Chapter 13

Kinetics of Particles: Newton's Second Law

13.1 Introduction:

So far in the earlier chapter we did the motion analysis of moving particles without taking into account the forces responsible for the motion. From this chapter we begin our motion analysis involving the forces responsible for the motion. This analysis is known as *kinetics*. Here we will analyse motion of moving cars, elevators, blocks, airplanes, rockets etc. treating them as a particle, since rotation of these bodies about their own centre of gravity, if any, is neglected.

In this chapter we will extensively use the Newton's Second Law approach to kinetics. As stated further, this approach involves determination of acceleration of the moving particle by knowing the forces acting on the particle. Having determined the acceleration, the analysis is completed using kinematic relations which we have studied in previous chapter.

13.2 Newton's Second Law of Motion:

Newton's second law of motion states "*The rate of change of momentum of a body is directly proportional to the resultant force and takes place in the direction of the force*".

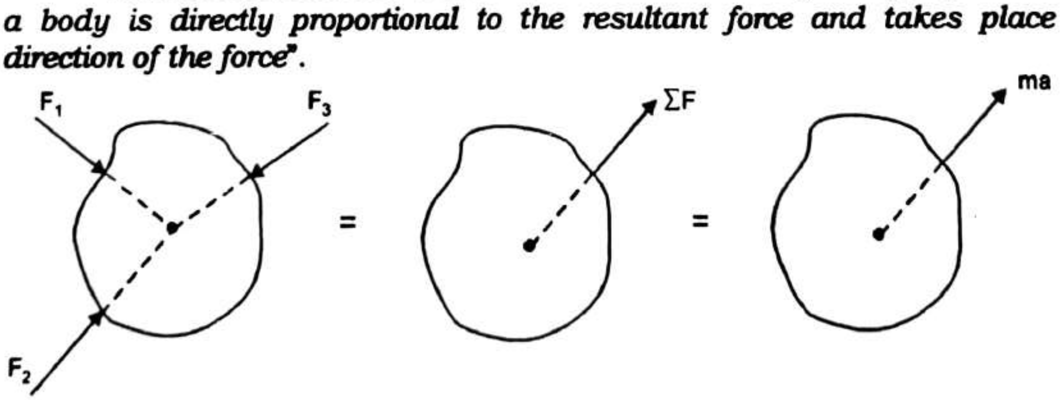


Fig. 13.1

Consider a particle acted upon by several forces as shown in Fig. 13.1. Let $\sum F$ be the resultant force. Because of the resultant force, the particle would move in the direction of the resultant force. If u is the initial velocity, v is final velocity and this change takes place in t sec, we have from Newton's second law:

Rate of change of Momentum = Resultant Force

i.e. $$\frac{\text{Final momentum - Initial momentum}}{\text{Time}} = \sum F$$

$$\frac{mv - mu}{t} = \sum F$$

$$\sum F = m\frac{(v-u)}{t}$$

or $$\boldsymbol{\sum F = ma} \qquad \text{.......[13.1]}$$

Equation 13.1 is the mathematical expression of Newton's Second Law and this gives rise to another statement of Newton's Second Law, which is "*If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of the resultant force*".

Equation 13.1 is a vector relation since both the force and acceleration are vector quantities. The scalar relations from 13.1 can be developed as,

$$\boldsymbol{\sum F_x = ma_x} \qquad \text{....... [13.2 (a)]}$$
$$\boldsymbol{\sum F_y = ma_y} \qquad \text{....... [13.2 (b)]}$$
$$\boldsymbol{\sum F_z = ma_z} \qquad \text{....... [13.2 (c)]}$$

Since we will normally restrict our analysis to coplanar forces, the equations 13.2 (a) and 13.2 (b) will be mainly used.

13.3 Kinetics Using Newton's Second Law

Application of Newton's equations to determine the acceleration should be carried out by a systematic approach as explained below.

Step (1) Draw the FBD showing all the forces acting on the moving particle. If more than one particle is involved, the particles may be isolated and separate FBD may be drawn.

