Electrodynamic

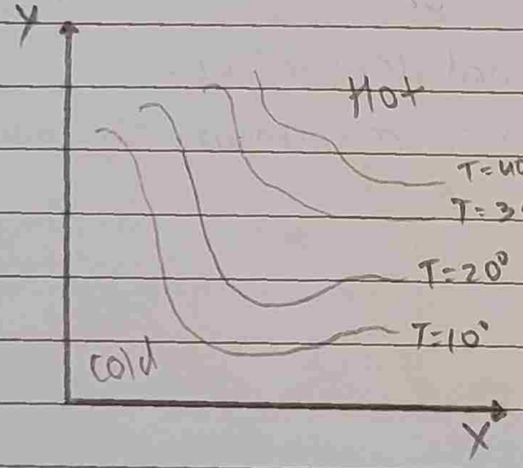
Fields

→ A field is a region od space where some physical quantity takes different values at different points in the region.
→ At each point od the region there exists a corresponding value od physical quantity.
→ A field is a mathematical function od position and time

Depending upon the type od physical quantity

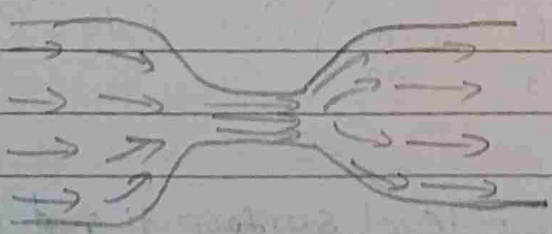
1) **Scalar field:**

→ Value used is scalar quantity at each point
→ Eg : Temperature.
→ The temperature field can be represented as below:



2) **Vector field:**

→ Value used is vector quantity at each point
→ It has both magnitude & direction.
→ Eg. the field od liquid flowing is a constricted pipe



The operator Del ($\vec{\nabla}$)

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

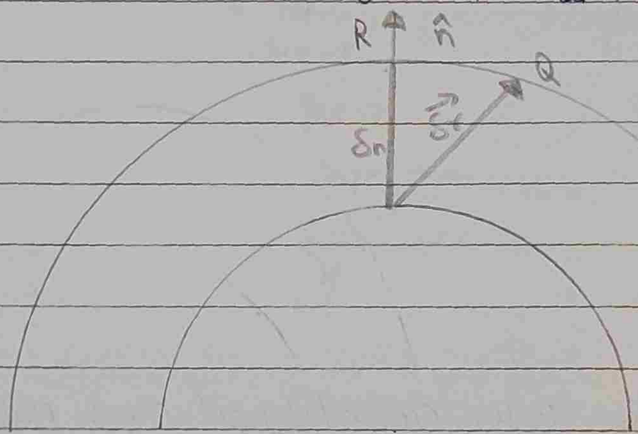
→ The operator $\vec{\nabla}$ can be operated on scalar & vector.
→ When scalar → called Gradient
→ When vector → called Divergence (Dot product)
→ When vector → called Curl (cross product)

Gradient

→ If $\phi(x,y,z)$ is a scalar function then,

$$\vec{\nabla}\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

→ Is also called Directional derivative.
→ It gives the direction of maximum change in ϕ



Let $\vec{r}$ and $\vec{r}+\vec{dr}$ be the position vector P and Q
$\therefore \vec{PQ} = \delta\vec{r}$

$$\vec{\nabla}\phi\cdot\vec{dr} = \left(\hat{i}\frac{\partial\theta}{\partial x} + \hat{j}\frac{\partial\theta}{\partial y} + \hat{k}\frac{\partial\theta}{\partial z}\right)(\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$= d\phi$$

If P & Q lies on same level surface then $d\phi = 0$,
$\vec{\nabla}\phi\cdot\vec{dr} = 0$
and $\vec{\nabla}\phi$ is normal to $\vec{dr}$ (tangent)
i.e. $\vec{\nabla}\phi$ is normal to surface $\phi(x,y,z) = C$

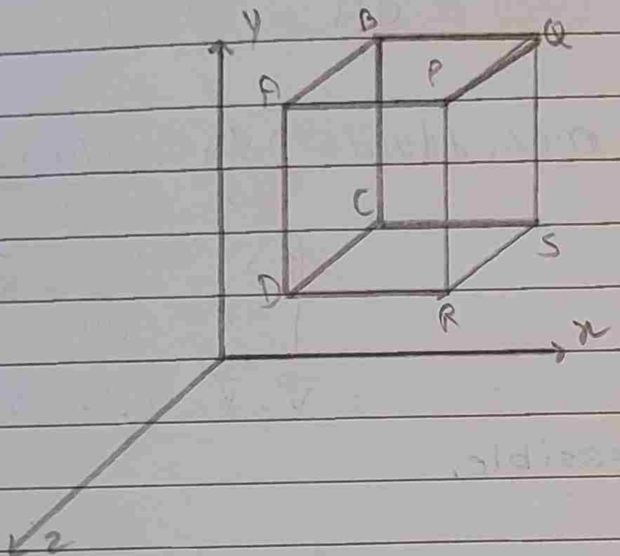
Divergence:

