

# THEORIES AS ORDINAL CONSTRAINT

A BAYES FACTOR APPROACH

**Julia M. Haaf**



University of Missouri

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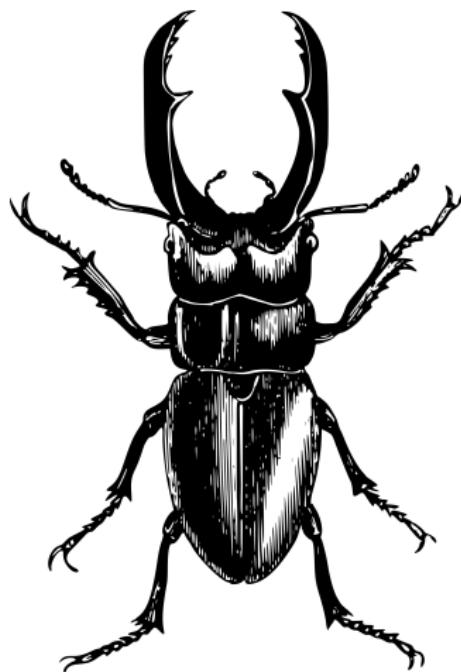


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- **Idea: Stating theory as multiple, simultaneous ordinal constraint**



# A PSYCHOLOGIST'S FAVORITE DESIGN WITH UGLY BUGS



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RYAN, WILDE, & CRIST (2013)

		Disgust	
		Low	High
Fear	Low	Low/Low	Low/High
	High	High/Low	High/High



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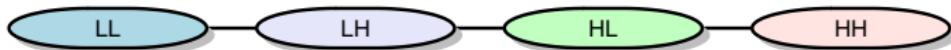
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Fear	Low	Low/Low	Low/High
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"How willing are you to kill/get rid of this bug?"



# SYSTEMS OF ORDERS WITH BUGS

$M_0$ : Null



$M_1$ : Consistent +

$M_2$ : + Equality

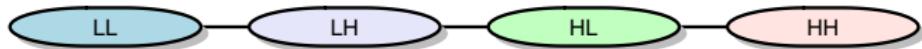
$M_3$ : Fear Only

$M_4$ : Disgust Only

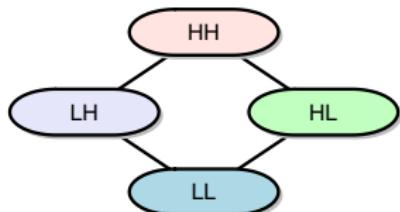
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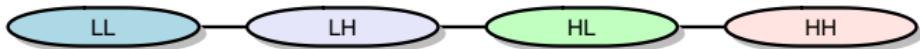
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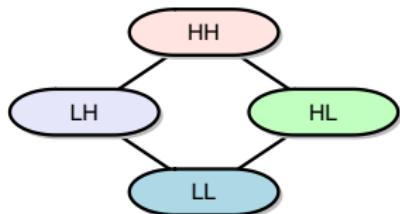
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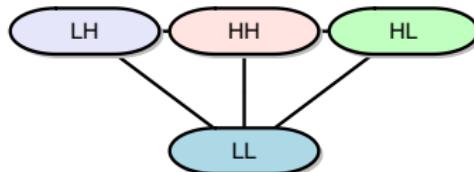
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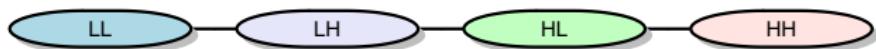
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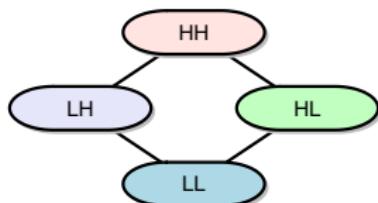
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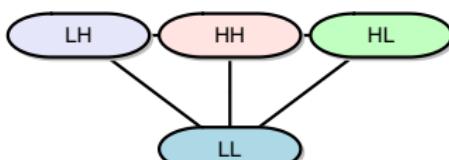
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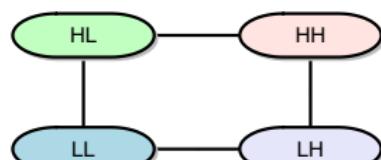
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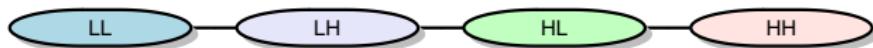


