step to 3-step unsmoothing are roughly similar to the gains of unsmoothing in the first place.

4 Conclusion

In this paper, we find that traditional return unsmoothing methods used to recover economic return estimates from observed returns of illiquid funds do not fully unsmooth the systematic component of the returns, and thus understate systematic risk exposures and overstate risk-adjusted performance. To address this issue, we provide a novel 3-step return unsmoothing method and apply it to hedge funds and private CRE funds.

In doing so, we find that the measurement of risk exposures and risk-adjusted performance substantially improves. Overall, the improvement in risk adjusted performance is stronger for more illiquid funds and the increase in the estimated risk exposures is particularly strong when we evaluate private CRE funds, which invest in highly illiquid assets.

Our results demonstrate that it is economically important to properly unsmooth the returns of illiquid assets before measuring risk exposures. They also raise the possibility that some previously estimated alphas of funds that invest in illiquid assets are partially due to mismeasured systematic risk. We provide initial evidence consistent with this argument in the context of hedge funds and private CRE funds and leave further explorations in this direction to future research.

Data Appendix

A.1 Database of Hedge Funds

We combine data from two major commercial hedge fund databases to build our hedge fund dataset. Specifically, we merge the Lipper Trading Advisor Selection System database (hereafter TASS), accessed in June 2018, with the BarclayHedge database, accessed in April

2018, which produces a representative coverage of the hedge fund universe.[35] In 1994, both data providers started keeping a so-called graveyard database of funds that stopped reporting their returns. Hence, following the literature, we start our empirical analysis in 1995, which avoids issues associated with survivorship bias.

We apply some standard screens before including observations in the sample. We start by excluding observations with stale (for more than one quarter) Assets Under Management (AUM) or that have missing return or AUM. We then restrict the sample to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach \$5 million in AUM at some point in the sample.

To minimize the impact of small and idiosyncratic funds and mitigate incubation bias and backfill bias, we perform two standard data screens utilized in the literature. First, funds are only included after reaching the \$5 million AUM threshold for the first time, and they are not dropped from the sample in case they fall below this threshold after reaching it. Second, after unsmoothing the returns and estimating factor regressions to obtain each fund's risk loadings, we drop all backfilled returns for each fund before calculating average excess returns, volatilities, Sharpe ratios, and alphas. We use the algorithm proposed by Jorion and Schwarz (2019) in order to identify backfilled observations.[36]

After these initial screens, we merge the data from TASS and BarclayHedge and eliminate duplicate fund observations that exist when the same fund reports to both data providers.

[35] Joenväärä et al. (2019) combine and compare five different hedge fund databases that have been used in academic studies. Their analysis shows that the two datasets used in this study (TASS and BarclayHedge) together with Hedge Fund Research, have the most complete data in terms of the number of funds included and the lack of survivorship bias (after 1994). Joenväärä et al. (2019) also find that the average fund performance is similar across the five databases.

[36] Jorion and Schwarz (2019) find that dropping the first 12 monthly returns is the most common procedure used in the literature to deal with hedge fund incubation bias and backfill bias, but that this adjustment alone is not sufficient to properly measure performance and propose an algorithm that allows researchers to impute each fund's initial reporting date and thus address the entirety of backfilled returns. We follow their algorithm to input the initial reporting date and drop returns prior to it before calculating performance measures (dropping the first 12 monthly returns instead yield similar results as we demonstrate in Internet Appendix D). We still use the entire history of returns (as it is standard the literature) to estimate the unsmoothing process and factor models since the literature has found that autocorrelations and risk exposures are not affected by backfilled returns.

In order to do so, we start by identifying possible duplicate funds by fuzzy-matching fund names and fund company names across the two data sources. Then, following Joenväärä et al. (2019), we calculate the correlation of returns for each potential duplicate pair and identify it as a duplicate if the correlation is 99% or higher. Finally, for each duplicate pair identified, we keep the one that has the longest series of valid return and AUM data. The final sample starts in January 1995 and ends in December 2017.

Many of our results separate hedge funds based on their strategies. We identify strategies using the "primary strategy" variable reported by TASS and BarclayHedge. We exclude funds whose strategy is classified as "other" or whose primary strategy does not fall into any of the 12 investment styles identified by Joenväärä et al. (2019). There are only a few funds whose strategy is classified as short bias, hence we group them together with long/short funds. Finally, we exclude funds of funds, because these funds often invest in different fund categories and therefore they cannot be considered a homogeneous group. Table 1 (discussed earlier) provides the final list of strategies used in our analysis as well as the number of funds in each strategy.

A.2 Database of Commercial Real Estate Funds

Commercial Real Estate (CRE) includes all major real estate product types except owner-occupied, single-family homes. As an investment asset class, the importance of CRE has increased significantly over the last four decades. For instance, the average CRE target allocations for institutional investors have grown from around 2% in the 1980s to around 10% to 12% in 2019 (PREA (2019)).

Institutional investors can invest in public and private CRE in several ways. They can own and manage the assets directly (direct investments) or they can invest through intermediaries (separate accounts, joint ventures, club deals, commingled funds, or publicly traded REITs). Our analysis focuses on US private CRE funds, which are a subset of commingled funds. Our private CRE fund dataset comes from the National Council of Real Estate Investment Fiduciaries (NCREIF), which is the leading collector of institutional real estate investment information for properties within the US. We have quarterly data and the sample period goes from Q1 1994 through Q4 2017, with the starting date selected because there are few funds available before 1994 (starting in 1994 also makes the private CRE analysis period roughly consistent with the hedge fund analysis).

Our sample includes all private CRE funds that report return data to NCREIF and have at least 36 quarterly observations.[37] The sample consists of 66 funds that are observed, on average, for 56 quarters within the 96 quarters studied. At the end of Q4 2017, the sample contains 37 funds with approximately $233 billion in assets under management.

Our final dataset is composed of 29 open-end funds and 37 closed-end funds.[38] Open-end private CRE funds are similar to mutual funds and some hedge funds in the sense that they are open to issuing and redeeming shares on a regular basis (quarterly) at stated Net Asset Values (NAVs). In contrast, investors in closed-end private CRE funds typically only have their positions liquidated as the fund sells its underlying assets and returns capital. Besides asset sales, investors are primarily rewarded through cash distributions. In both types of funds, NAVs are based on the cumulative appraised values of the individual assets they hold, and thus NAV-based (i.e., observed) returns reflect (highly) smoothed returns. Therefore, these private CRE funds provide a natural asset class to explore the effects of our 3-step AR unsmoothing method.

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[37] We require at least 36 observations to be consistent with the standard used in the Hedge Fund literature and to assure the smoothing process is estimated with some precision. However, including all funds available in the dataset regardless of the number of observations yields similar results to the ones we report.

[38] Our results are similar if we focus on the 29 open-end private CRE funds in our dataset (out of the 66 private CRE funds available), as demonstrated in Internet Appendix D.

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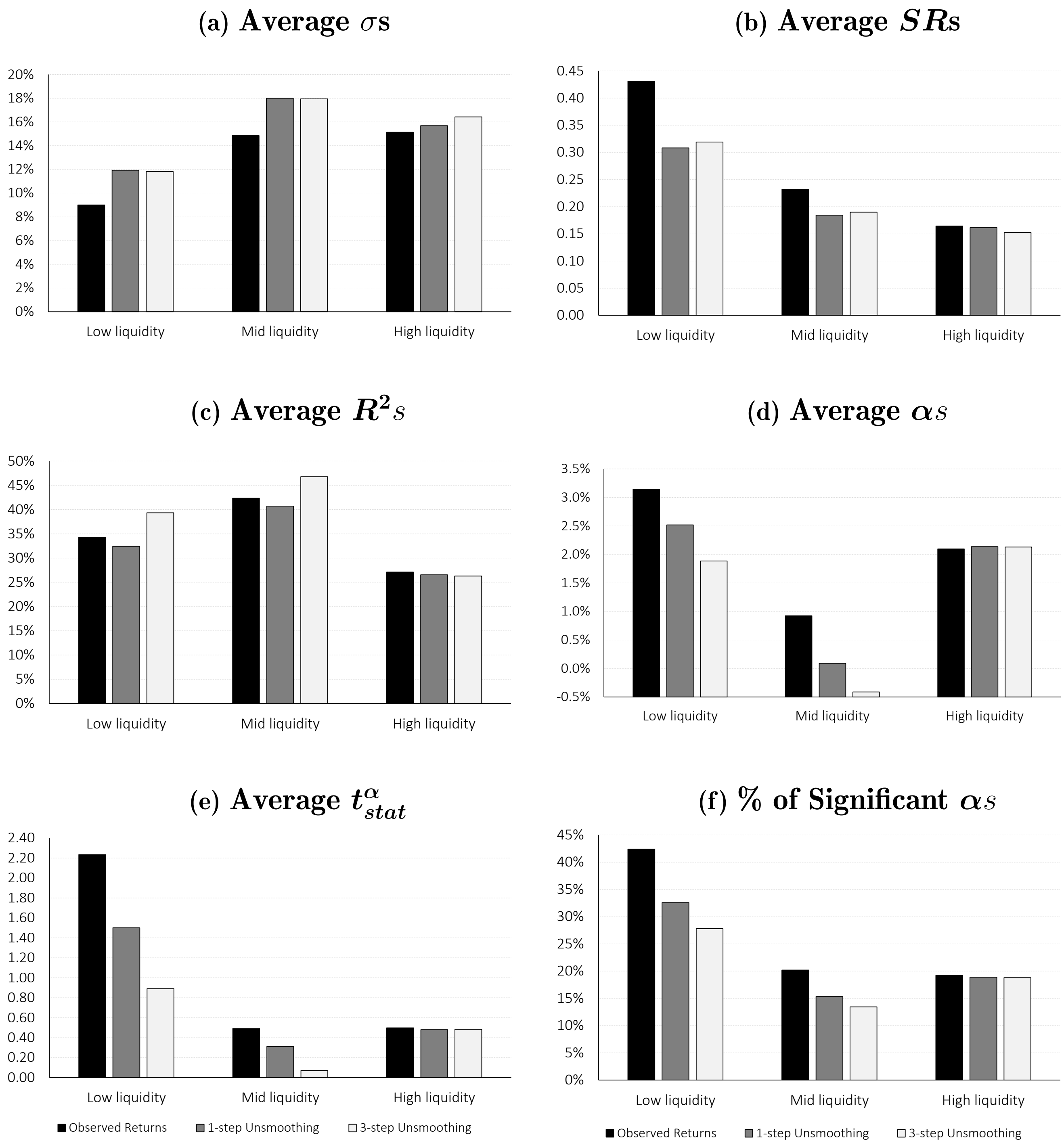


Figure 2
Hedge Fund Risk and Performance by Strategy Liquidity

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

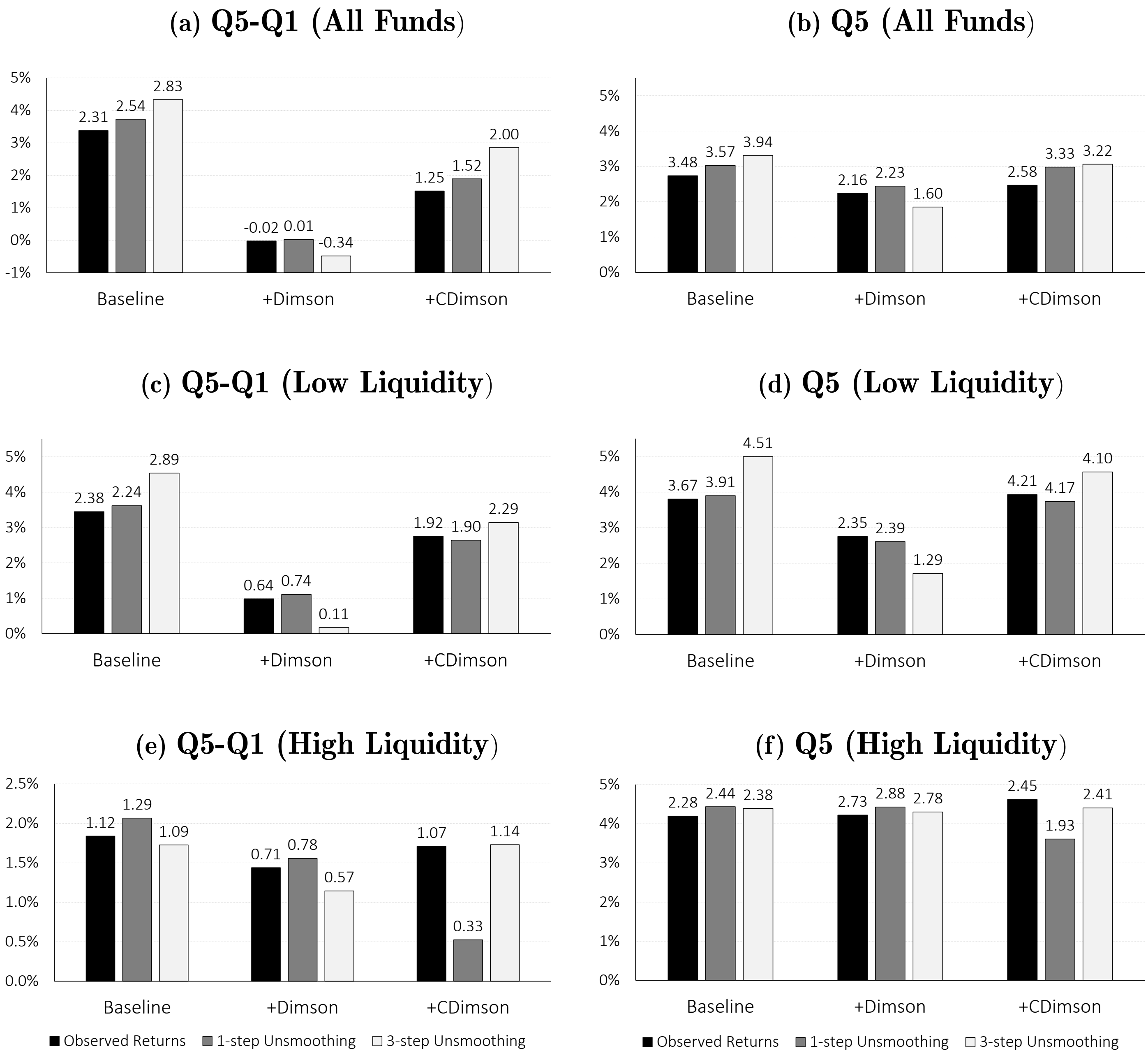


Figure 3

αs of Hedge Fund Quintiles Portfolios formed Based on the t-stat of Past αs

The figure plots αs (with their t_{stat} on the top of each bar) of quintile portfolios formed by sorting hedge funds based on the t-stat of their past αs (on a 24-month rolling window) measured using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns (with "+Dimson" further applying Dimson (1979) and "+CDimson" further applying our constrained Dimson method). Panels (a), (c), and (e) focus on αs for a strategy that buys the highest and sells the lowest past α quintiles. Panels (b), (d), and (f) focus only on the highest past α quintiles. Panels (a) and (b) use all funds during the sorting procedure whereas Panels (c) and (d) use only funds in the low liquidity strategies and Panels (e) and (f) use only funds in the high liquidity strategies. αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017, but the first portfolio formation is on December 2000 so that we have at least six years of data to unsmooth the hedge fund returns. See Section 1 for unsmoothing methods and Subsections 2.1 and 2.2 for further empirical details.

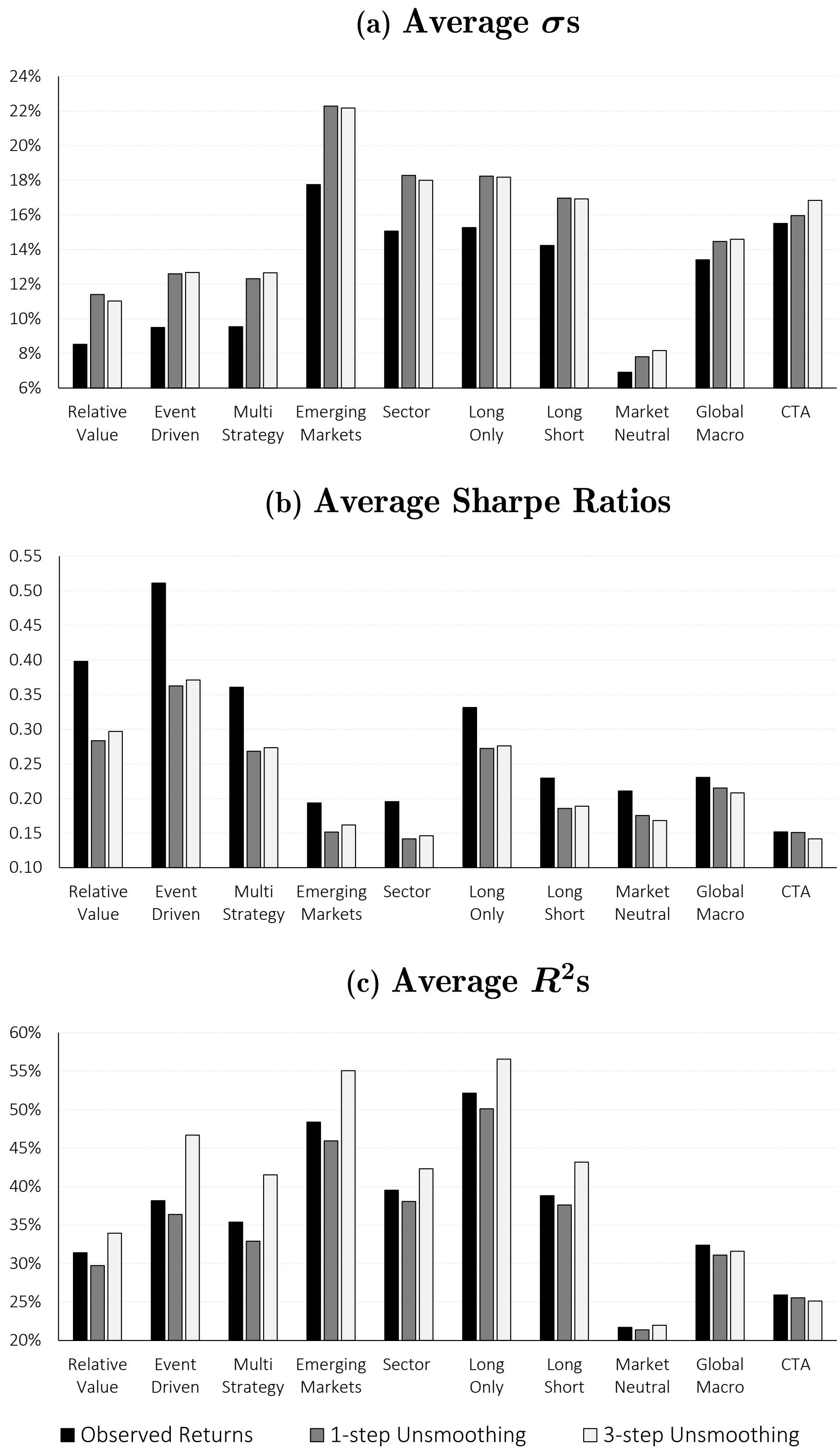
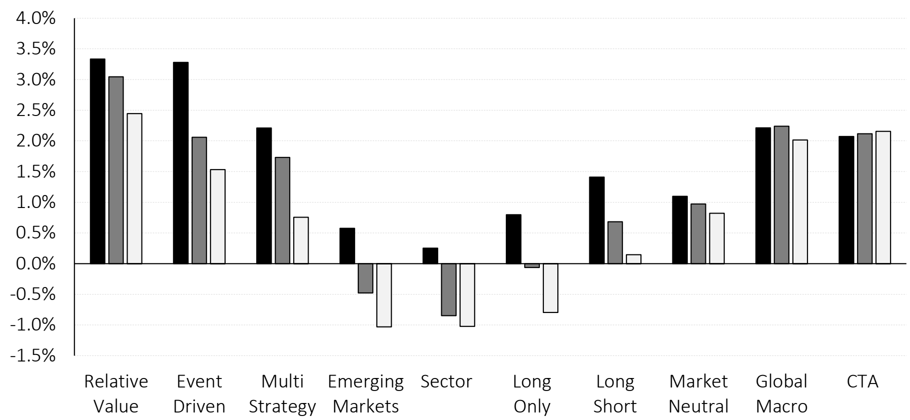


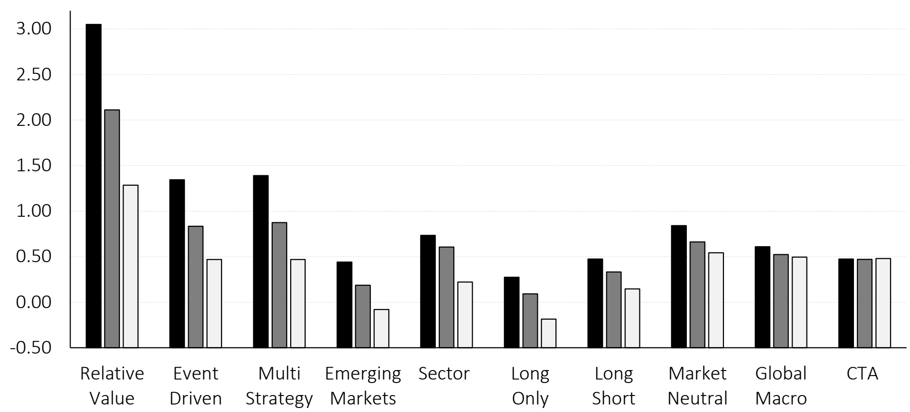
Figure 4
Hedge Fund Risk and Performance by Strategy

The figure plots average fund-level results by hedge fund strategy using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

