Electric field :- A region of space around a charge in which any other charge experiences force of attraction or repulsion is called electric field.

The electric field of a charge is measured in terms of vector quantity called Electric field Intensity ($\vec{E}$).

The electric field intensity of a charge at any given point (P) is defined as force acting on unit positive charge at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

SI unit. N/C

for point charge q, electric intensity at distance 'r' is given by

$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0\epsilon_r}\right)\frac{q}{r^2}\hat{r}$$

$\hat{r} \Rightarrow$ unit vector

In magnitude

$$E = \left(\frac{1}{4\pi\epsilon_0\epsilon_r}\right)\left(\frac{q}{r^2}\right)$$

Electric field due to a continuous charge distribution:

(a) for line charge:

$\because$ linear charge density $\lambda = \dfrac{dq}{dl}$

$\therefore dq = (\lambda)(dl)$

$\therefore q = \int_{line} \lambda \, dl$

$$\vec{E} = \frac{1}{4\pi\epsilon_0\epsilon_r}\int_{line} \frac{\hat{r}}{r^2}\lambda \, dl$$

(b) for Surface charge:

$\because$ Surface charge density, $\sigma = \dfrac{dq}{ds}$

$\therefore dq = \sigma \, ds$

$q = \int_{surface} \sigma \, ds$

$$\therefore \vec{E} = \left(\frac{1}{4\pi\epsilon_0\epsilon_r}\right)\int_{Surface} \frac{\hat{r}}{r^2}\sigma \, ds$$

(c) for Volume charge:-

$\because$ Volume charge density $\rho = \dfrac{dq}{dv}$

$\therefore dq = \rho \, dv$

$q = \int_{Vol} \rho \, dv$

$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0\epsilon_r}\right)\int_{Vol} \frac{\hat{r}}{r^2}\rho \, dv$$

Gauss's theorem in differential and Integral form:

Gauss's thm.

$$\phi_E = \frac{q}{\epsilon} \longrightarrow ①$$

$$\because \phi_E = \int_{Surface} \vec{E}\cdot\vec{ds} \longrightarrow ②$$

$$\rho = \frac{dq}{dv} \Rightarrow dq = \rho\, dv$$

$$q = \int_{Volume} dq = \int_{Vol} \rho\, dv \longrightarrow ③$$

$\therefore$ from ①, ② and ③

$$\int_{Surface} \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon}\int_{Vol} \rho\, dv \qquad \because \epsilon\vec{E} = \vec{D}$$

$$\therefore \boxed{\int_{Surface} \vec{D}\cdot\vec{ds} = \int_{Vol} \rho\, dv} \Rightarrow \text{Integral form.}$$

using fundamental thm. of div.

$$\int_{Surface} \vec{D}\cdot\vec{ds} = \int_{Vol} (\vec{\nabla}\cdot\vec{D})\, dv$$

$$\therefore \int_{Vol} (\vec{\nabla}\cdot\vec{D})\, dv = \int_{Vol} \rho\, dv$$

$$\therefore \boxed{\vec{\nabla}\cdot\vec{D} = \rho} \Rightarrow \text{Differential form}$$

Electric Potential :. It is a scalar quantity used to measure strength of a charge at a given point.

It is defined as, work done to bring unit +ve charge from ∞ to the given point.

It is also defined as a quantity whose rate of change in any direction is the electric intensity in that direction.

$$E = -\frac{dV}{dx} \quad \text{along } x\text{-axis}$$

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \quad \text{---- (in 3D)}$$

$$\boxed{\vec{E} = -\vec{\nabla} V}$$

$$\therefore \vec{E} = -\frac{dV}{dr}\hat{r}$$

$$dV = -\vec{E}\cdot\vec{dr}$$

$$V = -\int \vec{E}\cdot\vec{dr}$$

The electric potential difference between two points 'a' and 'b'

$$V(b) - V(a) = -\int_a^b \vec{E}\cdot\vec{dr}$$

Magnetic field :.

Magnetic field is defined as a space in which a moving charge experiences a velocity dependent force.

The science of time-independent magnetic fields caused by steady currents is known as magnetostatics.

