Logic

> Proposition is a declarative sentence / statement that is either true / false but not both at the same time.

> Logic operators are used to form new propositions from existing ones.

Conjunction

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Disjunction

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

Negation

p	$\sim p$
F	T
T	F

consequent

Conditional : If p then q $[p \rightarrow q] \equiv \sim p \vee q$

↳ p only if q

↳ p implies q

antecedent / hypothesis

Biconditional : p if and only if q $[p \leftrightarrow q]$

$p \leftrightarrow q$

p	q	$\sim p$	$\sim p \vee q$ [$p \to q$]
F	F	T	T
F	T	T	T
T	F	F	F
T	T	F	T

p	q	$p \to q$	$q \to p$	$p \to q \wedge q \to p$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

★ $p \to q = \sim p + q$

$p \leftrightarrow q = (p \to q) \wedge (q \to p)$

$p \to q$

Converse	Inverse	Contrapositive
$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$

Precedence of Logical Operators :

$\sim \quad \wedge \quad \vee \quad \to \quad \leftrightarrow$

Q. p : Raju is rich q : Raju is happy

1. Raju is poor but happy

$\sim p \wedge q$

2. Raju is neither rich nor happy

$\sim p \wedge \sim q$

3. Raju is either rich or unhappy

$p \vee \sim q$

4. Raju is poor or else he is rich and un.

$\sim p \vee (p \wedge \sim q)$

Q. 'Either my program runs and it contains no bugs, or my program contains bugs'.

p : My program runs q : My program contains bugs

$(p \wedge \sim q) \vee q$

1. If I am not in a good mood I will go to a movie

↳ $\sim r \to q$

2. I will not go to a movie & I will study dsm.

↳ $\sim q \wedge p$

3. I will go to a movie only if I will not study dsm.

↳ $q \to \sim p$

p	q	$\sim q$	$p \vee \sim q$	$p \wedge q$	$\sim(p \vee \sim q)$	$(p \vee \sim q) \to (p \wedge q)$
F	F	T	T	F	F	F
F	T	F	F	F	T	T
T	F	T	T	F	F	F
T	T	F	T	T	F	T

$((p \wedge q) \vee \sim q)$

p	q	$p \wedge q$	$\sim q$	$(p \wedge q) \vee \sim q$
F	F	F	T	T
F	T	F	F	F
T	F	F	T	T
T	T	T	F	T

$\sim [p \wedge (p \vee \sim q)]$

p	q	$\sim q$	$p \vee \sim q$	$p \wedge (p \vee \sim q)$	$\sim [p \wedge (p \vee \sim q)]$
F	F	T	T	F	T
F	T	F	F	F	T
T	F	T	T	T	F
T	T	F	T	T	F

Tautology – A stament form is called Tautology if it assumes the truth value 'T' irrespective of the truth values assigned to its variables

Contradiction – " 'F' "

Contingency – A statement form that is neither a tautology nor a contradiction.

1. Commutative

$p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

2. Associative

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

3. Distributive

$p \vee (q \wedge r) \equiv p \vee q \wedge p \vee r$

$p \wedge (q \vee r) \equiv p \wedge q \vee p \wedge r$

4. Idempontent

$p \vee p \equiv p$

$p \wedge p \equiv p$

5. Identity

$p \vee F \equiv p$

$p \wedge F \equiv F$

$p \vee T \equiv T$

$p \wedge T \equiv p$

6. Implication

$p \rightarrow q \equiv \sim p \vee q$

7. Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

8. Complementary

$p \vee \sim p \equiv T$

$p \wedge \sim p \equiv F$

$\sim(\sim p) = p$

9. De Morgan's

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

Q. $a \vee (\bar{a} \wedge b) = a \vee b$

Dist.

$\hookrightarrow (a \vee \bar{a}) \wedge (a \vee b)$

$(1) \wedge (a \vee b) \Rightarrow a \vee b$

Q. $[(p \to q) \wedge \sim q] \to \sim p$

$[(\sim p \vee q) \wedge \sim q] \to \sim p$ Implication

$[(\sim p \wedge \sim q) \vee (q \wedge \sim q)] \to \sim p$ Distr.

$[(\sim p \wedge \sim q) \vee F] \to \sim p$ Compl.

$(\sim p \wedge \sim q) \to \sim p$ Identity

$\sim (\sim p \wedge \sim q) \vee \sim p$ Imp.