(d) Average αs



(e) Average t^{α}_{stat}



(f) % of Funds with α Significant at 10%

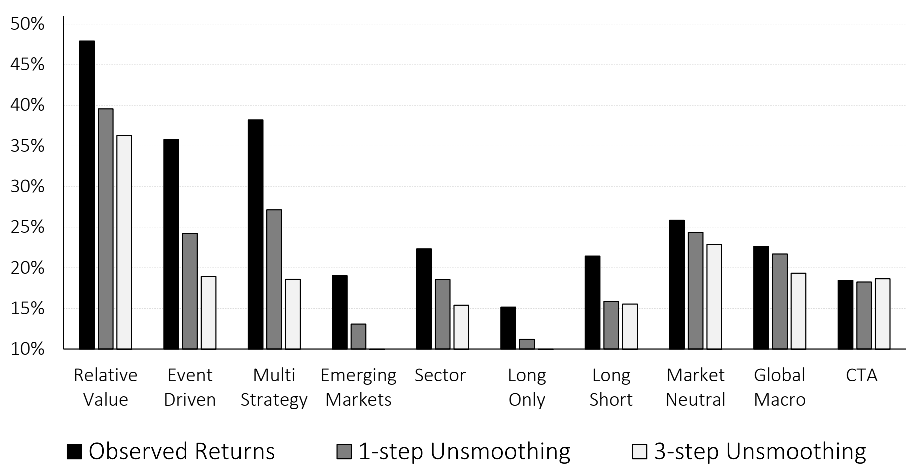


Figure 4 (Cont'd)
Hedge Fund Risk and Performance by Strategy

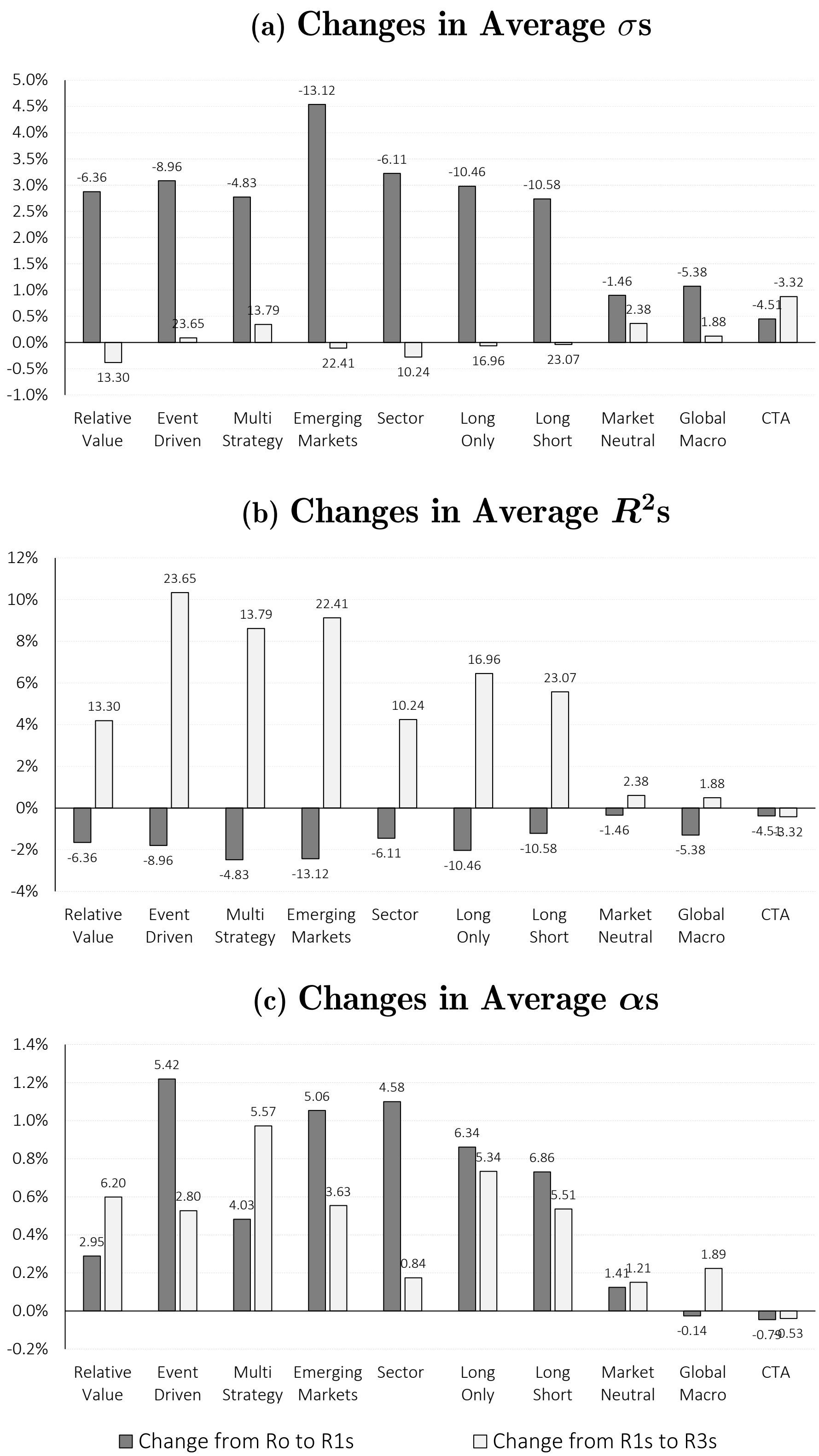


Figure 5
Changes in Hedge Fund Risk and Performance by Strategy

The figure plots increases in R^2 (and declines in Sharpe Ratios and α) with their t_{stat} by hedge fund strategy as we move (i) from observed to 1-step unsmoothed returns and (ii) from 1-step to 3-step unsmoothed returns. R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

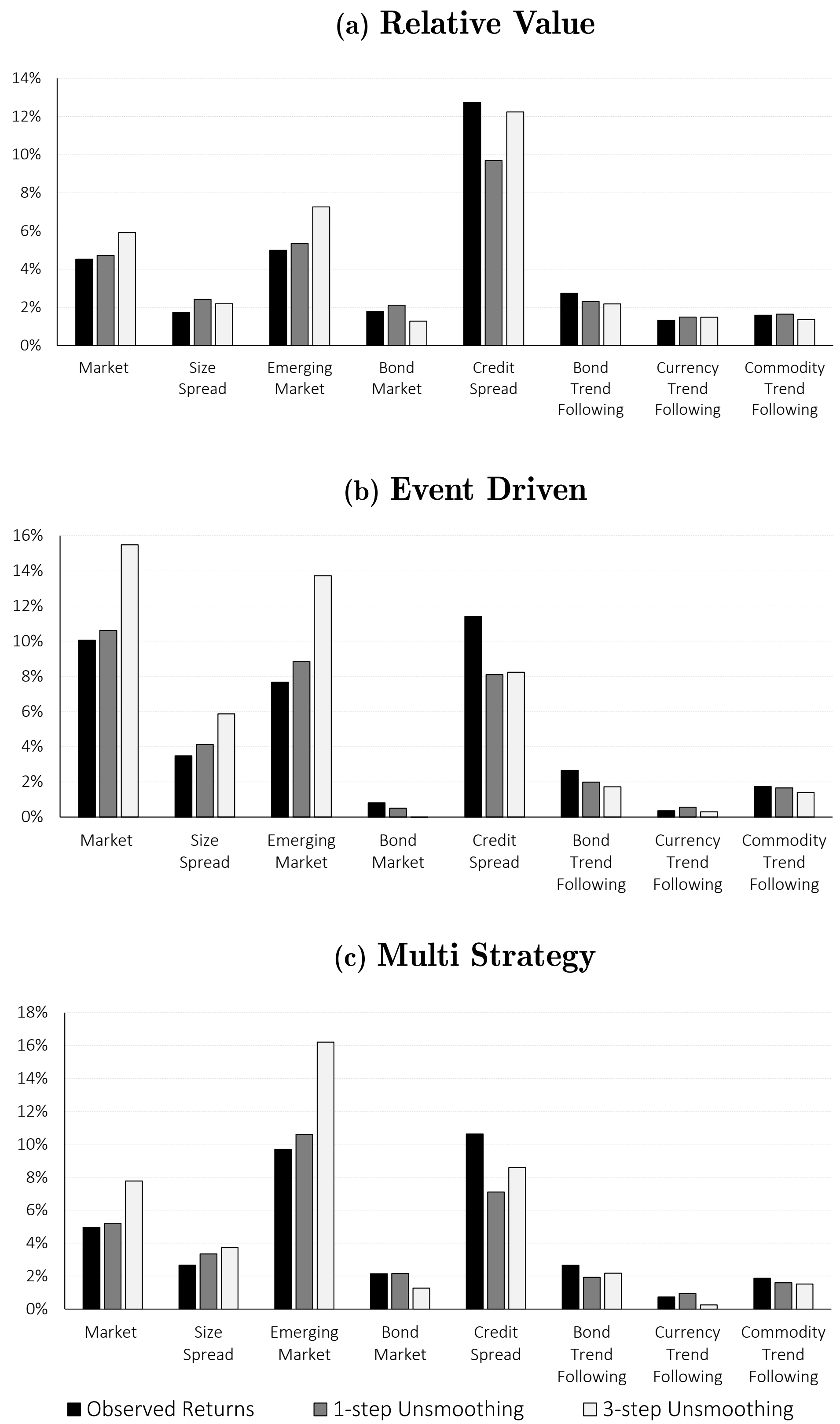
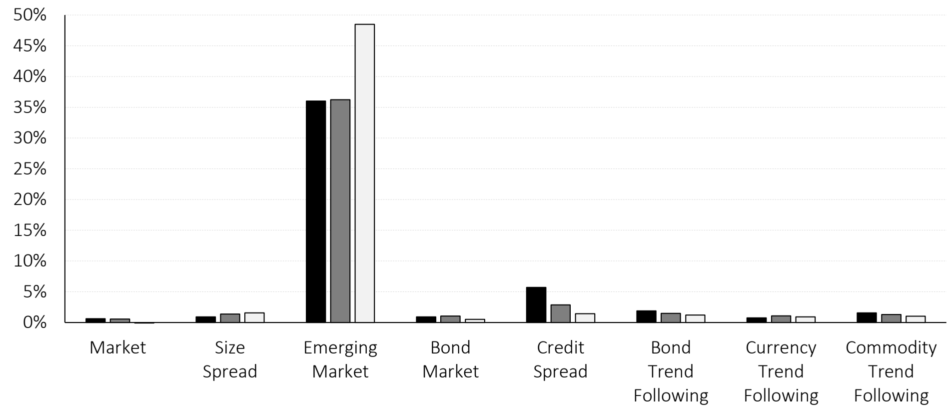


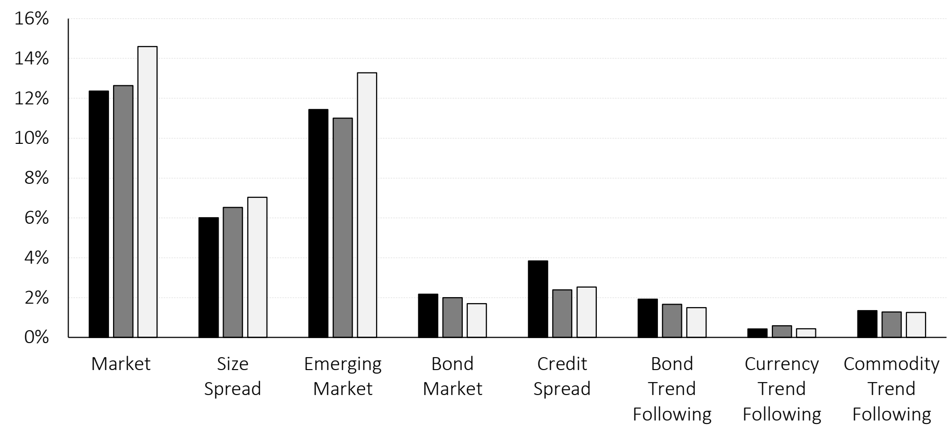
Figure 6
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

The figure plots, for each hedge fund strategy, the average R^2 from factor regressions decomposed into the effect of each risk factor (see Equation 18). We use the risk factors in the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

(d) Emerging Markets



(e) Sector



(f) Long Only

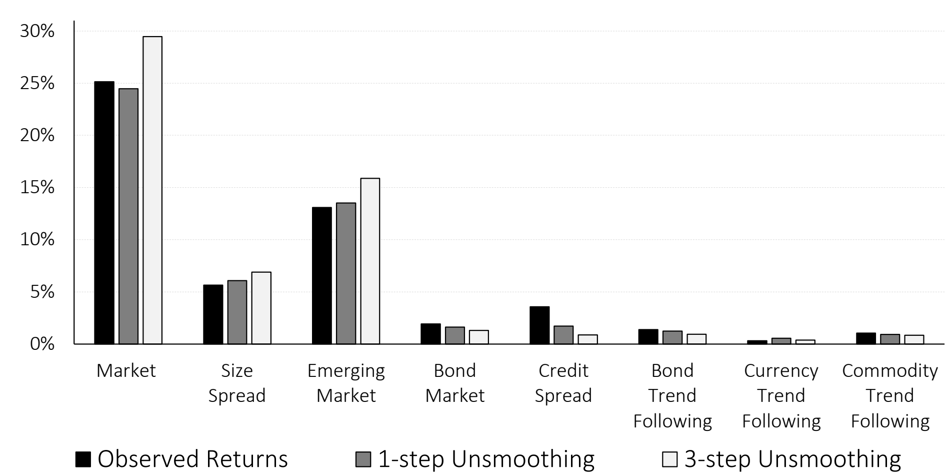
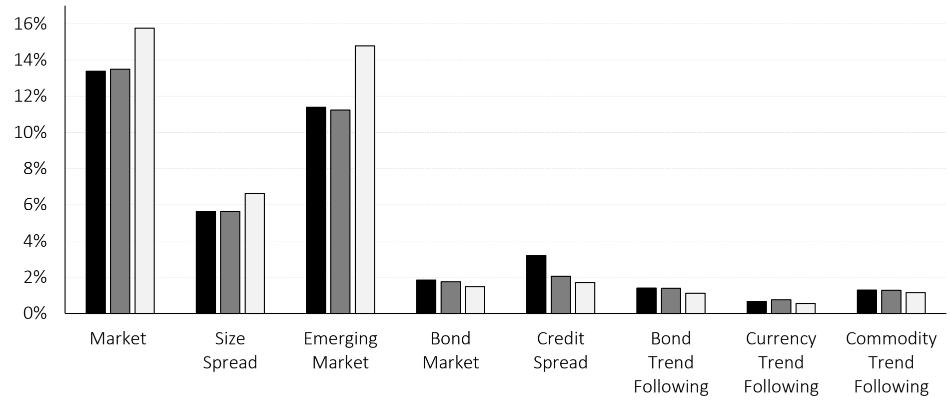
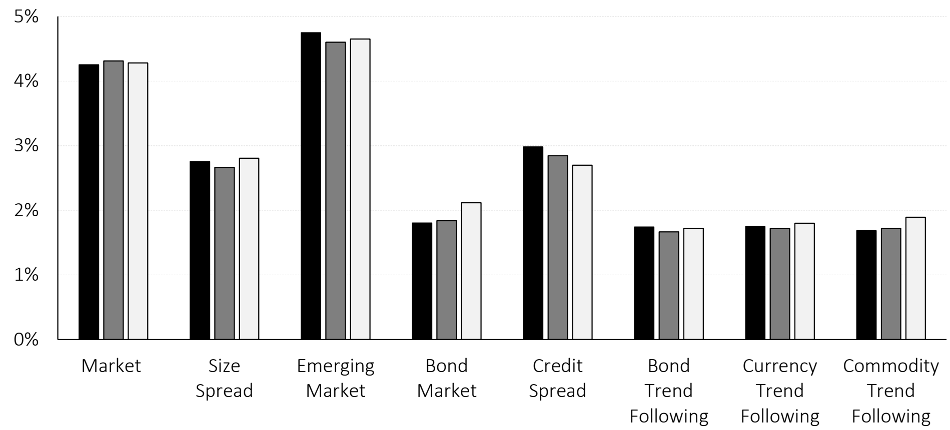


Figure 6 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

(g) Long Short



(h) Market Neutral



(i) Global Macro

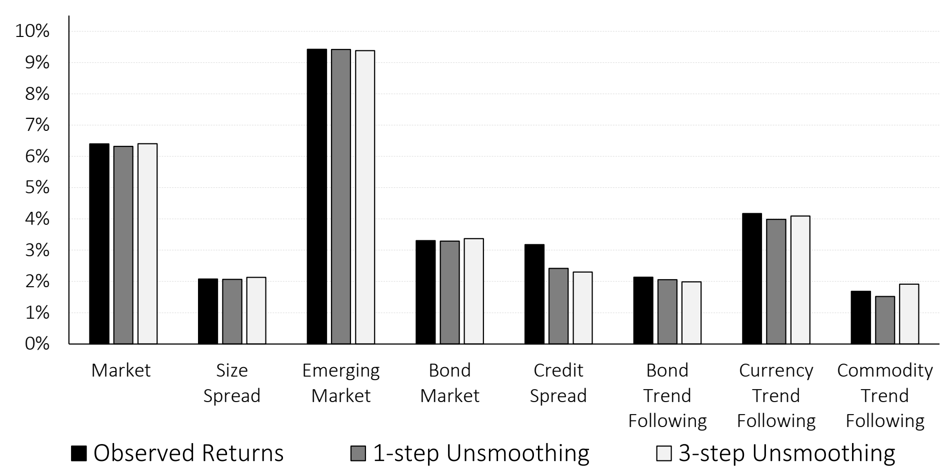


Figure 6 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

(j) CTA

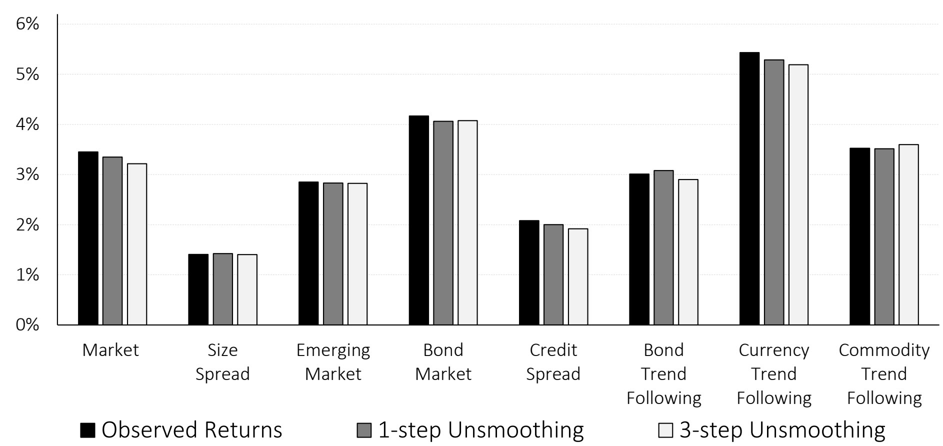


Figure 6 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

Table 1
Hedge Fund Strategies and Summary Statistics

The table reports the total number of hedge funds (N), the average number of months per hedge fund ($\overline{T}$) and other average fund-level statistics for hedge fund returns by strategy, with strategies sorted based on the 1st order (average fund-level) return autocorrelation. All statistics are based on observed returns. The sample goes from January 1995 to December 2017 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Subsection 2.1 for further empirical details.

Hedge Fund	Sample Size		Fund-level			
Strategies	N	$\overline{T}$	Cor_1	σ	$\mathbb{E}[r]$	$\mathbb{E}[r]/\sigma$
Relative Value	670	85	0.29	8.5%	3.4%	0.40
Event Driven	433	96	0.25	9.5%	4.9%	0.51
Multi Strategy	199	97	0.22	9.5%	3.4%	0.36
Emerging Mkts	657	92	0.18	17.7%	3.4%	0.19
Sector	318	90	0.13	15.1%	2.9%	0.20
Long Only	455	102	0.11	15.3%	5.1%	0.33
Long-Short	965	92	0.10	14.2%	3.3%	0.23
Market Neutral	201	79	0.09	6.9%	1.5%	0.21
Global Macro	212	93	0.06	13.4%	3.1%	0.23
CTA	959	94	0.00	15.5%	2.4%	0.15

Table 2
Autocorrelations of Hedge Fund Returns

The table reports average fund-level autocorrelations (from 1 to 3 months) for hedge fund returns by strategy, with strategies sorted based on the 1st order (average fund-level) observed return autocorrelation. Reported autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. For the columns under "Two Group 3-step Unsmoothing", we rank funds based on the alphabetic order of their names within strategy at the year of the fund inception, create two groups based on even vs odd ranks, and use the aggregate index from the second (first) group to unsmooth the returns of the first (second) group, with the table reporting results based on the unsmoothed returns from the first group (the results from the second group are similar). The numbers in parentheses reflect the fraction of funds with the respective autocorrelation being significant at 10% level. The sample goes from January 1995 to December 2017 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

Hedge Fund Strategies	Observed Returns			1-step Unsmoothing			3-step Unsmoothing			Two Group 3-step Unsmoothing		
	Cor_1	Cor_2	Cor_3	Cor_1	Cor_2	Cor_3	Cor_1	Cor_2	Cor_3	Cor_1	Cor_2	Cor_3
Relative Value	0.29	0.18	0.19	0.02	0.04	0.09	0.04	-0.03	0.10	0.03	-0.02	0.07
	(61%)	(38%)	(35%)	(3%)	(3%)	(4%)	(17%)	(7%)	(17%)	(14%)	(6%)	(8%)
Event Driven	0.25	0.13	0.10	0.01	0.01	0.01	-0.01	0.01	0.01	-0.02	-0.01	0.01
	(62%)	(37%)	(32%)	(2%)	(2%)	(3%)	(11%)	(7%)	(9%)	(10%)	(4%)	(9%)
Multi Strategy	0.22	0.11	0.07	-0.01	-0.01	0.00	0.00	-0.03	0.00	0.00	-0.04	-0.02
	(57%)	(37%)	(25%)	(1%)	(2%)	(1%)	(11%)	(5%)	(6%)	(10%)	(4%)	(2%)
Emerging Mkts	0.18	0.07	0.06	0.00	-0.01	0.00	-0.04	0.00	0.03	-0.05	0.00	0.02
	(47%)	(24%)	(16%)	(1%)	(0%)	(1%)	(7%)	(8%)	(8%)	(5%)	(10%)	(6%)
Sector	0.13	0.05	0.02	0.00	-0.01	-0.02	-0.01	0.01	-0.04	-0.02	0.01	-0.02
	(36%)	(19%)	(10%)	(1%)	(0%)	(1%)	(7%)	(5%)	(2%)	(5%)	(4%)	(1%)
Long Only	0.11	0.03	0.04	0.00	-0.01	-0.01	-0.03	-0.01	-0.01	-0.01	-0.01	-0.01
	(37%)	(10%)	(10%)	(1%)	(0%)	(0%)	(6%)	(3%)	(2%)	(6%)	(4%)	(3%)
Long-Short	0.10	0.03	0.04	-0.01	-0.02	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	-0.02
	(30%)	(14%)	(12%)	(1%)	(1%)	(2%)	(5%)	(5%)	(3%)	(5%)	(5%)	(3%)
Market Neutral	0.09	0.03	0.02	-0.01	-0.02	-0.02	-0.01	-0.03	-0.04	0.00	-0.06	-0.04
	(27%)	(13%)	(14%)	(1%)	(1%)	(1%)	(4%)	(3%)	(2%)	(1%)	(0%)	(4%)
Global Macro	0.06	0.00	-0.02	-0.01	-0.02	-0.03	-0.03	-0.01	-0.03	-0.04	-0.01	-0.02
	(23%)	(11%)	(7%)	(2%)	(2%)	(1%)	(3%)	(5%)	(2%)	(1%)	(3%)	(2%)
CTA	0.00	0.00	-0.02	-0.03	-0.02	-0.03	-0.04	-0.02	-0.04	-0.04	-0.02	-0.03
	(9%)	(9%)	(5%)	(1%)	(1%)	(0%)	(1%)	(4%)	(3%)	(1%)	(3%)	(2%)