Step (2) By the side of FBD, draw the Kinetic Diagram (KD) which shows the particle with a ma vector acting on it. The magnitude of ma vector is the product of particle's mass and its acceleration. ma vector acts in the direction of particle's acceleration.

Step (3) Equations of Newton's second law viz 13.2 (a) and 13.2 (b) are now used, taking help of the FBD and KD drawn earlier. The particle's acceleration is thus obtained.

To understand the above outlined steps, let us find out the acceleration of a block of mass m which is being pulled up an inclined plane by force P applied parallel to the plane. Let μ_k be the kinetic coefficient of friction between the block and plane. As the block moves up, the rough inclined surface develops a frictional force = $\mu_k N$ down the plane.

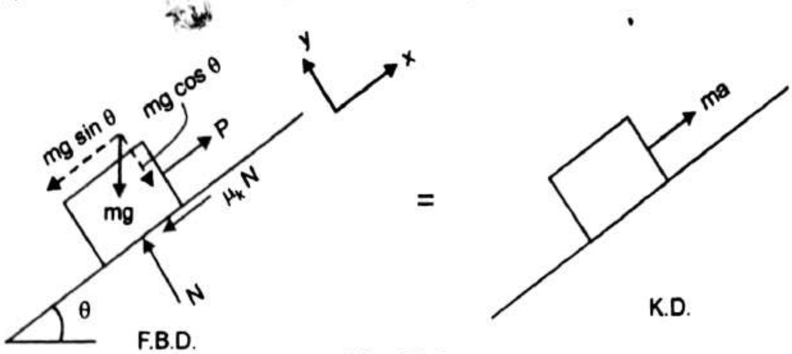


Fig. 13.2

The FBD of the block showing the forces and the corresponding Kinetic Diagram showing the ma vector is drawn as shown in figure. Taking the x and y axes as shown and using equation 13.2 (a) of the Newton's Second Law we have,

$\sum F_x = ma_x$

$P - m\,g\sin\theta - \mu_k N = ma$ .... (1)

$\sum F_y = ma_y$

$N - m\,g\cos\theta = 0$ ....(2) Since there is no component of ma vector in the y direction.

Substituting the value of N from equation (2) in (1)

$$P - m\,g\sin\theta - \mu_k\,(m\,g\cos\theta) = ma$$

$$\therefore \quad a = \frac{P - mg(\sin\theta - \mu_k\cos\theta)}{m}$$

Thus the particles acceleration is found out.

13.4 D' Alembert's Principle

Referring to equation 13.1 we have

$$\sum F = ma$$

This is Newton's Second Law equation and relates the particle's acceleration to the resultant of the external forces acting on the particle. The equation is a vector equation where the resultant force vector is equated to the ma vector.

Transposing the R.H.S. of the equation 13.1 we get,

$$\sum \mathbf{F} - \mathbf{ma} = 0 \qquad ....[13.3]$$

The above equation is a dynamic equilibrium equation put forth by D' Alembert. The ma vector is treated as an inertia force and when added with a negative sign to all other forces, results in equilibrium state of particle.

Figure shows the dynamic equilibrium state of
1) a block sliding down a rough inclined plane
2) a sphere tied to a string, swinging as a pendulum to a lower position.

Actual forces acting on the block

Actual forces + Inertia force creates a state of dynamic equilibrium.

Fig. 13.3 (a)

Actual forces acting on the pendulum

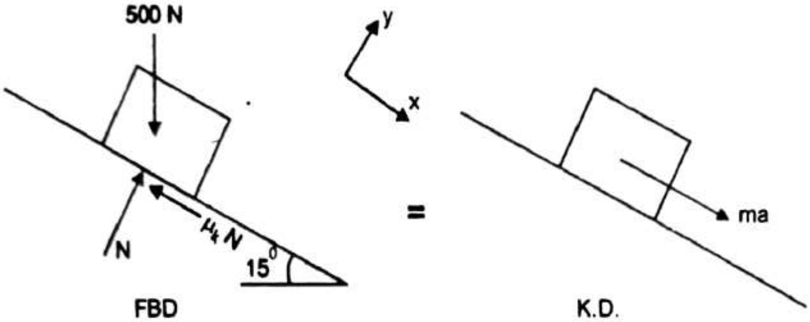
Actual forces + Inertia forces create a state of dynamic equilibrium

