Transforming to Worldspace

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What we want

$$\hat{\mathbf{x}}' = \hat{\mathbf{x}}$$

$$\hat{\mathbf{y}}' = -\hat{\mathbf{n}}$$

$$\hat{\mathbf{z}}' = -\hat{\mathbf{n}} \times \hat{\mathbf{x}}$$

$$\mathcal{O}' = \mathcal{O} + (\text{camera height}) \cdot \hat{\mathbf{n}}$$

Starting with the general affine transformation,

$$
\begin{aligned}
\hat{\mathbf{x}}' &= M_{0,0}\hat{\mathbf{x}} + M_{0,1}\hat{\mathbf{y}} + M_{0,2}\hat{\mathbf{z}} \\
\hat{\mathbf{y}}' &= M_{1,0}\hat{\mathbf{x}} + M_{1,1}\hat{\mathbf{y}} + M_{1,2}\hat{\mathbf{z}} \\
\hat{\mathbf{z}}' &= M_{2,0}\hat{\mathbf{x}} + M_{2,1}\hat{\mathbf{y}} + M_{2,2}\hat{\mathbf{z}} \\
\mathcal{O}' &= \mathcal{O} + M_{3,0}\hat{\mathbf{x}} + M_{3,1}\hat{\mathbf{y}} + M_{3,2}\hat{\mathbf{z}}
\end{aligned}
$$

reference: http://www.uio.no/studier/emner/matnat/ifi/INF3320/h03/undervisningsmateriale/lecture3.pdf

Or, in matrix form, with the entries filled in:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ n_x & n_y & n_z & 0 \\ (-\hat{\mathbf{n}} \times \hat{\mathbf{x}})_x & (-\hat{\mathbf{n}} \times \hat{\mathbf{x}})_y & (-\hat{\mathbf{n}} \times \hat{\mathbf{x}})_z & 0 \\ t_x & t_y & t_z & 1 \end{pmatrix}$$

This is the change of basis matrix

So the same point in space can be referred to in either of the 2 coordinate systems

$$\begin{pmatrix} p_0 & p_1 & p_2 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \\ \mathcal{O} \end{pmatrix} = \begin{pmatrix} p'_0 & p'_1 & p'_2 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}}' \\ \hat{\mathbf{y}}' \\ \hat{\mathbf{z}}' \\ \mathcal{O}' \end{pmatrix}$$

$$= \begin{pmatrix} p'_0 & p'_1 & p'_2 & 1 \end{pmatrix} M \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \\ \mathcal{O} \end{pmatrix}$$

Standard basis frame, of the camera

New basis, in the plane, below the camera

Which means that

$$p_i = M^T p_i'$$

$$p_i' = (M^T)^{-1} p_i$$

the part we want

…the world space transform

$$\begin{pmatrix} 1 & n_x & (-\hat{\mathbf{n}} \times \hat{\mathbf{x}})_x & t_x \\ 0 & n_y & (-\hat{\mathbf{n}} \times \hat{\mathbf{x}})_y & t_y \\ 0 & n_z & (-\hat{\mathbf{n}} \times \hat{\mathbf{x}})_z & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

The Change of Basis Matrix

$$u_0 = \gamma_{00} v_0 + \gamma_{01} v_1 + \gamma_{02} v_2$$

$$u_1 = \gamma_{10} v_0 + \gamma_{11} v_1 + \gamma_{12} v_2$$

$$u_2 = \gamma_{20} v_0 + \gamma_{21} v_1 + \gamma_{22} v_2$$

$$Q_0 = \gamma_{30} v_0 + \gamma_{31} v_1 + \gamma_{32} v_2 + P_0$$

$$M = \begin{bmatrix} \gamma_{00} & \gamma_{01} & \gamma_{02} & 0 \\ \gamma_{10} & \gamma_{11} & \gamma_{12} & 0 \\ \gamma_{20} & \gamma_{21} & \gamma_{22} & 0 \\ \gamma_{30} & \gamma_{31} & \gamma_{32} & 1 \end{bmatrix}$$

image credit: http://cs.brynmawr.edu/Courses/cs312/fall2010/lectures/312-04-coords.pdf