$(p \vee q) \vee \sim p$ Demorgan

$(p \vee \sim p) \vee (\sim p \vee q)$ Distr.

$T \vee (\sim p \vee q)$ Comp

$= T$ Identity

* Syllogism is the process of drawing logical conclusions from given premises

Quantifier – is a logical term used to indicate the quantity of objects that a statement pretains to.
How many elements in a given domain satisfy a particular condⁿ.

$\forall$ $\exists$

$p(x) \rightarrow$ prop funcn. defined on set A

$(\forall x \in A)\ p(x)$ or $\forall x\ p(x)$

↳ for all / for every / for each (universal Quant.)

Q. $(\forall n \in N)\ (n+4>3)$

$\hookrightarrow p(n)$

$p(n) = n+4>3$

Predicate $\rightarrow$ an assertion that contains one or more variables. Its truth value is det. after ass. values to the variables

$M(x) \rightarrow$ "x is a man" $C(x) \rightarrow$ "x is clever"

(i) $\exists x\ (M(x) \rightarrow C(x))$

There exists x such that if x is a man then x is clever.

$\hookrightarrow$ There exists a man who is clever.

(ii) $\forall x\ (M(x) \wedge C(x))$

For all/ every/ each x x is a man and x is clever

Ev

$(p \leftrightarrow q)$: $(p \rightarrow q) \wedge (q \rightarrow p)$

$(p' + q) \wedge (q' + p)$

$p'q' + \cancel{qq'} + \cancel{pp'} + pq$

$p'q' + pq$

$P(x, y) \rightarrow y = x + 1$

$\forall x \; \exists y \; P(x, y)$ $\exists y \; \forall x \; P(x, y)$

$\forall x \; [x^2 \geq 0]$

$x \cdot y = 1 \rightarrow P(x, y)$ $\forall x \; \exists y \; P(x, y)$

$\exists x \exists y [(x>0) \wedge (y>0) \wedge (x \cdot y > 0)]$

$\exists x \forall y [(y>0) \rightarrow (x+y<0)]$

Q. $\exists x\ Q(x)$

$\exists x [P(x) \wedge Q(x)]$

$\forall x [Q(x) \rightarrow \sim T(x)]$

$\exists x [Q(x) \wedge T(x)]$

$\forall x [(Q(x) \wedge R(x)) \rightarrow S(x)]$

$\exists x (\sim C(x))$

There exists an animal x such that x does not live in water

There

$\exists x [(B(x) \wedge \sim A(x)]$

There exists an ~~an~~ animal x such that x is a fish and x is not a whole

There exits a

DNF → SOP (Disjunctive Normal Form)

Q. $(p \wedge q) \vee \sim q$ SOP → DNF

$(p \wedge \sim q) \vee (p \wedge \sim q)$

Q. $p \wedge (p \rightarrow q) \equiv p \wedge (\sim p \vee q)$

$\equiv (p \wedge \sim p) \vee (p \wedge q)$

$\equiv F \vee (p \wedge q)$

$\equiv \underline{\underline{p \wedge q}}$

$p \vee (\sim p \rightarrow (q \vee (\sim q \rightarrow r)))$ DNF SOP

$p \vee (\sim p \rightarrow (q \vee (q \vee r)))$

$p \vee (\sim p \rightarrow (q \vee q \vee q \vee r))$

$p \vee (\sim p \rightarrow (q \vee (q \vee r)))$

$p \vee (p \vee (q \vee r)) \rightarrow p \vee q \vee r$

TT

$(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

$\sim p \rightarrow r \equiv p \vee r$

p	q	r	$\sim p$	$\sim q$	$\sim p \rightarrow r$	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	
F	F	F	T	T	F	T	T	T	F
F	F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F	F	F
F	T	T	T	F	T	T	F	F	F
T	F	F	F	T	T	F	T	F	F
T	F	T	F	T	T	F	T	F	F
T	T	F	F	F	T	T	T	T	T
T	T	T	F	F	T	T	T	T	T

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r)$ //

DNF (SOP)

POS

$\sim(p \vee q) \leftrightarrow (p \wedge q)$

$[\sim(p \vee q) \rightarrow (p \wedge q)] \wedge [(p \wedge q) \rightarrow \sim(p \vee q)]$

$[\sim\sim(p \vee q) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$

$[(p \vee q) \vee (p \wedge q)] \wedge [(\sim p \vee \sim q) \vee (\sim p \wedge \sim q)]$