Fig. 13.3 (b)

Particle's acceleration can be found out by D'Alembert's principle, by developing the figure representing a dynamic state and using the equilibrium equations used in static viz., $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Newton's Second Law approach to kinetics is more realistic than the D'Alembert's principle since it does not involve inertia forces and does nor refer to an equilibrium state of a moving particle. We would therefore solve the problems in kinetics involving forces and acceleration using Newton's Second Law equation.

Ex. 13.1 A 500 N crate kept on the top of a 15° sloping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu_s = 0.5$ and $\mu_k = 0.4$, determine the distance traveled by the block and the time it will take as it comes to rest.



FBD K.D.

Solution: Applying equation of Newton's Second Law

$$\sum F_y = ma_y$$
$$N - 500\cos 15 = 0$$
$$\therefore \quad N = 482.96 \text{ Newton}$$

$$\sum F_x = ma_x$$
$$500\sin 15 - \mu_k N = ma$$
$$500\sin 15 - 0.4\,(482.96) = \left(\frac{500}{9.81}\right)a$$
$$\therefore \quad a = -1.25 \text{ m/s}^2$$

The block travels down the slope with a negative acceleration i.e deceleration of 1.25 m/s².

Kinematics

This is a case of Rectilinear motion – Uniform acceleration

$u = 20$ m/s, $v = 0$, $s = ?$, $a = -1.25$ m/s², $t = ?$

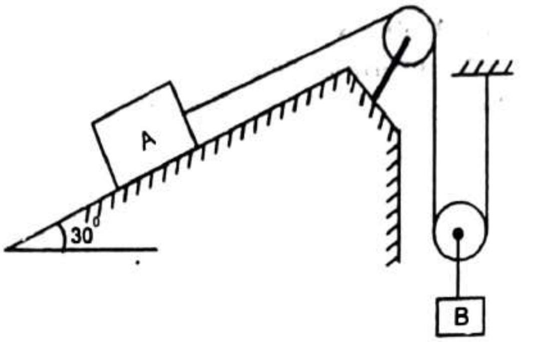
Using $v^2 = u^2 + 2as$

$$0 = (20)^2 + 2 \times (-1.25) \times s$$
$$\therefore \quad s = 160 \text{ m}$$ .... **Ans.**

using $v = u + at$

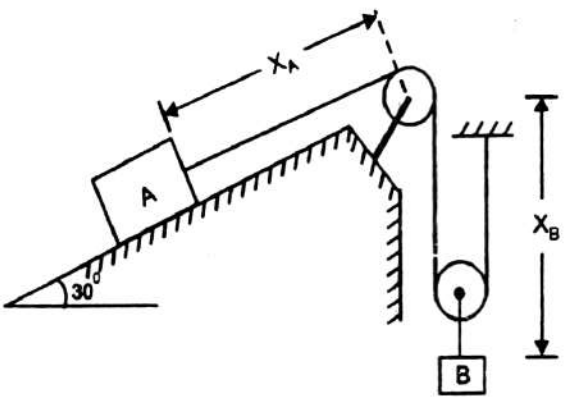
$$0 = 20 - 1.25 \times t$$
$$t = 16 \text{ sec}$$ .... **Ans.**

Ex. 13.2 A package A of mass 25 kg is being pulled up the incline by a load B of mass 60 kg connected to it by an inextensible rope passing over frictionless pulleys. Determine the accelerations of the two blocks and the tension in the connecting rope. Take μ_s = 0.4 and μ_k = 0.3 between the incline and A.



Solution: Downward movement of load B causes package A to slide up the plane. Let us develop the relation between the accelerations of A and B using Constant String Length Method (CSLM).

