SOMAIYA
VIDYAVIHAR UNIVERSITY

Batch: CS-3 Roll No.: 16010123325
Name: Shreyans Tatiya
Course: AM-1
Experiment / assignment / tutorial No. 9
Grade: Signature of the Faculty with date

Tut - 9

Q1] If α, β are the roots of the eqn. $x^2 - 2x + 2 = 0$, P.T. $\alpha^n + \beta^n = 2 \cdot 2^{n/2} \cos n\frac{\pi}{4}$, Hence deduce, $\alpha^8 + \beta^8 = 32$

→

Given eqn:

$$x^2 - 2x + 2 = 0$$

$$\therefore x = 1 \pm i$$

Let $\alpha = 1 + i$, $\beta = 1 - i$

$\therefore$ α & β can also be written as:

$$\alpha = \sqrt{2}\cos\frac{\pi}{4} + i\sqrt{2}\sin\frac{\pi}{4}$$

$$\beta = \sqrt{2}\cos\frac{\pi}{4} - i\sqrt{2}\sin\frac{\pi}{4}$$

for $\alpha^n + \beta^n = \left[\sqrt{2}\cos\frac{\pi}{4} + i\sqrt{2}\sin\frac{\pi}{4}\right]^n + \left[\sqrt{2}\cos\frac{\pi}{4} - i\sqrt{2}\sin\frac{\pi}{4}\right]^n$

By DeMoivre's Theorem,

$$\alpha^n + \beta^n = 2\left[\sqrt{2}^n \cos n\frac{\pi}{4}\right] = 2 \cdot 2^{n/2} \cos n\frac{\pi}{4}$$

Hence,

$$\alpha^8 + \beta^8 = 2 \cdot 2^{8/2} \cos 8\frac{\pi}{4}$$

$$= 2 \cdot 2^4 \cos 2\pi \qquad (\because \cos 2\pi = 1)$$

$$\alpha^8 + \beta^8 = 32$$

Q2] Find all the values of $(1+i)^{3/4}$ & find continued product of all the roots.

$\longrightarrow$

$$[1+i]^{3/4} = \left(\sqrt{2}\cos\frac{\pi}{4} + i\sqrt{2}\sin\frac{\pi}{4}\right)^{3/4}$$

$$= (\sqrt{2})^{3/4}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]^{3/4}$$

$$= (\sqrt{2})^{3/4}\left[\cos\left(2k\pi + \frac{\pi}{4}\right) + i\sin\left(2k\pi + \frac{\pi}{4}\right)\right]^{3/4}$$

$$= (\sqrt{2})^{3/4}\left[\cos\left(\frac{24k\pi + 3\pi}{16}\right) + i\sin\left(\frac{24k\pi + 3\pi}{16}\right)\right]$$

for values $k = 0, 1, 2, 3$

$$x_0 = (\sqrt{2})^{3/4}\left[\cos\frac{3\pi}{16} + i\sin\frac{3\pi}{16}\right]$$

$$x_1 = (\sqrt{2})^{3/4}\left[\cos\frac{27\pi}{16} + i\sin\frac{27\pi}{16}\right]$$

$$x_2 = (\sqrt{2})^{3/4}\left[\cos\frac{51\pi}{16} + i\sin\frac{51\pi}{16}\right]$$

$$x_3 = (\sqrt{2})^{3/4}\left[\cos\frac{75\pi}{16} + i\sin\frac{75\pi}{16}\right]$$

$$x_0 x_1 x_2 x_3 = 2\sqrt{2}\left[\cos\left(\frac{156\pi}{15}\right) + i\sin\left(\frac{156\pi}{18}\right)\right]$$

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Q3] If $u = \log(\tan(\frac{\pi}{4} + \frac{\theta}{2}))$, P.T 1) $\cos u = \sec\theta$, 2) $\sin u = \tan\theta$
3) $\tanh u = \sin\theta$, 4) $\tan u/2 = \tan\theta/2$

→

$u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

taking anti-log

$e^u = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{1 + \tan\theta/2}{1 - \tan\theta/2}$ — ①

$e^{-u} = \frac{1 - \tan\theta/2}{1 + \tan\theta/2}$ — ②

Let $\cos u = \frac{e^u + e^{-u}}{2} = \frac{2 + 2\tan^2\theta/2}{2 - 2\tan^2\theta/2}$