→ $\vec{V}(x,y,z) = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$ is a vector function

$$\vec{\nabla}\cdot\vec{V} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right](V_x\hat{i} + V_y\hat{j} + V_z\hat{k})$$

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

→ Physical significance



Let $\vec{V} = \hat{i}V_x + \hat{j}V_y + \hat{k}V_z$ represent velocity of fluid.
Fluid enters through face ABCD and comes out from PQRS
Mass of fluid ~~enters~~ flowing through face ABCD per unit time $= (V_x)(dy)(dz)$ — (1)

Mass of fluid flowing ~~through~~ out across face PQRS per unit time $= \left[V_x + \frac{\partial V_x}{\partial x}dx\right](dy)(dz)$ — (2)

Net decrease in x axis $= V_x(dy)(dz) - V_x(dy)(dz) - \frac{\partial V_x}{\partial x}dx\,dy\,dz$

$$= -\frac{\partial V_x}{\partial x}dx\,dy\,dz$$

Simiarly,

Along y-axis $= -\frac{\partial V_y}{\partial y} \, dx \, dy \, dz$

Along z-axis $= -\frac{\partial V_z}{\partial z} \, dx \, dy \, dz$

Total decrease $= \left[\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right] dx \, dy \, dz$

Rate of loss of volume of fluid $= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[\hat{i}V_x + \hat{j}V_y + \hat{k}V_z\right]$

$= \vec{\nabla} \cdot \vec{V}$ [divergence of $\vec{V}$]

If fluid is incompressible,

$\vec{\nabla} \cdot \vec{V} = 0$

→ $\vec{V}$ is also called solenoid vector function.

Curl:

→ The curl of a vector is a vector point function

→ If $\vec{V}(x, y, z)$ is a differentiable vector field, then curl of $\vec{V}$ (also called rotation of $\vec{V}$) is ~~call bottle~~ written as $\vec{\nabla} \times \vec{V}$

$$\vec{\nabla} \times \vec{V} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(\hat{i}V_x + \hat{j}V_y + \hat{k}V_z\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix} = \hat{i}\left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right] + \hat{j}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) + \hat{k}\left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]$$

→ Physical interpretation

- A vector field $\vec{V}$ is called irrotional if $\vec{\nabla} \times \vec{V} = 0$
- If $\vec{\nabla} \times \vec{V} \neq 0$ the $\vec{V}$ is not a conservative field
- for any scalar function

$$\text{curl}(\text{grad}.f) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_x & f_y & f_z \end{vmatrix} = 0$$

ie gradient field describing the motion are irrotional.

Fundamental theorems & continuity Equation:

(i) Fundamental theorems.

a) Gradient

$$\int_a^b (\vec{\nabla}\phi).\vec{dl} = \phi(b) - \phi(a)$$

b) Divergence

$$\int_{Vol} (\vec{\nabla}.\vec{V})\,dv = \int_{Surface} \vec{V}.\vec{ds}$$

c) Curl

$$\int_{Surface} (\vec{\nabla}\times\vec{V}).\vec{ds} = \int_{line} \vec{V}.\vec{dl}$$ [Also called stoke's theorem]

Continuity Equation

$$\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Electric field

→ ~~A~~ It is a region of space around a charge in which any other charge experience force of attraction or replusion.

→ Measured in vetoriquantity called Electrical field Intensity ($\vec{E}$)

$$\vec{E} = \frac{\vec{F}}{q_0}, \text{ SI Unit} = N/C$$

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0\varepsilon_r}\right)\frac{q}{r^2}\hat{r} \qquad \hat{r} \rightarrow \text{Unit vector}$$

In magnitude,

$$E = \left[\frac{1}{4\pi\varepsilon_0\varepsilon_r}\right]\frac{q}{r^2}$$

Electric field due to a continuous charge distribution

a) **line charge**

$$q = \int_{line} \lambda\, dl$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\int_{line}\frac{\hat{r}}{r^2}\lambda\, dl$$

b) **Surface charge**

$$q = \int_{Surface} \sigma\, ds$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\int_{Surface}\frac{\hat{r}}{r^2}\sigma\, ds$$

c) **Volume charge**

$$q = \int_{Vol} \rho\, dv$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\int_{Vol}\frac{\hat{r}}{r^2}\rho\, dv$$