Let variables x_A and x_B define the positions of A and B. As x_B increases, x_A would decrease. If L is the length of string, then the length L is the sum of string portions in terms of x_A and x_B and plus/minus constants (constants are the string portions which don't change during motion like the length of cord wrapped over the pulleys).



$\therefore$ $L = (2\,x_B) + (-\,x_A) \pm$ constants (x_A is – ve because it reduces with increase in x_B)

Differentiating w.r.t time

$$0 = 2\,v_B - v_A$$

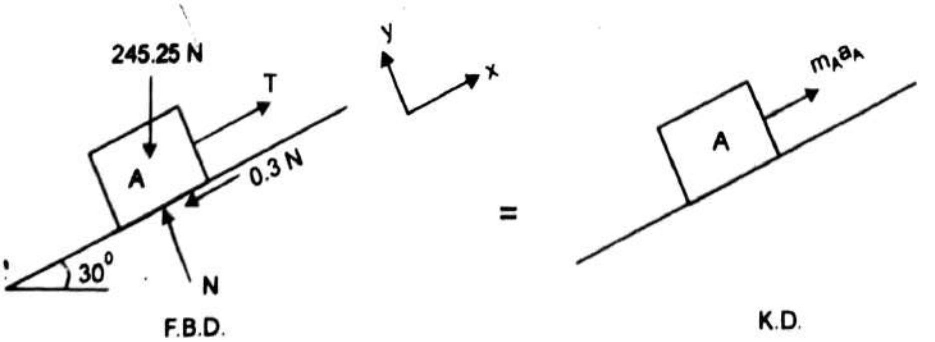
Differentiating again w.r.t time

$$0 = 2\,a_B - a_A$$

or $\quad a_A = 2\,a_B \quad$ .... (1)

Let us isolate A and B and perform kinetic analysis of each of them

<u>Kinetics of package A</u>



F.B.D. K.D.

Applying equations of Newton's second law to A

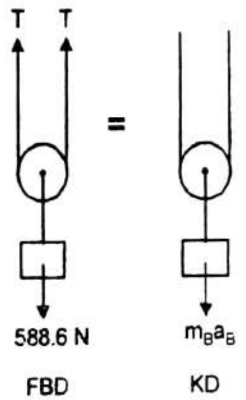
$$\sum F_y = ma_y$$
$$N - 245.25\cos 30 = 0$$
$$N = 212.39 \text{ Newton}$$

$$\sum F_x = ma_x$$
$$T - 245.25 \sin 30 - 0.3N = m_A a_A$$
$$T - 245.25 \sin 30 - 0.3(212.39) = 25\ a_A$$
$$T - 186.34 = 25 a_A \qquad ....(2)$$

<u>Kinetics of load B</u>

Applying equations of Newton's Second Law to B

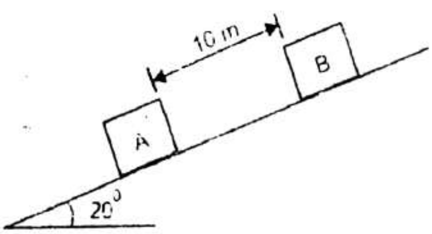
$$\sum F_y = ma_y \uparrow \quad +\text{ve}$$
$$2T - 588.6 = -m_B a_B$$
$$2T - 588.6 = -60\ a_B \qquad ....(3)$$