$(p \vee q) \wedge$

$(p \leftrightarrow q) \rightarrow (\sim p \wedge r)$

POS

$[(p \rightarrow q) \wedge (q \rightarrow p)] \rightarrow (\sim p \wedge r)$

$$[(\sim p \vee q) \wedge (\sim q \vee p)] \rightarrow (\sim p \wedge r)$$

$$\sim [(\sim p \vee q) \wedge (\sim q \vee p)] \vee (\sim p \wedge r)$$

$$[\sim (\sim p \vee q)] \vee [\sim (\sim q \vee p)] \vee (\sim p \wedge r)$$

POS

$$(p \wedge \sim q) \vee (q \wedge \sim p) \vee (\sim p \wedge r)$$

$$(p \wedge q) \vee (\sim p \wedge q \wedge r)$$

$$[p \vee (\sim p \wedge q \wedge r)] \wedge [q \vee (\sim p \wedge q \wedge r)]$$

$$[(p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r)]$$

$$(p \vee q) \wedge (p \vee r) \wedge (q \vee \sim p) \wedge \underline{\underline{q}} \wedge (q \vee r)$$

p	q	r	$\sim p$	$\sim q$	$p \leftrightarrow q$	$\sim p \wedge r$	$(p \leftrightarrow q)'$	Ans
F	F	F	T	T	T	F	F	F •
F	F	T	T	T	T	T	F	T
F	T	F	T	F	F	F	T	T
F	T	T	T	F	F	T	T	T
T	F	F	F	T	F	F	T	T
T	F	T	F	T	F	F	T	T
T	T	F	F	F	T	F	F	F •
T	T	T	F	F	T	F	F	F •

$(\sim p \vee \sim q \vee \sim r) \wedge (p \vee q \vee \sim r) \wedge (p \vee q \vee r)$ [POS]

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1+2+\dots n)^2$$

Basis of induction

$P(1) \Rightarrow 1^3 = 1^2$

$1 = 1$ $\rightarrow$ LHS = RHS

Induction step

$P(k) \Rightarrow 1^3 + 2^3 + 3^3 + \dots k^3 = (1+2+\dots+k)^2$

Assumed to be true

$P(k+1) \Rightarrow 1^3 + 2^3 + 3^3 + \dots + (k+1)^3 = (1+2+\dots+(k+1))^2$ — ①

$P(k) + (k+1) \Rightarrow$ should be

$1^3 + 2^3 + 3^3 + \dots k^3 + (k+1)^3 =$

$(1+2+\dots+k+(k+1))^2$

— ②

① = ②

$\therefore$ true

$7^n - 1$ is divisible

$P(n) \Rightarrow 7^n - 1$ is divisible by 6

Basis of induction :

$P(1) \Rightarrow 7^1 - 1 = 6$

divisible by 6 $\therefore$ true

Induction step :

$P(k) \Rightarrow 7^k - 1$ is assumed to be True

$P(k+1) \Rightarrow \underline{7^{k+1} - 1}$

$(7^k \cdot 7) - 1$

$(7^k \cdot 7) - 7 + \underline{7 - 1}$

$(7^k \cdot 7 - 7) + 6$

$7(7^k - 1) + 6 \Rightarrow$

divisible by 6   divisible 6

1

$\therefore$ divisible

$3^n - 1$ divisible by 2

$3^1 - 1 = 2$

$3^k - 1$

$(3^{k+1} - 1) \Rightarrow (3^k \cdot 3) - 1$

$= (3^k \cdot 3) + 3 - 3 - 1$

$\Rightarrow (3^k \cdot 3 - 3) + 2$

$\Rightarrow 3(3^k - 1) + 2$

divisible 2

$n^3 + 2n \rightarrow$

$P(1) \Rightarrow 1^3 + 2$

$= 3 \rightarrow$ true

1 3 3 1

$P(k) \rightarrow k^3 + 2k \rightarrow$ true

$P(k+1) \rightarrow (k+1)^3 + 2(k+1)$

$\Rightarrow K^3 + 3k^2 + 3k + 1 + 2k + 2$

$= \underbrace{k^3 + 2k}_{\text{divisible}} + (3k^3 + 3k + 3) \rightarrow 3(K^2 + k + 1)$