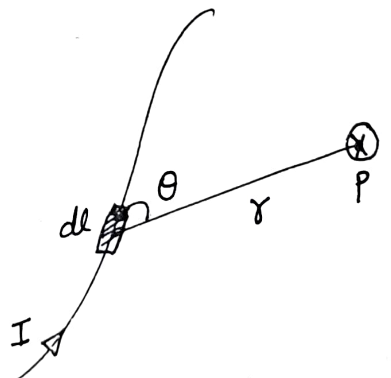
In 1819 Oerested observed that a current carrying wire produces magnetic field around it. This phenomenon is called Magnetic Effect of electric Current.

Biot-Savart's law

for length element dl, carrying current I, the magnetic induction dB

$$\vec{dB} = \left(\frac{\mu_0}{4\pi}\right) I \left(\frac{\vec{dl} \times \vec{r}}{r^3}\right)$$

The direction of $\vec{dB}$ is given by Right Hand Rule.



$$\vec{B} = \int \vec{dB} = \int \left(\frac{\mu_0}{4\pi}\right) \frac{I\,(\vec{dl} \times \vec{r})}{r^3}$$

Ampere's law in Integral and Differential form

Ampere's law $\oint_{line} \vec{B} \cdot \vec{dl} = \mu_0 I$

$\because I = \int \vec{J} \cdot \vec{ds}$

$\therefore \oint_{line} \vec{B} \cdot \vec{dl} = \mu_0 \int_{surface} \vec{J} \cdot \vec{ds}$

$$\boxed{\oint_{line} \vec{H} \cdot \vec{dl} = \int_{surface} \vec{J} \cdot \vec{ds}}$$ $\Rightarrow$ Integral form

From fundamental thm. of curl

$$\int_{line} \vec{H} \cdot \vec{dl} = \int_{surface} (\vec{\nabla} \times \vec{H}) \cdot \vec{ds}$$

$$\therefore \int_{surface} (\vec{\nabla} \times \vec{H}) \cdot \vec{ds} = \int_{surface} \vec{J} \cdot \vec{ds}$$

$$\therefore \boxed{\vec{\nabla} \times \vec{H} = \vec{J}}$$ $\Rightarrow$ Differential form

Gauss's thm in Magnetism

$$\phi_M = \oint_{surface} \vec{B} \cdot \vec{ds}$$

$\therefore$ Lines of mag. field have neither begning or ending

$$\therefore \int_{surface} \vec{B} \cdot \vec{ds} = 0$$

$$\int_{Vol} (\vec{\nabla} \cdot \vec{B})\, dv = 0$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

Faraday's law in Integral and Differential form :.

$$e = -\frac{d\phi_M}{dt} \longrightarrow ①$$

$$e = \oint_{line} \bar{E} \cdot \vec{dl} \longrightarrow ②$$

$$\phi_M = \int_{Surface} \vec{B} \cdot \vec{ds} \longrightarrow ③$$

$$\therefore \boxed{\int_{line} \bar{E} \cdot \vec{dl} = -\int_{surface} \frac{d\vec{B}}{\partial t} \cdot \vec{ds}} \Rightarrow \text{Integral form}$$

from stoke's thm

$$\int_{line} \vec{E} \cdot \vec{dl} = \int_{surface} (\bar{\nabla} \times \bar{E}) \cdot \vec{ds}$$

$$\therefore \int_{surface} (\bar{\nabla} \times \bar{E}) \cdot \vec{ds} = \frac{-\partial}{\partial t} \int_{surface} \vec{B} \cdot \vec{ds}$$

$$\therefore \boxed{\bar{\nabla} \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}} \Rightarrow \text{Differential form}$$

Displacement Current :. From continuity eq^n

$$\bar{\nabla} \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\bar{\nabla} \cdot \bar{J} = \frac{-\partial \rho}{\partial t} \longrightarrow ①$$

Ampere's law is, $\bar{\nabla} \times \bar{H} = \bar{J}$

Taking div. of both sides

$$\bar{\nabla} \cdot (\bar{\nabla} \times \bar{H}) = \bar{\nabla} \cdot J$$

$$0 = \bar{\nabla} \cdot \bar{J}$$

$$\bar{\nabla} \cdot \bar{J} = 0 \longrightarrow ②$$

but $\bar{\nabla} \cdot \bar{J} \neq 0$ according to continuity eqn.
Maxwell modified Ampere's law by adding time varying electric field.