Solving equations (1), (2) and (3), we get

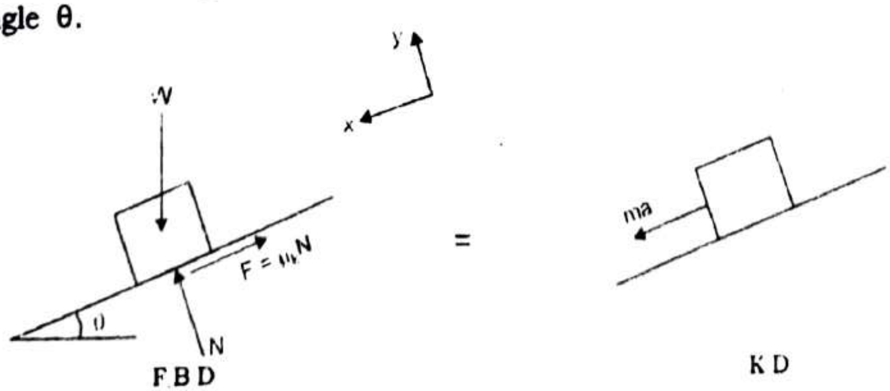
$a_A = 2.7\ m/s^2$ .... **Ans.**
$a_B = 1.35\ m/s^2$ .... **Ans.**
$T = 253.84\ N$ .... **Ans.**

Ex. 13.3 Two blocks A and B are held stationary 10 m apart on a 20° inclined plane as shown. The kinetic coefficient of friction between A and plane is 0.3 and between B and plane is 0.1. If the blocks are released simultaneously, calculate the time taken and distance traveled by each block before they are on the verge of collision.



Solution: The motion of a particle down an inclined plane under the action of its weight component is a case of rectilinear motion with uniform acceleration. The acceleration depends on the value of coefficient of kinetic friction μ_k and on the angle of inclination θ of the plane and is independent of the weight of the body.

Let us consider a general case of a block of weight W moving down an inclined plane of angle θ.



Applying Newton's Second Law

$$\sum F_y = ma_y$$
$$N - W\cos\theta = 0$$
$$\therefore \quad N = W\cos\theta \qquad \ldots\ldots\ (1)$$

$$\sum F_x = ma_x$$
$$W\sin\theta - \mu_k N = ma \qquad \ldots\ldots\ (2)$$

Substituting for N from (1) in (2)

$$W\sin\theta - \mu_k(W\cos\theta) = \frac{W}{g}\,a$$
$$\therefore \quad a = g(\sin\theta - \mu_k\cos\theta) \qquad \ldots\ldots\ (3)$$

The above is a general relation of acceleration of a freely sliding particle on a rough inclined plane.

$\therefore$ For block A, $\mu_k = 0.3$ and $\theta = 20°$

We get $a_A = 9.81\,(\sin 20 - 0.3\cos 20)$
$= 0.5897\ \text{m/s}^2$

Also for block B, $\mu_k = 0.1$ and $\theta = 20°$

We get $a_B = 9.81\,(\sin 20 - 0.1\cos 20)$
$= 2.433\ \text{m/s}^2$

Since the acceleration of B is more than of A, the two blocks will soon collide. We therefore need to perform kinematics analysis. Let A travel x metres before B collides. Therefore B travels (x + 10) metres during the same time interval.

<u>Kinematics</u>

Block A

Rectilinear motion – Uniform acceleration
$u = 0$
$v = -$
$s = x$ metres
$a = 0.5897\ \text{m/s}^2$
$t = t$ sec

Using $s = ut + \frac{1}{2}at^2$
$x = 0 + \frac{1}{2}\times 0.5897 \times t^2 \ldots\ldots\ldots\ldots\ (4)$

Block B

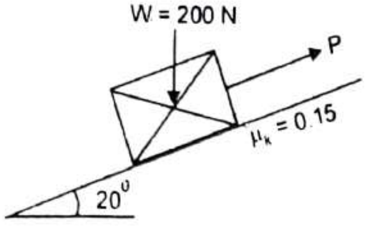
Rectilinear motion – uniform acceleration
$u = 0$
$v = -$
$s = x + 10$ metres
$a = 2.433\ \text{m/s}^2$
$t = t$ sec

