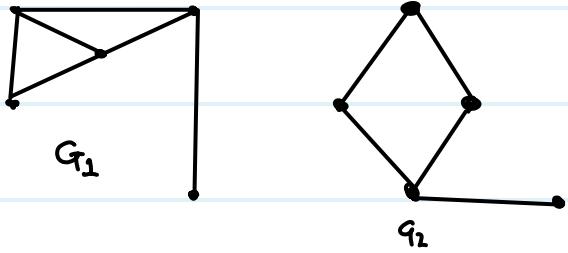
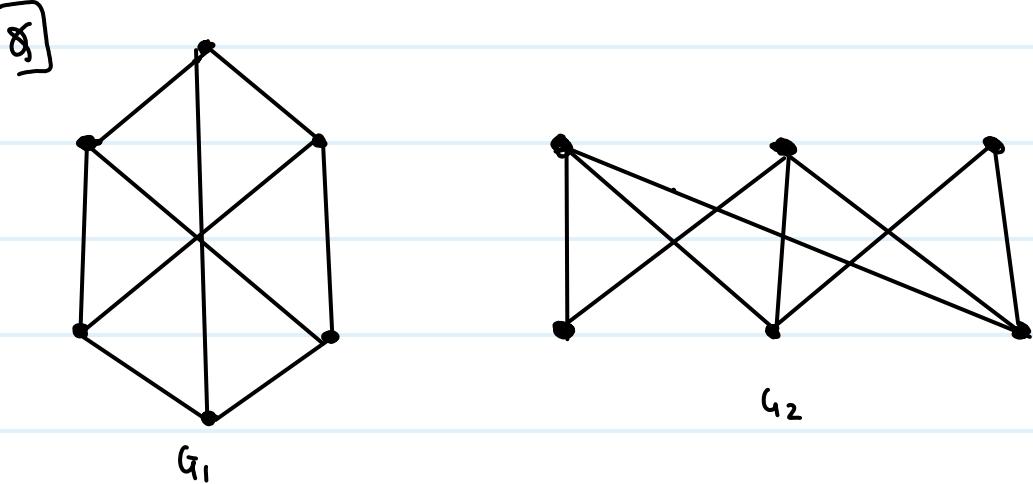


* Isomorphic Graph



\rightarrow	Graph G_1	Graph G_2
No. of vertices	5	5
No. of edges	6	5

Both Graph G_1 & G_2 having diff no. of edges



G_1

No. of V

6

 G_2

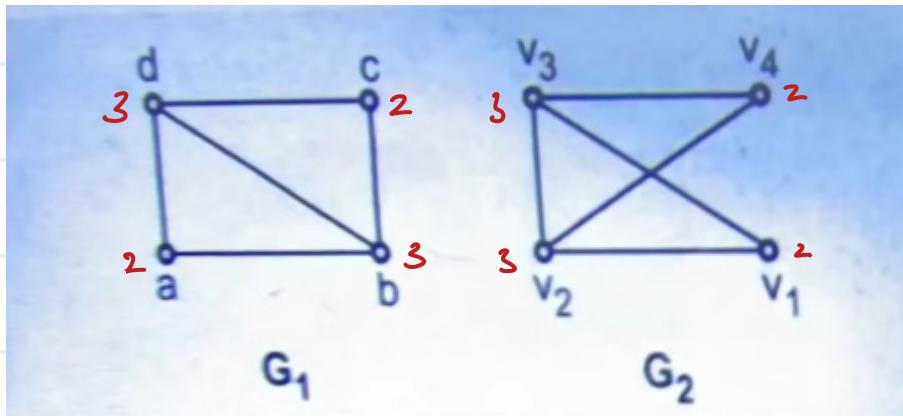
6

No. of E

9

8

Q]