Gauss's theorem

1) Integral form:

$$\phi_E = \frac{q}{\varepsilon} \quad \text{— ①}$$

$$\phi_E = \int_{\text{Surface}} \vec{E} \cdot \vec{ds} \quad \text{— ②}$$

$$q = \int_{\text{Vol}} \rho \, dv \quad \text{— ③} \qquad \left[\rho = \frac{dq}{dv}\right]$$

from ①, ②, & ③

$$\int_{\text{Surface}} \vec{E} \, \vec{ds} = \frac{1}{\varepsilon} \int_{\text{Vol}} \rho \, dv \qquad \because \varepsilon \vec{E} = \vec{D}$$

$$\boxed{\int_{\text{Surface}} \vec{D} \, \vec{ds} = \int_{\text{Vol}} \rho \, dv}$$

2) Differential form

Using fundamental thm of divergence

$$\int_{\text{Surface}} \vec{D} \cdot \vec{ds} = \int_{\text{Vol}} (\vec{\nabla} \cdot \vec{D}) \, dv$$

$$\int_{\text{Vol}} (\vec{\nabla} \cdot \vec{D}) \, dv = \int_{\text{Vol}} \rho \, dv$$

$$\boxed{\therefore \vec{\nabla} \cdot \vec{D} = \rho}$$

Electric Potential

→ It is a scalar quantity used to measure strength of charge of a given point.

$$E = -\frac{dv}{dx} \quad \text{along } x\text{-axis}$$

$$\vec{E} = -\left(\hat{i} \frac{\partial v}{\partial x} + \hat{j} \frac{\partial v}{\partial y} + \hat{k} \frac{\partial v}{\partial z}\right) \quad \text{---} \quad \boxed{\text{in 3D}}$$

$$\vec{E} = \cancel{-\vec{\nabla} V} \; -\vec{\nabla} V$$

$$V = -\int \vec{E} \cdot \vec{dr}$$

The eletric potential didference between two points 'a' and 'b'

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot \vec{dr}$$

Magnetic Field

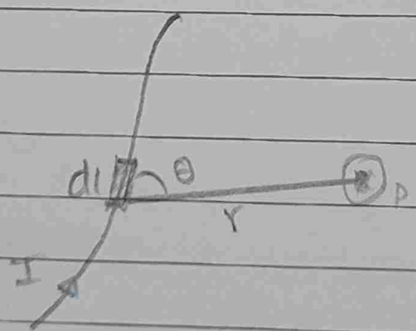
→ It is defined as a space in which a moving charge experiences a velocity dependent force.

→ The science of time - independent magnetic field caused by steady currents is known as magnetostatics

Biot-savart's law

For length element dl, carrying current I, the magnetic induction dB

$$\vec{dB} = \left(\frac{\mu_0}{4\pi}\right) I \left[\frac{\vec{dl} \times \vec{r}}{r^3}\right]$$



$$\vec{B} = \int \vec{dB} = \int \left(\frac{\mu_0}{4\pi}\right) \frac{I(\vec{dl} \times \vec{r})}{r^3}$$

Ampere's law

1) Integral form

$$\oint_{line} \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\because I = \int \vec{J} \cdot \vec{ds}$$

$$\boxed{\oint_{line} \vec{B}\,\vec{dl} = \mu_0 \int_{surface} \vec{J}\cdot\vec{ds}}$$

2) Didderential form

Using fundamental thm od Curl

$$\int_{line} \vec{H}\cdot\vec{dl} = \int_{Surface} (\vec{\nabla}\times\vec{H})\cdot\vec{dS}$$

$$\therefore \int_{surface} (\vec{\nabla}\times\vec{H})\cdot\vec{dS} = \int_{Surface} \vec{J}\cdot\vec{dS}$$

$$\boxed{\therefore \vec{\nabla}\times\vec{H} = \vec{J}}$$

Gauss's thm in magnetism

$$\phi_m = \oint_{Surface} \vec{B}\cdot\vec{dS}$$

~~∴ lines od magnetic field have neither beg.~~

$$\therefore \int_{Surface} \vec{B}\cdot\vec{dS} = 0$$

$$\therefore \int_{Vol} (\vec{\nabla}\cdot\vec{B})\,dv = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

Faraday's law

1) Integral form

$$e = -\frac{d\phi_m}{dt} \rightarrow ①$$

$$e = \oint_{line} \vec{E}\cdot\vec{dl} \rightarrow ②$$

$$\phi_m = \int_{Surface} \vec{B}\cdot\vec{dS} \rightarrow ③$$

$$\boxed{\therefore \int_{line} \vec{E}\cdot\vec{dl} = -\int \frac{d\vec{B}}{dt}\cdot\vec{dS}}$$

Data
Page

2) Differential form

From Stoke's thm

$$\int_{line} \vec{E}.\vec{dl} = \int_{surface} (\vec{\nabla}.\vec{E}).\vec{ds}$$

$$\int_{surface} (\vec{\nabla}.\vec{E})\vec{ds} = -\frac{\partial}{\partial t}\int \vec{B}.\vec{ds}$$

$$\boxed{\therefore \vec{\nabla}.\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Displacement current

From continuity eq^n

$$\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t} \quad — ①$$

From Ampere's law

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$\vec{\nabla}.(\vec{\nabla} \times \vec{H}) = \vec{\nabla}.\vec{J}$$

$$\vec{\nabla}.\vec{J} = 0 \quad — ②$$

but $\vec{\nabla}.\vec{J} \neq 0$ according to continuity eq^n.