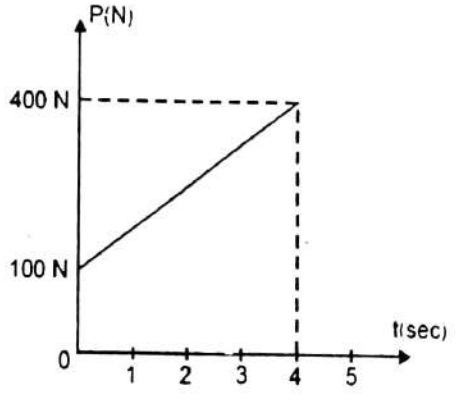
Using $s = ut + \frac{1}{2}at^2$
$(x + 10) = 0 + \frac{1}{2} \times 2.433 \times t^2$

Solving equations 4 and 5, we get
$x = 3.198$ m
$t = 3.294$ sec

Ans. The two blocks collide 3.294 sec after release. Block A travels 3.198 m and block B travels 13.198 m during this time.

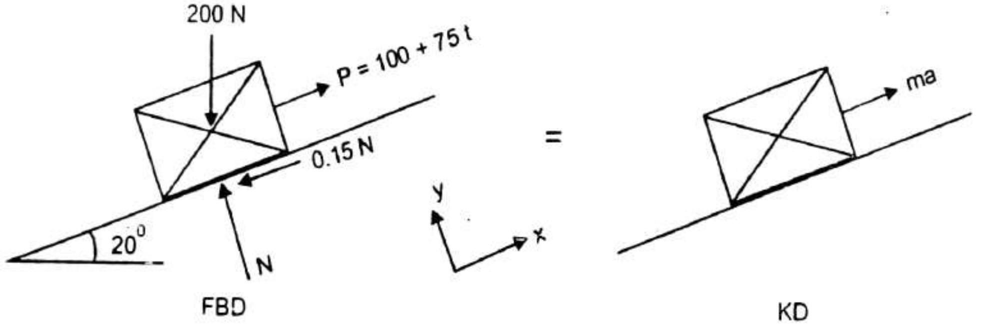
Ex. 13.4 A package is being pulled up the incline by a force P which varies as per the graph shown. Find the acceleration and velocity of the package at t = 4 sec knowing that the particle's velocity was 5 m/s at t = 0.





Solution: The graph indicates a linear variation of force P with time of the type y = mx + c. Here $m = \dfrac{400-100}{4} = 75$ and c = 100

$\therefore$ P = 75 t + 100 N



Applying equations of Newton's Second Law.

$\sum F_y = ma_y$
N – 200 cos 20 = 0
N = 187.94 Newton.

$\sum F_x = ma_x$
(100 + 75 t) – 200 sin 20 – 0.15 N = m a
(100 + 75 t) – 200 sin 20 – 0.15 (187.94) = 20.39 a
$\therefore$ a = 3.68 t + 0.167 m/s²

Kinematics

This is a case of rectilinear motion with variable acceleration.

Using $a = \dfrac{dv}{dt}$

or dv = a dt

$\therefore$ dv = (3.68 t + 0.167) dt

Integrating both sides, taking lower limits as v = 5 m/s and t = 0.