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J_D} \longrightarrow (3)$$

J_D is called displacement current density

Taking div. of eqn (3)

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{J_D} \longrightarrow (4)$$

$$\because \bar{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0$$

$$\therefore \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

from (4)

$$\therefore 0 = -\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J_D}$$

$$\therefore \vec{\nabla} \cdot \vec{J_D} = \frac{\partial \rho}{\partial t} \longrightarrow (5)$$

$$\because \bar{\nabla} \cdot \bar{D} = \rho$$

$$\vec{\nabla} \cdot \vec{J_D} = \frac{\partial}{\partial t} (\bar{\nabla} \cdot \vec{D})$$

$$\vec{\nabla} \cdot \vec{J_D} = \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\vec{J_D} = \frac{\partial \vec{D}}{\partial t} \longrightarrow (6)$$

$\therefore$ from (3), modified Ampere's law is

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Maxwell's equations:-

The field equations which govern the time-varying electric and magnetic fields are now written as

(A) Differential form:-

(i) Gauss's law $\vec{\nabla}\cdot\vec{D} = \rho$

(ii) Gauss's law for magnetism, $\vec{\nabla}\cdot\vec{B} = 0$

(iii) Faraday's law, $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

(iv) Ampere's law $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

(B) Integral form:-

(i) $$\int_{surface} \vec{D}\cdot\vec{ds} = \int_{vol} \rho\, dv$$

(ii) $$\oint_{surface} \vec{B}\cdot\vec{ds} = 0$$

(iii) $$\int_{line} \vec{E}\cdot\vec{dl} = -\int_{surface} \frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$$

(iv) $$\int_{line} \vec{H}\cdot\vec{dl} = \int_{surface} \left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)\cdot\vec{ds}$$

Physical Significance:

(1) Maxwell's first equation shows that the total electric flux density $\vec{D}$ through the surface enclosing a volume is equal to the charge density ρ within the volume. It means charge distribution generates a steady electric field.

(2) Maxwell's second equation tells us that the net mag. flux through a closed surface is zero. It implies that mag. Mono-poles do not exist.

(3) The third equation shows that the emf around a closed path is equal to the time derivative of mag. flux density

through the surface bounded by the path. It means an electric field can also be generated by a time-varying mag. field.

④ Fourth equation shows that the magneto-motive force around a closed path is equal to conduction current plus time-derivative of electric flux density through any surface bounded by the path. It also shows that the mag. field is generated by time-varying electric field.

The Wave Equation: for free space $\rho = 0$ and $\vec{J} = 0$. Maxwell's equations for free space can be written as

$$\vec{\nabla} \cdot \vec{D} = 0$$

$$\because \vec{D} = \epsilon_0 \vec{E}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \longrightarrow (1)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \longrightarrow (2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow (3)$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \longrightarrow (4)$$

Taking curl of eq^n ③, we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla}\left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B}) \longrightarrow (5)$$

Sub ④ in ⑤, we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow (6)$$

but $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 E$

$\because \vec{\nabla} \cdot \vec{E} = 0$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow (7)$$

Similarly for mag. field

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \longrightarrow (8)$$

Eqns (7) and (8) are wave equations. Any function satisfying such an eqn describes a wave. The square root of quantity is the reciprocal of the coeff. of time derivative that gives phase velocity.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

$\therefore$ It indicates that em waves propagate with velocity

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

sub. values of μ_0 and ϵ_0

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.9 \times 10^{-12}}} = 3.0 \times 10^8 \text{ m/s}$$

$$\therefore \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \text{ (speed of light)}$$

The emergence of speed of light from em wave is great achievement of Maxwell's theory. Maxwell predicted that em disturbance should propagate in free space with a speed equal to speed of light hence light waves are em in nature.

—X—X—X—