VECTOR INTEGRATION

SOMAIYA VIDYAVIHAR UNIVERSITY

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Course: ITVC

Experiment / assignment / tutorial No. 10

Grade: Signature of the Faculty with date

Q.1) $\int_c \vec{F}\cdot d\vec{r}$ $\vec{F} = x^2\hat{i} + xy\hat{j}$

C is a straight line joining O(0,0) to A(1,1)

→ Let $C \rightarrow x = t,\ y = t,\ 0 \leq t \leq 1$

$$\vec{F}\cdot d\vec{r} = (x^2\hat{i} + xy\hat{j})(\hat{i}dt + \hat{j}dt)$$

$$= x^2 dt + xy\, dt.$$

$$= t^2 dt + t^2 dt = 2t^2 dt$$

$$\therefore \int_0^1 2t^2 dt = 2\int_0^1 t^2 dt$$

$$= 2\left[\frac{t^3}{3}\right]_0^1$$

$$= 2 \times \frac{1}{3}$$

$$= \boxed{\frac{2}{3}}$$

2) $\oint_c \left(\frac{1}{y}dx + \frac{1}{x}dy\right)$

C is boundary of region defined by $x = 1$, $x = 4$, $y = 1$, & $y = \sqrt{x}$.

→ By Vector Green's Theorem

$$\oint_c (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$

→ Partial Derivatives, $P = 1/y$, $Q = 1/x$

$$\frac{\partial Q}{\partial x} = \frac{-1}{x^2},\quad \frac{\partial P}{\partial y} = \frac{-1}{y^2},\ \cancel{\frac{\partial Q}{\partial x}}.$$

$\Rightarrow$ R.H.S

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{-1}{x^2} + \frac{1}{y^2}$$

$$\iint_R \left(\frac{1}{y^2} - \frac{1}{x^2} \right) dx\, dy.$$ By Green's theorem,

$$\int_1^4 \int_1^{\sqrt{x}} \left(\frac{1}{y^2} - \frac{1}{x^2} \right) dy\, dx$$

$\rightarrow$ $$\int_1^{\sqrt{x}} \frac{1}{y^2}\, dy = \left[\frac{-1}{y} \right]_1^{\sqrt{x}}$$

$$= \frac{-1}{\sqrt{x}} + 1$$

$$\int_1^{\sqrt{x}} \frac{1}{x^2}\, dy = \frac{\sqrt{x} - 1}{x^2}$$

$$\therefore \int_1^{\sqrt{x}} \left(\frac{-1}{x^2} + \frac{1}{y^2} \right) dy = \frac{-1}{x^{3/2}} + \frac{1}{x^2} + 1 - \frac{1}{\sqrt{x}}$$

$\rightarrow$ $$\int_1^4 \left(\frac{-1}{x^{3/2}} + \frac{1}{x^2} + 1 - \frac{1}{\sqrt{x}} \right) dx$$

$$= -\frac{x^{-1/2}}{-1/2} + \frac{x^{-1}}{-1} + x - \frac{x^{1/2}}{1/2}$$

$$= \frac{1}{2\sqrt{x}} + \frac{1}{x} + x - \frac{1}{2\sqrt{x}} \Rightarrow = \frac{2\sqrt{x}}{\sqrt{x}} + \frac{1}{x} + x - \frac{1}{2\sqrt{x}}$$

$$= \left[\frac{1}{x} + x \right]_1^4 \qquad = \left[\frac{3}{2\sqrt{x}} + \frac{1}{x} + x \right]_1^4$$

$$= \frac{1}{4} + 4 - 1 - 1 =$$

$$= \left[\frac{2}{\sqrt{x}} + \frac{1}{x} + x - 2\sqrt{x} \right]_1^4 = \boxed{5.75}$$

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$\Rightarrow$ L.H.S

Boundary C consists of 4 segments:

1) $x=1$ from $y=1$ to $y=\sqrt{1}=1$
no contribution.