$$\int_5^v dv = \int_0^t 3.68\,t + 0.167\,dt$$

$$[v]_5^v = \left[3.68\frac{t^2}{2} + 0.167\,t\right]_0^t$$

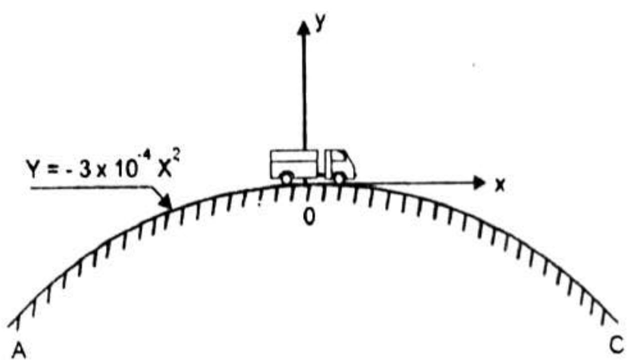
$$v - 5 = 1.84\,t^2 + 0.167\,t$$

$\therefore$ $$v = 1.84\,t^2 + 0.167\,t + 5 \quad m/s$$

at $t = 4$ sec

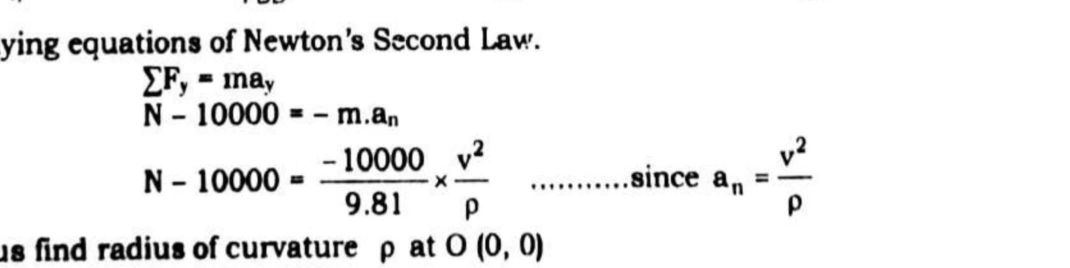
$v = 35.1$ m/s .... **Ans.**

Ex. 13.5 A 10 kN weight car traveling on a vertical curve AOC of parabolic shape increases its speed at a uniform rate of 2 m/s². At the top most point O on the curve its speed is 54 kmph. At this instant determine the reaction force it receives from the ground. Also determine the thrust force developed by the engine.



Solution: The car has a uniform curvilinear motion
Let T be the thrust force developed by the engine.

Let N be the reaction force exerted by the ground.



Applying equations of Newton's Second Law.

$$\sum F_y = ma_y$$

$$N - 10000 = -m.a_n$$

$$N - 10000 = \frac{-10000}{9.81} \times \frac{v^2}{\rho} \quad \text{...........since } a_n = \frac{v^2}{\rho}$$

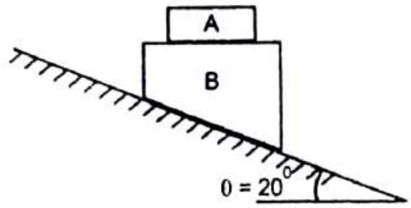
Let us find radius of curvature ρ at O (0, 0)

Equation of path $y = -3 \times 10^{-4} x^2$

$$\frac{dy}{dx} = -6 \times 10^{-4} x$$

also $$\frac{d^2y}{dx^2} = -6 \times 10^{-4}$$

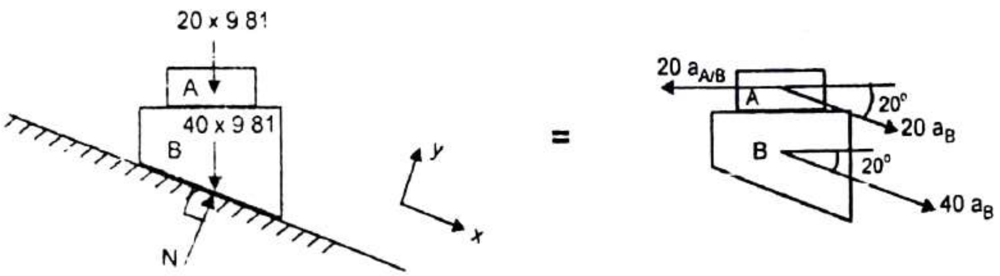
Ex. 13.8 Block A of mass 20 kg is kept on another block B of mass 40 kg. The system is released from rest. Determine the acceleration of the blocks A and B. Neglect friction.