 G_1

No. of V

4

 G_2

4

No. of E

5

5

Degree Seq. $3, 2, 3, 2$ $3, 2, 2, 3$

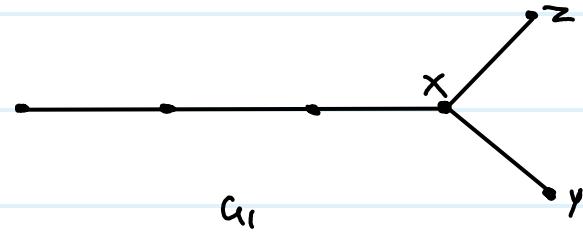
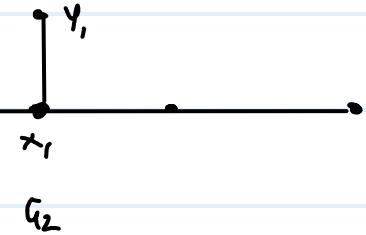
1-to-1 correspondence of V & E

 $a \rightarrow v_1$

Graph is isomorphic

 $b \rightarrow v_2$ $c \rightarrow v_4$ $d \rightarrow v_3$

Q)

 G_1  G_2 G_1 G_2

V

6

6

E

5

5

D

1, 2, 2, 3, 1, 1

1, 2, 3, 2, 1, 1

For graph G_1 , Degree 3 connected to $\text{Deg}(1), \text{Deg}(1), \text{Deg}(2)$
 For graph G_2 , Degree 3 connected to $\text{Deg}(2), \text{Deg}(2), \text{Deg}(1)$

Mismatch

* Eulerian Graph 4 Hamiltonian Graph

Eulerian Path: Every edges of graph appears exactly once in the path

\rightarrow 0 or 2 vertices of odd degree

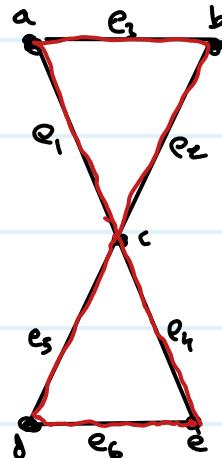
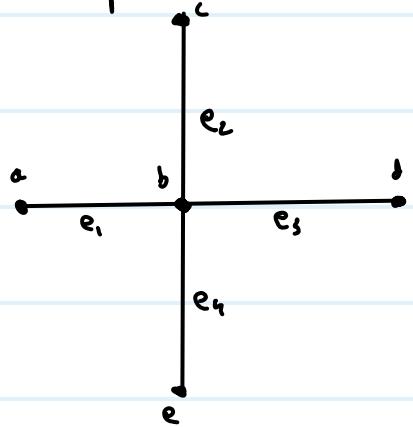
Eulerian Circuit: The circuit which contains every edge of the graph exactly once

Starting & Ending point should be same

\rightarrow Each vertices must be even degree.

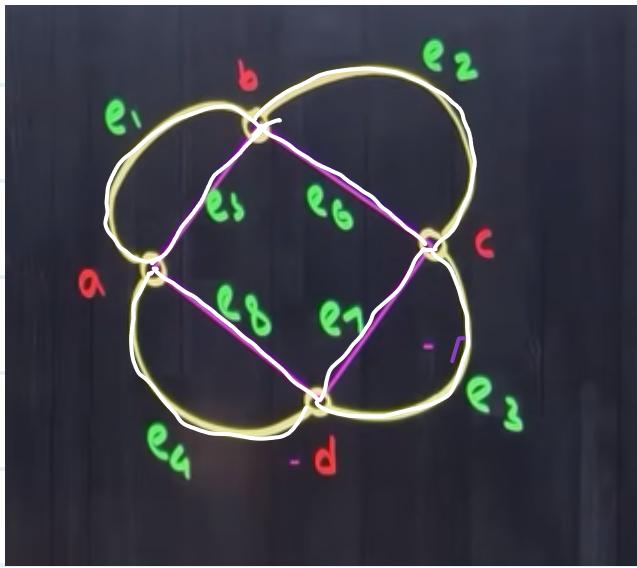
Eulerian Graph: A graph which has an Eulerian circuit

Nope



$e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}$

Q) Check whether the graph has Euler circuit, path justify:



