Quick Sort

Divide And Conquer

Module 2

Quick Sort

- Quick Sort uses Divide and Conquer Strategy.
- There are three steps:
  1. **Divide:**
     - Splits the array into sub arrays.
     - Splitting of array is based on **pivot element**.
     - Each element in left sub array is less than and equal to middle (pivot) element.
     - Each element in right sub array is greater than the middle (pivot) element.

  2. **Conquer:** Recursively sort the two sub arrays
  3. **Combine:** Combine all sorted elements in a group to form a list of sorted elements.

Example

Low							High
50	30	10	90	80	20	40	70
i / **Pivot**							j

Example

Step 1:

Low High

50	30	10	90	80	20	40	70

i / **Pivot** j

Step 2:

Increment `i` if `A[i]<= Pivot` and continue to increment it until element pointed by i is greater than `A[Low]`

Low High

50	30	10	90	80	20	40	70

Pivot i j

Step 3:

Decrement j if `A[j]> Pivot` and continue to decrement it until element pointed by j is less than `A[High]`

High

50	30	10	90	80	20	40	70

Pivot i j

Example

Step 4:

As `A[i] > A[Low],` stop incrementing i

Low High

50	30	10	90	80	20	40	70

i

Pivot j

Step 5: Increment i if A[i]<= Pivot and continue to increment it until element pointed by i is greater than A[Low]

High

50	30	10	90	80	20	40	70

Pivot i j

Step 6:

Decrement j if A[j]> Pivot and continue to decrement it until element pointed by j is less than A[Low]

High

50	30	10	90	80	20	40	70

Pivot i j

Example

Step 7:

As **A[j] > A[Low]**, stop decrementing j

Low High

50	30	10	**90**	80	20	**40**	70

i j

Pivot

Step 8:

Since i and j cannot be further incremented and decremented, we will **swap A[i] and A[j]**

Low High

50	30	10	**40**	80	20	**90**	70

Pivot i j

Step 9:

Continue incrementing i and decrementing j until false conditions are obtained

Low High

50	30	10	40	80	20	90	70

i j

Pivot

Example

Step 7:

Low

High

50	30	10	40	**80**	**20**	90	70
				i	j		

Pivot

Step 8:

swap A[i] and A[j]

Low

High

50	30	10	40	**20**	**80**	90	70
				i	j		

Pivot

Step 9:

Again, Increment i and decrement j. **As soon as i > j, swap A[Low] and A[j]**

Low

High

50	30	10	40	**20**	80	90	70
				j	i		

Pivot

Example

Step 10:

swap A[Low] and A[j]

Low				Pivot			High
20	30	10	40	**50**	80	90	70
				j	i		

Step 11:

Low							High
20	30	10	40	**50**	80	90	70
i / Pivot			j	**mid**	i/Pivot		j

Left Sub Array

Right Sub Array

Algorithm

```
Algorithm QuickSort(A[0…n], low, high)
{
        if(low<high) then
        mid ← partition(A[low…high])
        QuickSort(A[low…mid-1])
        QuickSort(A[mid+1…high])
}
```

```
Algorithm Partition(A[0…n], low, high)
{
        pivot←A[low];
        i ← low;
        j ← high+1;
        while(i ≤ j)do
        {
                while(A[i] ≤ pivot)do
                { i++;}
                while(A[j] ≥ pivot)do
                { j--;}
                if(i <= j) then
                    swap(A[i],A[j])
        }
        swap(A[low],A[j])
        return j;
}
```

Analysis

1. Best Case:
   - If array is partitioned at the mid
   - The Recurrence relation for quick sort for obtaining best case time complexity.

$$T(n) = T(n/2) + T(n/2) + cn \qquad \text{for } n > 1 \quad \ldots\ldots\ldots\ldots\ (1)$$

$$= 0 \qquad \text{for } n = 1 \quad \ldots\ldots\ldots\ldots\ (2)$$

Using Master Theorem:

