46755 - Renewables in Electricity Markets

Assignment 1: Market Clearing (System Perspective)

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Step 0: Build a Relevant Case Study

Please select an electric power network from the following options:

1. IEEE 24-bus reliability test system: link.
2. IEEE reliability test system (2019 update): link.
3. IEEE power systems test cases (various cases with 14, 30, 57, 118, and 300 buses): link.
4. Case studies available in the open-source Julia platform PowerModels.jl: link.

You are also free to choose another case study. If some data is missing, please select reasonable arbitrary values.

For technical details on conventional generators and transmission lines, this link may be helpful (it corresponds to the IEEE 24-bus case study, but similar data can be used for other cases).

Additional assumptions:

- Assume that the price bids of all producers are non-negative and equal to their marginal production cost. In particular, the production cost of renewable units is assumed to be zero. Additionally, these units offer their forecasted capacity, meaning their offer quantities vary over time.
- For the bid price of price-elastic demands, use comparatively high values (relative to the generation cost of conventional units) to ensure that most demands are supplied. For inspiration, check the real bid price data in Nord Pool [link].
- A potential source for wind power forecast data is available at this link (you may normalize the data to fit your case study). Another potential source for the renewable power generation data is renewables.ninja.
- For transmission lines, you may assume a uniform reactance for all lines (e.g., 0.002 p.u., leading to a susceptance of 500 p.u.).

Step 1: Copper-Plate, Single Hour

In Lecture 2, you learn how to develop a market-clearing optimization model for a copper-plate power system (i.e., without modeling the transmission network) in a single-hour setting.

Please determine the following market-clearing outcomes:

- The market-clearing price under a uniform pricing scheme.
- The total operating cost and social welfare of the system.
- The profit of each producer, including both conventional units and wind farms.
- The utility of each demand, defined as:

$$\text{Utility} = \text{Power Consumption} \times (\text{Bid Price} - \text{Market-Clearing Price}).$$

Additionally, please verify the market-clearing price using the KKT conditions, by linking it to the dual variables of the market-clearing optimization problem, and using the stationarity + complementary slackness conditions to explain how this price relates to the bids of the accepted/rejected/marginal producers and consumers.

Step 2: Copper-Plate, Multiple Hours

This step extends Step 1 by incorporating multiple time periods (here, 24 hours). To achieve this, introduce a new index, t, running from 1 to 24. Some input data vary across hours, such as wind power generation, demand bid prices, and demand bid quantities. Arbitrarily generate demand data so that demand bids are comparatively higher during peak hours. Furthermore, we now enforce intertemporal constraints by introducing a large-scale energy storage unit (e.g., pumped hydro storage) into the case study. Note that the objective function of the market-clearing problem is now the maximization of total social welfare over 24 hours.

Storage modeling: We use lowercase symbols for variables and Greek or uppercase symbols for parameters (input data). Let us define the following variables for storage:

- p_t^{ch} and p_t^{dis}: Charging and discharging power (MW) in hour t.
- e_t: Energy stored in the battery (MWh) in hour t.

The storage operation is subject to the following constraints:

$$0 \leq p_t^{\text{ch}} \leq P^{\text{ch}}, \quad \forall t \tag{1}$$

$$0 \leq p_t^{\text{dis}} \leq P^{\text{dis}}, \quad \forall t \tag{2}$$

$$0 \leq e_t \leq E, \quad \forall t \tag{3}$$

$$e_t = e_{t-1} + p_t^{\text{ch}} \eta^{\text{ch}} - \frac{p_t^{\text{dis}}}{\eta^{\text{dis}}}, \quad \forall t, \tag{4}$$

where η^{ch} and η^{dis} represent the charging and discharging efficiencies, respectively, with η^{ch} and η^{dis}. Additionally, P^{ch}, P^{dis}, and E represent the charging capacity, discharging capacity, and energy storage capacity, respectively.

Note 1: The storage size is assumed to be on the order of other conventional units.

Note 2: For simplicity, we assume that this storage submits zero bid and offer prices to the market. Consequently, its presence does not alter the objective function formulation of the market-clearing problem. In reality, from the perspective of the storage owner, determining optimal hourly bid/offer prices for storage is a complex task that involves opportunity cost considerations.

Note 3: We consider $0 < \eta^{\mathrm{ch}} < \eta^{\mathrm{dis}} < 1$ to avoid simultaneous charging and discharging. Otherwise, we would need to introduce binary variables, which would prevent us from deriving prices as dual variables.