$=$

$$1^2+2^2+3^2 \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$P(1) \Rightarrow 1 = \frac{1\cancel{(2)}(3)}{\cancel{6}}$$

$$1 = 1$$

$2(k+1)+$

$2k+2+1$

$$P(k) \Rightarrow 1^2+2^2+3^2+ \dots k^2 = \frac{k(k+1)(2k+1)}{6}$$

$$P(k+1) \Rightarrow 1^2+2^2+3^2+ \dots \underline{(k+1)^2} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$P(k) + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= (k+1)\frac{[k(2k+1) + 6k+6]}{6}$$

$$= (k+1)\frac{[2k^2+7k+6]}{6}$$

$$= (k+1)\frac{[2k^2+4k+3k+6]}{6}$$

$$= (k+1)\frac{2k(k+2)+3(k+2)}{6}$$

$1+3+5+\dots 2n-1 = n^2$

$P(1) = 2(1)-1 = 1$

$\Rightarrow 1 = 1$

$P(K) = 1+3+5+\dots 2K-1 = K^2$

$P(K+1) \Rightarrow 1+3+5+\dots [2(K+1)-1] = (K+1)^2$

$1+3+5+\dots + \underline{\underline{(2K+1)}} = (K+1)^2$

$P(K) + K+1$

$K^2 + (2K+1) \Rightarrow K^2 + 2K + 1$

$= \underline{(K+1)^2}$

Relations

Q. $A = \{1, 2, 3\}$ $B = \{r, s\}$

$R = \{(1, r), (2, s), (3, r)\}$

$Dom(R) = \{1, 2, 3\}$

$Ran(R) = \{r, s\}$

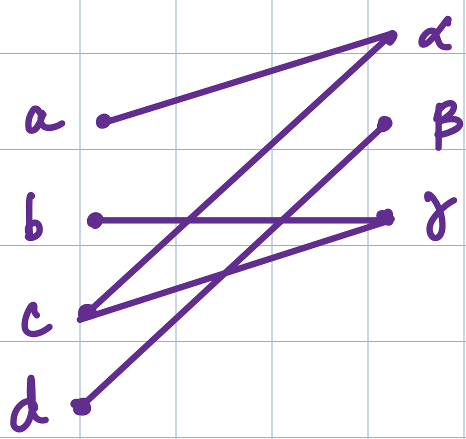
* Diagraph is for relations defined on just one set

eg: $R \subseteq A \times A$

Relation → Graphical, Tabular, Diagraph

$A = \{a, b, c\}$ $B = \{\alpha, \beta, \gamma\}$

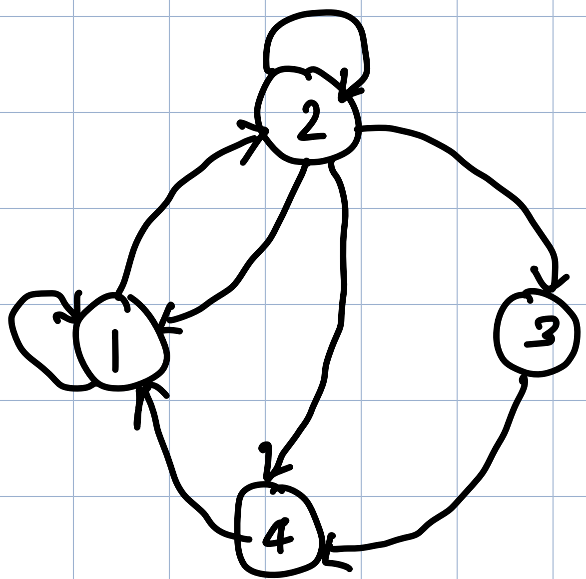
$R = \{(a, \alpha), (b, \gamma), (c, \alpha), (d, \beta), (c, \gamma)\}$



	α	β	γ
a	✓		
b			✓
c	✓		✓
d		✓	

$A = \{1, 2, 3, 4\}$

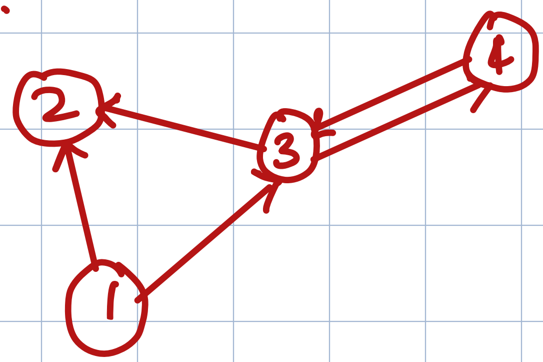
$R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\}$



In-degree : coming into the v

Out-degree : going out of the v

Q.