Maxwell modified Ampere's law by adding time varying electric field.

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J_D} \quad — ③$$

J_D is called displacement current density

$$\underbrace{\vec{\nabla}.(\vec{\nabla} \times \vec{H})}_{0} = \underbrace{\vec{\nabla}\vec{J}}_{-\frac{\partial \rho}{\partial t}} + \vec{\nabla}.J_D \quad — ④$$

— From ①

$$\therefore \vec{\nabla}.\vec{J_D} = \cancel{\partial e}\frac{\partial \rho}{\partial t} \quad — ⑤$$

$$\because \vec{\nabla}.\vec{D} = \rho$$

$$\vec{\nabla}\cdot\vec{J}_D = \vec{\nabla}\cdot\frac{\partial\vec{D}}{\partial t}$$

$$\vec{J}_D = \frac{\partial\vec{D}}{\partial t} \quad —⑤$$

From ③,

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

Maxwell's equation

The field equations which govern the time-varying electric & magnetic field are now written as.

A) Differential form

i) Gauss's law, $\vec{\nabla}\cdot\vec{D} = \rho$

ii) for magnetism, $\vec{\nabla}\cdot\vec{B} = 0$

iii) Faraday's law, $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$

iv) Ampere's law, $\vec{\nabla}\times\vec{H} = \vec{J} + \partial\vec{D}/\partial t$

B) Integral form

i) $\int_{surface} \vec{D}\cdot\vec{ds} = \int_{vol} \rho\, dv$

ii) $\oint_{surface} \vec{B}\cdot\vec{ds} = 0$

iii) $\int_{line} \vec{E}\cdot\vec{dl} = -\int_{surface} \frac{\partial\vec{B}}{\partial t}\cdot ds$

iv) $\int_{line} \vec{H}\cdot\vec{dl} = \int_{surface} \left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)\cdot\vec{ds}$

Physical significance

i) Maxwell's first equation shows that the total electric flux density $\vec{D}$ through the surface enclosing a volume is equal to the charge density ρ within the volume. It means charge distribution generates a steady electric field.

ii) Maxwell's second equation tells us that the net mag. flux through a close surface is zero. It implies that mag. ~~monopole~~ monopoles do not exist.

iii) The third equation shows that the emf around a closed path is equal to time derivative of mag. flux density through the surface bounded by the path. It means an electric field can also be generated by a time-varying mag. field.

iv) Fourth equation shows that the magneto-motive force around a closed path is equal to conduction current plus time-derivative of electric flux density through any surface bounded by the path. It also shows that the mag. field is generated by time-varying electric field.

The wave Equation

For free space.

$\rho = 0$

$\vec{J} = 0$

Maxwell's equations for free space can be written as

$\vec{\nabla} \cdot \vec{D} = 0$

$\vec{D} = \varepsilon_0 \vec{E}$

$\vec{\nabla} \cdot \vec{E} = 0$ — ①

$\vec{\nabla} \cdot \vec{B} = 0$ — ②

$\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ — ③

$\vec{\nabla} \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t}$

$\vec{H} = \dfrac{\vec{B}}{\mu_0}$

$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$ — ④

Taking curl of eq^n ③, we get

$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla}\left(\dfrac{\partial \vec{B}}{\partial t}\right) = \cancel{\dfrac{-\partial (\vec{\nabla} \times \vec{B})}{\partial t}} \quad \dfrac{-\partial (\vec{\nabla} \times \vec{B})}{\partial t}$ — ⑤

Sub (4) in (5), we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}\left\{\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$$

but $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{E}}_{0}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad — (7)$$

Similarly for mag. field

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \quad — (8)$$

→ Eq^{n}s (7) & (8) are wave equation

→ Any function satisfying such an eq^n describes a wave

→ The $\sqrt{\ }$ of quantity is the reciprocal of the coeff. of time derivative that gives phase velocity

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

∴ It indicates the EM waves propagate with velocity

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

→ The emergence of speed of light from EM wave is great achiement of Maxwell's theory

→ Maxwell predicted that EM disturbance should propagate in free space with a speed equal to speed of light.

→ Hence light waves are EM ~~wave~~ in nature.