Table 3
Autocorrelations of Aggregated Hedge Fund Returns

The table reports autocorrelations (from 1 to 3 months) for returns of each hedge fund strategy index (i.e., equal-weighted portfolio of all funds following the given strategy), with strategies sorted based on the 1st order (average fund-level) observed return autocorrelation. Reported autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. For the columns under "Two Group 3-step Unsmoothing", we rank funds based on the alphabetic order of their names within strategy at the year of the fund inception, create two groups based on even vs odd ranks, and use the aggregate index from the second (first) group to unsmooth the returns of the first (second) group, with the table reporting results based on the unsmoothed returns from the first group (the results from the second group are similar). The numbers in parentheses reflect the p-value for the test of whether the respective autocorrelation differs from zero. The sample goes from January 1995 to December 2017 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

Hedge Fund Strategies	Observed Returns			1-step Unsmoothing			3-step Unsmoothing			Two Group 3-step Unsmoothing		
	Cor_1	Cor_2	Cor_3	Cor_1	Cor_2	Cor_3	Cor_1	Cor_2	Cor_3	Cor_1	Cor_2	Cor_3
Relative Value	0.51	0.28	0.14	0.27	0.12	0.04	0.01	0.03	0.10	0.00	0.05	0.08
	(0%)	(0%)	(2%)	(0%)	(5%)	(51%)	(85%)	(57%)	(9%)	(97%)	(42%)	(17%)
Event Driven	0.46	0.25	0.18	0.21	0.09	0.08	0.03	0.03	0.05	-0.02	0.01	0.01
	(0%)	(0%)	(0%)	(0%)	(15%)	(20%)	(62%)	(68%)	(46%)	(70%)	(82%)	(83%)
Multi Strategy	0.48	0.34	0.24	0.23	0.15	0.11	0.06	0.03	0.05	0.02	-0.02	0.01
	(0%)	(0%)	(0%)	(0%)	(1%)	(8%)	(33%)	(62%)	(41%)	(78%)	(72%)	(83%)
Emerging Mkts	0.32	0.11	0.06	0.14	0.04	0.02	0.01	0.02	0.03	-0.02	0.03	0.03
	(0%)	(7%)	(30%)	(2%)	(48%)	(70%)	(93%)	(69%)	(64%)	(74%)	(66%)	(67%)
Sector	0.21	0.05	0.04	0.07	-0.01	0.03	0.02	0.00	0.00	0.04	0.04	-0.05
	(0%)	(39%)	(49%)	(28%)	(87%)	(64%)	(74%)	(96%)	(97%)	(55%)	(48%)	(38%)
Long Only	0.22	0.05	0.04	0.09	0.02	0.00	0.00	0.00	0.01	0.02	-0.03	0.00
	(0%)	(41%)	(56%)	(12%)	(71%)	(96%)	(99%)	(100%)	(92%)	(76%)	(64%)	(94%)
Long-Short	0.23	0.11	0.08	0.10	0.06	0.04	0.00	0.00	0.02	-0.02	-0.02	0.00
	(0%)	(7%)	(18%)	(12%)	(36%)	(52%)	(98%)	(95%)	(74%)	(70%)	(80%)	(97%)
Market Neutral	0.18	0.10	0.06	0.09	0.07	0.03	0.03	0.04	0.04	0.01	-0.02	0.13
	(0%)	(9%)	(30%)	(14%)	(25%)	(61%)	(57%)	(54%)	(47%)	(92%)	(73%)	(4%)
Global Macro	0.05	-0.03	0.00	-0.01	-0.03	0.01	0.01	-0.01	0.01	0.01	-0.08	0.04
	(37%)	(63%)	(99%)	(93%)	(64%)	(90%)	(87%)	(84%)	(93%)	(87%)	(18%)	(47%)
CTA	-0.02	-0.03	0.00	-0.04	-0.01	0.02	-0.01	0.03	0.01	-0.05	0.00	0.02
	(71%)	(61%)	(98%)	(54%)	(91%)	(70%)	(88%)	(57%)	(86%)	(37%)	(98%)	(79%)

Table 4
Autocorrelations of β Sorted Hedge Fund Portfolio Returns

The table reports first order return autocorrelations of 24 portfolios. To construct the portfolios, we start by estimating the Fung and Hsieh (2001) factor model betas for each fund using observed returns. We then sort all funds into three groups based on their exposures to each of the eight factors in the model, which yields $3 \times 8 = 24$ portfolios. Autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns, with fund-level unsmoothed returns being the same ones used in Table 3. The numbers in parentheses reflect the p-value for the test of whether the respective autocorrelation differs from zero. The sample goes from January 1995 to December 2017 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach \$5 million in AUM at some point in the sample, with fund observations included only after reaching the \$5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Subsection 2.1 for further empirical details.

Hedge Fund	**Observed Returns**			**1-step Unsmoothing**			**3-step Unsmoothing**		
Strategies	Low β	Mid β	High β	Low β	Mid β	High β	Low β	Mid β	High β
Market	0.15	0.33	0.21	0.06	0.16	0.10	-0.01	0.05	0.03
	(1%)	(0%)	(0%)	(30%)	(1%)	(11%)	(86%)	(44%)	(66%)
Size Spread	0.08	0.27	0.24	0.00	0.13	0.11	-0.05	0.01	0.04
	(18%)	(0%)	(0%)	(94%)	(4%)	(7%)	(39%)	(92%)	(51%)
Emerging Market	0.20	0.30	0.23	0.12	0.16	0.08	-0.03	0.04	0.02
	(0%)	(0%)	(0%)	(6%)	(1%)	(17%)	(67%)	(50%)	(73%)
Bond Market	0.16	0.32	0.25	0.08	0.16	0.10	0.02	0.03	0.02
	(1%)	(0%)	(0%)	(20%)	(1%)	(8%)	(80%)	(64%)	(79%)
Credit Spread	0.35	0.26	0.15	0.20	0.12	0.05	0.12	0.02	-0.04
	(0%)	(0%)	(1%)	(0%)	(4%)	(38%)	(5%)	(80%)	(50%)
Bond Trend Following	0.23	0.33	0.13	0.10	0.17	0.05	0.00	0.04	0.00
	(0%)	(0%)	(3%)	(11%)	(1%)	(39%)	(100%)	(49%)	(97%)
Currency Trend Following	0.35	0.28	0.08	0.18	0.14	0.01	0.07	0.02	-0.03
	(0%)	(0%)	(18%)	(0%)	(3%)	(85%)	(25%)	(77%)	(58%)
Commodity Trend Following	0.36	0.33	0.04	0.20	0.16	-0.03	0.11	0.03	-0.08
	(0%)	(0%)	(54%)	(0%)	(1%)	(65%)	(7%)	(61%)	(19%)

Table 5
Unsmoothing Techniques in Simulations

The table reports autocorrelations and alphas in simulations. In Panel A, we simulate returns for a panel of 670 funds over a 85 month period. The monthly economic returns of each fund j satisfy $R_{j,t} = \alpha_j + \beta_j \cdot f_t + \varepsilon_{j,t}$, where $\alpha_j \sim N(\mu_\alpha, \sigma_\alpha^2)$, $\beta_j \sim N(1, \sigma_\beta^2)$, $f_t \sim N(\mu_f, \sigma_f^2)$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. We then smooth these returns according to the underlying smoothing process of our 3-step method (i.e., Equation 5) with $H = L = 1$ (i.e., MA(1) smoothing), $\phi_j^{(1)} = \phi^{(1)}$, and $\pi_j^{(1)} = \pi^{(1)}$ for simplicity. Finally, we estimate economic returns for each fund in the panel using the 1- and 3-step unsmoothing methods and study the properties of observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. Columns with "+Dimson" apply the Dimson (1979) method (with one lag) to the given return measure. We report the average results obtained from 1,000 simulations of this panel of funds. The first column shows results for a specification in which the aggregate and fund-specific components of returns are smoothed with the same intensity ($\phi^{(1)} = \pi^{(1)} = 0.3$), the second column considers a specification in which the fund-level component of returns is smoothed less than the aggregate-level component ($\phi^{(1)} = 0.2$ and $\pi^{(1)} = 0.4$), and the third column considers an alternative scenario in which the fund-level component of returns is smoothed more than the aggregate-level component ($\phi^{(1)} = 0.4$ and $\pi^{(1)} = 0.2$). See Section 1 for unsmoothing methods and Subsection 1.4 for further simulation details.

PANEL A: MA(1) with 1-Factor Model

	$\phi^{(1)} = \pi^{(1)}$		$\phi^{(1)} < \pi^{(1)}$		$\phi^{(1)} > \pi^{(1)}$	
$\phi^{(1)}$	0.30		0.20		0.40	
$\pi^{(1)}$	0.30		0.40		0.20	
$Cor_1(R_o)$	0.35		0.31		0.32	
$Cor_1(R_{1s})$	0.00		0.00		0.00	
$Cor_1(R_{3s})$	-0.01		-0.01		-0.02	
$Cor_1(\overline{R}_o)$	0.36		0.46		0.24	
$Cor_1(\overline{R}_{1s})$	0.01		0.24		-0.14	
$Cor_1(\overline{R}_{3s})$	0.00		-0.01		0.00	
	Standard	+Dimson	Standard	+Dimson	Standard	+Dimson
$\widehat{\alpha}_o$	2.1%	0.0%	2.8%	0.0%	1.4%	0.0%
$\widehat{\alpha}_{1s}$	0.1%	0.0%	1.3%	-0.4%	-0.7%	0.3%
$\widehat{\alpha}_{3s}$	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
$\lvert\widehat{\alpha}_o\rvert$	2.3%	0.6%	3.0%	0.6%	1.6%	0.6%
$\lvert\widehat{\alpha}_{1s}\rvert$	0.5%	0.7%	1.4%	0.8%	0.9%	0.7%
$\lvert\widehat{\alpha}_{3s}\rvert$	0.3%	0.7%	0.4%	0.8%	0.4%	0.8%
$100 \times \widehat{\alpha}_o^2$	0.074	0.006	0.124	0.007	0.042	0.006
$100 \times \widehat{\alpha}_{1s}^2$	0.004	0.009	0.029	0.014	0.014	0.008
$100 \times \widehat{\alpha}_{3s}^2$	0.002	0.011	0.003	0.011	0.002	0.011

Table 5 (Cont'd)
Unsmoothing Techniques in Simulations

The table reports autocorrelations and alphas in simulations. In Panel B, we simulate returns for a panel of 670 funds over a 85 month period or a 170 month period. The monthly economic returns of each fund j satisfy $R_{j,t} = \alpha_j + \beta_j' f_t + \varepsilon_{j,t}$ with β_j representing a vector of eight fund-specific risk exposures and f_t representing a vector of eight risk factors. Simulated fund-level parameters (i.e., β_j and the vectors of smoothing parameters, π_j and ϕ_j) are bootstrapped with replacement from the corresponding joint empirical distribution of parameters based on relative value funds estimated under the 3-step method with three lags. The panel of factor returns is common to all funds within a given simulation run and is bootstrapped with replacement from the time-series of the 8 FH factors. For simplicity, "true alpha" (α_j) is set to zero for all funds. Finally, the standard deviation of $\varepsilon_{j,t}$ is set to 2.2%, which ensures that the smoothed simulated fund returns have the same average volatility of the reported returns of relative value funds. Finally, we estimate economic returns for each fund in the panel using the 1- and 3-step unsmoothing methods and study the properties of observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. Columns with "+Dimson" also apply the Dimson (1979) method (with three lags) while columns with "+CDimson" also apply the constrained Dimson method (see Footnote 20). We report the average results obtained from 1,000 simulations of this panel of funds. See Section 1 for unsmoothing methods and Subsection 1.4 for further simulation details.

PANEL B: MA(3) with 8-Factor Model

	$T = 85$ Months			$T = 170$ Months		
$Cor_1(R_o)$	0.13			0.14		
$Cor_1(R_{1s})$	-0.01			0.00		
$Cor_1(R_{3s})$	-0.01			-0.01		
$Cor_1(\overline{R}_o)$	0.46			0.47		
$Cor_1(\overline{R}_{1s})$	0.28			0.28		
$Cor_1(\overline{R}_{3s})$	0.00			0.01		
	Standard	+Dimson	+CDimson	Standard	+Dimson	+CDimson
$\widehat{\alpha}_o$	0.7%	0.0%	0.1%	0.7%	0.0%	0.2%
$\widehat{\alpha}_{1s}$	0.4%	0.0%	0.1%	0.4%	0.1%	0.1%
$\widehat{\alpha}_{3s}$	0.0%	0.0%	-0.3%	0.0%	0.0%	-0.2%
$\lvert\widehat{\alpha}_o\rvert$	1.1%	2.4%	1.3%	1.0%	1.7%	1.0%
$\lvert\widehat{\alpha}_{1s}\rvert$	0.9%	2.6%	1.4%	0.7%	1.9%	1.0%
$\lvert\widehat{\alpha}_{3s}\rvert$	0.8%	2.7%	1.3%	0.6%	1.9%	0.9%
$100 \times \widehat{\alpha}_o^2$	0.024	0.115	0.039	0.019	0.074	0.026
$100 \times \widehat{\alpha}_{1s}^2$	0.016	0.141	0.043	0.011	0.091	0.027
$100 \times \widehat{\alpha}_{3s}^2$	0.012	0.147	0.039	0.008	0.095	0.027

Table 6
Autocorrelations of Private CRE Fund Returns

The table reports (average fund-level and aggregate) autocorrelations (from 1 to 4 quarters) for US private commercial real estate (CRE) funds. Autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. In the upper panel, the numbers in parentheses reflect the fraction of funds with the respective autocorrelation being significant at 10% level. In the lower panel, the numbers in parentheses reflect the p-value for the test of whether the respective autocorrelation differs from zero. Partial autocorrelations refer to coefficients from a multivariate regression that includes lagged returns from up to 4 quarters. The sample goes from Q1 1994 through Q4 2017 and is restricted to private CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used and Subsection 3.2 for further empirical details.

		Autocorrelations				**Partial Autocorrelations**			
	Returns	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4
Fund Level	**Observed**	0.45	0.39	0.23	0.21	0.45	0.10	0.01	0.01
		(63.6%)	(74.2%)	(51.5%)	(40.9%)	(59.1%)	(21.2%)	(9.1%)	(12.1%)
	1-step	-0.03	0.02	0.04	0.10	-0.02	0.03	0.05	0.08
		(0.0%)	(12.1%)	(7.6%)	(21.2%)	(0.0%)	(12.1%)	(7.6%)	(21.2%)
	3-step	-0.04	0.13	-0.05	0.17	-0.02	0.11	-0.04	0.14
		(0.0%)	(27.3%)	(0.0%)	(31.8%)	(0.0%)	(12.1%)	(0.0%)	(21.2%)
Aggregate Level	**Observed**	0.75	0.67	0.42	0.31	0.65	0.42	-0.33	0.00
		(0.0%)	(0.0%)	(0.0%)	(0.3%)	(0.0%)	(0.1%)	(0.9%)	(99.7%)
	1-step	0.46	0.31	0.12	0.07	0.41	0.16	-0.09	0.02
		(0.0%)	(0.3%)	(26.6%)	(48.3%)	(0.0%)	(18.3%)	(45.0%)	(86.8%)
	3-step	-0.02	0.24	-0.09	0.19	0.02	0.20	-0.09	0.14
		(86.7%)	(2.3%)	(37.7%)	(6.9%)	(84.2%)	(6.0%)	(38.5%)	(18.5%)

Table 7
Risk and Performance of Private CRE Funds

The table reports (average fund-level) statistics related to the risk and performance of US private commercial real estate (CRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from Q1 1994 through Q4 2017 and is restricted to private CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used and Subsection 3.2 for further empirical details.

Statistics are	**Raw Performance**			**1-Factor Model**			**2-Factor Model**			
Related to	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}	R^2	α	β_{re}	β_e	R^2
Observed, R_o	5.0%	13.1%	0.38	4.3%	0.07	3.2%	4.0%	0.02	0.10	4.6%
				(53.0%)	(27.3%)		(51.5%)	(9.1%)	(7.6%)	
1-step, R_{1s}	5.0%	25.3%	0.20	2.7%	0.22	7.9%	2.4%	0.15	0.15	9.9%
				(13.6%)	(47.0%)		(13.6%)	(30.3%)	(10.6%)	
3-step, R_{3s}	5.0%	24.1%	0.21	1.6%	0.34	13.7%	0.8%	0.21	0.26	16.0%
				(9.1%)	(89.4%)		(9.1%)	(30.3%)	(13.6%)	
From R_o to R_{1s}	0.0%	12.1%	-0.18	-1.6%	0.16	4.8%	-1.7%	0.13	0.05	5.3%
				[-8.71]	[8.40]		[-7.36]	[6.44]	[1.62]	
From R_{1s} to R_{3s}	0.0%	-1.1%	0.01	-1.2%	0.12	5.7%	-1.6%	0.06	0.12	6.0%
				[-3.93]	[3.70]		[-4.42]	[1.95]	[4.03]	
From R_o to R_{3s}	0.0%	11.0%	-0.17	-2.7%	0.27	10.5%	-3.2%	0.19	0.17	11.3%
				[-11.73]	[11.01]		[-11.34]	[9.62]	[8.38]	

Internet Appendix

“Unsmoothing Returns of Illiquid Funds”

By Spencer Couts, Andrei S. Gonçalves, and Andrea Rossi

This Internet Appendix contains further results that supplement the main findings in the paper. Section A shows that our 3-step unsmoothing method produces consistent alpha and beta estimates despite abstracting away from the exact structure of our Bayesian economic framework, Section B explores a method to unsmooth returns without pre-specified fund categories (e.g., hedge fund strategies), Section C explores the impact of time-varying expected returns on our 3-step unsmoothing method, and Section D performs several robustness checks that modify some of our empirical specifications related to sample construction and econometric design.

A The Impact of $\xi_{j,t} \neq 0$ in the 3-Step Unsmoothing Method

Subsection 1.5 in the main text shows that, in a simple Bayesian learning model, observed returns can be written as

$$\begin{aligned} r^o_{j,t} &= \pi_j \cdot \overline{r}_t \;+\; (1-\pi_j)\cdot \overline{r}_{t-1} \;+\; \phi_j \cdot \widetilde{r}_{j,t} \;+\; (1-\phi_j)\cdot \widetilde{r}_{j,t-1} \;+\; \xi_{j,t} \\ &= \mu_j \;+\; \pi_j \cdot \overline{\eta}_t \;+\; (1-\pi_j)\cdot \overline{\eta}_{t-1} \;+\; \phi_j \cdot \widetilde{\eta}_{j,t} \;+\; (1-\phi_j)\cdot \widetilde{\eta}_{j,t-1} \;+\; \xi_{j,t} \end{aligned} \tag{IA.1}$$

where
$\overline{\eta}_t \overset{iid}{\sim} N(0, \overline{\sigma}^2)$ is the aggregate return shock
$\widetilde{\eta}_{j,t} \overset{iid}{\sim} N(0, \widetilde{\sigma}^2_j)$ is the fund j relative return shocks
$\overline{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\overline{\sigma}}^2_j)$ is the noise of the $\overline{\eta}_t$ signal
$\widetilde{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\widetilde{\sigma}}^2_j)$ is the noise of the $\widetilde{\eta}_{j,t}$ signal
$\pi_j = (1/\widehat{\overline{\sigma}}^2_j)/(1/\widehat{\overline{\sigma}}^2_j + 1/\overline{\sigma}^2)$
$\phi_j = (1/\widehat{\widetilde{\sigma}}^2_j)/(1/\widehat{\widetilde{\sigma}}^2_j + 1/\widetilde{\sigma}^2_j)$
$\xi_{j,t} = \pi_j \cdot (\overline{u}_{j,t} - \overline{u}_{j,t-1}) + \phi_j \cdot (\widetilde{u}_{j,t} - \widetilde{u}_{j,t-1})$

We use this Bayesian framework as an economic motivation for our 3-step unsmoothing process. However, for our unsmoothing process to be a direct generalization of the 1-step unsmoothing method of Getmansky, Lo, and Makarov (2004), our econometric framework specifies smoothed returns as

$$\begin{aligned} r^o_{j,t} &= \pi_j \cdot \overline{r}_t \;+\; (1-\pi_j)\cdot \overline{r}_{t-1} \;+\; \phi_j \cdot \widetilde{r}_{j,t} \;+\; (1-\phi_j)\cdot \widetilde{r}_{j,t-1} \\ &= \mu_j \;+\; \pi_j \cdot \overline{\eta}_t \;+\; (1-\pi_j)\cdot \overline{\eta}_{t-1} \;+\; \phi_j \cdot \widetilde{\eta}_{j,t} \;+\; (1-\phi_j)\cdot \widetilde{\eta}_{j,t-1} \end{aligned} \tag{IA.2}$$

which implies our econometric framework effectively ignores the $\xi_{j,t}$ term in Equation IA.1.

This section shows that the $\xi_{j,t}$ term can in fact be safely ignored for the purpose of measuring αs and βs, which is the focus of our paper.[IA.1]

[IA.1] Another mismatch between our Bayesian model and our econometric framework is that we use regular returns instead of log returns in our empirical analysis of hedge funds to be consistent with the prior literature. Subsection D.2 shows that our baseline hedge fund results are very similar if we use log returns instead.

A.1 Alphas and Betas with Known Smoothing Parameters

Suppose the parameters $\overline{\sigma}$, $\widetilde{\sigma}_j$, $\widehat{\overline{\sigma}}_j$, $\widehat{\widetilde{\sigma}}_j$ (and thus π_j and ϕ_j) are known. In this case, our 3-step unsmoothing method obtains unsmoothed returns from three steps.