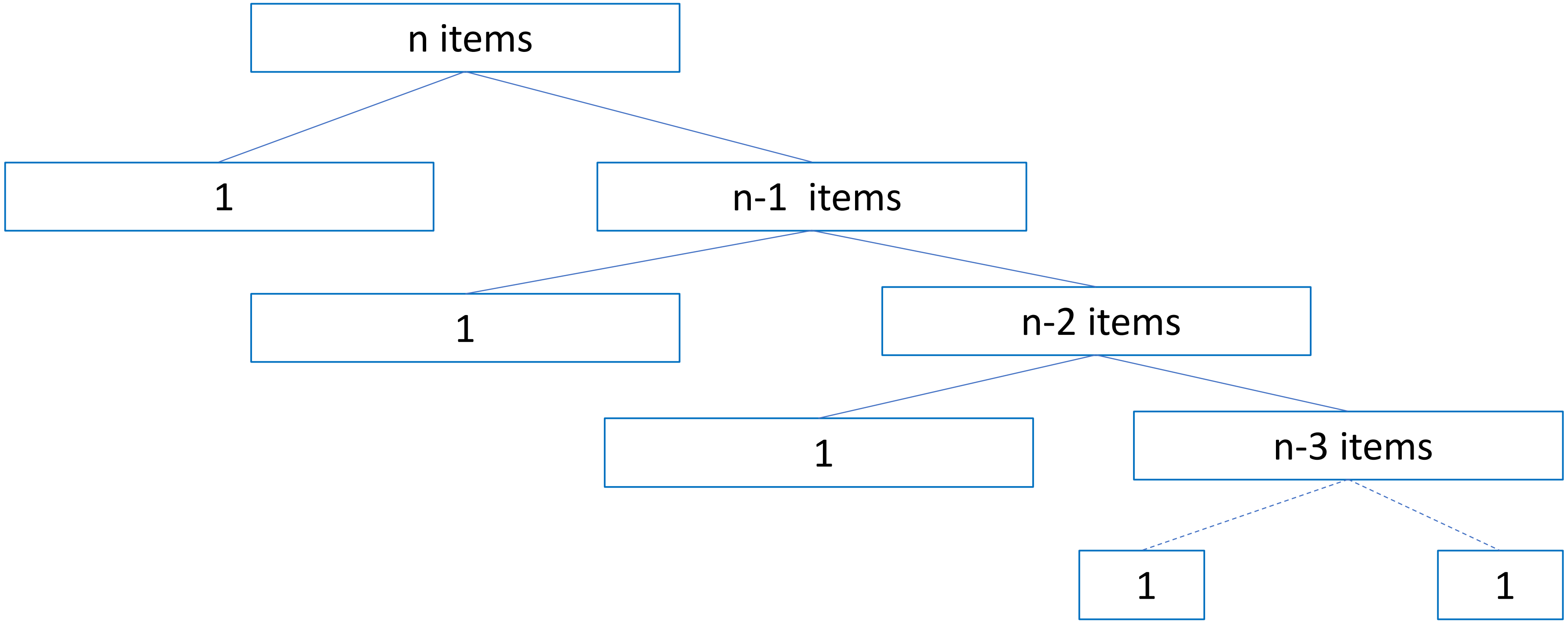
$$T(n) = 2 * T(n/2) + cn$$

$$\boxed{T(n) = \Theta(nlogn)}$$

Analysis

2. Worst Case:

- If pivot is a maximum or minimum of all the elements in the sorted list.
- This can be graphically represented as follows



Time Required

n

n-1

n-2

…

1

Analysis

2. Worst Case:

- If pivot is a maximum and minimum of all the elements in the sorted list.
- The Recurrence relation for quick sort for obtaining best case time complexity.

$$T(n) = T(n-1) + cn \qquad \text{for n >1} \qquad \text{...............} \ 1$$
$$= 0 \qquad \text{for n = 1} \qquad \text{...............} \ 2$$

$$T(n) = \Theta(n^2)$$

Analysis

3. Average Case:
   - For any pivot position i; where $i \in \{0,1,2,3 \ldots n-1\}$
     - Time for partition an array: cn
     - Head and Tail sub-arrays contain i and $n\text{-}1\text{-}i$ items.
     - So,

$$T(n) = T(i) + T(n-1-i) + cn$$

     - Average running time for sorting:

$$\boxed{T(n) = \frac{1}{n}\sum_{i=0}^{n-1}\left(T(i) + T(n-1-i)\right) + cn}$$