2) $x=4$ from $y=1$ to $y=\sqrt{4}=2$
3) curve $y=\sqrt{x}$ from $x=1$ to $x=4$
4) $y=1$ from $x=1$ to $x=4$

$\rightarrow$ For (1) no segment
$\rightarrow$ For (2) $dx=0$

$$\int_1^2 \frac{1}{4}\,dy = \frac{1}{4}[y]_1^2 = \frac{1}{4}$$

$\rightarrow$ For (3),

$$\int_1^4 \frac{1}{\sqrt{x}}\,dx + \frac{1}{x}\cdot\frac{1}{2\sqrt{x}}\,dx$$

$$= [2\sqrt{x}]_1^4 + \frac{-1}{2}\left[\frac{2}{\sqrt{x}}\right]_1^4$$

$$= 2 + \frac{1}{2}$$

$$= 2.5$$

$\rightarrow$ For (4) $dy=0$

$$\int_1^4 \frac{1}{1}\,dx = [x]_1^4$$

$$= 3$$

$\therefore$ sum $= \boxed{5.75}$ L.H.S = R.H.S

Hence verified.

3) $\int_C \vec{F}\cdot d\vec{r} = \iint_S (\nabla \times \vec{F})\cdot d\vec{s}$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 4xz & -y^2 & yz \end{vmatrix}$$

$\vec{F} = (4xz, -y^2, yz)$

curl,

$$\nabla \times \vec{F} = \left(\frac{\partial yz}{\partial y} - \frac{\partial(-y^2)}{\partial z}\right)\hat{i} - \left(\frac{\partial yz}{\partial x} - \frac{\partial 4xz}{\partial z}\right)\hat{j} + \left(\frac{\partial(-y^2)}{\partial x} - \frac{\partial 4xz}{\partial y}\right)\hat{k}$$

$$= z\hat{i} - 4x\hat{j}$$

surface in plane $z=0$

$\nabla \times \vec{F} = 0$

$\int_S (\nabla \times \vec{F})\cdot d\vec{s} = 0$

$\therefore$ By Stokes' Theorem

$$\int_C \vec{F}\cdot d\vec{r} = \boxed{0}$$

4) $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$

$\iint_S \vec{N}\cdot\vec{F}\,ds$

S is bounded by $x^2+y^2=4$, $z=0$, $z=3$

By Gauss's Divergence Theorem

$$\iint_S \vec{N}\cdot\vec{F}\,ds = \iiint_V (\nabla\cdot\vec{F})\,dV$$

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$$\nabla \cdot \vec{F} = \frac{\partial 4x}{\partial x} + \frac{\partial(-2y^2)}{\partial y} = \frac{\partial z^2}{\partial z}$$

$$\nabla \cdot \vec{F} = 4 + (-4y) + 2z$$

$$= 4 - 4y + 2z$$

→ The region bounded by $x^2+y^2=4$ (cylinder)

$x = r\cos\theta$, $y = r\sin\theta$, $r \in [0,2]$, $\theta \in [0, 2\pi]$, $z \in [0,3]$

$dV = r,\ dr,\ d\theta,\ dz$

∴ Volume integral

$$\iiint_V (4-4y+2z)\,dV = \int_0^3 \int_0^{2\pi} \int_0^2 (4 - 4r\sin\theta + 2z)\, r\, dr\, d\theta\, dz$$

$$I = \int_0^3 \int_0^{2\pi} \int_0^2 4r\, dr\, d\theta\, dz - \int_0^3 \int_0^{2\pi} \int_0^2 4r^2 \sin\theta\, dr\, d\theta\, dz + \int_0^3 \int_0^{2\pi} \int_0^2 2zr\, dr\, d\theta\, dz.$$

$$= \int_0^3 \int_0^{2\pi} [2r^2]_0^2\, d\theta\, dz - 0 + \int_0^3 \int_0^{2\pi} [zr^2]_0^2\, d\theta\, dz$$

$$= \int_0^3 \int_0^{2\pi} 8\, d\theta\, dz + \int_0^3 \int_0^{2\pi} 8z\, d\theta\, dz$$

$$= \int_0^3 16\pi\, dz + \int_0^3 16\pi z\, dz$$

$$= 48\pi + 72\pi$$

$$= \boxed{120\pi}$$