In the first step, aggregate returns are given by

$$
\begin{aligned}
\overline{r}_t^o &= \overline{\mu} \;+\; \overline{\pi} \cdot \overline{\eta}_t \;+\; (1-\overline{\pi}) \cdot \overline{\eta}_{t-1} \;+\; \overline{\pi} \cdot \widehat{\mathbb{E}}[\overline{u}_{j,t} - \overline{u}_{j,t-1}] \;+\; \overline{\phi} \cdot \widehat{\mathbb{E}}[\widetilde{u}_{j,t} - \widetilde{u}_{j,t-1}] \\
&\quad + \widehat{Cov}(\phi_j, \widetilde{\eta}_{j,t} - \widetilde{\eta}_{j,t-1}) \;+\; \widehat{Cov}(\pi_j, \overline{u}_{j,t} - \overline{u}_{j,t-1}) \;+\; \widehat{Cov}(\phi_j, \widetilde{u}_{j,t} - \widetilde{u}_{j,t-1}) \\
&\approx \overline{\mu} \;+\; \overline{\pi} \cdot \overline{\eta}_t \;+\; (1-\overline{\pi}) \cdot \overline{\eta}_{t-1}
\end{aligned} \tag{IA.3}
$$

which implies that aggregate MA(1) residuals that ignore $\xi_{j,t}$,

$$
\begin{aligned}
resid &= \frac{1}{\overline{\pi}} \cdot \left[(r_{j,t}^o - \overline{\mu}) - (1-\overline{\pi}) \cdot \overline{\eta}_{t-1} \right] \\
&= \overline{\eta}_t
\end{aligned} \tag{IA.4}
$$

in fact recover aggregate return shocks, $\overline{\eta}_t$.

In the second step, relative fund-level returns are given by

$$
\begin{aligned}
\widetilde{r}_{j,t}^o &= \psi_j \cdot \overline{r}_t \;+\; (1-\psi_j) \cdot \overline{r}_{t-1} \;+\; \phi_j \cdot \widetilde{r}_{j,t} \;+\; (1-\phi_j) \cdot \widetilde{r}_{j,t-1} \;+\; \xi_{j,t} \\
&= \mu_j \;+\; \psi_j \cdot \overline{\eta}_t \;+\; (1-\psi_j) \cdot \overline{\eta}_{t-1} \;+\; \phi_j \cdot \widetilde{\eta}_{j,t} \;+\; (1-\phi_j) \cdot \widetilde{\eta}_{j,t-1} \;+\; \xi_{j,t}
\end{aligned} \tag{IA.5}
$$

where $\psi_j = \pi_j - \overline{\pi}$, which implies that fund-level MA(1) relative residuals that ignore $\xi_{j,t}$,

$$
\begin{aligned}
resid &= \frac{1}{\phi_j} \cdot \left[\widetilde{r}_{j,t}^o \;-\; \left(\mu_j \;-\; \psi_j \cdot \overline{\eta}_t \;+\; (1-\psi_j) \cdot \overline{\eta}_{t-1} \;+\; (1-\phi_j) \cdot \widetilde{\eta}_{j,t-1} \right) \right] \\
&= \widetilde{\eta}_{j,t} \;+\; \frac{1}{\phi_j} \cdot \xi_{j,t}
\end{aligned} \tag{IA.6}
$$

recover a measure of relative returns that is distorted by $\xi_{j,t}/\phi_j$.

In the third step, we obtain

$$
\begin{aligned}
\widehat{r}_{j,t} &= \mu_j \;+\; \overline{\eta}_t \;+\; \widetilde{\eta}_{j,t} \;+\; \frac{1}{\phi_j} \cdot \xi_{j,t} \\
&= r_{j,t} \;+\; \frac{1}{\phi_j} \cdot \xi_{j,t}
\end{aligned} \tag{IA.7}
$$

so that our unsmoothed return estimates also have the $\xi_{j,t}/\phi_j$ distortion.

Our objective is to estimate the α and β parameters from the projection

$$r_{j,t} = \alpha_j + \beta_j' f_t + \varepsilon_{j,t} \tag{IA.8}$$

where $\mathbb{E}[f_t \cdot \varepsilon_{j,t}] = 0$. However, we rely on $\widehat{r}_{j,t}$ instead of $r_{j,t}$, which implies we effectively estimate α and β from the projection

$$\begin{aligned} \widehat{r}_{j,t} &= \widehat{\alpha}_j + \widehat{\beta}_j' f_t + (\varepsilon_{j,t} + \frac{1}{\phi_j} \cdot \xi_{j,t}) \\ &= \widehat{\alpha}_j + \widehat{\beta}_j' f_t + \widehat{\varepsilon}_{j,t} \end{aligned} \tag{IA.9}$$

Since $\xi_{j,t} = \pi_j \cdot (\overline{u}_{j,t} - \overline{u}_{j,t-1}) + \phi_j \cdot (\widetilde{u}_{j,t} - \widetilde{u}_{j,t-1})$ is composed of shocks that are uncorrelated with risk factors (i.e., they reflect the noise in the fund-specific signals of our Bayesian framework), we still have $\mathbb{E}[f_t \cdot \widehat{\varepsilon}_{j,t}] = 0$ so that our $\widehat{\alpha}_j$ and $\widehat{\beta}_j$ estimates in fact converge in probability to α_j and β_j. Intuitively, while $\widehat{r}_{j,t}$ is a distorted measure of $r_{j,t}$, the distortion is idiosyncratic so that it does not affect the estimation of the comovement between fund returns and risk factors (and consequently α and β).

A.2 Alphas and Betas with Unknown Smoothing Parameters

The prior subsection shows that the 3-step α_j and β_j can be consistently estimated when $\widehat{r}_{j,t}$ is obtained from known smoothing parameters. However, in practice we need to estimate the smoothing parameters. We now show through simulations that α_j and β_j estimated through the 3-step unsmoothing method remain effectively unbiased if we ignore $\xi_{j,t}$ even when $\overline{\sigma}$, $\widetilde{\sigma}_j$, $\widehat{\overline{\sigma}}_j$, $\widehat{\widetilde{\sigma}}_j$ (and thus π_j and ϕ_j) are estimated during the unsmoothing process (instead of being known ex-ante as in the analytical results of the prior subsection).

Table IA.1 reports results from simulations analogous to those in Panel A of Table 5, except that true economic returns now have a $\xi_{j,t}$ term that is ignored during the unsmoothing process. The Table IA.1 header provides the details of the simulation. The results indicate that αs estimated from observed returns or 1-step unsmoothed returns have a strong bias and that αs estimated from 3-step unsmoothed returns are effectively unbiased. Note also that alphas estimated using the Dimson method are unbiased and have similar efficiency to the 3-step method. Thus, consistent with the results presented in Panel A of Table 5, the Dimson method appears to perform similarly to the 3-step method in the context of a simple one-factor model with only one smoothing factor.

In Panel B of Table IA.1, we consider a more realistic simulation based on eight factors, with factors and fund-level parameters obtained from the empirical distribution of relative value funds (analogous to Panel B of Table 5). In this more realistic simulation, the 3-step unsmoothing method (as well as the Dimson and CDimson methods) remain unbiased, but the 3-step unsmoothing method is significantly more efficient than the Dimson and CDimson methods (especially when we consider mean squared pricing errors).

Overall, the results indicate that our 3-step unsmoothing method yields consistent α estimates if the economic returns are driven by the Bayesian framework outlined above even though the 3-step unsmoothing process ignores the $\xi_{j,t}$ component.

A.3 A Method to Unsmooth Returns Accounting for $\xi_{j,t}$

The prior two subsections jointly show (through analytical results and simulations) that our 3-step unsmoothing method yields unbiased alpha estimates even if $\xi_{j,t} \neq 0$ and the econometrician does not know the unsmoothing parameters. However, one may want to obtain estimates of economic returns directly to estimate other quantities beyond alpha (and betas). In this subsection, we develop an unsmoothing procedure that recognizes that the smoothing process in our economic model is given by Equation IA.1. We also use this method to validate empirically our analytical and simulation findings (in Subsections A.1 and A.2) that the 3-step unsmoothing method yields unbiased α estimates even if $\xi_{j,t} \neq 0$.

In a nutshell, we write the model in state space form, which allows us to use a conditional maximum likelihood estimation (CMLE) framework to estimate the smoothing parameters and recover unsmoothed returns (through a Kalman filter algorithm). The details are below.

The measurement equation associated with Equation IA.1 is given by

$$r^o_{j,t} = \mu_j + \omega_j' x_{j,t} + \nu_j' z_t \tag{IA.10}$$

and its respective transition equation is given by

$$x_{j,t} = \Omega x_{j,t-1} + \epsilon_{j,t} \tag{IA.11}$$

where $\epsilon_{j,t} \sim N(0, \Sigma_j)$.

We then have that $x_{j,t} = [\widetilde{\eta}_{j,t}\ \widetilde{\eta}_{j,t-1}\ \widetilde{u}_{j,t}\ \widetilde{u}_{j,t-1}\ \overline{u}_{j,t}\ \overline{u}_{j,t-1}]'$ captures the latent state variables while $z_t = [\overline{\eta}_t\ \overline{\eta}_{t-1}]'$ reflects the observable state variables.[IA.2] Moreover, the state space parameters are given by

$$\nu_j = \begin{bmatrix} \pi_j \\ 1-\pi_j \end{bmatrix} \quad \omega_j = \begin{bmatrix} \phi_j \\ 1-\phi_j \\ \phi_j \\ -\phi_j \\ \pi_j \\ -\pi_j \end{bmatrix} \quad \Omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \Sigma_j = \begin{bmatrix} \widetilde{\sigma}_j^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \widehat{\widetilde{\sigma}}_j^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \widehat{\overline{\sigma}}_j^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{IA.12}$$

and satisfy the restrictions $\pi_j = (1/\widehat{\overline{\sigma}}_j^2)/(1/\widehat{\overline{\sigma}}_j^2 + 1/\overline{\sigma}^2)$ and $\phi_j = (1/\widehat{\widetilde{\sigma}}_j^2)/(1/\widehat{\widetilde{\sigma}}_j^2 + 1/\widetilde{\sigma}_j^2)$ so that there are only three free parameters $\Theta_j = (\widetilde{\sigma}_j, \widehat{\widetilde{\sigma}}_j, \widehat{\overline{\sigma}}_j)$ given that $\overline{\sigma}$ can be directly measured from the $\overline{\eta}_t$ standard deviation and μ_j from the average $r^o_{j,t}$.

From the state space structure in Equations IA.10 to IA.12, we apply a Kalman filter algorithm to estimate the parameters in Θ_j by CMLE and obtain estimates (i.e., filtered values) for the latent processes in $x_{j,t}$. Specifically, for a given fund j and estimates for μ_j, z_t, and $\overline{\sigma}$ (with the last two obtained as described in Footnote IA.2), we follow the algorithm detailed below to evaluate the log likelihood based on any given guess for the vector of parameters (Θ_j) and estimate Θ_j as the parameter vector that maximizes the log likelihood function in Equation IA.13.

[IA.2] Note that $z_t = [\overline{\eta}_t\ \overline{\eta}_{t-1}]'$ is treated as observable since $\overline{\eta}_t$ can be recovered from an MA(1) estimation on aggregate observed returns (see Equation IA.3).

1. Compute $\pi_j = (1/\widehat{\widehat{\sigma}}_j^2)/(1/\widehat{\widehat{\sigma}}_j^2 + 1/\overline{\sigma}^2)$ and $\phi_j = (1/\widehat{\widetilde{\sigma}}_j^2)/(1/\widehat{\widetilde{\sigma}}_j^2 + 1/\widetilde{\sigma}_j^2)$

2. Compute ν_j, ω_j, Ω, and Σ_j based on the Equations in IA.12

3. Apply the Kalman filter algorithm

   (a) Initiate $x_{j,0|0} = 0_{6\times 1}$

   (b) Initiate $P_{j,0|0} = 0_{6\times 6}$[IA.3]

   (c) $x_{j,t|t-1} = \Omega x_{t-1,t-1}$

   (d) $P_{j,t|t-1} = \Omega P_{j,t-1|t-1}\Omega' + \Sigma_j$

   (e) $\zeta_{j,t} = r^o_{j,t} - (\mu_j + \omega_j' x_{j,t|t-1} + \nu_j' z_t)$

   (f) $\sigma^2_{\zeta,j,t} = \omega_j' P_{j,t|t-1}\omega_j$

   (g) $K_{j,t} = P_{j,t|t-1}\omega_j \cdot \sigma^2_{\zeta,j,t}$

   (h) $x_{j,t|t} = x_{j,t|t-1} + K_{j,t} \cdot \zeta_{j,t}$

   (i) $P_{j,t|t} = (I_{6\times 6} - K_{j,t} \cdot \omega_j')P_{j,t|t-1}$

4. Evaluate the log likelihood function,

$$logL \;=\; -\left[\sum_{t=1}^{T_j} log(\sigma^2_{\zeta,j,t}) + \frac{\zeta^2_{j,t}}{\sigma^2_{\zeta,j,t}}\right] \tag{IA.13}$$

After estimating Θ_j by CMLE as described above, we estimate economic returns based on $r_{j,t} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$ (with $\overline{\eta}_t$ and $\widetilde{\eta}_{j,t}$ obtained from the $x_{j,t|t}$ vector at the estimated Θ_j). Finally, we treat these economic return estimates as true returns to estimate alphas and betas based on the FH 8-Factor model (just as we do with our baseline unsmoothing methods).

Since our Bayesian framework (for simplicity) only accounts for one smoothing lag, we cannot directly compare the alphas obtained from this Kalman filter unsmoothing with the

[IA.3] Note that initiating the state vector uncertainty at zero is analogous to how unsmoothed returns are recovered in the MA processes underlying the 1-step and 3-step unsmoothing processes (which treat the first shocks as zero).

ones obtained in our baseline analysis (which accounts for three smoothing lags). So, we instead reestimate the alphas of all funds using our baseline unsmoothing methods (both 1-step and 3-step) but with one smoothing lag. Figure IA.1 reports, for each fund category, the average alphas and their t-statistics obtained using observed returns, 1-step unsmoothed returns, 3-step unsmoothed returns, and the Kalman filter unsmoothed returns from this subsection. As the results demonstrate, 3-step unsmoothing and Kalman filter unsmoothing lead to very similar alpha estimates. This empirical result confirms that our analytical and simulation findings (in Subsections A.1 and A.2) also hold in the data. That is, ignoring $\xi_{j,t}$ in the unsmoothing process (as in our 3-step unsmoothing) does not lead to biased alpha estimates in the sense that the average alphas are similar for the 3-step unsmoothing and the Kalman filter unsmoothing (the largest difference, which is still small, is in the high liquidity group and even that difference is small).

B Estimating Unsmoothed Returns without Fund Categories

In the main text, we assume that observed returns of fund j are given by

$$
\begin{aligned}
R^o_{j,t} &= \Sigma^H_{h=0}\,\phi^{(h)}_j \cdot \widetilde{R}_{j,t-h} \;+\; \Sigma^L_{h=0}\,\pi^{(h)}_j \cdot \overline{R}_{j,t-h} \\
&= \mu_j \;+\; \Sigma^H_{h=0}\,\phi^{(h)}_j \cdot \widetilde{\eta}_{j,t-h} \;+\; \Sigma^L_{h=0}\,\pi^{(h)}_j \cdot \overline{\eta}_{j,t-h}
\end{aligned} \tag{IA.14}
$$

where $\overline{R}_{j,t} = \Sigma^J_{i=1} w_{j,i} \cdot R_{i,t}$ are aggregate economic returns, $\widetilde{R}_j = R_{j,t} - \overline{R}_{j,t}$ are relative economic returns, and the parameters satisfy $\Sigma^H_{h=0}\phi^{(h)}_j = \Sigma^L_{h=0}\pi^{(h)}_j = 1$.[IA.4] . We then provide a 3-step unsmoothing method to estimate $\phi^{(h)}_j$ and $\pi^{(h)}_j$, and thus recover economic returns, $R_{j,t} = \widetilde{R}_j + \overline{R}_{j,t}$. However, our 3-step unsmoothing method implicitly assumes that the researcher knows how to aggregate fund-level observed returns to obtain $\overline{R}^o_{j,t} = \Sigma^J_{i=1} w_{j,i} \cdot R^o_{i,t}$. This implicit assumption is natural when funds have pre-specified categories, such as the different hedge fund trading strategies we study in our empirical analysis (and the assumption is in line with our Bayesian framework, which links the aggregate index to the overall asset class the fund belongs to). Nevertheless, researchers may face applications in which the underlying grouping of funds into categories is not clear. As such, this section provides an extended 3-step method to consistently estimate Equation IA.14 without knowing the $w_{j,i}$ values underlying the aggregate economic return, $\overline{R}_{j,t}$, that should be used for fund j.

To estimate Equation IA.14 without knowing the group of funds underlying $\overline{R}_{j,t}$, we make one key assumption. Namely, we assume that the covariance matrix of all N funds under

[IA.4]Note that the $\overline{R}_{j,t}$ notation emphasizes that the index/aggregation depends on fund j through the weights. While the notation in the main text did not emphasize this aspect for simplicity, it is also true there. For instance, if j reflects a relative value fund, then in the main text we have $w_{j,i} = 1/N_j$ for all i reflecting relative value funds and$w_{j,i} = 0$ for all other i.

analysis can be summarized by $K << N$ factors such that:[IA.5]

$$\eta_{j,t} \ = \Sigma_{k=1}^{K}\, \beta_{k,j} \cdot f_{k,t} \ + \ \epsilon_{j,t} \tag{IA.15}$$

where $\epsilon_{j,t} \sim IID(0, \sigma_j^2)$ and we normalize the factors such that $f_{k,t} \sim IID(0, 1)$.

Under our factor structure assumption, we have:

$$\begin{aligned}
\overline{\eta}_{j,t} \ &= \Sigma_{i=1}^{J}\, w_{j,i} \cdot \left(\Sigma_{k=1}^{K}\, \beta_{k,i} \cdot f_{k,t}\right) \ + \ \Sigma_{i=1}^{J} w_i \cdot \epsilon_{i,t} \\
&\approx \Sigma_{i=1}^{J}\, w_{j,i} \cdot \left(\Sigma_{k=1}^{K}\, \beta_{k,i} \cdot f_{k,t}\right) \\
&= \Sigma_{k=1}^{K}\, (\Sigma_{i=1}^{J} w_{j,i} \beta_{k,i}) \cdot f_{k,t} \\
&= \Sigma_{k=1}^{K}\, \overline{\beta}_{k,j} \cdot f_{k,t}
\end{aligned} \tag{IA.16}$$

where the second equality follows from $\underset{J\to\infty}{Plim}\ \Sigma_{i=1}^{J} w_i \cdot \epsilon_{i,t} = 0$ and the last equality defines $\overline{\beta}_{k,j} = \Sigma_{k=1}^{K}(\Sigma_{i=1}^{J} w_{j,i}\beta_{k,i})$, which clarifies that the group underlying $\overline{\eta}_{j,t}$ depends on fund j only through the betas.

Substituting Equation IA.16 into Equation IA.14, we have

$$R_{j,t}^{o} = \mu_j \ + \ \Sigma_{h=0}^{H}\, \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^{L}\, \pi_j^{(h)} \cdot (\Sigma_{k=1}^{K}\, \overline{\beta}_{k,j} \cdot f_{k,t-h}) \tag{IA.17}$$

which can be estimated as long as we have estimates for $f_{k,t-h}$.

Consequently, we propose the following extended 3-step return unsmoothing method:

1. Use Principle Components Analysis (PCA) to obtain $\{f_{k,t}\}_{k=1}^{K}$ (see Subsection B.1);

2. For each fund j, estimate Equation IA.17 by Maximum Likelihood (see Subsection B.2);

3. Obtain unsmoothed returns as $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \Sigma_{k=1}^{K}\overline{\beta}_{k,j} \cdot f_{k,t}$.

[IA.5]This factor structure is a common assumption in asset pricing (for its early foundation, see Ross (1976)). Moreover, Equation IA.15 is not an asset pricing model (i.e., it does not require alphas to be zero relative to the latent factors). It is simply a characterization of the covariance matrix of returns (i.e., our assumption is simply that $K << N$ because if we had $K = N$ our framework would hold by construction). Without imposing economic structure, it is possible that a subset of these latent factors are not priced or even that some other factors are priced (and neither of these two possibilities is a problem for our extended 3-step method).

B.1 Latent Factors from PCA

This subsection describes how we obtain $\{f_{k,t}\}_{k=1}^{K}$ using PCA, which is Step 1 of our extended 3-step unsmoothing method.

To obtain $\{f_{k,t}\}_{k=1}^{K}$, we first aggregate fund-level returns into portfolios based on their level of smoothness. Specifically, we sort funds into N portfolios based on the smoothness index proposed by Getmansky, Lo, and Makarov (2004), $SI_j = \Sigma_{h=0}^{H}(\theta_j^{(h)})^2$, where $\theta_j^{(h)}$ are the MA(H) coefficients from Equation 2. We then 1-step unsmooth the returns of these portfolios to obtain economic returns (since they are aggregate returns, the 3-step method is not necessary). Finally, we extract K standardized PCs based on the economic return covariance matrix of these N portfolios, which yields $\{f_{k,t}\}_{k=1}^{K}$. For our empirical application in Subsection B.3, we use $N = 50$, $H = 3$, and $K = 10$.

It is important to note that extracting $\{f_{k,t}\}_{k=1}^{K}$ directly from the panel of fund-level returns would introduce serious problems. First, fund-level returns result in an umbalanced panel, which creates difficulties in imposing a common rotation matrix across the balanced blocks necessary for PCA (for an early discussion of this issue, see Connor and Korajczyk (1991)). Second, the typical hedge fund stays in the sample for a relatively short period, and thus the PCs extracted from each balanced block of fund-level returns would rely on a large number of funds and a short time series, which renders PCA unreliable in detecting latent factors (see, e.g., Lettau and Pelger (2020)). And third, we do not observe fund-level economic returns (estimating them is the objective of our 3-step unsmoothing method), and thus PCs extracted from fund-level returns would reflect the latent factors for the covariance matrix of observed returns, not economic returns as we need during our unsmoothing process.

Given the above paragraph, our method relies on $N = 50$ portfolios of hedge funds to extract PCs. However, since our portfolios are created based on a single fund characteristic (the smoothness index), one may worry that these portfolios have low dimentionality, which would make it hard to identify the factors in Equation IA.15.[IA.6] This is not the case as we

[IA.6] Formally, if our N portfolios have no exposure to factor k (i.e., $\beta_{k,p} = 0$ for each portfolio p), then the PCs extracted from these N portfolios do not identify factor k from Equation IA.15 (i.e., factor k is a weak need ten PCs to explain 89% of the variance of our 50 portfolios whereas Giglio, Kelly, and Kozak (2023) show that two PCs are sufficient to explain more than 90% of the variance of 100 portfolios based on fifty different equity anomalies (i.e., the long and short legs of each anomaly). In other words, our 50 portfolios have much higher dimentionality than a dataset of 100 portfolios capturing a large set of equity anomalies. Intuitively, funds with different smoothness index tend to invest in different types of assets and thus are exposed to different risk factors so that extracting PCs from portfolios sorted on the smoothness index provides a good way to recover the risk factors driving the covariance matrix of hedge fund returns.

B.2 Smoothing Parameters by Maximum Likelihood

This subsection describes how we estimate Equation IA.17 given $\{f_{k,t}\}_{k=1}^{K}$, which is Step 2 of our extended 3-step unsmoothing method.