$M_4$ : Disgust Only

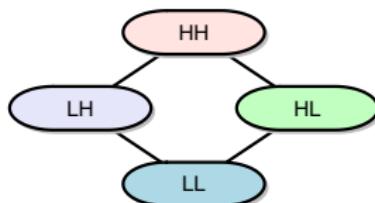
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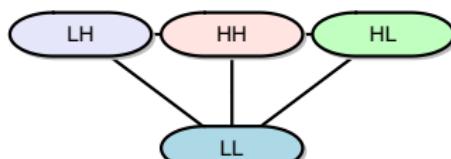
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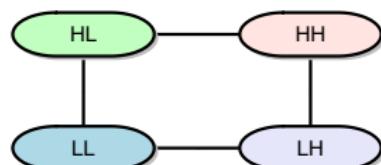
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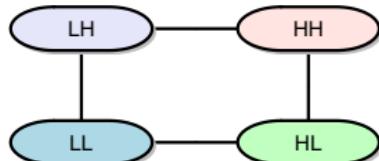
$M_2$ : + Equality



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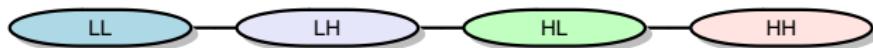
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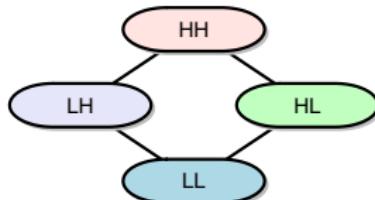
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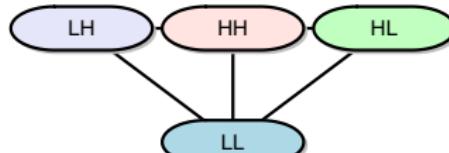
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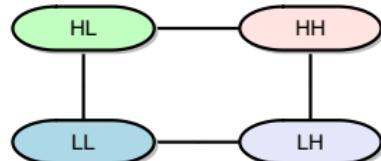
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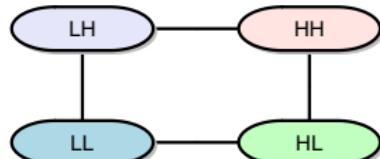
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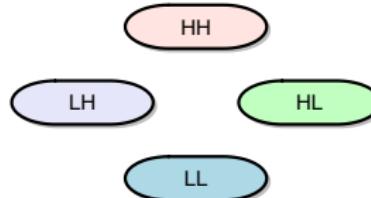
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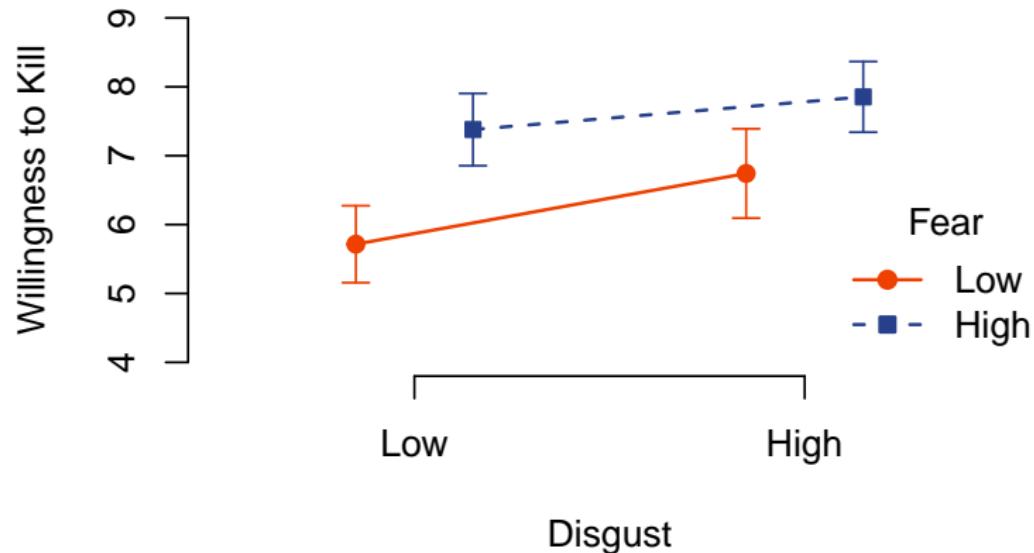