Degree of each vertex
is even

\therefore Eulerian circuit

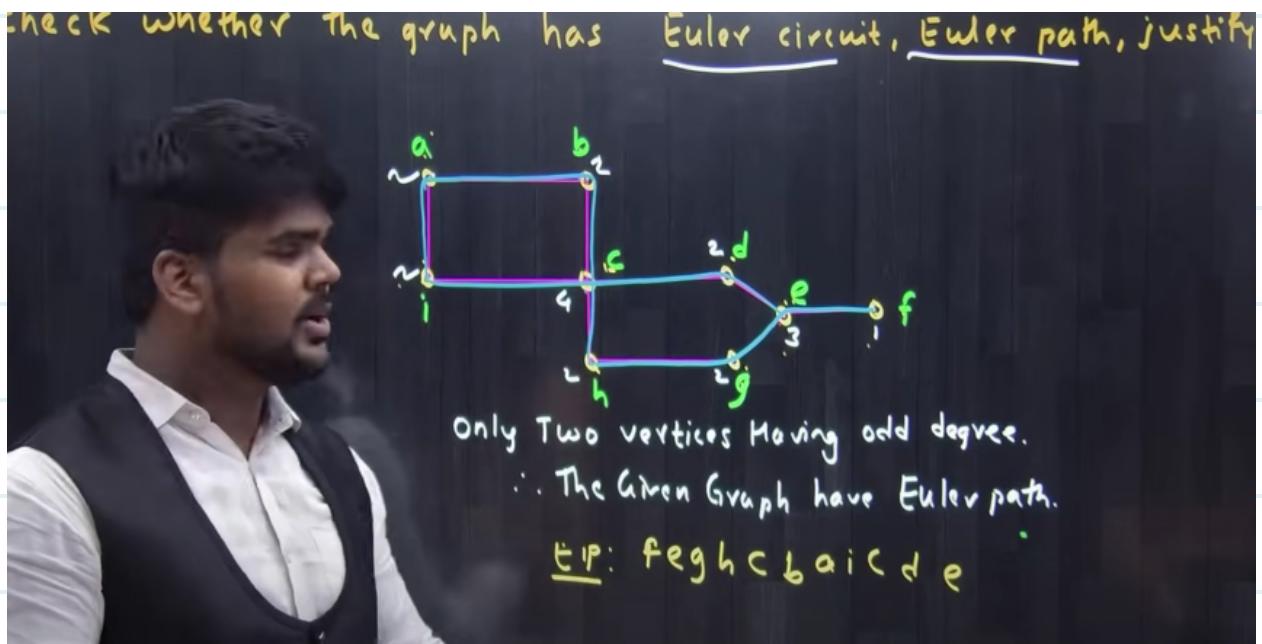
Here zero vertex has
odd degree

\therefore Eulerian Path

E.P.: ae, be₂, (e₃, d)e₄, ae₅, be₆, (e₇, de₈)

E.C.: ae, be₂, (e₃, d)e₄, ae₅, be₆, (e₇, de₈), a

Q)



No E.C. \rightarrow Not all vertices even degree.

* Hamiltonian Graph

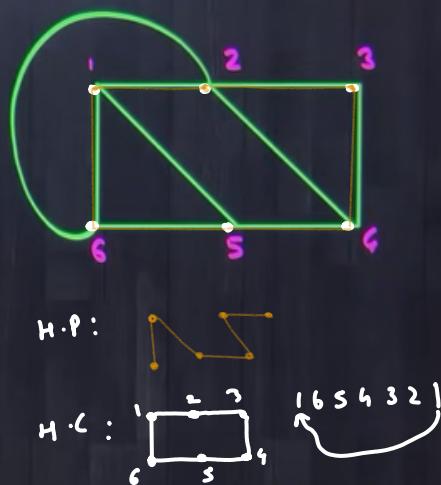
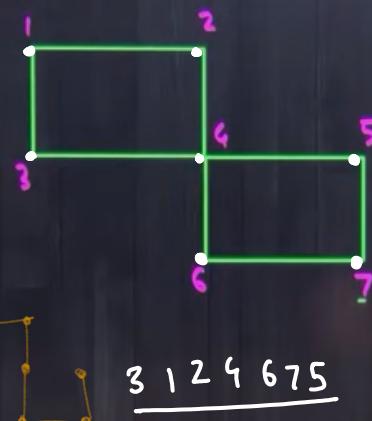
Hamiltonian Path: Every vertex of Graph appears exactly once in path

Hamiltonian Circuit: The circuit which contains every vertex of graph exactly once.