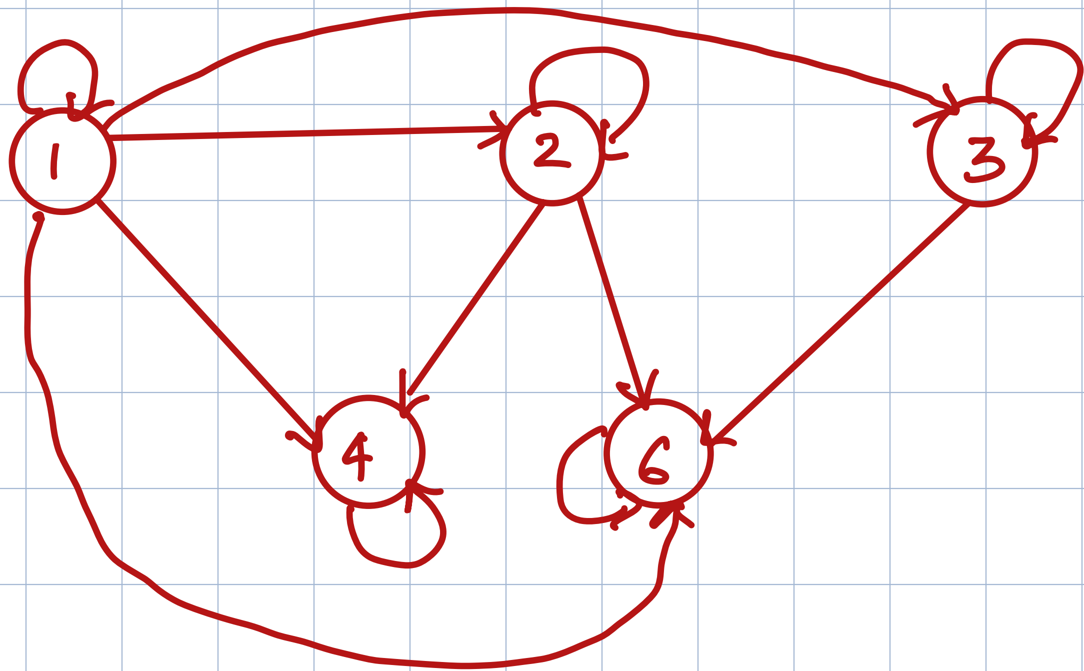


Vertex	1	2	3	4
In-deg	0	2	2	1
Out-deg	2	0	2	1

Q. $A = \{1, 2, 3, 4, 6\}$ x divides y

$R = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)$
$(2,2)\ (2,4)\ (2,6)$
$(3,3)\ (3,6)$
$(4,4)\ (6,6) \}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



$$R^{-1} = \{(1,1)\ \ (2,1)\ \ (3,1)\ \ (4,1)\ \ (6,1)\ \ (2,2)$$
$$(4,2)\ \ (6,2)\ \ (3,3)\ \ (6,3)$$
$$(4,4)\ \ (6,6)\}$$

Q. $A = \{1, 2, 3, 4, 6\}$

$a \rightarrow$ multiple of b

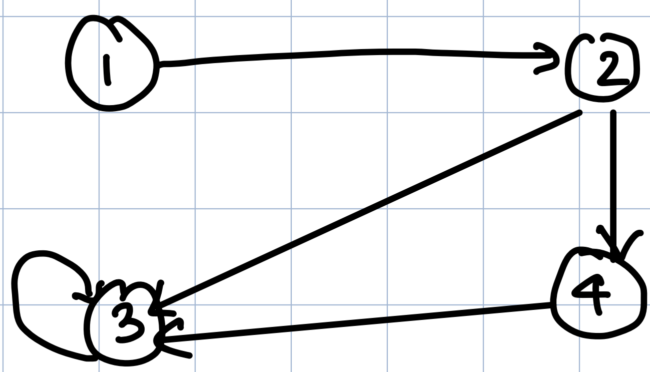
$R = \{ (1,1)\ (2,1)\ (2,2)$
$(3,1)\ (3,3)\ (4,1)\ (4,2)\ (4,4)$
$(6,1)\ (6,2)\ (6,3)\ (6,6) \}$