$\therefore \cosh u = \frac{1}{\cos\theta}$

$\therefore \underline{\cosh u} = \underline{\sec\theta}$

we know that:

$\sinh u = \sqrt{\cosh^2 u - 1}$

$\sinh u = \sqrt{\sec^2\theta - 1}$

$\sinh u = \sqrt{\tan^2\theta}$

$\underline{\sinh u} = \underline{\tan\theta}$

$\tanh u = \frac{\sinh u}{\cosh u} = \frac{\tan\theta}{\sec\theta}$

$\tanh u = \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}$

$\therefore \underline{\tanh u} = \underline{\sin\theta}$

Now, $\tanh\frac{u}{2} = \frac{\sinh u/2}{\cosh u/2}$

multiply by $2\cosh u/2$

$$\therefore \tanh\frac{u}{2} = \frac{\sinh u}{1+\cosh u}$$

$$\therefore \tanh\frac{u}{2} = \frac{\tan\theta}{1+\sec\theta}$$

$$\therefore \tanh\frac{u}{2} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{1+\sec\theta}} = \frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\cos\theta+1}$$

$$\tanh\frac{u}{2} = \frac{\sin\theta/2}{\cos\theta/2}$$

$$\therefore \underline{\tanh\frac{u}{2}} = \underline{\tan\frac{\theta}{2}}$$

Q4) if $\cos(x+iy) = \cos\alpha + i\sin\alpha$. P.T.

i) $\sin\alpha = \pm\sin^2 x = \pm\sinh^2 y$

ii) $\cos 2x = +\cosh 2y = 2$

$\longrightarrow$

$$\cos(x+iy) = \cos x\cos iy - \sin x\sin iy$$

$$\cos\alpha + i\sin\alpha = \cos x\cosh y - i\sin x\sinh y$$

Hence,

$$\cos\alpha = \cos x\cosh y$$
$$\sin\alpha = -\sin x\sinh y$$

$\therefore$ Squarring & Adding:

$$\cos^2\alpha + \sin^2\alpha = \cos^2 x\cosh^2 y + \sin^2 x\sinh^2 y$$
$$1 = \cos^2 x\cosh^2 y + \sin^2 x\sinh^2 y$$
$$0 = \sinh^2 y - \sin^2 x$$
$$\therefore \pm\underline{\sinh^2 y} = \pm\underline{\sin^2 x}$$

Now, $\cos 2x + \cosh 2y = 1 - 2\sin^2 x + 1 + 2\sinh^2 y$ $\quad(\because \sin^2 x = \sinh^2 y)$

$$\underline{\cos 2x} + \underline{\cosh^2 y} = \underline{2}$$

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Q5 P.T.

$$\cosh^{-1}(\sqrt{1+x^2}) = \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

→ $\cosh^{-1}(\sqrt{1+x^2}) = a \quad = LHS$

$\therefore \sqrt{1+x^2} = \cosh a$

$1 + x^2 = \cosh^2 a$

But, $1 + \sinh^2 a = \cosh^2 a$

$x = \sinh a$

Now, $\dfrac{x}{\sqrt{1+x^2}} = \dfrac{\sinh a}{\cosh a} = \tanh a$

$\tanh^{-1}\left(\dfrac{\sinh a}{\cosh a}\right) = a$

$\tanh^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = a \quad = RHS$

$\therefore$ Hence Proved

Q6] Find value of $\log[\sin(x+iy)]$

$\sin(x+iy) = \sin x \cos iy + \cos x \sin iy$
$= \sin x \cosh y + i \cos x \sinh y$

$\therefore \log[\sin(x+iy)] = \log[\sin x \cosh y + i \sinh y \cos x]$

we know,

$\log[x+iy] = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{y}{x}\right)$

$\log(\sin x \cosh y + i \cos x \sinh y)$
$= \frac{1}{2}\log[\cosh^2 y - \cos^2 x] + i\tan^{-1}(\cot x . \tanh y)$
$= \frac{1}{2}\log\left[\frac{1}{2}(\cosh 2y - \cos 2x)\right] + i\tan^{-1}(\cot x . \tanh y)$

Hence,

$\log(\sin(x+iy)) = \frac{1}{2}\log\left[\frac{1}{2}(\cosh 2y - \cos 2x) + i\tan^{-1}(\cot x . \tanh y)\right]$