Hamiltonian Graph: A Graph which has an Hamiltonian circuit

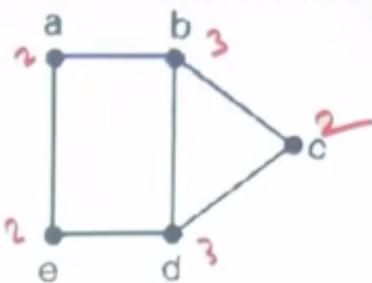
Q)

Determine whether the following graph has a Hamiltonian circuit or Hamiltonian path.



Q)

EX. 6.4.17 : Determine whether the given graph has a Hamilton circuit or Eulerian circuit. If it does, find such a circuit.



odd degree

↳ X Euler circuit X

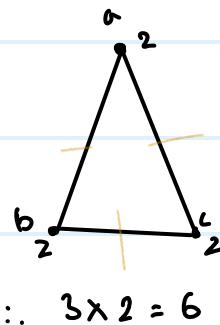
Fig. Ex. 6.4.17

Hamilton ✓



* Handshaking Lemma

$$\sum_{i=1}^n d(V_i) = 2e$$



Max degree of any vertex in α

Simple graph with n vertices is $(n-1)$

Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$

Max degree of Any vertex in simple graph is $(n-1)$.

Acc. Handshaking lemma

$$\sum_{i=1}^n d(V_i) = 2 \times e$$

$$d(V_1) + d(V_2) + d(V_3) + \dots + d(V_n) = 2 \times e$$

$$(n-1) + (n-1) + \dots + (n-1) = 2e$$

$$\frac{n(n-1)}{2} = e$$

Determine the number of edges in a graph with 6 nodes, 2 of degree 4 and 4 of degree 2. Draw two such graphs.

$$\begin{aligned}d(V_1) &= 4 & d(V_3) &= d(V_4) = d(V_5) = d(V_6) = 2 \\d(V_2) &= 4\end{aligned}$$

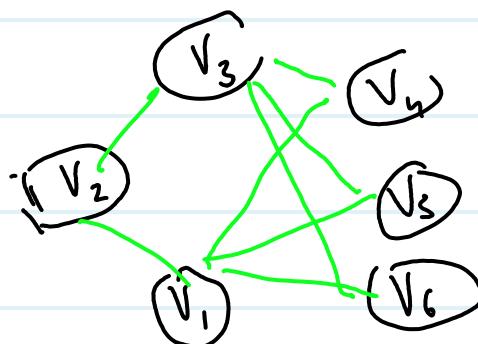
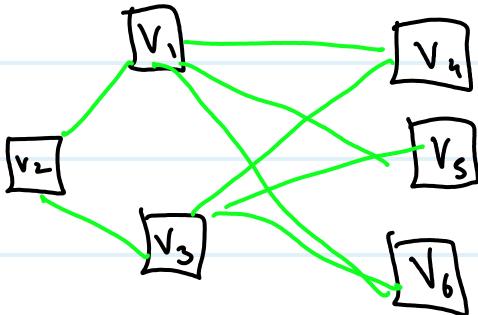
Handshaking lemma

$$\sum_{i=1}^6 d(V_i) = 2e$$

$$d(V_1) + d(V_2) + d(V_3) + d(V_4) + d(V_5) + d(V_6) = 2 \times e$$

$$\therefore 4 + 4 + 2 + 2 + 2 + 2 = 2e$$

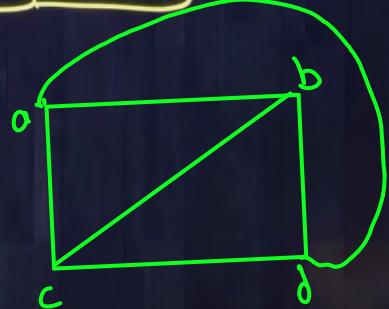
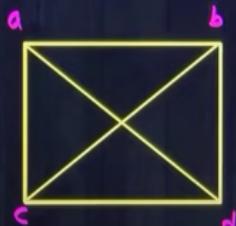
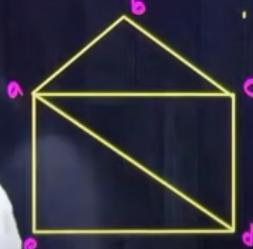
$$\therefore e = 8$$



* Planar Graph

Planar Graph:

graph is said to be planar if it can be drawn on a plane such a way that no edges cross one another.