$R(3) \rightarrow \{1, 3\}$ $\quad$ $R(4) = \{1, 2, 4\}$

$R(6) = \{1, 2, 3, 6\}$

Q. $R = \{(1,2)\ (2,3)\ (2,4)\ (3,3)\}$

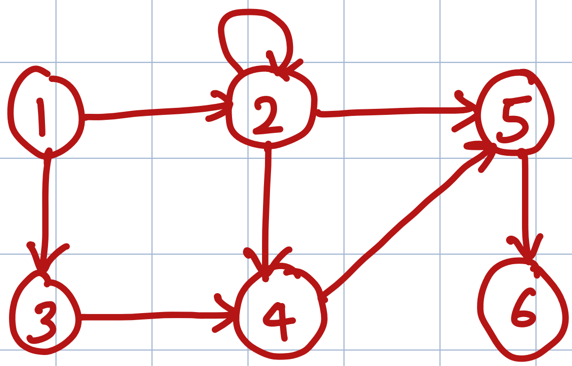
$A = \{1, 2, 3, 4\}$



$R^1 = \{(1,2)\ (2,3)\ (4,3)\ (3,3)\ (2,4)$

$R^2 = \{(1,3)\ (2,3),\ (4,3)\ (3,3)\ (1,4)\}$

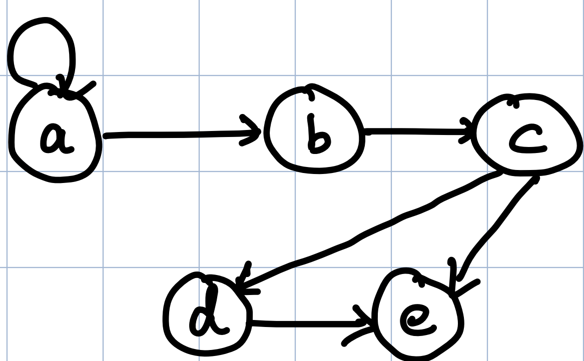
Q. $A = \{1, 2, 3, 4, 5, 6\}$



$R^1 = \{ (1,3)\ (1,2)$
$(2,2)\ (2,4)\ (2,5)$
$(3,4)\ (4,5)\ (5,6) \}$

$R^2 = \{ (1,2)\ (1,5)\ (1,4)$
$(2,2)\ (2,5)\ (2,6)\ (2,4)$
$(3,5)\ (4,6) \}$

Q.



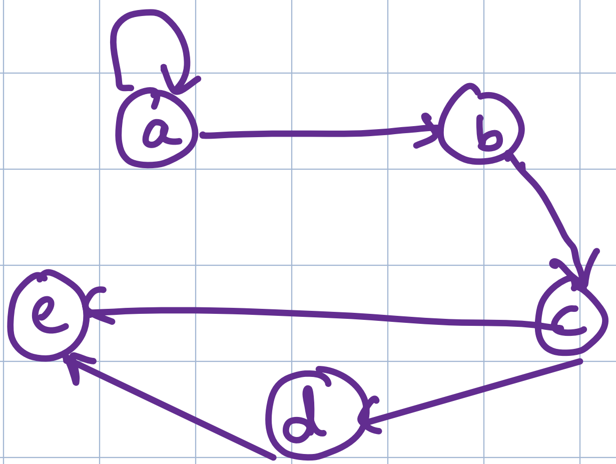
$R^2 = \{ (a,a)\ (a,b)\ (a,c)$
$(b,d)\ (b,e)\ (c,e) \}$

$R^{\infty} = \{ (a,a)\ \ (a,b)\ (a,c)\ (a,d)\ (a,e)$
$(b,c)\ (b,d)\ (b,e)$
$(c,d)\ (c,e)$
$(d,e) \}$

Q. $A = \{a, b, c, d, e\}$

$R = \{(a,a)\ (a,b)\ (b,c)\ (c,e)\ (c,d)\ (d,e)\}$

$M_R^2 \Rightarrow$ we can find R^2 & then compute



$R^2 = \{ (a,a)\ (a,b)\ (a,c)$
$(b,e)\ (b,d)$
$(c,e) \}$

$M_R \odot M_E =$

> Reflexive
> Symmetric
> Transitive
> Antisymm.
> Asymm.

Reflexive	$(a,a) \in R$		
Symmetric	$(a,b) \in R$	$(b,a) \in R$	
Asymmetric	$(a,b) \in R$	$(b,a) \notin R$	$(a,a) / (b,b)^{\sim}$
Antisymmetric	aRb	bRa	$a=b$
	aRb	$b \not R a$	$a \neq b$

Q. $A = \mathbb{Z}$ $R = \{(a,b) \in A \times A \mid a < b\}$

$a < b$ $b \nless a$ $\therefore aRb$ $b \not R a$ $\rightarrow$ not symm.

