Ex. 12.16 A car starts from rest at t =0 on a circular curve of 300 m radius. The speed of the car is uniformly increased to 54 kmph in 60 sec. Determine the normal and tangential components of acceleration and the distance traveled at t = 120 sec.

Solution: The car is in curvilinear motion w: h uniform tangential acceleration.

Motion 0 – 60 sec

$u = 0$
$v = 54$ kmph $= 15$ m/s
$s = -$
$a_t = ?$
$t = 60$ sec.

using

$v = u + a_t \times t$
$15 = 0 + a_t \times 60$
$a_t = 0.25\ m/s^2$ .... **Ans.**

Motion 0 – 120 sec

$u = 0$
$v = ?$
$s = ?$
$a_t = 0.25\ m/s^2$
$t = 120$ sec

using

$v = u + a_t \times t$
$v = 0 + 0.25 \times 120$
$v = 30\ m/s$

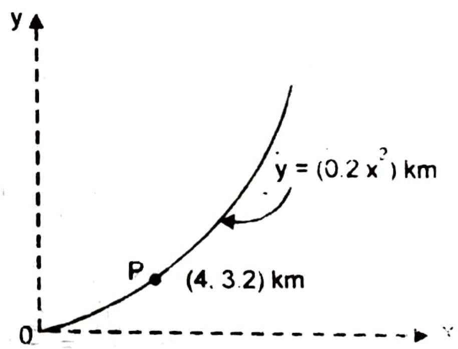
Now

$$a_n = \frac{v^2}{\rho} = \frac{(30)^2}{300} = 3\ m/s^2$$ .... **Ans.**

using

$v^2 = u^2 + 2\ a_t \times s$
$(30)^2 = 0 + 2 \times 0.25 \times s$
$s = 1800$ m .... **Ans.**

Ex. 12.17 An airplane travels on a curved path. At P it has a speed of 360 kmph which is increasing at a rate of 0.5 m/s². Determine at P
a) the magnitude of total acceleration.
b) angle made by the acceleration vector with the positive x axis. Refer figure.



Solution: Given equation of path as $y = 0.2\ x^2$

$$\frac{dy}{dx} = 0.4x \qquad \left(\frac{dy}{dx}\right)_{x=4km} = 1.6$$

$$\frac{d^2y}{dx^2} = 0.4 \qquad \left(\frac{d^2y}{dx^2}\right)_{x=4km} = 0.4$$

using $$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+(1.6)^2\right]^{3/2}}{0.4}$$

$$= 16.792 \text{ km} = 16792 \text{ m}$$

Now $$a_n = \frac{v^2}{\rho} = \frac{(100)^2}{16792} = 0.595 \text{ m/s}^2$$

also $a_t = 0.5 \text{ m/s}^2$ ....given

∴ total acceleration $a = \sqrt{a_n^{\,2} + a_t^{\,2}}$

$$a = 0.777 \text{ m/s}^2 \text{ ......} \textbf{Ans.}$$

Let θ be the angle made by the acceleration vector with the tangent at x = 4 km.

$$\tan\theta = \frac{a_n}{a_t} = \frac{0.595}{0.5}$$

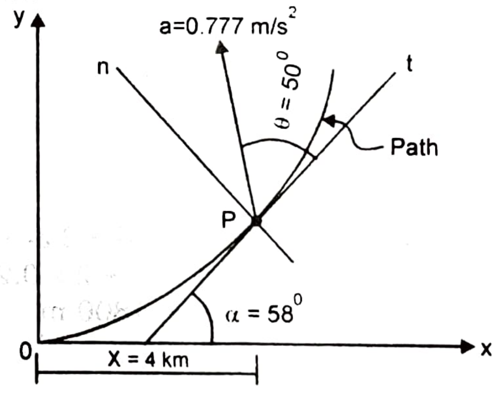
$$\therefore \quad \theta = 50°$$

Let α be the angle made by the tangent with the x axis then

$$\tan\alpha = \frac{dy}{dx}$$

$$\tan\alpha = 1.6$$

$$\therefore \quad \alpha = 58°$$



The total angle made by the acceleration vector with the positive x axis
= 50 + 58 = 108° .... **Ans.**