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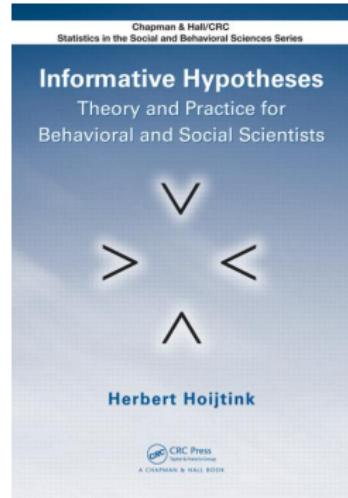
# SYSTEMS OF ORDER ANALYSIS WITH BUGS

## DATA



# ASSESSING SYSTEMS OF ORDERS WITH BAYESIAN ANALYSIS

- Assessing ordinal constraints is straight-forward in Bayesian analysis
- Take posterior samples from the unconstrained model
- What proportion obeys the ordinal constraint in the posterior as compared to the prior?



# SYSTEMS OF ORDER ANALYSIS WITH BUGS

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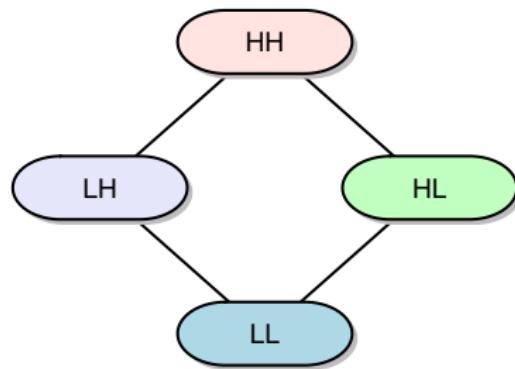
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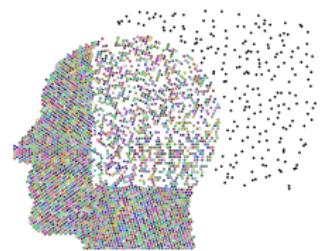
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# TAKE-AWAY FOR THE PRAGMATIC PSYCHOLOGIST

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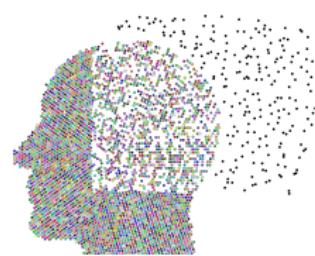
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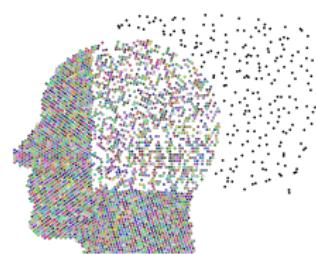
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- Are you convinced by a Bayes factor of 3-to-1?



# A STEP FURTHER: ORDINAL CONSTRAINT ACROSS PEOPLE



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- Handedness: Does everyone have a true right handed advantage? (Clearly no)
- **Symbolic distance: Does everyone truly represent numbers as analog quantities? (IDK)**

# DOES EVERYONE?

- Different Structures Yield Different Processing Interpretations



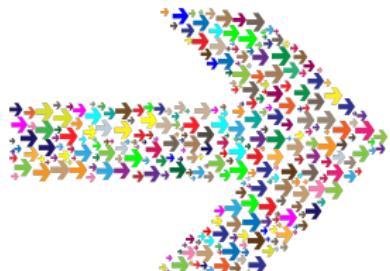
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- Mixed effects → strategies, perhaps complex. What determines who is truly positive and who is truly negative?