To start, we estimate μ_j from the time-series average of $R^o_{j,t}$ and then rewrite Equation IA.17 as:

$$\begin{aligned} R^o_{j,t} - \mu_j &= \Sigma_{h=0}^{H}\, \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \;+\; \Sigma_{h=0}^{L}\, \pi_j^{(h)} \cdot (\Sigma_{k=1}^{K}\, \overline{\beta}_{k,j} \cdot f_{k,t-h}) \\ &= \Sigma_{h=0}^{H}\, \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \;+\; \Sigma_{k=1}^{K}\, \overline{\beta}_{k,j} \cdot (\Sigma_{h=0}^{L}\, \pi_j^{(h)} \cdot f_{k,t-h}) \\ &= \Sigma_{h=0}^{H}\, \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \;+\; \Sigma_{k=1}^{K}\, \overline{\beta}_{k,j} \cdot \overline{f}_{k,t} \end{aligned} \tag{IA.18}$$

where $\overline{f}_{k,t} = \Sigma_{h=0}^{L}\, \pi_j^{(h)} \cdot f_{k,t-h}$.

We then define $\Theta_j = \left[\{\phi_j^{(h)}\}_{h=0}^{H}, \{\pi_j^{(h)}\}_{h=0}^{L}, \{\overline{\beta}_{k,j}\}_{k=1}^{K}\right]$ to be the set of parameters to be estimated, let $logL(\Theta_j)$ reflect the log Likelihood function of Equation IA.18, and write the MLE estimator as

$$\begin{aligned} \Theta_j^{MLE} &= \underset{\Theta_j}{argmax}\; logL(\Theta_j) \\ &= \underset{\{\pi_j^{(h)}\}_{h=0}^{L}}{argmax} \left(\underset{\{\phi_j^{(h)}\}_{h=0}^{H}, \{\overline{\beta}_{k,j}\}_{k=1}^{K},}{argmax} \; logL(\Theta_j) \right) \end{aligned} \tag{IA.19}$$

Writing the estimator as in Equation IA.19 simplifies the estimation since the inner

factor with respect to our set of N portfolios, see Giglio, Xiu, and Zhang (2022)).

optimization can be solved as a MA(H) with $\{\overline{f}_{k,t}\}_{k=1}^{K}$ as exogenous variables, where $\overline{f}_{k,t} = \Sigma_{h=0}^{L}\ \pi_j^{(h)} \cdot f_{k,t-h}$ is obtained from the outer generic optimization (which effectively has only L parameters since $\Sigma_{h=0}^{L}\pi_j^{(h)} = 1$). That said, the outer objective function is highly non-linear, and thus finding the global optimum of Equation IA.19 for each hedge fund is non-trivial. To further help numerical stability, we constraint the parameter space of $\pi_j^{(h)}$ and $\phi_j^{(h)}$. Specifically, we take the 1-step unsmoothing $\theta_j^{(h)}$ as a benchmark since in this case $\pi_j^{(h)} = \phi_j^{(h)} = \theta_j^{(h)}$ and constrain the extended 3-step estimation to satisfy $\pi_j^{(h)} \in [\theta_j^{(h)} - 0.5\ ;\ \theta_j^{(h)} + 0.5]$ and $\phi_j^{(h)} \in [\theta_j^{(h)} - 0.5\ ;\ \theta_j^{(h)} + 0.5]$. We find that such a constraint solves the numerical issues we encounter while estimating Equation IA.18.

After obtaining Θ_j^{MLE}, we recover $\widetilde{\eta}_{j,t}$, which allows us to calculate unsmoothed returns as $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \Sigma_{k=1}^{K}\overline{\beta}_{k,j} \cdot f_{k,t}$ in Step 3 of our extended 3-step unsmoothing method.

B.3 Empirical Results

We apply our extended 3-step unsmoothing method to our hedge fund database, and find results that are similar to the ones we report in the main text using our baseline 3-step unsmoothing method. In particular, Figure IA.2 replicates Figure 2 in the main text and shows that the risk-adjusted performance of hedge funds based on our extended 3-step method is consistent with their risk-adjusted performance based on our baseline 3-step method. That is, despite some quantitative differences relative to 1-step unsmoothing (e.g., R^2s increase little relative to 1-step unsmoothing), alphas and their statistical significance tend to decline relative to 1-step unsmoothing, which is our core result. Moreover, this effect is concentrated in illiquid funds just as with the baseline 3-step unsmoothing method.

C The Impact of Time-Varying Expected Returns

It is standard in the return unsmoothing literature to assume that economic returns are not autocorrelated. Our return unsmoothing method, just like 1-step unsmoothing methods, makes this assumption as well. However, time-varying expected returns necessarily induce autocorrelation in economic returns. As such, in this subsection we provide a method to adjust return unsmoothing techniques to account for time-varying expected returns (see Subsections C.1 and C.2). We then apply our new method to show that our 3-step unsmoothing technique is robust to controlling for momentum (see Subsection C.3). Finally, we show that the bias (in the estimation of alphas and betas) that arises from ignoring short-term autocorrelation in true returns is generally small relative to the bias that arises from ignoring return smoothing (see Subsection C.4) and that the same bias is present when using the Dimson (1979) method (see Subsection C.5).

C.1 Formally Defining αs and βs

To start, we want to formally define the quantities we seek to estimate (α_j and β_j). For that, let M_t represent the Stochastic Discount Factor (SDF) in the economy and note that the Euler condition $\mathbb{E}_t[M_{t+1} \cdot (R_{j,t+1} - R_{f,t+1})] = 0$ can be written as

$$
\begin{aligned}
0 \;&=\; \mathbb{E}_t\left[M_{t+1} \cdot (R_{j,t+1} - R_{f,t+1})\right] - \mathbb{E}_t\left[M_{t+1} \cdot \mathbb{E}_t[R_{j,t+1} - R_{f,t+1}]\right] + \mathbb{E}_t\left[M_{t+1} \cdot \mathbb{E}_t[R_{j,t+1} - R_{f,t+1}]\right] \\
&=\; \mathbb{E}_t\left[M_{t+1} \cdot (R_{j,t+1} - \mathbb{E}_t[R_{j,t+1}])\right] \;+\; (1/R_{f,t+1}) \cdot \mathbb{E}_t\left[R_{j,t+1} - R_{f,t+1}\right] \\
&=\; \mathbb{E}_t\left[R_{f,t+1} \cdot M_{t+1} \cdot (R_{j,t+1} - \mathbb{E}_t[R_{j,t+1}])\right] \;+\; \mathbb{E}_t\left[R_{j,t+1} - R_{f,t+1}\right] \\
&=\; \mathbb{E}\left[R_{f,t+1} \cdot M_{t+1} \cdot (R_{j,t+1} - \mathbb{E}_t[R_{j,t+1}])\right] \;+\; \mathbb{E}\left[R_{j,t+1} - R_{f,t+1}\right] \\
&\Downarrow \\
\mathbb{E}\left[R_{j,t+1} - R_{f,t+1}\right] \;&=\; -Cov\left[R_{f,t+1} \cdot M_{t+1}\,,\; R_{j,t+1} - \mathbb{E}_t[R_{j,t+1}]\right]
\end{aligned}
\tag{IA.20}
$$

where the second line uses $\mathbb{E}_t[M_{t+1}] = 1/R_{f,t+1}$ and the fourth line takes unconditional expectation on both sides of the equation.

Now, we specify the linear SDF

$$M_{t+1} = \frac{1}{R_{f,t+1}} \cdot \left[1 - b^{'} \left(f_{t+1} - \mathbb{E}_t[f_{t+1}]\right)\right] \tag{IA.21}$$

so that $\mathbb{E}_t[M_{t+1}] = 1/R_{f,t+1}$ is satisfied and Equation IA.20 yields:

$$\begin{aligned}
\mathbb{E}\left[R_{j,t+1} - R_{f,t+1}\right] &= b^{'} Cov\left[f_{t+1} - \mathbb{E}_t[f_{t+1}]\, ,\, R_{j,t+1} - \mathbb{E}_t[R_{j,t+1}]\right] \\
&= Cov\left[f_{t+1}\, ,\, R_{j,t+1} - \mathbb{E}_t[R_{j,t+1}]\right]^{'} b \\
&= Cov\left[f\, ,\, \ddot{R}_j\right]^{'} \Sigma_f^{-1} \Sigma_f b \\
&= \beta_{f,j}^{'} \lambda
\end{aligned} \tag{IA.22}$$

where the second line follows from the fact that $\ddot{R}_{j,t+1} \equiv R_{j,t+1} - \mathbb{E}_t[R_{j,t+1}]$ is orthogonal to any time t information, the third line defines Σ_f as the unconditional covariance matrix of f_t, and the fourth line defines risk prices $\lambda \equiv \Sigma_f b$ and multivariate betas $\beta_{f,j} \equiv Cov[f\, ,\, \ddot{R}_j]^{'} \Sigma_f^{-1}$.

Finally, if the factors are excess returns then Equation IA.22 applies to them as well so that $\lambda = \mathbb{E}[f]$ and we have a factor model

$$\mathbb{E}\left[R_j - R_f\right] = \alpha_j + \beta_{f,j}^{'} \mathbb{E}\left[f\right] \tag{IA.23}$$

that implies $\alpha_j = 0$.

The derivation of Equation IA.23 is useful because it clarifies that to properly measure risk exposures ($\beta_{f,j}$) and consequently risk-adjusted performance (α_j) we need estimates of economic return innovations, $\ddot{R}_{j,t}$, not of economic returns, $R_{j,t}$. When expected returns are time-invariant (as we assume in our baseline return unsmoothing framework) then this distinction is irrelevant as either can be used to estimate $\beta_{f,j}$. However, if economic returns are autocorrelated, then expected returns vary over time and we need $\ddot{R}_{j,t}$ to estimate $\beta_{f,j}$.

C.2 A Method to Unsmooth Returns Accounting for $\mathbb{E}_t[R]$

We now develop a method to account for time-varying expected returns. To do so, we replace the assumptions that $\overline{R}_t = \overline{\mu} + \overline{\eta}_t$ and $\widetilde{R}_{j,t} = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$ with $\overline{R}_t = \overline{\mu}_{t-1} + \overline{\eta}_t$ and $\widetilde{R}_{j,t} = \widetilde{\mu}_{j,t-1} + \widetilde{\eta}_{j,t}$, where

$$\overline{\mu}_t = a \ + \ b'\overline{x}_t \tag{IA.24}$$

$$\widetilde{\mu}_{j,t} = a_j \ + \ b_j'\widetilde{x}_{j,t} \tag{IA.25}$$

where $\widetilde{x}_{j,t} = x_{j,t} - \overline{x}_t$ and $\overline{x}_t$ are exogenous observable variables, and $\overline{\eta}_t \sim IID$ and $\widetilde{\eta}_{j,t} \sim IID$ are iid shocks. Note that since $\Sigma_{j=1}^J w_j \cdot \widetilde{\mu}_{j,t} = 0$, we have $\Sigma_{j=1}^J w_j \cdot a_j = \Sigma_{j=1}^J w_j \cdot b_j = 0$.

Combining the economic returns above with the smoothing process underlying our 3-step method, we have the following observed returns:

$$\begin{aligned} R_{j,t}^o &= \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} \ + \ \Sigma_{h=0}^L \ \pi_j^{(h)} \cdot \overline{R}_{t-h} \\ &= \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{\mu}_{j,t-1-h} \ + \ \Sigma_{h=0}^L \ \pi_j^{(h)} \cdot \overline{\mu}_{t-1-h} \ + \ \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^L \ \pi_j^{(h)} \cdot \overline{\eta}_{t-h} \\ &= a_j \ + \ a \ + \ b_j'\widetilde{z}_{j,t-1} \ + \ b'\overline{z}_{j,t-1} \ + \ \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^L \ \pi_j^{(h)} \cdot \overline{\eta}_{t-h} \end{aligned} \tag{IA.26}$$

where $\widetilde{z}_{j,t} = \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{x}_{j,t-h}$ and $\overline{z}_{j,t} = \Sigma_{h=0}^L \ \pi_j^{(h)} \cdot \overline{x}_{t-h}$.

Aggregating Equation IA.26 yields:

$$\overline{R}_t^o \approx a \ + \ b'\overline{z}_{t-1} \ + \ \Sigma_{h=0}^L \ \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h} \tag{IA.27}$$

where $\overline{z}_{j,t} = \Sigma_{h=0}^L \ \overline{\pi}^{(h)} \cdot \overline{x}_{t-h}$ and the derivation is analogous to Equation 7 in the main text, with the approximation converging to equality as the number of funds grow.

Then, subtracting Equation IA.27 from Equation IA.26, we have observed relative returns:

$$\begin{aligned} \widetilde{R}_{j,t}^o &= \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} \ + \ \Sigma_{h=0}^L \ \psi_j^{(h)} \cdot \overline{R}_{t-h} \\ &= a_j \ + \ b_j'\widetilde{z}_{j,t-1} \ + \ b'\widetilde{\overline{z}}_{j,t-1} \ + \ \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^L \ \psi_j^{(h)} \cdot \overline{\eta}_{t-h} \end{aligned} \tag{IA.28}$$

where $\psi_j^{(h)} = \pi_j^{(h)} - \overline{\pi}_j^{(h)}$ and $\widetilde{\overline{z}}_{j,t-1} = \Sigma_{h=0}^L \ \psi_j^{(h)} \cdot \overline{x}_{t-h}$.

To recover economic return innovations, we follow a procedure analogous to the main text. First, we estimate aggregate economic returns innovations, $\overline{\eta}_t$, as residuals of a MA(L) fit to $\overline{R}_t^o$ with $\overline{z}_{t-1}$ as covariates (Equation IA.27). Second, we obtain fund-level economic relative return innovations, $\widetilde{\eta}_{j,t}$, as residuals from a MA(H) fit with $\widetilde{z}_{j,t-1}$, $\widetilde{\overline{z}}_{j,t-1}$, $\overline{\eta}_t$, $\overline{\eta}_{t-1}$, ..., $\overline{\eta}_{t-L}$ as covariates. Third, we recover recover fund-level economic return innovations as

$\ddot{R}_{j,t} = \overline{\eta}_t + \widetilde{\eta}_{j,t}$. This procedure summarizes our 3-step unsmoothing process with time-varying expected returns.

Note that $\overline{z}_{t-1}$ in Equation IA.27 is not observable since it depends on the $\overline{\pi}^{(h)}$ parameters. As such, to estimate Equation IA.27, we start with the guess $\overline{\pi}^{(0)} = 1$ to calculate $\overline{z}_{t-1}$ and update the $\overline{\pi}^{(h)}$ parameters (and consequently the variables in $\overline{z}_{t-1}$) until the $\overline{\pi}^{(h)}$ parameters converge. Similarly, $\widetilde{z}_{j,t-1}$ and $\widetilde{\widetilde{z}}_{j,t-1}$ in Equation IA.28 are not observable since they depends on the $\phi_j^{(h)}$ and $\psi^{(h)}$ parameters. As such, to estimate Equation IA.28, we start with the guess $\phi_j^{(0)} = 1$ and $\psi_j^{(h)} = 0$ and update these parameters (and consequently the variables in $\widetilde{z}_{j,t-1}$ and $\widetilde{\widetilde{z}}_{j,t-1}$) until the $\phi_j^{(h)}$ and $\psi^{(h)}$ parameters converge. Finally, note that b in Equation IA.28 is already estimated from Equation IA.27. As such, we simply use $\widetilde{R}^o_{j,t} - b'\widetilde{\widetilde{z}}_{j,t-1}$ as the dependent variable in this equation in each iteration of the estimation.

C.3 Unsmoothing when Returns Display Momentum

We now use the extended 3-step unsmoothing method developed in the prior subsection to show that our results are robust to momentum since Jegadeesh and Titman (1993) and Ehsani and Linnainmaa (2022) show that momentum predicts both firm-level and portfolio-level (economic) returns.

Specifically, we define $MOM_{j,t} = \Sigma_{h=2}^{12} R^o_{j,t-h}$ and use $x_{j,t-1} = MOM_t$ as the expected return instrument in Equations IA.24 and IA.25. Note that we are using observed returns to define momentum so that (with $H = 3$) we are effectively putting some small weight on economic returns lagged by 13, 14, and, 15 months. However, the results we describe below are very similar if we use $MOM_{j,t} = \Sigma_{h=2}^{9} R^o_{j,t-h}$, in which case (with $H = 3$) no weight is assigned to economic returns lagged by more than 12 months.

As Figure IA.3 demonstrates, accounting for momentum in economic returns when unsmoothing (through the 1-step or 3-step method) has little effect in our findings. In particular, the αs estimated from 3-step unsmoothing are still substantially lower than the ones obtained directly from observed returns or using 1-step unsmoothed returns.

C.4 Unsmoothing when Returns Display Monthly Autocorrelation

While accounting for momentum is relatively straightforward given that we can construct a reasonable momentum signal from observed returns, accounting for monthly autocorrelation in economic returns is much more complicated because we cannot use observed returns as the signal (since observed returns reflect economic returns over H months). So, instead of trying to control for monthly autocorrelation, we derive the bias that arises from it in our baseline 3-step unsmoothing method and argue that, given reasonable levels of monthly autocorrelation, this bias is small relative to the bias that arises from ignoring return smoothing in the first place.

To start, we assume that monthly (short-term) autocorrelation in economic returns is well captured by a MA(1) process so that we have

$$\overline{R}_t - \overline{\mu} = \ddot{\overline{R}}_t \ + \ \rho \cdot \ddot{\overline{R}}_{t-1} \tag{IA.29}$$

$$\widetilde{R}_{j,t} - \widetilde{\mu}_j = \ddot{\widetilde{R}}_{j,t} \ + \ \rho \cdot \ddot{\widetilde{R}}_{j,t-1} \tag{IA.30}$$

which also implies

$$R_{j,t} - \mu_j = \ddot{R}_{j,t} \ + \ \rho \cdot \ddot{R}_{j,t-1} \tag{IA.31}$$

where $R_{j,t} = \overline{R}_{j,t} + \widetilde{R}_{j,t}$ and $\ddot{R}_{j,t} = \ddot{\overline{R}}_{j,t} + \ddot{\widetilde{R}}_{j,t} = R_{j,t} - \mathbb{E}_{t-1}[R_{j,t}]$. While a stylized assumption, the homogeneity of the ρ coefficient makes it tractable to derive the bias in our 3-step method in closed form.

Then, under the smoothing process of the 3-step unsmoothing method, observed returns are given by

$$\begin{aligned} R^o_{j,t} &= \Sigma_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} \ + \ \Sigma_{h=0}^{L} \pi_j^{(h)} \cdot \overline{R}_{t-h} \\ &= \mu_j \ + \ \Sigma_{h=0}^{H} \phi_j^{(h)} \cdot (\ddot{\widetilde{R}}_{j,t-h} + \rho \cdot \ddot{\widetilde{R}}_{j,t-1-h}) \ + \ \Sigma_{h=0}^{L} \pi_j^{(h)} \cdot (\ddot{\overline{R}}_{j,t-h} + \rho \cdot \ddot{\overline{R}}_{j,t-1-h}) \\ &= \mu_j \ + \ \Sigma_{h=0}^{H+1} (\phi_j^{(h)} + \rho \cdot \phi_j^{(h-1)}) \cdot \ddot{\widetilde{R}}_{j,t-h} \ + \ \Sigma_{h=0}^{L+1} (\pi_j^{(h)} + \rho \cdot \pi_j^{(h-1)}) \cdot \ddot{\overline{R}}_{j,t-h} \end{aligned} \tag{IA.32}$$

where $\phi_j^{(-1)} = \phi_j^{(H+1)} = \pi_j^{(-1)} = \pi_j^{(H+1)} = 0$.

Now, if we define $H^* = H+1$, $L^* = L+1$, $\phi_j^{*(h)} = \frac{\phi_j^{(h)}+\rho\cdot\phi_j^{(h-1)}}{1+\rho}$, $\pi_j^{*(h)} = \frac{\pi_j^{(h)}+\rho\cdot\pi_j^{(h-1)}}{1+\rho}$, $\widetilde{\eta}_{j,t} = (1+\rho)\cdot\ddot{\widetilde{R}}_{j,t}$, and $\overline{\eta}_t = (1+\rho)\cdot\ddot{\overline{R}}_t$, then Equation IA.32 can be written as

$$R^o_{j,t} = \mu_j \ + \ \Sigma_{h=0}^{H^*}\ \phi_j^{*(h)}\cdot\widetilde{\eta}_{j,t-h} \ + \ \Sigma_{h=0}^{L^*}\ \pi_j^{*(h)}\cdot\overline{\eta}_{t-h} \tag{IA.33}$$

with parameters satisfying $\Sigma_{h=0}^{H^*}\ \phi_j^{*(h)} = 1$ and $\Sigma_{h=0}^{L^*}\ \pi_j^{*(h)} = 1$ as in our 3-step method. Consequently, our 3-step method recovers $\widetilde{\eta}_{j,t}$ and $\overline{\eta}_t$ and estimates innovations to returns as

$$\begin{aligned}\widehat{\ddot{R}}_{j,t} &= \widetilde{\eta}_{j,t} \ + \ \overline{\eta}_t \\ &= (1+\rho)\cdot(\ddot{\widetilde{R}}_{j,t} + \ddot{\overline{R}}_t) \\ &= (1+\rho)\cdot\ddot{R}_{j,t}\end{aligned} \tag{IA.34}$$

From Equation IA.34, we have that the estimated betas are given by $\widehat{\beta}_{f,j} = (1+\rho)\cdot\beta_{f,j}$ so that the estimated alphas are

$$\begin{aligned}\widehat{\alpha}_j &= \mathbb{E}[R_j] - \widehat{\beta}'_{f,j}\mathbb{E}[f] \\ &= \mathbb{E}[R_j] - (1+\rho)\cdot\beta'_{f,j}\mathbb{E}[f] \\ &= \alpha_j \ - \ \rho\cdot\beta'_{f,j}\mathbb{E}[f] \\ &= \alpha_j \ - \ \frac{\rho}{1+\rho}\cdot\widehat{\beta}'_{f,j}\mathbb{E}[f] \\ &\Downarrow \\ \alpha_{Bias} &= \ - \ \frac{\rho}{1+\rho}\cdot\widehat{\beta}'_{f,j}\mathbb{E}[f]\end{aligned} \tag{IA.35}$$

Equation IA.35 shows that if economic returns are positively (negatively) autocorrelated in the short-term, then the 3-step method underestimates (overestimates) α. Moreover, this bias also applies to the 1-step unsmoothing method even if the true smoothing process satisfies the $\phi_j^{(h)} = \pi_j^{(h)}$ condition of the 1-step method.