Draw a planar representation of graphs given below if possible.

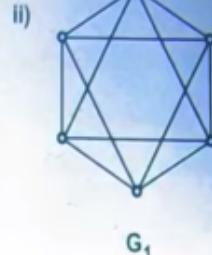
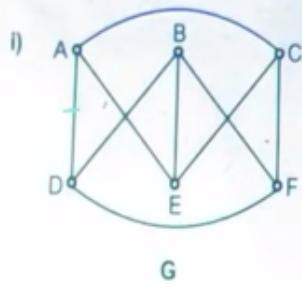
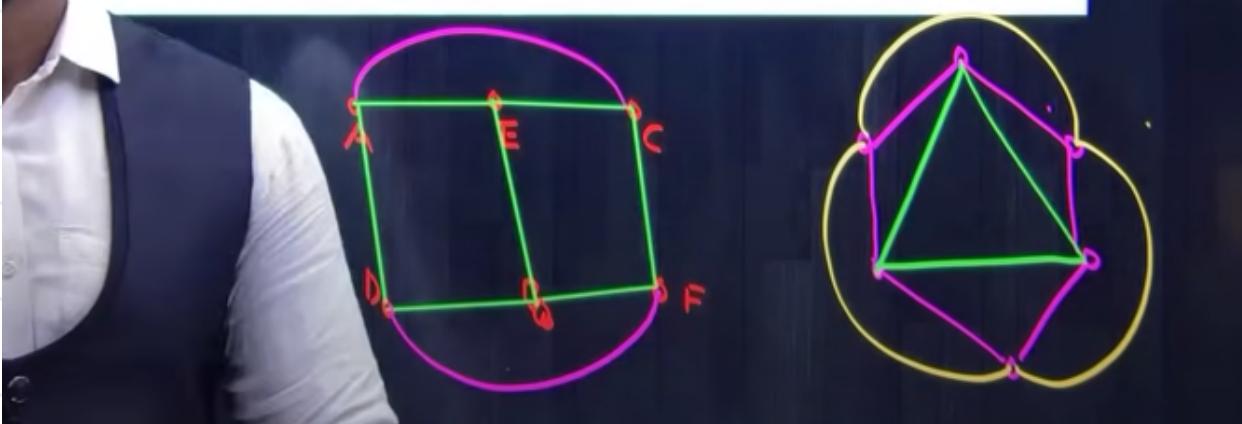
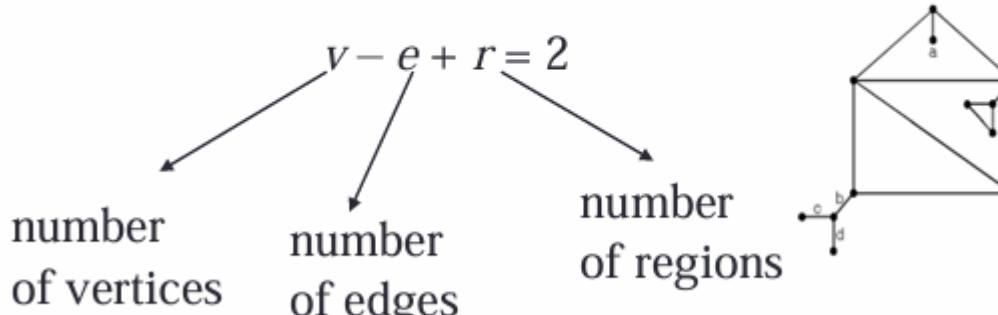


Fig. 5.15.7



- **Theorem : Euler's connected planar graph theorem**



Show that in a connected planar graph with 6 vertices, 12 edges each of region is bounded by 3 edges.

$V = 6$
 $E = 12$
 $R = ?$

Total Edges = $12 \times 2 = 24$ $\rightarrow R$

No. of Edges Required for Each Region = $\frac{24}{8} = 3$,

Euler's Formula.

$$\begin{aligned} V - E + R &= 2 \\ 6 - 12 + R &= 2 \\ -6 + R &= 2 \\ R &= 2 + 6 \\ R &= 8 \end{aligned}$$

For Subgraphs → Refer Notes