# SYMBOLIC DISTANCE

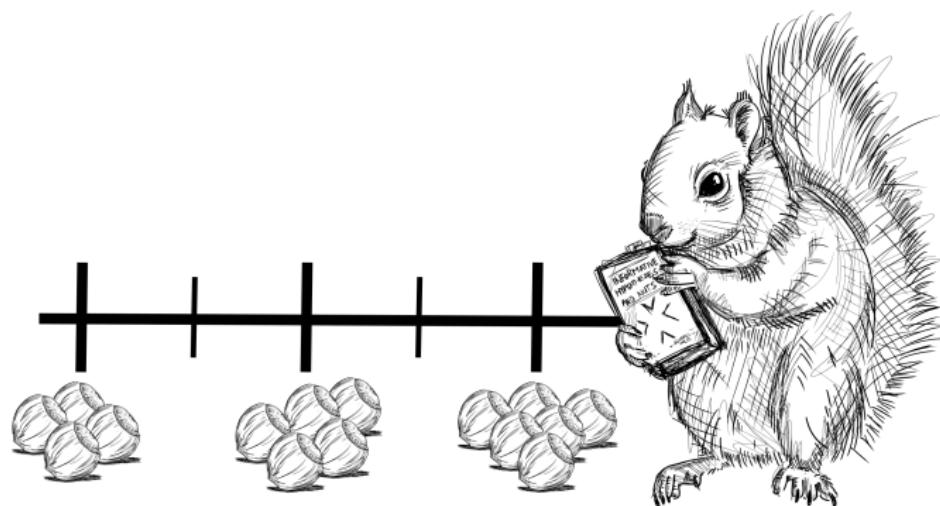
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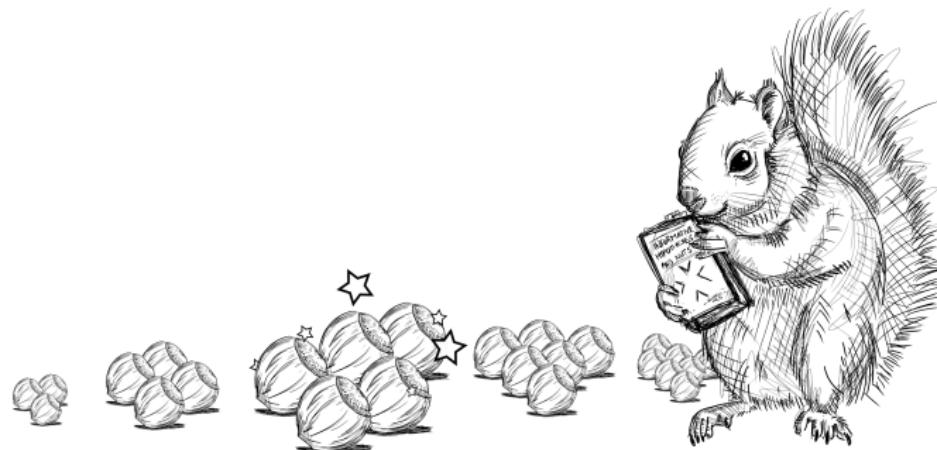
$$\begin{array}{ccc} \text{nuts} & < & \text{nuts} \\ & & = \text{true} \end{array}$$



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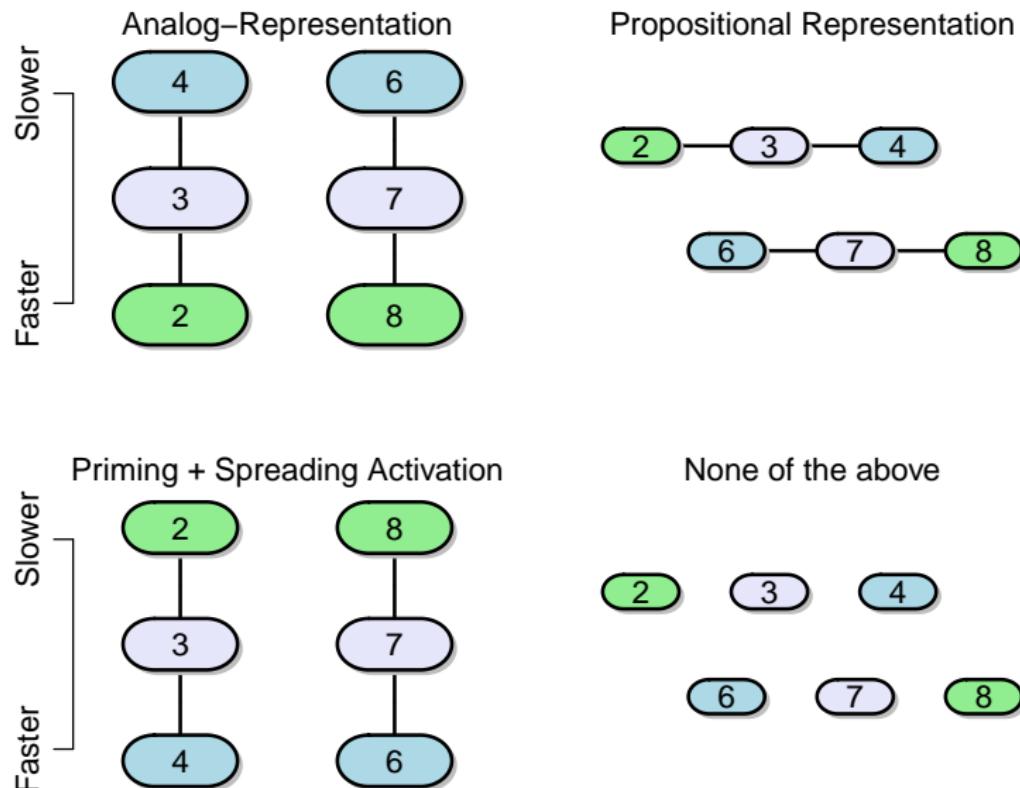
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- ④ None of the above.