Table IA.2 shows the estimated α for each hedge fund strategy as well as its α_{Bias} under different assumptions for ρ. To calibrate ρ, we consider different forecasting R^2 values and also show the corresponding first order autocorrelations in true returns.[IA.7] We consider R^2

[IA.7] Specifically, $R^2 = Var(\mathbb{E}_t[R_j])/Var(R_j) = \rho^2/(1+\rho^2)$, which allows us to get $\rho = \pm\sqrt{R^2/(1-R^2)}$ and also the implied first order autocorrelation in economic returns, $Cor_1 = \rho/(1+\rho^2)$. We then use the ranging from 1% to 3% since monthly forecasting R^2 values beyond 3% would be implausibly high given the prior literature on forecasting aggregate returns (e.g., Campbell and Thompson (2008)).

As can be seen from Table IA.2, α_{Bias} is very small even when the forecasting R^2 and the corresponding autocorrelation in economic returns are quite high ($R^2 = 3\%$ and $Cor_1 = \pm 0.17$). Moreover, for all low and mid liquidity hedge fund strategies, the alpha bias in observed returns (displayed in the first column) is substantially higher than the alpha bias due to autocorrelation. For instance, in the case of relative value funds, the alpha bias in observed returns is nine times the alpha bias due to autocorrelation.

C.5 The Dimson Method when Economic Returns are Autocorrelated

The main alternative to return unsmoothing is the Dimson method (Dimson (1979)). While this discussion is not explicit in the literature, the Dimson method also assumes no time-varying expected returns. If expected returns are time-varying, then the Dimson method has a bias just like the unsmoothing method we rely on in the main text. We now demonstrate this aspect in a very simple setting.

For simplicity, we assume that there is no smoothing so that observed returns equal economic returns, which follow the same MA(1) process we outline in Equation IA.31 (i.e., $R^o_{j,t} = R_{j,t} = \mu_j \; \ddot{R}_{j,t} \; + \; \rho \cdot \ddot{R}_{j,t-1}$). Moreover, we assume that there is a single risk factor that is not autocorrelated. In this case, the Dimson beta estimate is given by:

$$\begin{aligned}
\widehat{\beta}_{f,j} &= \frac{Cov(f_t, R_{j,t})}{Var(f_t)} + \frac{Cov(f_{t-1}, R_{j,t})}{Var(f_{t-1})} \\
&= \frac{Cov(f_t, \ddot{R}_{j,t} + \rho \cdot \ddot{R}_{j,t-1})}{Var(f_t)} + \frac{Cov(f_{t-1}, \ddot{R}_{j,t} + \rho \cdot \ddot{R}_{j,t-1})}{Var(f_{t-1})} \\
&= \frac{Cov(f_t, \ddot{R}_{j,t})}{Var(f_t)} + \rho \cdot \frac{Cov(f_{t-1}, \ddot{R}_{j,t-1})}{Var(f_{t-1})} \\
&= (1+\rho) \cdot \beta_{f,j} \qquad \text{(IA.36)}
\end{aligned}$$

calibrated ρ value (together with $\widehat{\beta}'_{f,j}\mathbb{E}[f]$) to calculate the α bias separately for each hedge fund strategy based on Equation IA.35.

which leads to the same α_{Bias} present in return unsmoothing methods when economic returns are autocorrelated (in Equation IA.35).

D Robustness Analyses

This section performs a series of robustness analyses that modify different aspects of our empirical design. To conserve space, we only report the core results for each alternative specification studied, but other results are also similar to what we report in the main text.

D.1 3-step Unsmoothing with Value-Weighted Aggregate Returns

While developing our 3-step unsmoothing method, we relied on time-invariant weights, w_j. For this reason, we use equal-weights in our empirical analysis (as opposed to value-weights) when unsmoothing aggregate (or strategy-level) returns. For robustness, we repeat our analysis here after replacing equal-weights with value-weights (i.e., weights based on NAV). All other aspects of the analysis are unchanged, including the fact that we report equal-weighted averages of statistics, not value-weighted averages.

Figure IA.4 replicates Figure 2 in the main text after replacing equal-weights with value-weights to construct strategy indexes. It is clear that all results presented in the main text remain valid after relying on value-weighted strategy indexes. In particular, the 3-step method has little effect on volatility, but it increases R^2s and decreases αs.

Table IA.3 replicates Table 7 in the main text after replacing equal-weights with value-weights to construct the aggregate private CRE fund return. Similar to what we report in the main text, the 3-step method continues to substantially increase risk exposures. Consequently, the 3-step method drives the average α of private CRE funds to (close to) zero.

D.2 3-step Unsmoothing with Regular vs Log Returns

In the main text, we use regular returns for the hedge fund analysis and log returns for the private CRE analysis to be consistent with the respective literatures. For robustness, we repeat our analysis here after replacing regular (log) returns with log (regular) returns in the case of hedge funds (private CRE funds). All other aspects of the analysis are unchanged, including the fact that the statistics reported are based on regular returns (even when we obtain regular returns from unsmoothed log returns).

Figure IA.5 replicates Figure 2 in the main text after replacing regular returns with log returns, with all results being similar to what we report in the main text. In particular, the 3-step method has little effect on volatility, but it increases R^2s and decreases αs.

Table IA.4 replicates Table 7 in the main text after replacing log returns with regular returns, with all results being similar to what we report in the main text. In particular, the 3-step method continues to substantially increase risk exposures and to drive the average α of private CRE funds to (close to) zero.

D.3 MA 3-step Unsmoothing Dropping the First 12 Monthly Returns

Our baseline specification deals with backfilled returns by following Jorion and Schwarz (2019). However, their approach is relatively new. In earlier research, it was common to drop the first 12 monthly returns before calculating performance-related statistics. To provide results that are directly comparable to the previous literature, Figure IA.6 replicates Figure 2 in the main text after dropping the first 12 monthly returns when calculating performance-related statistics instead of following Jorion and Schwarz (2019)'s algorithm to identify backfilled observations. It is clear that all results presented in the main text remain valid after relying on this alternative backfilling adjustment. In particular, the 3-step method has little effect on volatility, but it increases R^2s and decreases αs.

D.4 MA 3-step Unsmoothing with AIC Criterion

In the main text, we use a MA(3) to unsmooth hedge fund returns since prior literature largely uses a fixed number of lags (e.g., Getmansky, Lo, and Makarov (2004) uses a MA(2), which our MA(3) specification nests). However, it can be more efficient to directly account for fund heterogeneity. As such, Figure IA.7 replicates Figure 2 in the main text using the AIC criterion to choose the number of smoothing lags (between 0, 1, 2, and 3) for each fund separately. The results are very similar to what we report in Figure 2.

D.5 AR 3-step Unsmoothing for Open-end Private CRE Funds

In the main text, we rely on all 66 private CRE funds (29 open-end and 37 closed-end) available in our dataset to maximize our coverage. However, one could argue that open-end funds are more appropriate for the type of analysis we perform. As such, Table IA.5 replicates Table 7 in the main text after subsetting the data to the 29 open-end funds available. Results are similar, with the exposure to the public real estate market increasing strongly (and the α decreasing strongly) after 3-step unsmoothing.

D.6 Ex-Ante Performance: Alternative Portfolio Alphas and Sorting Variable

For our ex-ante performance analysis in Subsection 2.2, we obtain portfolio-level alphas using 1-step unsmoothing (note that 3-step unsmoothing is not needed since these are not fund-level alphas). If the smoothing process is mispecified, these alphas could be biased and lead to different conclusions in terms of the relative performance of the different methods in sorting funds by alpha. Figures IA.8 and IA.9 consider two alternative alpha calculations that apply Dimson and constrained Dimson after 1-step unsmoothing. The results are qualitatively similar to what we report in the main text, with the main conclusion being that our 3-step unsmoothing method performs better than 1-step unsmoothing (and Dimson) in the ex-ante identification of α, with such result being driven by illiquid funds.

Another potential concern with the our ex-ante performance analysis in Subsection 2.2 is that alphas are estimated with noise and the use of alpha t-statistics for the sorting (following Bollen, Joenväärä, and Kauppila (2021)) may not provide the correct adjustment for the estimation noise. To deal with this issue, we consider an alternative sorting procedure that relies on past αs estimated using a shrinkage method analogous to Vasicek (1973). Specifically, we obtain OLS alpha estimates ($\widehat{\alpha}_j$) and their standard errors ($\sigma_{\widehat{\alpha},j}$), and then calculate $\alpha_{j,Shrunk} = (1 - w_j) \cdot \overline{\alpha}_j + w_j \cdot \widehat{\alpha}_j$, where $\overline{\alpha}_j$ is the average $\widehat{\alpha}$ of funds in the same category as fund j and $w_j = (1/\sigma^2_{\widehat{\alpha},j})/(1/\sigma^2_{\widehat{\alpha},j} + 1/\sigma^2_{\alpha,j})$ with $\sigma_{\alpha,j}$ representing the cross-sectional standard deviation of the $\widehat{\alpha}$ values for the funds in the same category as fund j. Figures IA.8 reports the results. While adjusting for noise through this shrinkage method generally improves the sorting procedures (in that the ex-post alphas tend to be higher in Figure IA.8 than in Figure 3), our core result remains similar. That is, alphas estimated through 3-step unsmoothing provide a better signal for future alphas than all other methods we consider, including the approach in Dimson (1979) and our CDimson modification to it.

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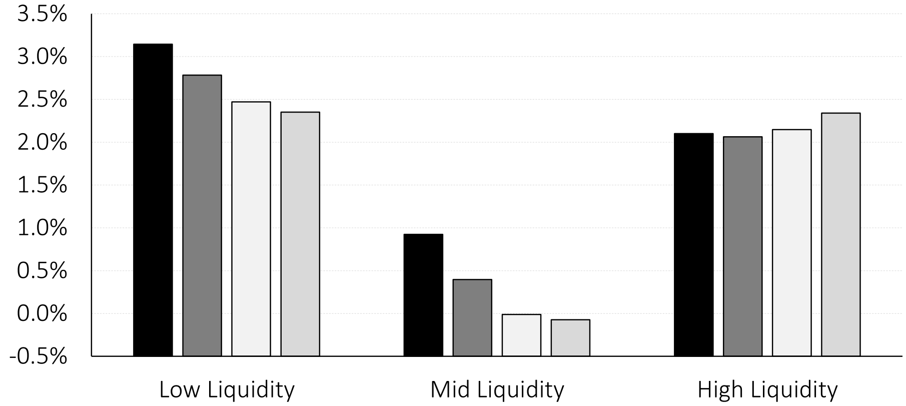
(a) Average αs



(b) Average t^{α}_{stat}

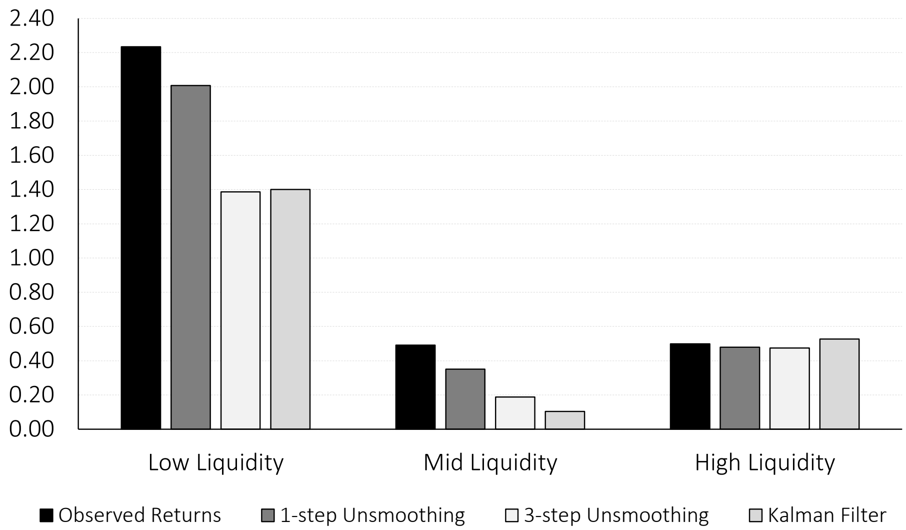


Figure IA.1

Hedge Fund Risk and Performance by Strategy Liquidity (Kalman Filter Unsmoothing)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), 3-step unsmoothed returns, and returns unsmoothed through a Kalman Filter that accounts for the $\xi_{j,t}$ term in our Bayesian model. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017. See Section 1 for a general description of unsmoothing methods, Subsection 2.1 for further empirical details, and Section A.3 for the Kalman Filter unsmoothing method we use in this figure, which accounts for the $\xi_{j,t}$ term in our Bayesian model.

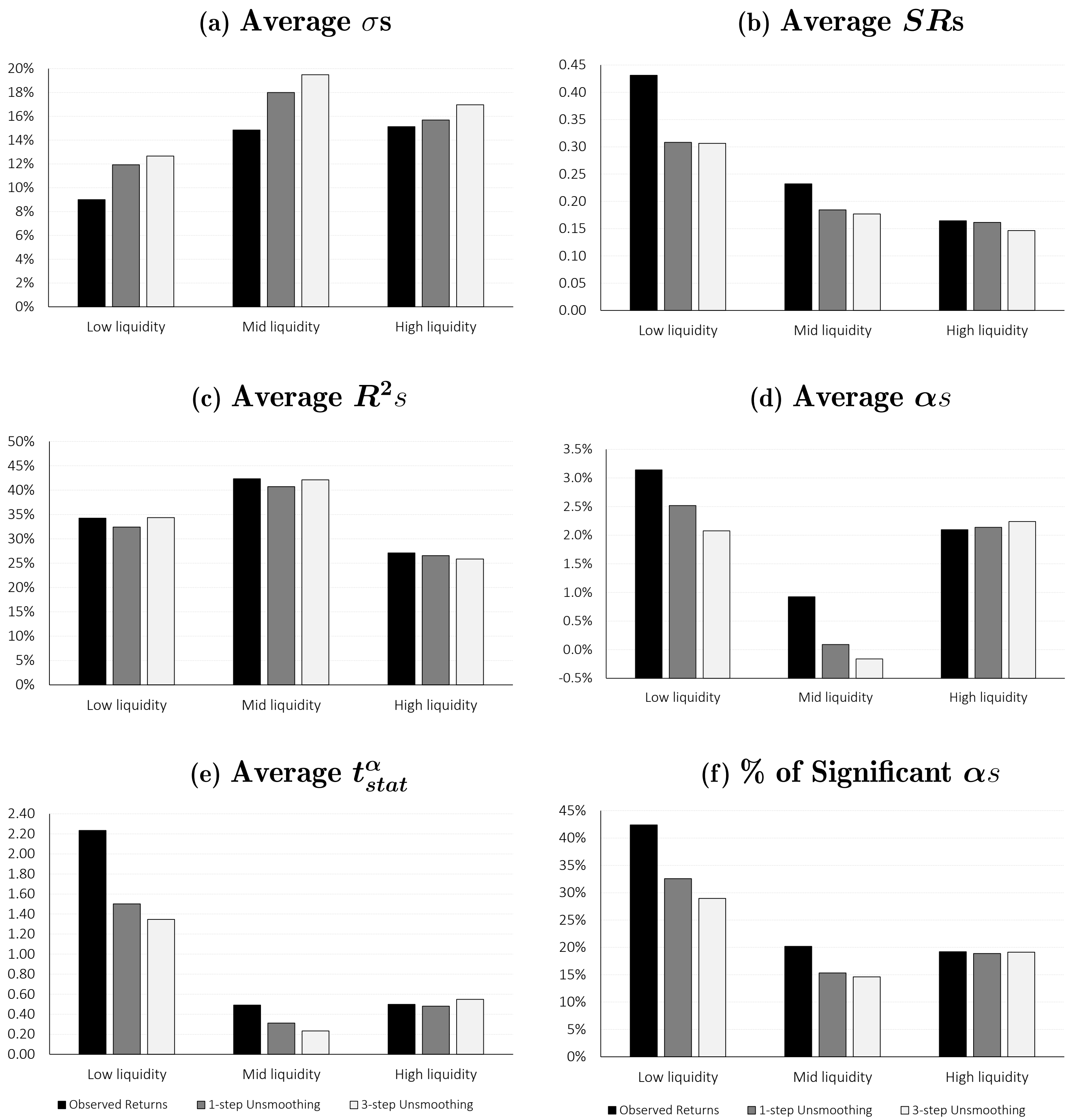


Figure IA.2
Hedge Fund Risk and Performance by Strategy Liquidity (Unknown Fund Categories)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for a general description of unsmoothing methods, Subsection 2.1 for further empirical details, and Section B for the extended 3-step unsmoothing method we use in this figure, which does not rely on fund categories.

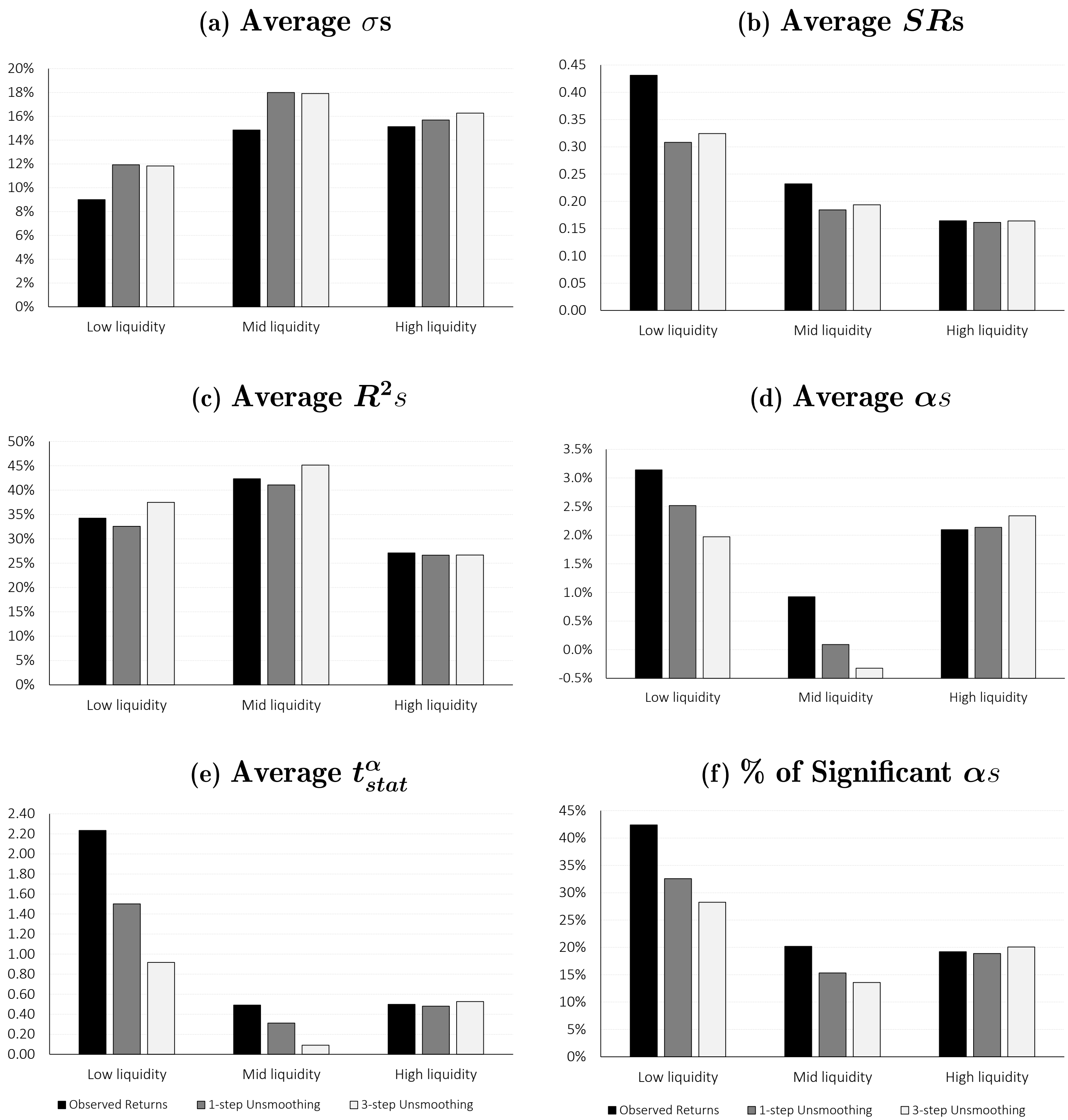


Figure IA.3

Hedge Fund Risk and Performance by Strategy Liquidity (Control for Momentum)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods, Subsection 2.1 for further empirical details, and Subsection C.3 for how we account for return momentum.

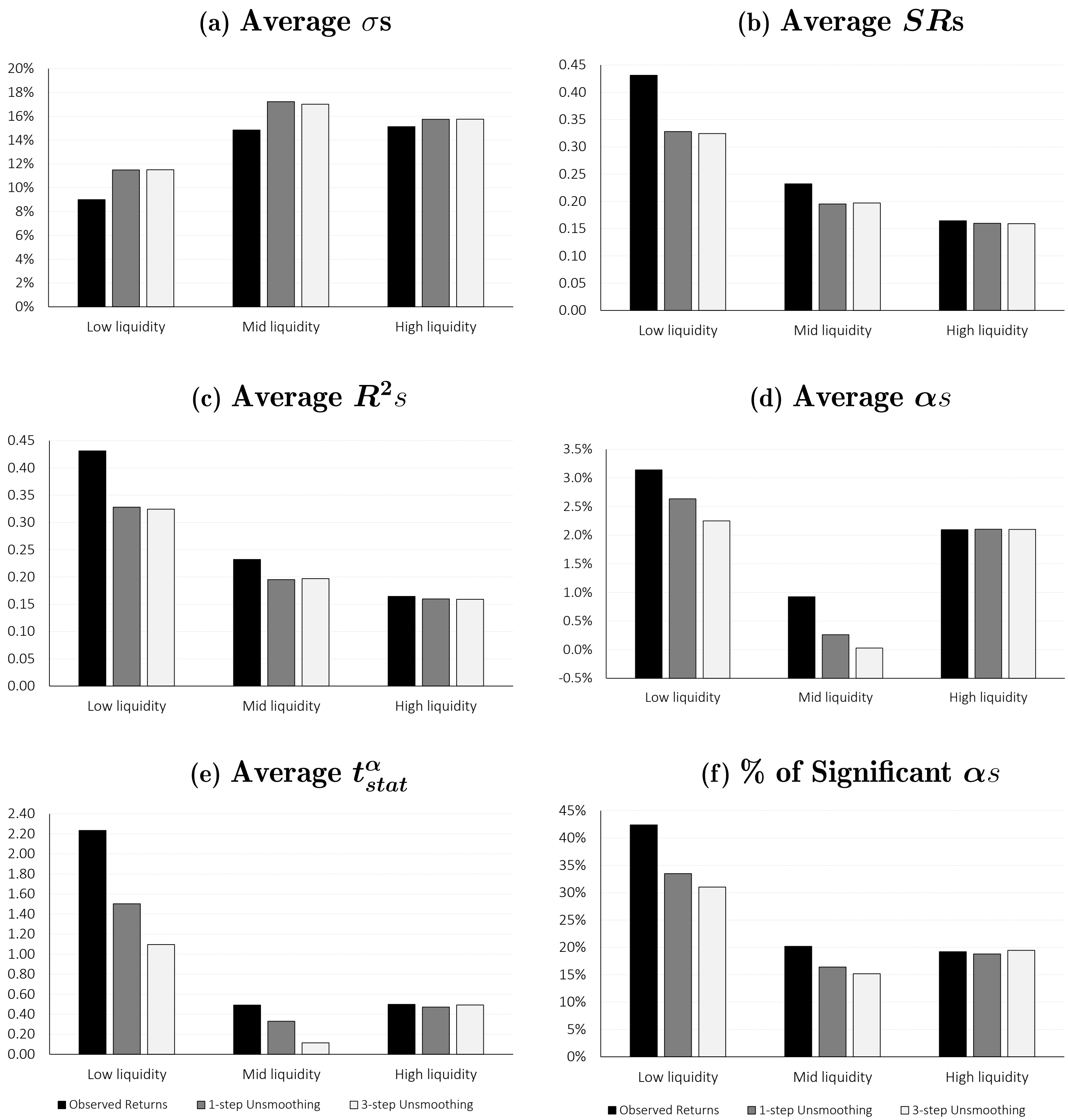


Figure IA.4

Hedge Fund Risk and Performance by Strategy Liquidity (Value-Weights)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods, Subsection 2.1 for further empirical details, and Subsection D.1 for a discussion of value-weighted returns.

