SOMAIYA VIDYAVIHAR UNIVERSITY

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Course: Applied Math - I
Experiment / assignment / tutorial No. 4
Grade: Signature of the Faculty with date

Assignment - 1

Partial Differentiation

Q1] If $u = e^{xyz}$, P.T. $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2) e^{xyz}$

$\rightarrow$ $\frac{\partial u}{\partial z} = xy\, e^{xyz}$

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial y}(xy e^{xyz})$$

$$\therefore \frac{\partial^2 u}{\partial y \partial z} = xy \frac{\partial}{\partial y}(e^{xyz}) + e^{xyz} \cdot \frac{\partial}{\partial y}(xy)$$

$$\therefore \frac{\partial^2 u}{\partial y \partial z} = (xy)(xz)\, e^{xyz} + e^{xyz} \cdot x = e^{xyz}(x^2yz + x)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial y \partial z}\right) = (x^2yz + x) yz \cdot e^{xyz} + e^{xyz}(2xyz + 1)$$

$$\therefore \frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}\left[x^2y^2z^2 + 3xyz + 1\right]$$

$$\therefore \frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}(1 + 3xyz + x^2y^2z^2)$$

Q2] $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ P.T. $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$

$\rightarrow$ $$\frac{\partial z}{\partial y} = x^2\left(\frac{x^2}{x^2+y^2} \times \frac{1}{x}\right) - \left(y^2 \cdot \frac{y^2}{x^2+y^2} \cdot \frac{-x}{y^2}\right) + 2y \tan^{-1}\left(\frac{x}{y}\right)$$

$$\frac{\partial z}{\partial y} = \frac{x^3}{x^2+y^2} + \frac{xy^2}{x^2+y^2} - 2y \tan^{-1}\left(\frac{x}{y}\right)$$

$$\frac{\partial z}{\partial y} = \frac{x\,\cancel{(x^2+y^2)}}{\cancel{x^2+y^2}} - 2y\tan^{-1}\left(\frac{x}{y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = 1 - 2y \cdot \frac{y^2}{x^2+y^2} \times \frac{1}{y}$$

$$\frac{\partial^2 z}{\partial x \partial y} = 1 - \frac{2y^2}{x^2+y^2} = \frac{x^2+y^2-2y^2}{x^2+y^2} = \frac{x^2-y^2}{x^2+y^2}$$

$$\frac{\partial z}{\partial x} = \left(2x\tan^{-1}\left(\frac{y}{x}\right) - x^2 \cdot \frac{x^2}{x^2+y^2} \times -\frac{y}{x^2}\right) - \left(y^2 \cdot \frac{y^2}{x^2+y^2} \times \frac{1}{y}\right)$$

$$= 2x\tan^{-1}\left(\frac{y}{x}\right) \cdot \frac{-x^2 y}{x^2+y^2} - \frac{y^2}{x^2+y^2}$$

$$= 2x\tan^{-1}\left(\frac{y}{x}\right) - \frac{y\,\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = 2x \cdot \frac{x^2}{x^2+y^2} \cdot \frac{1}{x} - 1$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{2x^2}{x^2+y^2} - 1 = \frac{2x^2-x^2-y^2}{x^2+y^2} = \frac{x^2-y^2}{x^2+y^2}$$

$$\therefore \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = \frac{x^2-y^2}{x^2+y^2}$$ Hence Proved..