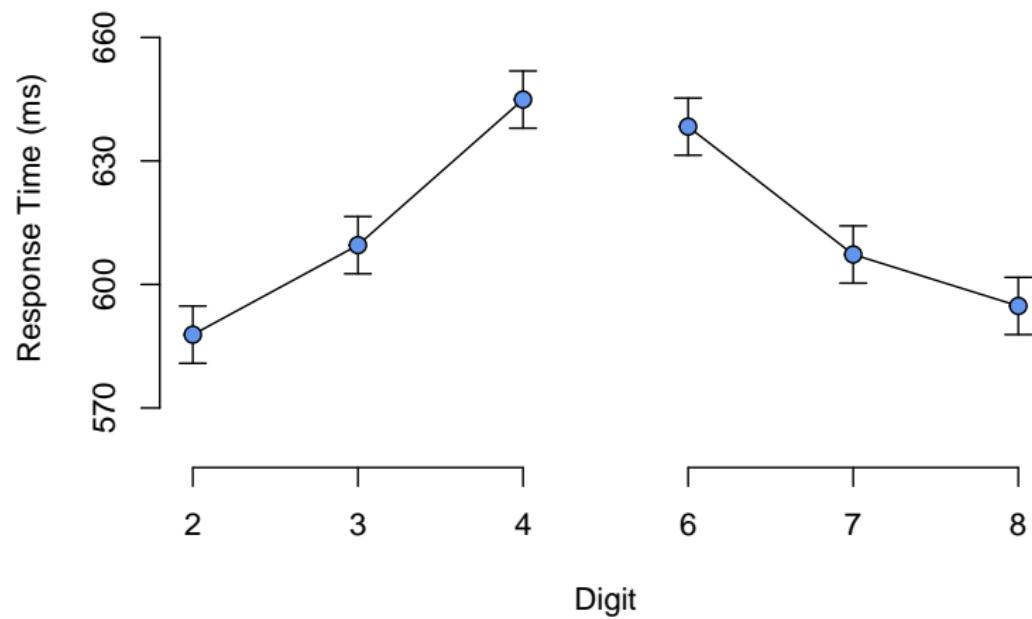
# SYMBOLIC DISTANCE

THEORETICAL POSITIONS AS SYSTEMS OF ORDERS



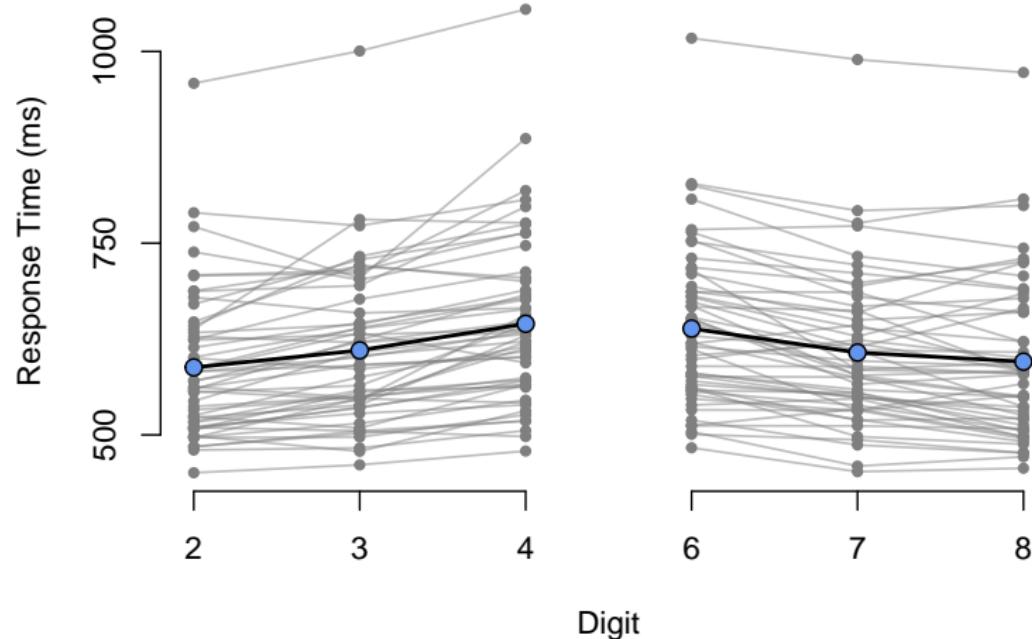
# RESULTS FOR “DOES EVERYONE” ANALYSIS

ROUDER, LU, SPECKMAN, SUN, & JIANG (2005)