Ex. 12.18 The position vector of a particle is given by $\bar{r} = \frac{1}{4} t^3 i + 3 t^2 j$ m. Determine at t = 2 sec

a) the radius of curvature of the path

b) the N – T components of acceleration

Solution: Given position $\bar{r} = \frac{1}{4} t^3 i + 3 t^2 j$ m

velocity $$\bar{v} = \frac{d\bar{r}}{dt} = \frac{3}{4}.t^2\, i + 6t\, j \text{ m/s}$$

acceleration $$\bar{a} = \frac{d\bar{v}}{dt} = 1.5t\, i + 6\, j \text{ m/s}^2$$

at $\quad t = 2$ sec

$\bar{v} = 3i + 12j$ m/s $\quad \therefore v = 12.369$ m/s......

$\bar{a} = 3i + 6j$ m/s^2 $\quad \therefore a = 6.708$ m/s

using the relation between rectangular and N- T system

$$\left|\bar{a} \times \bar{v}\right| = \frac{v^3}{\rho}$$

$$\left|(3i + 6j) \times (3i + 12j)\right| = \frac{(12.396)^3}{\rho}$$

$$18 = \frac{1892}{\rho}$$

$$\therefore \quad \rho = 105.1\,\text{m} \qquad \text{........ Ans.}$$

using $\quad a_n = \dfrac{v^2}{\rho} = \dfrac{(12.369)^2}{105.1} = 1.456$ m/s^2

also $\quad a_t = \sqrt{a^2 - a_n^2}$

$$= \sqrt{(6.708)^2 - (1.456)^2}$$

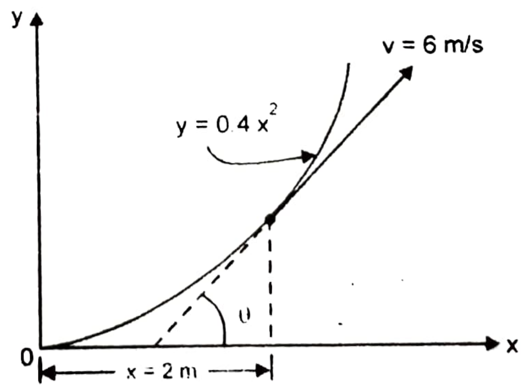
$$\therefore \; a_t = 6.548 \text{ m/s}^2 \qquad \text{........ Ans.}$$

Ex. 12.19 A point moves along a curved path $y = 0.4x^2$. At $x = 2$ m its speed is 6 m/s increasing at 3 m/s^2. At this instant find

a) velocity components along x and y direction
b) its acceleration.

Solution: We know that velocity is always tangent to the path.
The slope of any tangent to a curve defined as $y = f(x)$ is given by

$$\tan\theta = \frac{dy}{dx}$$



here equation of path of curve is
$$y = 0.4x^2$$

$$\frac{dy}{dx} = 0.8x \quad \therefore \left(\frac{dy}{dx}\right)_{x=2m} = 0.8 \times 2 = 1.6$$

$\therefore \quad \tan\theta = 1.6$

or $\quad \theta = 58°$

$\therefore \quad v_x = 6 \cos 58 = 3.18 \text{ m/s} \rightarrow$ .... **Ans.**

and $\quad v_y = 6 \sin 58 = 5.09 \text{ m/s} \uparrow$ .... **Ans.**

To find the normal component of acceleration we are required to find the radius of curvature ρ

from above $\quad \dfrac{dy}{dx} = 0.8\,x$

$$\therefore \quad \frac{d^2y}{dx^2} = 0.8$$

using
$$\rho = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$$

$$\rho_{x=2} = \frac{\left[1 + (1.6)^2\right]^{3/2}}{0.8} = 8.396 \text{ m}$$

using $\quad a_n = \dfrac{v^2}{\rho} = \dfrac{6^2}{8.396} = 4.288 \text{ m/s}^2$

Also $\quad a_t = 3 \text{ m/s}^2$............given

$$\therefore \quad a = \sqrt{(4.288)^2 + (3)^2}$$

$$= 5.233 \text{ m/s}^2$$ .... **Ans.**