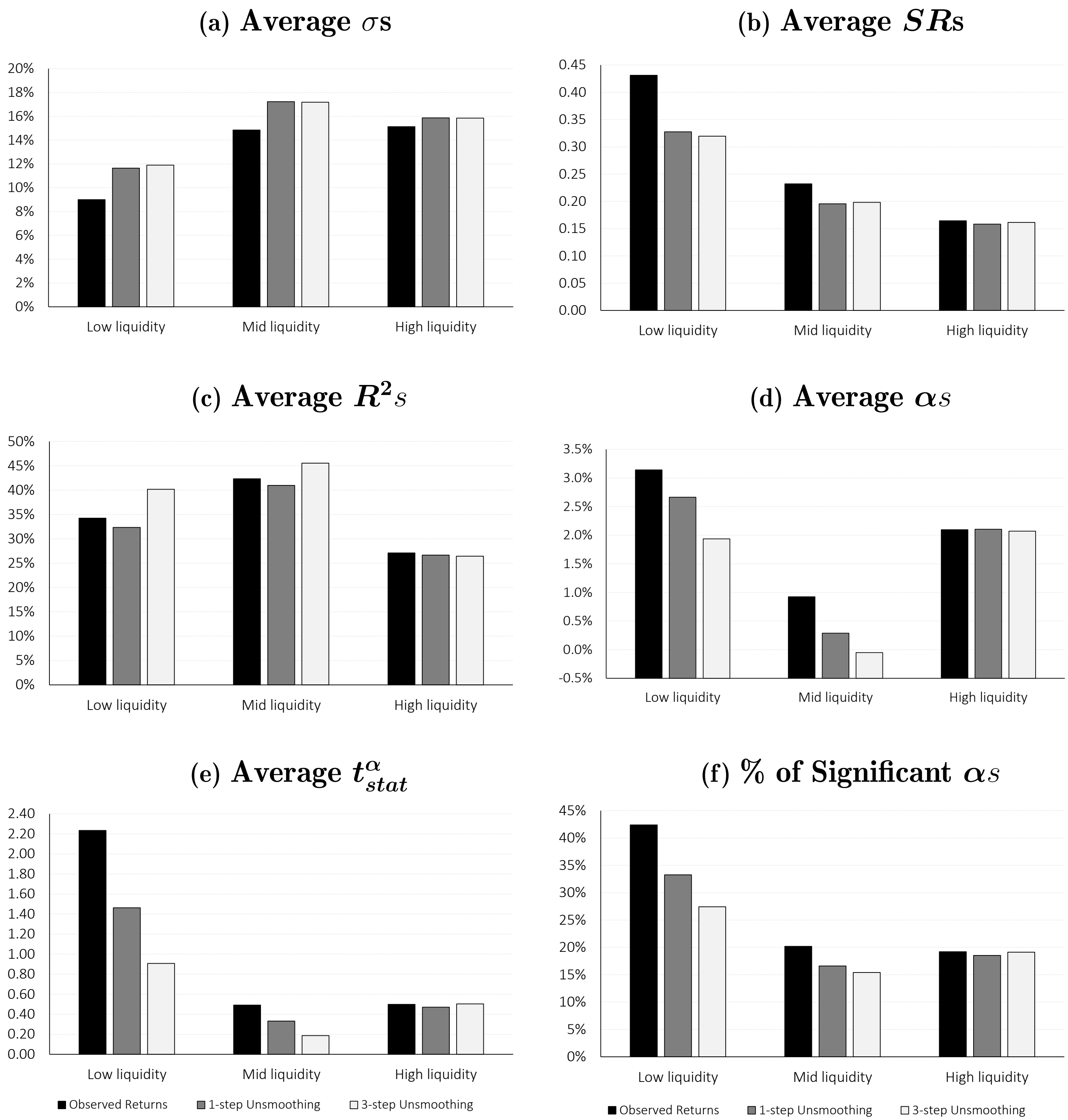


Figure IA.5
Hedge Fund Risk and Performance by Strategy Liquidity (Log Returns)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods, Subsection 2.1 for further empirical details, and Subsection D.2 for a discussion of regular versus log returns.

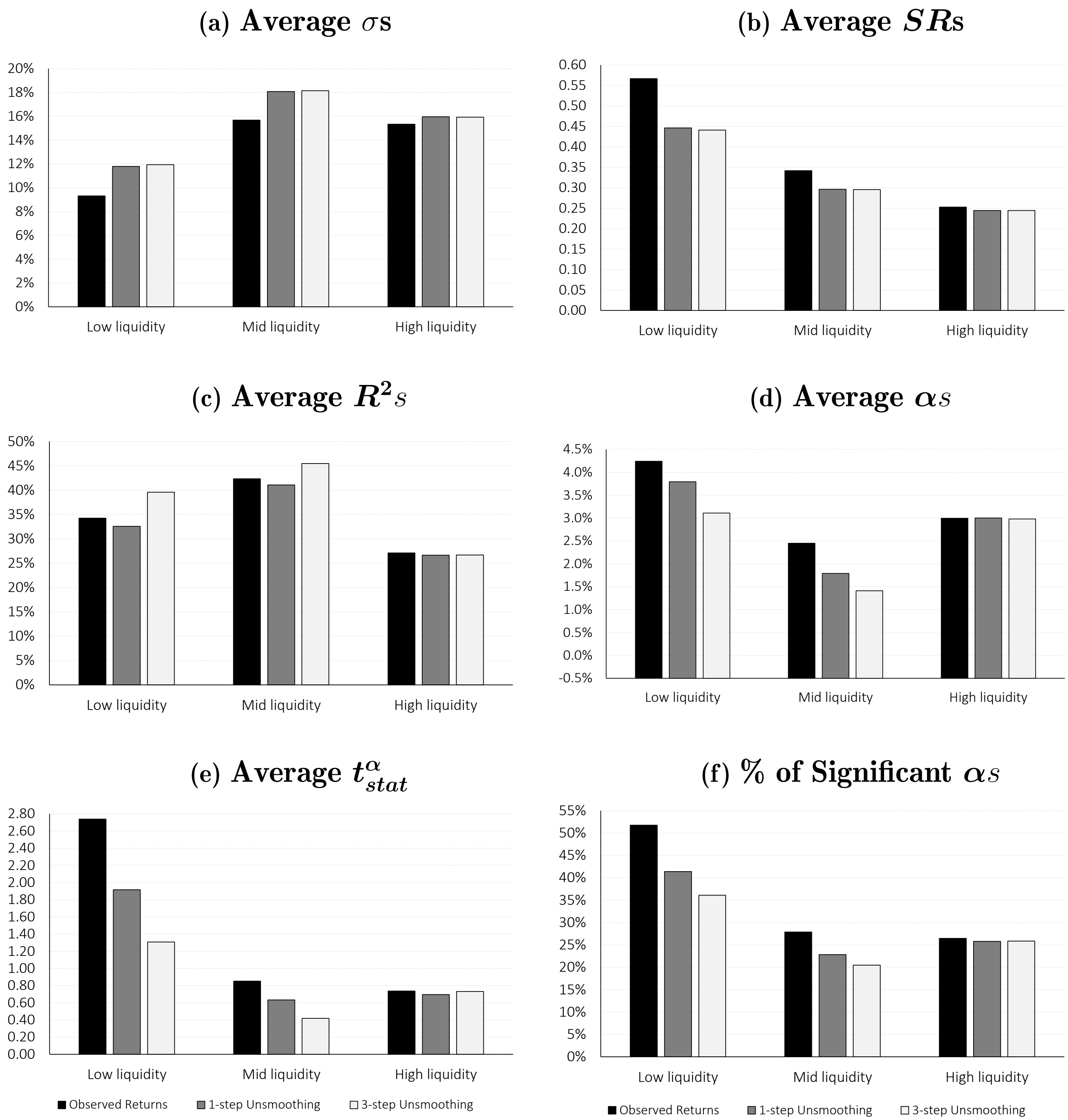


Figure IA.6

Hedge Fund Risk and Performance by Strategy Liquidity (Drop 12 Months)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods, Subsection 2.1 for further empirical details, and Subsection D.3 for a discussion of the removal of 12 months of returns.

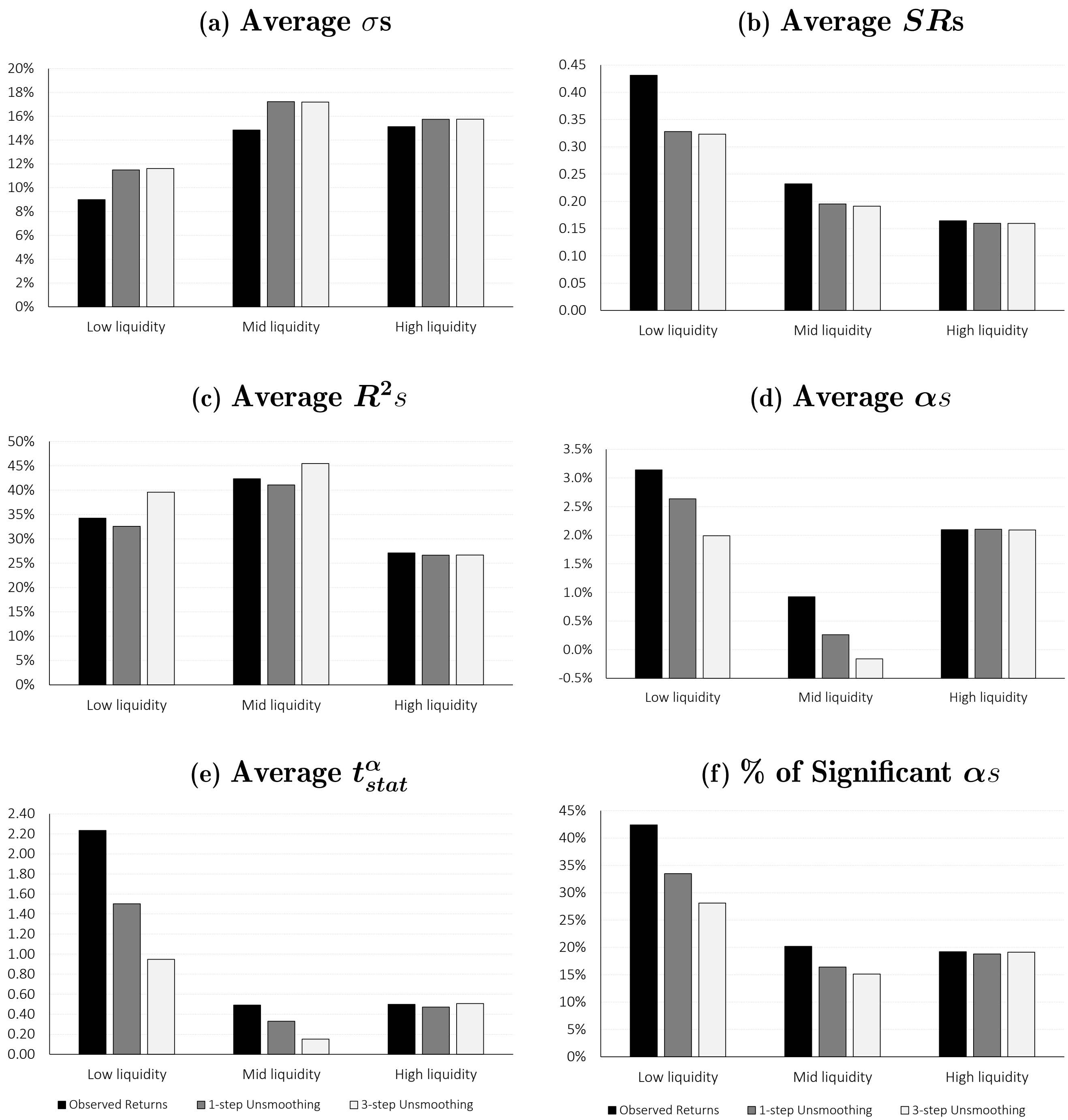


Figure IA.7

Hedge Fund Risk and Performance by Strategy Liquidity (AIC to Select H)

The figure plots average fund-level results for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2017. See Section 1 for unsmoothing methods, Subsection 2.1 for further empirical details, and Subsection D.4 for a discussion of the AIC criterium to select H.

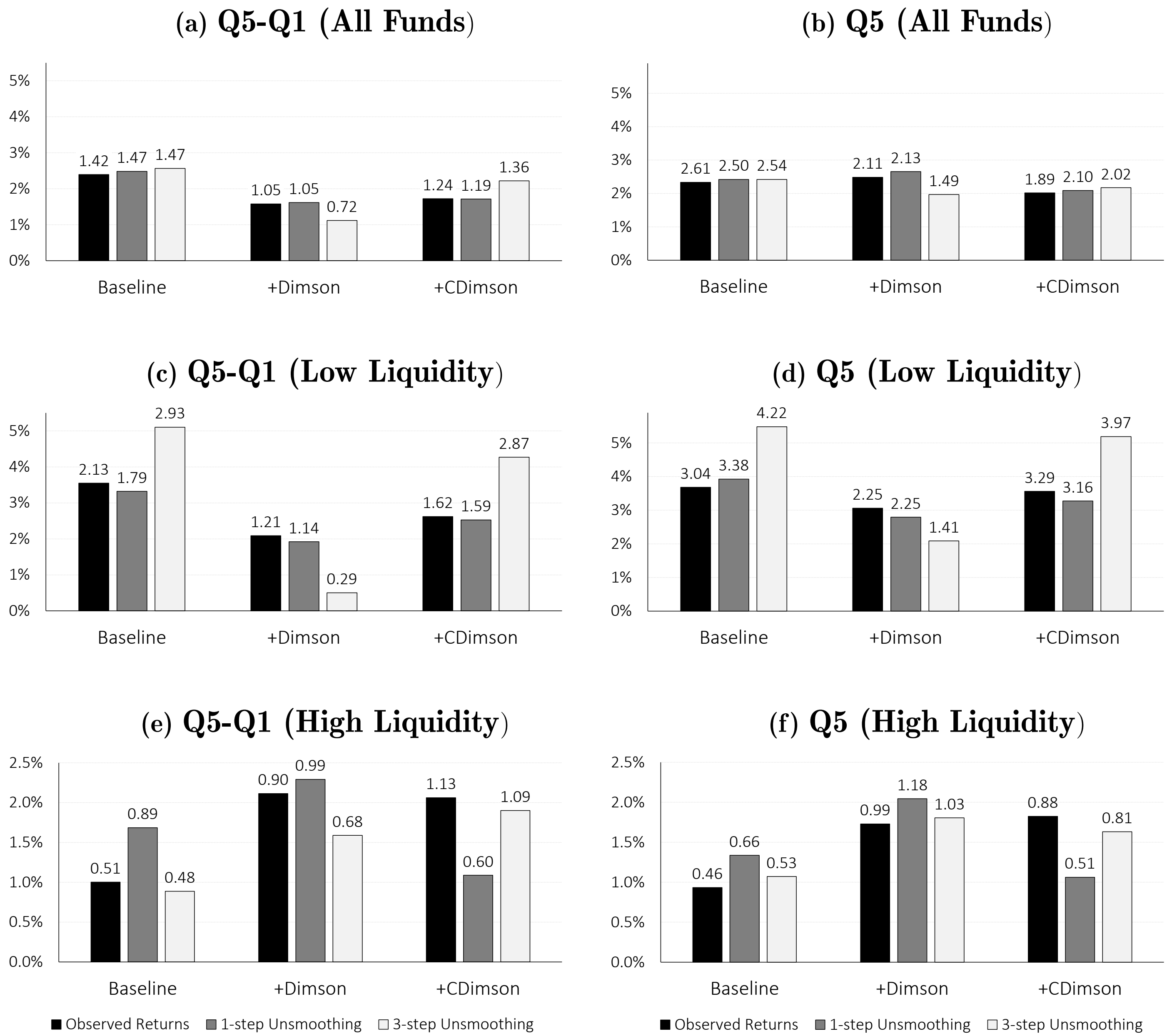


Figure IA.8
αs of Hedge Fund Quintile Portfolios formed Based on the t-stat of Past αs (Portfolio Alphas Based on Dimson)

The figure plots αs (with their t_{stat} on the top of each bar) of quintile portfolios formed by sorting hedge funds based on the t-stat of their past αs (on a 24-month rolling window) measured using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns (with "+Dimson" further applying Dimson (1979) and "+CDimson" further applying our constrained Dimson method). Panels (a), (c), and (e) focus on αs for a strategy that buys the highest and sells the lowest past α quintiles. Panels (b), (d), and (f) focus only on the highest past α quintiles. Panels (a) and (b) use all funds during the sorting procedure whereas Panels (c) and (d) use only funds in the low liquidity strategies and Panels (e) and (f) use only funds in the high liquidity strategies. αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017, but the first portfolio formation is on December 2000 so that we have at least six years of data to unsmooth the hedge fund returns. See Section 1 for unsmoothing methods and Subsections 2.1 and 2.2 for further empirical details. Subsection D.6 discusses the results.

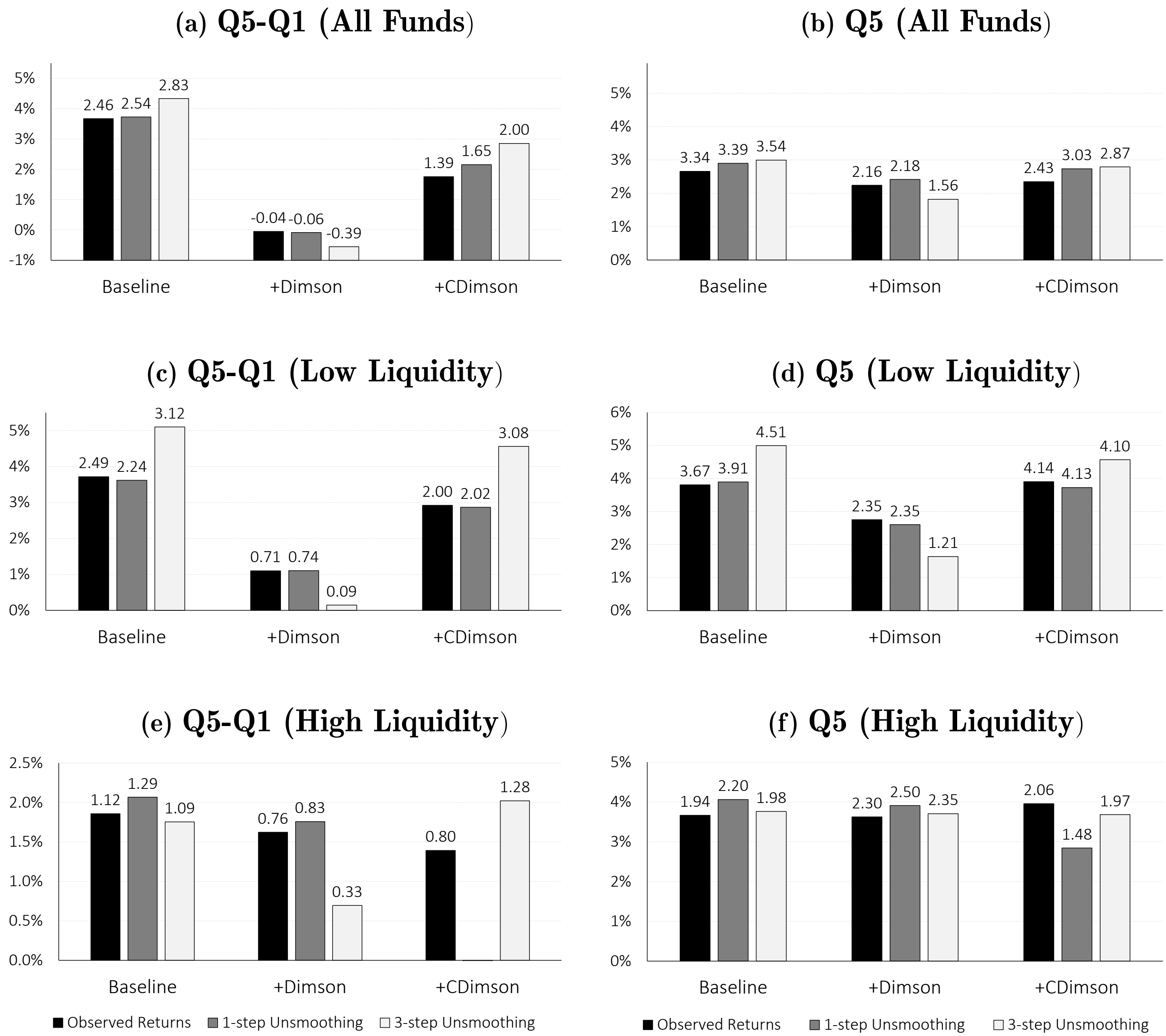


Figure IA.9
αs of Hedge Fund Quintile Portfolios formed Based on the t-stat of Past αs
(Portfolio Alphas Based on Constrained Dimson)

The figure plots αs (with their t_{stat} on the top of each bar) of quintile portfolios formed by sorting hedge funds based on the t-stat of their past αs (on a 24-month rolling window) measured using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns (with "+Dimson" further applying Dimson (1979) and "+CDimson" further applying our constrained Dimson method). Panels (a), (c), and (e) focus on αs for a strategy that buys the highest and sells the lowest past α quintiles. Panels (b), (d), and (f) focus only on the highest past α quintiles. Panels (a) and (b) use all funds during the sorting procedure whereas Panels (c) and (d) use only funds in the low liquidity strategies and Panels (e) and (f) use only funds in the high liquidity strategies. αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2017, but the first portfolio formation is on December 2000 so that we have at least six years of data to unsmooth the hedge fund returns. See Section 1 for unsmoothing methods and Subsections 2.1 and 2.2 for further empirical details. Subsection D.6 discusses the results.