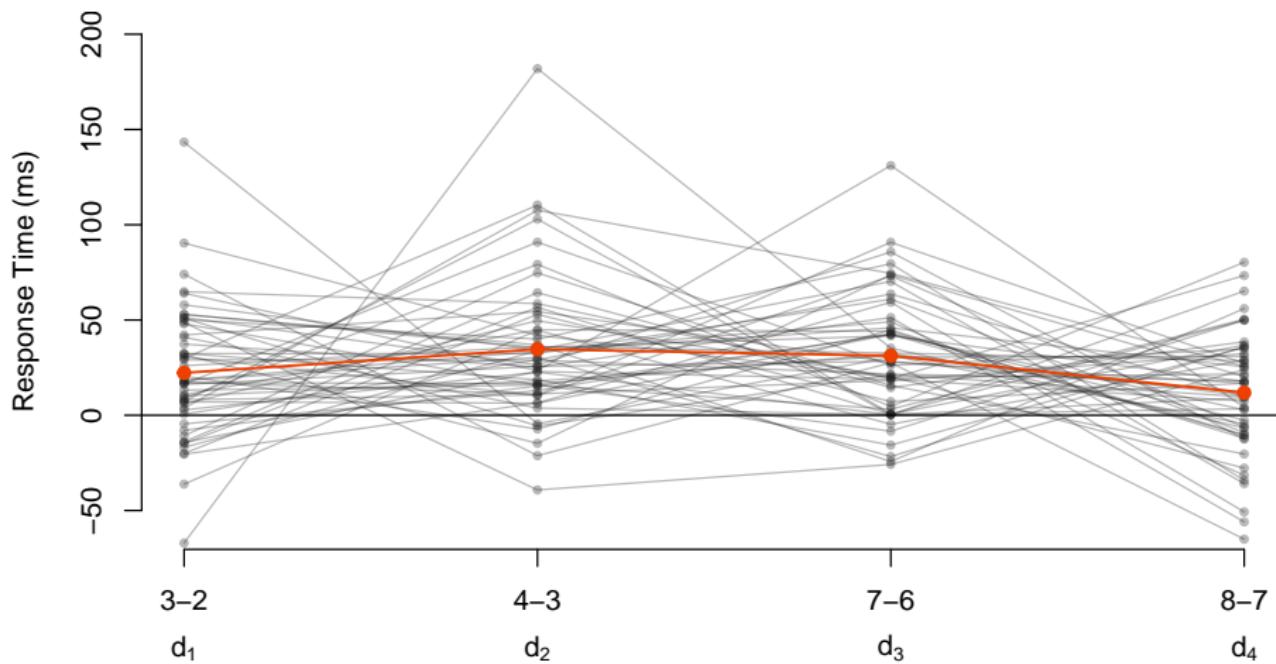
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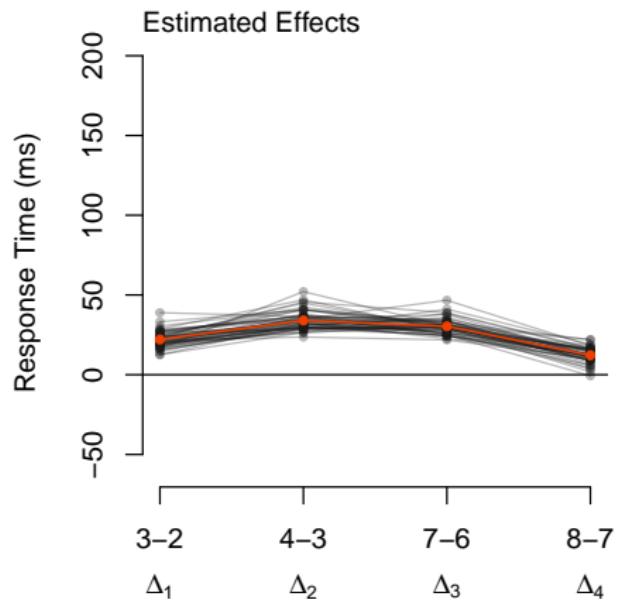
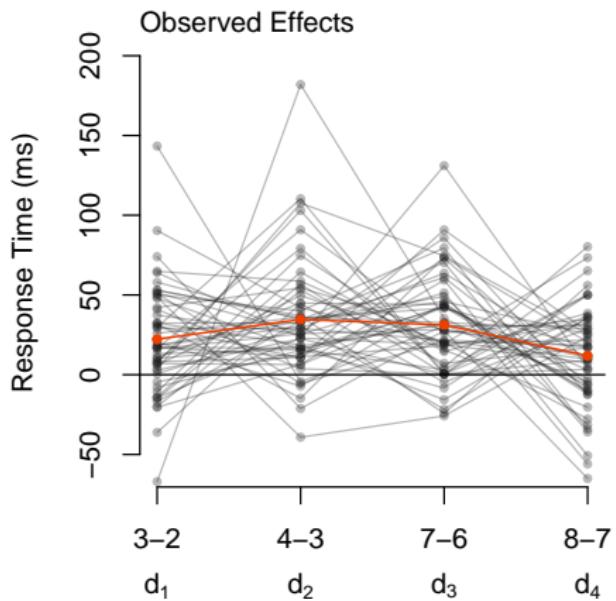