$a < b$ $b \nless a$ $\therefore$ assy.

$a \nless a$ $b \nless b$

$a \neq b$ $\therefore$ anti

Q. $A = Z^+$ set of +ve integers

$R = \{ (a, b) \in A \times A \mid a \text{ divides } b \}$

↳ b/a is R transitive?

Reflexive a/a $\therefore$ aRa → Reflexive

Symm b/a a/b → not possible $\therefore$ not symm.

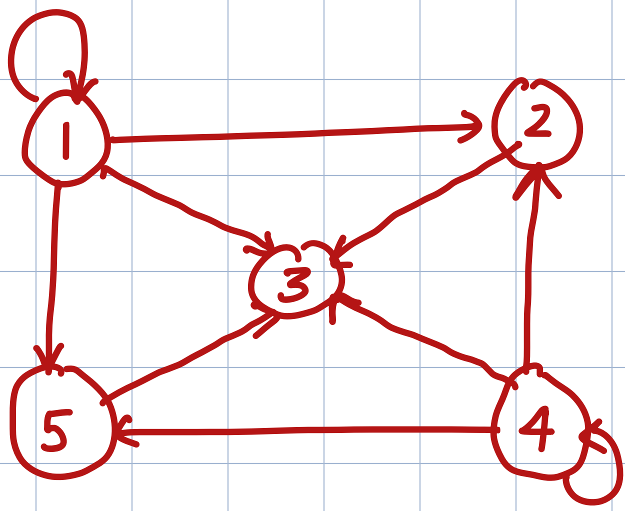
Asymm No

Anti-sym. $a \neq b$ aRb $b\not{R}a$ Yes

$a = b$ aRb bRa

Transitive b/a c/b $\Rightarrow$ c/a $\therefore$ tr.