Q3] $u = f\left(\frac{x^2}{y}\right)$ P.T. $x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = 0$

And $x^2\frac{\partial^2 u}{\partial x^2} + 3xy\frac{\partial^2 u}{\partial x \partial y} + 2y^2\frac{\partial^2 u}{\partial y^2} = 0$

→ a) $\frac{\partial u}{\partial x} = f'\left(\frac{x^2}{y}\right) \cdot \frac{2x}{y}$

$\frac{\partial y}{\partial x} = f'\left(\frac{x^2}{y}\right) \cdot \frac{-x^2}{y^2}$

$\therefore x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = x \cdot f'\left(\frac{x^2}{y}\right) \cdot \frac{2x}{y} - 2y\, f'\left(\frac{x^2}{y}\right) \cdot \frac{-x^2}{y} = 0$

$\therefore x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = 0$

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b) $\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = f''\left(\frac{x^2}{y}\right) \cdot \frac{2x}{y} \cdot \frac{2x}{y} = f''\left(\frac{x^2}{y}\right) \cdot \frac{4x^2}{y^2}$

$\frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y}\right) = -f''\left(\frac{x^2}{y}\right) \cdot \frac{x^2}{y^2} \cdot \frac{2x}{y} = -f''\left(\frac{x^2}{y}\right)\left(\frac{2x^3}{y^3}\right)$

$\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial y}\right) = f''\left(\frac{x^2}{y}\right) \frac{x^4}{y^4}$

$\therefore x^2 \cdot \frac{\partial^2 u}{\partial x^2} + 3xy \frac{\partial^2 u}{\partial x \partial y} + 2y^2 \frac{\partial^2 u}{\partial y^2} =$

$f''\left(\frac{x^2}{y}\right)\left[\frac{4x^2}{y^2} - 3xy\left(\frac{2x^3}{y^3}\right) + 2y^2 \cdot \frac{x^4}{y^4}\right]$

$= f''\left(\frac{x^2}{y}\right)\left[\frac{4x^2 - 6x^4 + 2x^4}{y^2}\right]$

$= 0$

$\therefore x^2 \frac{\partial^2 u}{\partial x^2} + 3xy \frac{\partial^2 u}{\partial x \partial y} + 2y^2 \frac{\partial^2 u}{\partial y^2} = 0$

Q4] $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ p.T. $U_x + U_y + U_z = 0$

$\rightarrow$ $y - z = t_1$, $z - x = t_2$, $x - y = t_3$

$u = f(e^{t_1}, e^{t_2}, e^{t_3})$

$\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial t_1} \cdot \frac{\partial t_1}{\partial x} + \frac{\partial u}{\partial t_2} \cdot \frac{\partial t_2}{\partial y} + \frac{\partial u}{\partial t_3} \frac{\partial t_3}{\partial z}$

$\frac{\partial u}{\partial x} = 0 - e^{t_2} \cdot \cancel{\frac{\partial t_2}{\partial y}} + e^{t_3} \cancel{\frac{\partial t_3}{\partial z}}$

$\frac{\partial u}{\partial y} = e^{t_1} \cdot \cancel{\frac{\partial t_1}{\partial y}} + 0 - e^{t_3} \cancel{\frac{\partial t_3}{\partial z}}$

$\frac{\partial u}{\partial z} = -e^{t_1} \cdot \cancel{\frac{\partial t_1}{\partial x}} + e^{t_2} \cancel{\frac{\partial t_2}{\partial y}} + 0$

$\therefore \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

Q5 If $z = f(x, y)$ $x = r\cos\theta$, $y = r\sin\theta$

P.T $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

→ R.H.S.

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$= f'(x, y)\cos\theta + f'(x, y)\sin\theta$$

$$= f'(x, y)(\cos\theta + \sin\theta)$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

$$= -f(x, y)\, r\sin\theta + f'(x, y)\, r\cos\theta$$

$$= rf'(x, y)(\cos\theta - \sin\theta)$$

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = (f'(x, y))^2(\cos\theta + \sin\theta)^2 + ((f'(x,y))^2(\cos\theta - \sin\theta)^2$$

$$= (f'(x, y))^2\left[(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2\right]$$

$$= (f'(x, y))^2 \, (2)$$

$$= 2(f'(x, y))^2$$

L.H.S.

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = f'(x, y)$$

$$\therefore \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 2(f'(x, y))^2$$

$\therefore$ L.H.S. = R.H.S.