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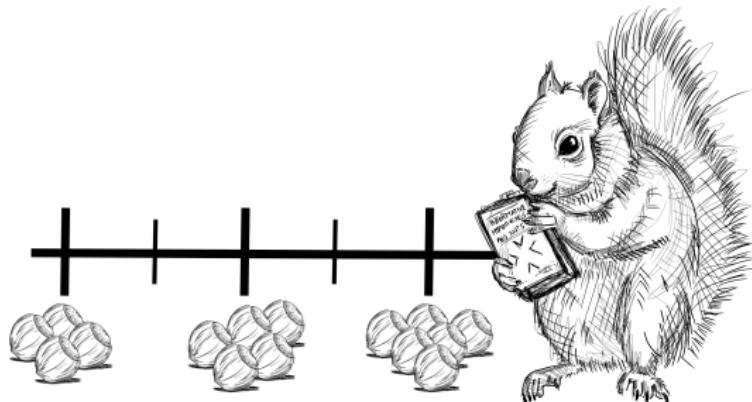


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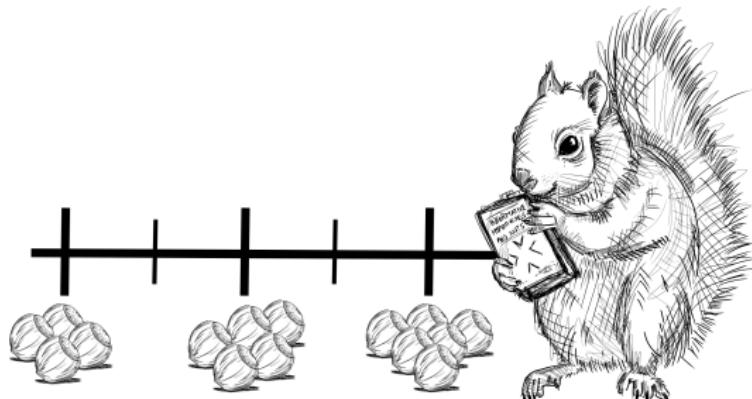
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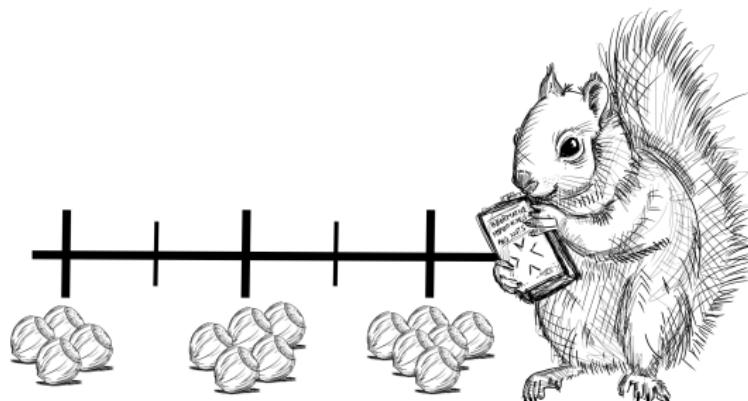
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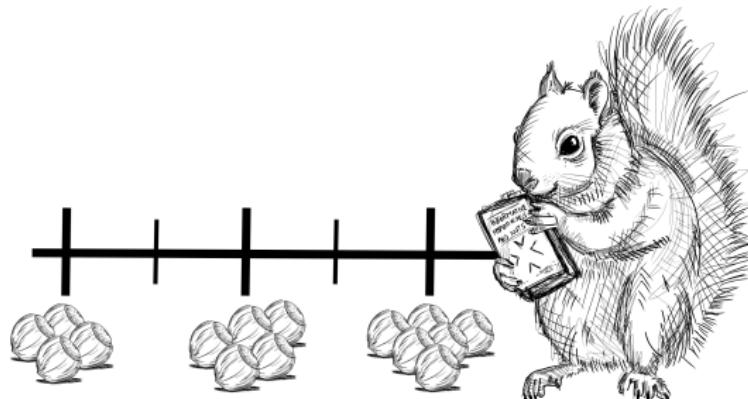
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- Bayes factor for **Priming + spreading activation** model cannot be estimated



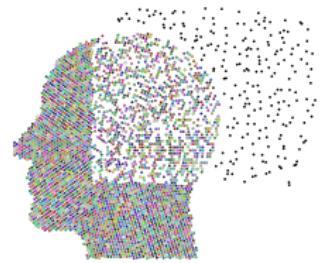
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- ③ Bayesian analysis is the perfect tool to:
  - Estimate individuals’ effects with hierarchical modeling (visualization, diagnosis)
  - Assess how well each model predicts observed data using Bayes factors



# FURTHER REOURCES FOR THE PRAGMATIC PSYCHOLOGIST

- Accessible book to get started with systems of orders: Hoijtink (2012)
- Software packages: `bain` or `BayesFactor` for R users
- “Does everyone” analysis: Haaf & Rouder (2017)
- Presentation and all code is available at: [tinyurl.com/OrdinalConstraint](http://tinyurl.com/OrdinalConstraint)

# THANK YOU



Fayette Klaassen  
(Utrecht University)

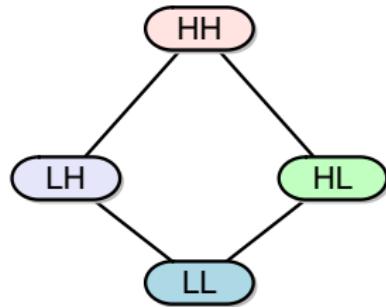


Jeff Rouder  
(University of California - Irvine)

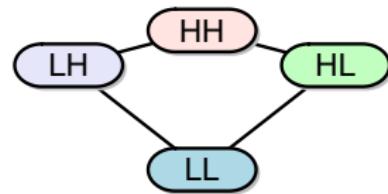
# ADDITIONAL SLIDES

# PREFERRED MODEL

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# REFERENCES

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