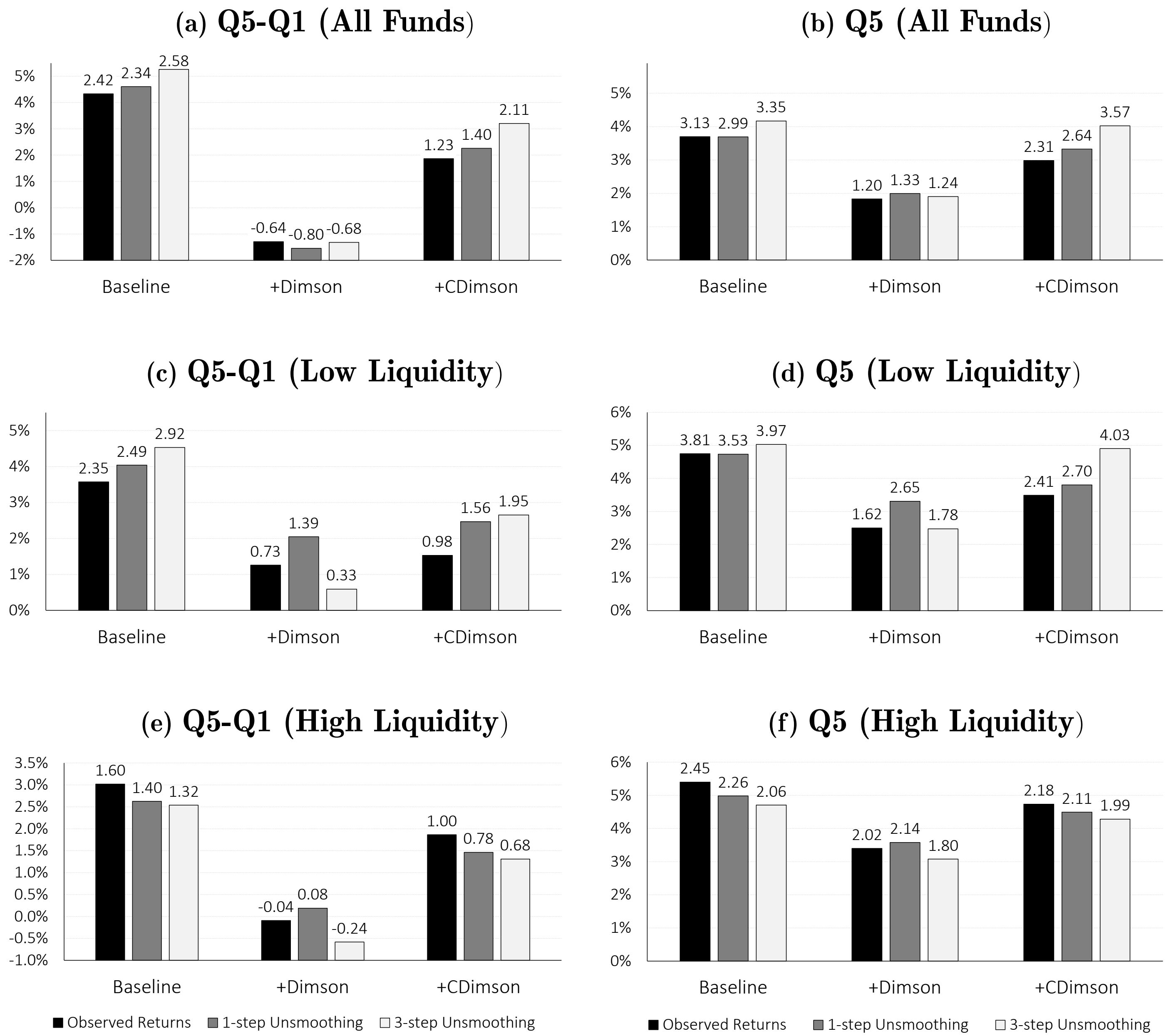


Figure IA.10

αs of Hedge Fund Quintile Portfolios formed from Past αs Estimated with Shrinkage

The figure plots αs (with their t_{stat} on the top of each bar) of quintile portfolios formed by sorting hedge funds based on their past αs (on a 24-month rolling window) measured using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns (with "+Dimson" further applying Dimson (1979) and "+CDimson" further applying our constrained Dimson method). Panels (a), (c), and (e) focus on αs for a strategy that buys the highest and sells the lowest past α quintiles. Panels (b), (d), and (f) focus only on the highest past α quintiles. Panels (a) and (b) use all funds during the sorting procedure whereas Panels (c) and (d) use only funds in the low liquidity strategies and Panels (e) and (f) use only funds in the high liquidity strategies. αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and are shrunk towards zero in a method analogous to Vasicek (1973). The sample goes from January 1995 to December 2017, but the first portfolio formation is on December 2000 so that we have at least six years of data to unsmooth the hedge fund returns. See Section 1 for unsmoothing methods and Subsections 2.1 and 2.2 for further empirical details. Subsection D.6 discusses the results.

Table IA.1
Unsmoothing in Simulations with $\xi_{j,t} \neq 0$

The table reports autocorrelations and alphas in simulations. In Panel A, we simulate returns on a panel of 670 funds over a 85 month period. The monthly economic returns of each fund j satisfy $R_{j,t} = \alpha_j + \beta_j \cdot f_t + \varepsilon_{j,t}$, where $\alpha_j \sim N(\mu_\alpha, \sigma^2_\alpha)$, $\beta_j \sim N(1, \sigma^2_\beta)$, $f_t \sim N(\mu_f, \sigma^2_f)$, and $\varepsilon_t \sim N(0, \sigma^2_\varepsilon)$, which is identical to the simulations in Panel A of Table 5. Within the Bayesian framework of Section A, this process implies $\overline{\eta}_t = (f_t - \mu_f) \overset{iid}{\sim} N(0, \overline{\sigma}^2)$ with $\overline{\sigma}^2 = \sigma^2_f$ and $\widetilde{\eta}_{j,t} = (R_{j,t} - (\alpha_j + \beta_j \cdot \mu_f) - (f_t - \mu_f)) \overset{iid}{\sim} N(0, \widetilde{\sigma}^2_j)$ with $\widetilde{\sigma}^2_j = (\beta_j - 1)^2 \cdot \sigma^2_f + \sigma^2_\varepsilon$. Then, given target π_j and ϕ_j values, we have $\widehat{\overline{\sigma}}^2_j = [(1-\pi_j)/\pi_j] \cdot \overline{\sigma}^2$ and $\widehat{\widetilde{\sigma}}^2_j = [(1-\phi_j)/\phi_j] \cdot \widetilde{\sigma}^2_j$, which allows us to simulate $\overline{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\overline{\sigma}}^2_j)$ and $\widetilde{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\widetilde{\sigma}}^2_j)$ and calculate $\xi_{j,t} = \pi_j \cdot (\overline{u}_{j,t} - \overline{u}_{j,t-1}) + \phi_j \cdot (\widetilde{u}_{j,t} - \widetilde{u}_{j,t-1})$ so that observed returns are given by $r^o_{j,t} = \pi_j \cdot \overline{r}_t + (1-\pi_j) \cdot \overline{r}_{t-1} + \phi_j \cdot \widetilde{r}_{j,t} + (1-\phi_j) \cdot \widetilde{r}_{j,t-1} + \xi_{j,t}$. Given observed returns, we then estimate economic returns for each fund in the panel using the 1- and 3-step unsmoothing methods and study the properties of observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. Columns with "+Dimson" apply the Dimson (1979) method to the given return measure. The table reports the average results obtained from 1,000 simulations of this panel of funds. See Section 1 for the unsmoothing methods and Section A for the Bayesian framework

PANEL A: 1-Factor Model

	$\phi^{(1)} = \pi^{(1)}$		$\phi^{(1)} < \pi^{(1)}$		$\phi^{(1)} > \pi^{(1)}$	
$\phi^{(1)}$	0.30		0.20		0.40	
$\pi^{(1)}$	0.30		0.40		0.20	
$Cor_1(R_o)$	-0.02		-0.02		-0.02	
$Cor_1(R_{1s})$	0.00		0.00		0.00	
$Cor_1(R_{3s})$	-0.02		-0.03		-0.01	
$Cor_1(\overline{R}_o)$	0.36		0.46		0.23	
$Cor_1(\overline{R}_{1s})$	0.37		0.46		0.24	
$Cor_1(\overline{R}_{3s})$	0.00		-0.01		0.00	
	Standard	+Dimson	Standard	+Dimson	Standard	+Dimson
$\widehat{\alpha}_o$	2.2%	0.0%	3.0%	0.0%	1.5%	0.0%
$\widehat{\alpha}_{1s}$	2.3%	0.1%	3.0%	0.1%	1.6%	0.1%
$\widehat{\alpha}_{3s}$	0.0%	0.0%	0.1%	0.1%	0.0%	0.0%
$\lvert\widehat{\alpha}_o\rvert$	2.3%	0.9%	3.1%	0.9%	1.7%	0.9%
$\lvert\widehat{\alpha}_{1s}\rvert$	2.4%	1.0%	3.1%	1.0%	1.7%	0.9%
$\lvert\widehat{\alpha}_{3s}\rvert$	1.0%	0.9%	0.9%	0.9%	1.1%	0.9%
$100 \times \widehat{\alpha}^2_o$	0.079	0.015	0.125	0.015	0.048	0.014
$100 \times \widehat{\alpha}^2_{1s}$	0.081	0.017	0.127	0.018	0.048	0.016
$100 \times \widehat{\alpha}^2_{3s}$	0.017	0.013	0.014	0.014	0.021	0.013

Table IA.1 (Cont'd)
Unsmoothing in Simulations with $\xi_{j,t} \neq 0$

The table reports autocorrelations and alphas in simulations. In Panel B, we simulate returns for a panel of 670 funds over a 85 month period or a 170 month period. The monthly economic returns of each fund j satisfy $R_{j,t} = \alpha_j + \beta_j^{'} f_t + \varepsilon_{j,t}$ with β_j representing a vector of eight fund-specific risk exposures and f_t representing a vector of eight risk factors. The construction of economic returns follows a process analogous to the simulations in Panel B of Table 5. Specifically, simulated fund-level parameters (i.e., β_j and the smoothing parameters, π_j and ϕ_j) are bootstrapped with replacement from the corresponding joint empirical distribution of parameters based on relative value funds estimated under the 3-step method with one lag. The panel of factor returns is common to all funds within a given simulation run and is bootstrapped with replacement from the time-series of the 8 FH factors. For simplicity, "true alpha" (α_j) is set to zero for all funds. We then calculate (in each simulation) $\overline{\eta}_t = \overline{R}_t - 1/T \cdot \sum_{t=1}^{T} \overline{R}_t$ to obtain $\overline{\sigma}^2$ as the variance of $\overline{\eta}_t$. Similarly, we calculate (in each simulation and for each fund) $\widetilde{\eta}_{j,t} = R_{j,t} - (\alpha_j + \beta_j^{'} f_t) - \overline{\eta}_t$ to obtain $\widetilde{\sigma}^2$ as the variance of $\widetilde{\eta}_t$. Then, given each fund π_j and ϕ_j values, we have $\widehat{\overline{\sigma}}_j^2 = [(1-\pi_j)/\pi_j] \cdot \overline{\sigma}^2$ and $\widehat{\widetilde{\sigma}}_j^2 = [(1-\phi_j)/\phi_j] \cdot \widetilde{\sigma}_j^2$, which allows us to simulate $\overline{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\overline{\sigma}}_j^2)$ and $\widetilde{u}_{j,t} \overset{iid}{\sim} N(0, \widehat{\widetilde{\sigma}}_j^2)$ and calculate $\xi_{j,t} = \pi_j \cdot (\overline{u}_{j,t} - \overline{u}_{j,t-1}) + \phi_j \cdot (\widetilde{u}_{j,t} - \widetilde{u}_{j,t-1})$ so that observed returns are given by $r_{j,t}^o = \pi_j \cdot \overline{r}_t + (1-\pi_j) \cdot \overline{r}_{t-1} + \phi_j \cdot \widetilde{r}_{j,t} + (1-\phi_j) \cdot \widetilde{r}_{j,t-1} + \xi_{j,t}$. Given observed returns, we then estimate economic returns for each fund in the panel using the 1- and 3-step unsmoothing methods and study the properties of observed returns, 1-step unsmoothed returns, and 3-step unsmoothed returns. Columns with "+Dimson" also apply the Dimson (1979) method (with one lag) while columns with "+CDimson" also apply the constrained Dimson method (see Footnote 20). The table reports the average results obtained from 1,000 simulations of this panel of funds. See Section 1 for the unsmoothing methods and Section A for the Bayesian framework.

PANEL B: 8-Factor Model

	$T = 85$ Months			$T = 170$ Months		
$Cor_1(R_o)$	0.01			0.01		
$Cor_1(R_{1s})$	-0.03			-0.03		
$Cor_1(R_{3s})$	-0.03			-0.02		
$Cor_1(\overline{R}_o)$	0.31			0.33		
$Cor_1(\overline{R}_{1s})$	0.25			0.27		
$Cor_1(\overline{R}_{3s})$	0.01			0.00		
	Standard	+Dimson	+CDimson	Standard	+Dimson	+CDimson
$\widehat{\alpha}_o$	0.3%	0.0%	0.1%	0.3%	0.0%	0.1%
$\widehat{\alpha}_{1s}$	0.3%	0.0%	0.1%	0.3%	0.0%	0.1%
$\widehat{\alpha}_{3s}$	0.0%	0.0%	-0.1%	0.0%	0.0%	-0.1%
$\lvert\widehat{\alpha}_o\rvert$	1.0%	1.4%	1.0%	0.8%	1.0%	0.8%
$\lvert\widehat{\alpha}_{1s}\rvert$	0.9%	1.4%	1.0%	0.7%	1.0%	0.8%
$\lvert\widehat{\alpha}_{3s}\rvert$	0.9%	1.4%	1.0%	0.6%	1.0%	0.7%
$100 \times \widehat{\alpha}_o^2$	0.016	0.034	0.019	0.011	0.021	0.012
$100 \times \widehat{\alpha}_{1s}^2$	0.014	0.035	0.020	0.009	0.022	0.012
$100 \times \widehat{\alpha}_{3s}^2$	0.013	0.035	0.018	0.008	0.022	0.011

Table IA.2
Alpha Bias due to Autocorrelation in Economic Returns

The table reports the bias in alpha estimates arising from different channels. The first column reports the alpha bias due to ignoring return smoothing (i.e., the average difference between alphas based on observed returns and 3-step unsmoothed returns). The other columns report the alpha bias from 3-step unsmoothing under different assumptions about ρ. Specifically, we note that the return forecasting R^2 is given by $R^2 = Var(\mathbb{E}_t[R_j])/Var(R_j) = \rho^2/(1+\rho^2)$, which implies $\rho = \pm\sqrt{R^2/(1-R^2)}$ and the first order autocorrelation $Cor_1 = \rho/(1+\rho^2)$. We then use different assumptions for R^2 to calibrate ρ and obtain the alpha bias separately for each hedge fund strategy based on Equation IA.35 using the calibrated ρ (together with $\widehat{\beta}'_{f,j}\mathbb{E}[f]$). See Subsection C.4 for further details.

Hedge Fund Strategies	**Alpha Bias in R_o**	**Alpha Bias after Unsmoothing**		
		$R^2 = 1\%$	$R^2 = 2\%$	$R^2 = 3\%$
		$Cor_1 = \pm 0.10$	$Cor_1 = \pm 0.14$	$Cor_1 = \pm 0.17$
Relative Value	0.9%	± 0.1%	± 0.1%	± 0.1%
Event Driven	1.7%	± 0.3%	± 0.4%	± 0.5%
Multi Strategy	1.5%	± 0.2%	± 0.3%	± 0.4%
Emerging Mkts	1.6%	± 0.4%	± 0.6%	± 0.7%
Sector	1.3%	± 0.3%	± 0.4%	± 0.5%
Long Only	1.6%	± 0.5%	± 0.7%	± 0.8%
Long-Short	1.3%	± 0.3%	± 0.4%	± 0.4%
Market Neutral	0.3%	± 0.1%	± 0.1%	± 0.1%
Global Macro	0.2%	± 0.1%	± 0.1%	± 0.1%
CTA	-0.1%	± 0.0%	± 0.0%	± 0.0%

Table IA.3
Risk and Performance of Private CRE Funds (Value-weights)

The table reports (average fund-level) statistics related to the risk and performance of US private commercial real estate (CRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from Q1 1994 through Q4 2017 and is restricted to private CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used, Subsection 3.2 for further empirical details, and Subsection D.1 for a discussion of value-weighted returns.

Statistics are	Raw Performance			1-Factor Model			2-Factor Model			
Related to	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}	R^2	α	β_{re}	β_e	R^2
Observed, R_o	5.2%	12.7%	0.41	4.6%	0.06	3.2%	4.4%	0.02	0.09	4.6%
				(54.5%)	(28.8%)		(53.0%)	(9.1%)	(7.6%)	
1-step, R_{1s}	5.2%	24.1%	0.22	3.0%	0.23	8.1%	2.7%	0.16	0.14	10.1%
				(13.6%)	(48.5%)		(13.6%)	(30.3%)	(10.6%)	
3-step, R_{3s}	5.2%	26.4%	0.20	1.2%	0.40	17.0%	-0.1%	0.18	0.44	21.6%
				(7.6%)	(89.4%)		(9.1%)	(27.3%)	(57.6%)	
From R_o to R_{1s}	0.0%	11.4%	-0.20	-1.6%	0.16	5.0%	-1.7%	0.14	0.05	5.5%
				[-9.19]	[8.90]		[-7.73]	[6.73]	[1.68]	
From R_{1s} to R_{3s}	0.0%	2.3%	-0.02	-1.8%	0.17	8.9%	-2.8%	0.02	0.30	11.5%
				[-4.56]	[4.37]		[-5.23]	[0.74]	[6.43]	
From R_o to R_{3s}	0.0%	13.7%	-0.21	-3.4%	0.34	13.9%	-4.5%	0.16	0.35	17.0%
				[-9.43]	[9.23]		[-9.18]	[6.70]	[9.17]	

Table IA.4
Risk and Performance of Private CRE Funds (Regular Returns)

The table reports (average fund-level) statistics related to the risk and performance of US private commercial real estate (CRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from Q1 1994 through Q4 2017 and is restricted to private CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used, Subsection 3.2 for further empirical details, and Subsection D.2 for a discussion of regular versus log returns.

Statistics are	**Raw Performance**			**1-Factor Model**			**2-Factor Model**			
Related to	$\mathbb{E}[r]$	$\boldsymbol{\sigma}$	$\mathbb{E}[r]/\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta_{re}}$	$\boldsymbol{R^2}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta_{re}}$	$\boldsymbol{\beta_e}$	$\boldsymbol{R^2}$
Observed, R_o	5.0%	13.1%	0.38	4.3%	0.07	3.2%	4.0%	0.02	0.10	4.6%
				(53.0%)	(27.3%)		(51.5%)	(9.1%)	(7.6%)	
1-step, R_{1s}	5.0%	24.8%	0.20	2.5%	0.25	8.7%	2.1%	0.17	0.17	10.6%
				(13.6%)	(53.0%)		(13.6%)	(30.3%)	(12.1%)	
3-step, R_{3s}	5.0%	26.0%	0.19	0.8%	0.42	16.1%	-0.1%	0.26	0.32	18.5%
				(9.1%)	(90.9%)		(9.1%)	(34.8%)	(13.6%)	
From R_o to R_{1s}	0.0%	11.7%	-0.18	-1.8%	0.18	5.5%	-2.0%	0.15	0.07	5.9%
				[-9.42]	[9.01]		[-8.32]	[7.47]	[2.70]	
From R_{1s} to R_{3s}	0.0%	1.1%	-0.01	-1.7%	0.17	7.4%	-2.2%	0.09	0.15	7.9%
				[-5.16]	[4.88]		[-5.67]	[3.12]	[5.55]	
From R_o to R_{3s}	0.0%	12.9%	-0.19	-3.5%	0.35	12.9%	-4.2%	0.24	0.22	13.8%
				[-11.22]	[10.43]		[-10.78]	[9.70]	[8.43]	

Table IA.5
Risk and Performance of Private CRE Funds (Open-end)

The table reports (average fund-level) statistics related to the risk and performance of US private commercial real estate (CRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1991, 1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from Q1 1994 through Q4 2017 and is restricted to private CRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Subsection 3.1 for the AR unsmoothing methods used, Subsection 3.2 for further empirical details, and Subsection D.5 for a discussion of the results based on Open-end funds.

Statistics are Related to	Raw Performance			1-Factor Model			2-Factor Model			
	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}	R^2	α	β_{re}	β_e	R^2
Observed, R_o	5.5%	7.9%	0.70	5.0%	0.05	3.4%	4.9%	0.04	0.02	3.8%
				(82.1%)	(35.7%)		(82.1%)	(17.9%)	(0.0%)	
1-step, R_{1s}	5.5%	20.0%	0.28	2.7%	0.28	12.3%	2.5%	0.24	0.08	13.5%
				(10.7%)	(71.4%)		(10.7%)	(64.3%)	(0.0%)	
3-step, R_{3s}	5.5%	17.3%	0.32	2.3%	0.32	15.7%	1.7%	0.25	0.15	17.0%
				(14.3%)	(96.4%)		(10.7%)	(60.7%)	(3.6%)	
From R_o to R_{1s}	0.0%	12.1%	-0.42	-2.3%	0.23	8.9%	-2.5%	0.20	0.06	9.7%
				[-11.05]	[11.43]		[-10.68]	[6.73]	[1.44]	
From R_{1s} to R_{3s}	0.0%	-2.7%	0.04	-0.4%	0.04	3.4%	-0.7%	0.01	0.07	3.5%
				[-1.73]	[1.72]		[-2.34]	[0.22]	[1.81]	
From R_o to R_{3s}	0.0%	9.4%	-0.38	-2.8%	0.27	12.3%	-3.2%	0.21	0.13	13.2%
				[-13.45]	[15.33]		[-12.55]	[17.19]	[7.52]	