Solution: Motion Analysis – It is a system of two particles having dependent motion. Block B is constrained to slide on the incline and has acceleration a_B whereas block A will move along with B having the same acceleration as of B and may also slide on the block B with an acceleration $a_{A/B}$

$\therefore$ acceleration of block A $\quad a_A = a_{A/B} + a_B$

<u>Motion of entire system</u>

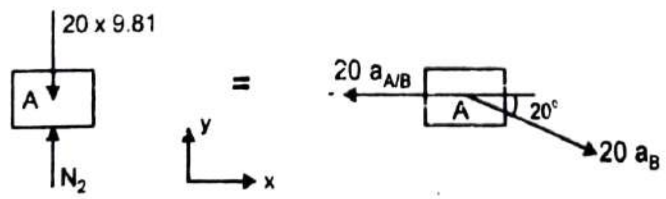


FBD KD

Applying equation of Newton's Second Law

$$\sum F_x = ma_x$$
$$(20 + 40) \times 9.81 \sin 20 = 20\ a_B + 40\ a_B - 20\ a_{A/B} \cos 20$$
$$201.31 = 60\ a_B - 18.794\ a_{A/B} \qquad \text{.......... (1)}$$

<u>Motion of block A after isolation</u>



FBD KD

Applying Newton's Second Law

$$\sum F_x = ma_x$$
$$0 = 20\ a_B \cos 20 - 20\ a_{A/B}$$
$$a_{A/B} = 0.9397\ a_B \qquad \text{........... (2)}$$

Solving equations (1) and (2)

$$201.31 = 60\, a_B - 18.794\,(0.9397\, a_B)$$

$$\therefore \quad a_B = 4.754\ m/s^2 \ \searrow 20^\circ \qquad \text{..........} \ \textbf{Ans.}$$

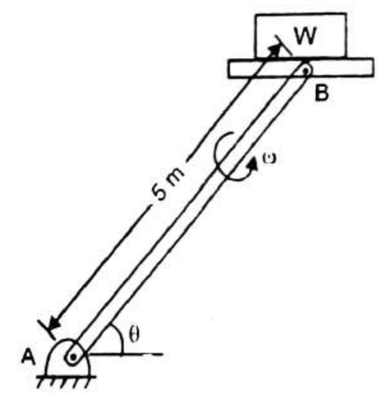
also $\quad a_{A/B} = 0.9397 \times 4.754$

$$\therefore \quad a_{A/B} = 4.468\ m/s^2 \leftarrow$$

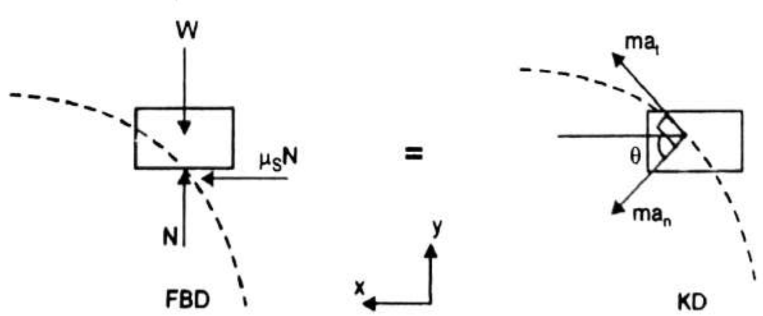
To find acceleration of A, using relative motion equation in vector form.

$$\bar{a}_A = \bar{a}_{A/B} + \bar{a}_B$$
$$= -4.468\,i + (4.468\ i - 1.626\,j)$$
$$= -1.626\,j$$
$$= 1.626\,m/s^2 \downarrow \qquad \text{............} \ \textbf{Ans.}$$

Ex. 13.9. A block of weight W kept on a horizontal platform is being raised up by a link AB which rotates about A in a vertical plane with a constant angular velocity ω rad/s. The platform is maintained in the horizontal position throughout the motion. If $\mu_s = 0.5$ and $\mu_k = 0.4$.
Determine (a) The maximum angular velocity of the link at which the block will tend to slide on the platform.
(b) The corresponding angle θ.



Solution: The block is undergoing curvilinear motion.



Applying Newton's Second Law

$$\sum F_x = ma_x$$

$\mu_s N = ma_t \sin\theta + ma_n \cos\theta$ since speed is constant $\therefore a_t = 0$

also $\quad a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{5}$

$$0.5\,N = 0 + \frac{W}{9.81} \times \frac{v^2}{5} \cos\theta \qquad \text{............ (1)}$$