Please compute the following market-clearing outcomes:

- The market-clearing prices (one per hour) under a uniform pricing scheme.
- The total operating cost and social welfare of the system over 24 hours.
- The total profit of each producer, including both conventional units and wind farms, over 24 hours.
- The total profit of the storage unit over 24 hours.

Furthermore, please analyze the following:

- Compare the hourly market-clearing prices with and without storage. Do you observe any interesting patterns?
- Conduct a sensitivity analysis: What happens when the storage size, charging/discharging power increase or decrease? Why?
- How are the hourly market-clearing prices determined in the presence of storage? Is the market-clearing price in every hour still equal to the offer price of the marginal producer? Please analyze your numerical findings.

Step 3: Network Constraints

In Lecture 3, you learn how to model power flow across a network and enforce power transmission network limits. Please extend your market-clearing optimization model from Step 1 (no storage, 1-hour) to incorporate these network constraints.

Nodal Market Prices: Once the model is extended to include the network constraints, you will derive the nodal market-clearing prices. These prices represent the marginal cost of supplying electricity at each node, considering both local generation and the transmission constraints. Are the nodal prices in the given hour necessarily identical?

Sensitivity Analysis: Perform a sensitivity analysis by adjusting the capacity of one or more transmission lines. This analysis will help you understand how changes in the transmission network affect the nodal prices. Discuss your results and examine how variations in transmission line capacity influence nodal price formation. When the network is congested, it would be useful to interpret how nodal prices are formed if they are not equal to the offer price of the local producer.

Zonal Market Prices: Consider switching to a zonal framework by dividing the power network into two or three zones. Within each zone, the electricity price is uniform. For simplicity, the Available Transfer Capacities (ATC) between two zones could be the total capacity of all lines between those two zones in the nodal system. How do the zonal market prices compare to the nodal prices? For different values of ATCs between the zones, derive the zonal market prices and analyze them.

Implications of Nodal vs. Zonal Frameworks: How are the profits of conventional and renewable generators affected? Compare the social welfare obtained in the nodal and zonal frameworks. Additionally, check whether the zonal market-clearing outcomes are feasible in terms of transmission line capacities within the zones. What is the magnitude of potential ex-post re-dispatch required to restore feasibility? You are not expected to solve a re-dispatch optimization problem, but a discussion on the magnitude of infeasibility is sufficient.

Step 4: Optimization vs. Equilibrium

In Lecture 4, you learned that there is an equivalent equilibrium problem for the market-clearing optimization problem, under which every market participant maximizes its own profit. Derive such a profit-maximization problem for the storage in Step 2. Then, derive its KKT conditions. There is no need for simulations or coding for this step.

Step 5: Balancing Market

In Lecture 5, you learn about the balancing market. Building upon the model in Step 1 (no storage, no transmission network, 1 hour), assume there is an unexpected failure (outage) in one of the conventional generators. Additionally, the actual production of some wind farms is lower than their day-ahead forecast (e.g., 15%), while that of the remaining wind farms is higher than their day-ahead forecast (e.g., 10%). A subset of conventional generators are potential balancing service providers. Demands are not flexible.

If possible (depending on their day-ahead schedule), each flexible conventional generator offers upward regulation service at a price equal to the day-ahead price plus 10% of its production cost. Similarly, it offers downward regulation service at a price equal to the day-ahead price minus 15% of its production cost. The load curtailment cost is €500/MWh.

Please clear the balancing market for the given hour and derive the balancing price. Then, calculate the total profit (in both day-ahead and balancing markets) of all conventional generators and wind farms in the given hour, assuming the imbalance settlement follows:

1. One-price scheme,
2. Two-price scheme.

By comparing these profits, please analyze the implications of these two schemes for those who provide balancing services and for those who cause the imbalance.

Step 6: Reserve Market

In Lecture 6, you learn about the reserve market. We again use the model in Step 1. Assume the TSO has determined that the upward reserve services to be procured in the reserve market is 15% of the total demand in the corresponding hour. This value is 10% for the downward reserve services. Consider the same subset of conventional generators in Step 5 as flexible resources.

Please clear the reserve and day-ahead markets sequentially, following the current practice in European electricity markets, and report the market-clearing prices for reserve and electricity. How does the reserve market change prices in the day-ahead market?

Optional Task: (with an additional 10% points, but the total grade will not exceed 100%. You can go beyond the page limit by half a page.) Please clear the U.S.-style market (the joint reserve and energy market in the day-ahead stage). Compare the market-clearing outcomes (schedules and prices) achieved with those of the European style.

Note: Please refer to the introduction slides presented on February 2, 2026, for the report template and page limit. For every page exceeding the page limit, there will be a grade deduction of 1.5%. This does not apply to the appendix (if included), but the appendix should only include input data.