Q. $A = \{1, 2, 3, 4, 5\}$



$R = \{ (1,1)\ (1,2)\ (1,3)\ (1,5)$
$(2,3)\ (4,2)\ (4,3)\ (4,5)\ (4,4)$
$(5,3) \}$

Reflexive — ✗

Symm. — ✗

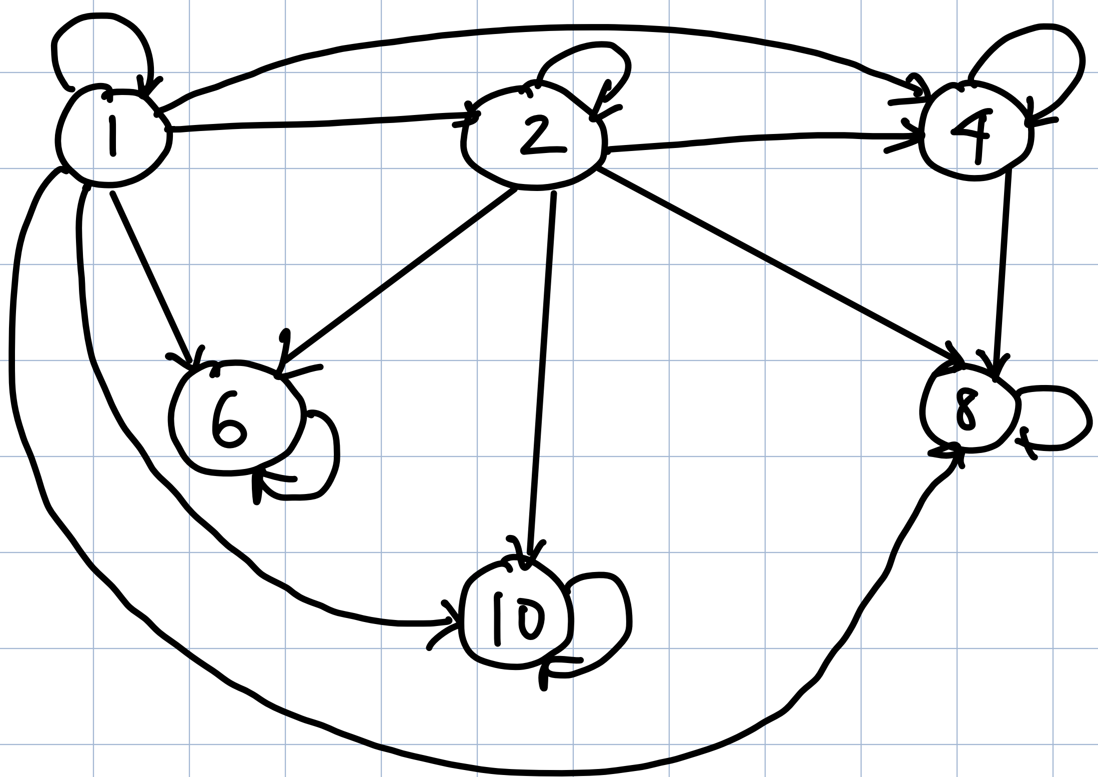
Asymm — ✗

Anti — ✓

Transitive — ✓

Q. $A = \{1, 2, 4, 6, 8, 10\}$ aRb a divides b

$R = \{ (1,1)\ (1,2)\ (1,4)\ (1,6)\ (1,8)\ (1,10)$
$(2,2)\ (2,4)\ (2,6)\ (2,8)\ (2,10)$
$(4,4)\ (4,8)\ (6,6)\ (8,8)\ (10,10) \}$



$R^2 = \{ (1,1)\ (1,2)\ (1,4)\ (1,6)\ (1,8)\ (1,10)$
$(2,2)\ (2,4)\ (2,8)\ (2,6)\ (2,10)$
$(4,4)\ (4,8)\ (6,6)\ (8,8)\ (10,10) \}$

$R^3 = \{ (1,1)\ (1,2)\ (1,4)\ (1,8)\ (1,10)\ (1,6)$
$(2$

Q. $A = \{1, 2, 3, 4, 5\}$ a is a mult. b

$\{(1,1)$
$(2,1)$ $(2,2)$
$(3,1)$ $(3,3)$
$(4,1)$ $(4,2)$ $(4,4)$
$(5,1)$ $(5,5)\}$

Reflexive ✓

Symm → x

Trans → ✓

Q. $R = \{(a,b) \mid a+b = \text{even}\}$

$[x] = \{y \mid (x,y) \in R\}$

$R = \{ (1,1) \quad (1,3) \quad (1,5)$
$(2,2) \quad (2,4)$
$(3,3) \quad (3,1) \quad (3,5)$
$(4,2) \quad (4,4)$
$(5,1) \quad (5,3) \quad (5,5) \}$

$[1] = \{1, 3, 5\} \qquad [5] = \{1, 3, 5\}$

$A|R = \{ \{1,3,5\} \quad \{2,4\} \}$

Q. $\{1, 2, 3, 4, 5\}$

$P = \{\{1, 3, 5\}\ \{2, 4\}\}$

eq. $R \Rightarrow \{(1,1)\ (3,3)\ (5,5)\ (1,3)\ (1,5)$
$(3,1)\ (3,5)\ (5,1)\ (5,3)\}$

Composite

$R = \{ (1,2) \ (2,4) \ (3,2) \ (4,3) \}$

$S = \{ (1,1) \ (2,4) \ (3,1) \ (4,4) \}$

$S \circ R$ =

R	S
$1 \to 2$	$1 \quad 1$
$2 \to 4$	$2 \to 4$
$3 \to 2$	$3 \to 1$
$4 \to 3$	$4 \to 4$

$(1,4) \ (2,4) \ (3,4) \ (4,1)$

$R \circ S$ =

S	R
$1 \to 1$	$1 \to 2$
$2 \to 4$	$2 \quad 4$
$3 \to 1$	$3 \quad 2$
$4 \to 4$	$4 \to 3$

$\{ (1,2) \ (2,3) \ (3,2) \ (4,3) \}$

$A = \{1, 2, 3\}$ $R = \{(1,1)\ (1,3)\ (2,1)(2,2)(2,3)\ (3,2)\}$

$S = \{\ (1,1)\ (2,2)\ (3,3)\ (3,1)\}$

SOR =

R	S
$1 \to 1$	$1 \to 1$
$1 \to 3$	$2 \to 2$
$2 \to 1$	$3 \to 3$
$2 \to 2$	$3 \to 1$
$2 \to 3$	
$3 \to 2$	

$\{(1,1)\ (1,3)$ $(2,1)\ (2,2)\ (2,3)$

$(3,2)\}$

$R = \{1, 2, 3, 4\}$

$R = \{ (1,1)\ (1,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\ (4,2) \}$

$S = \{ (3,1)\ (4,4)\ (2,3)\ (2,4)\ (1,1)\ (1,4) \}$

$S \circ R \rightarrow$

SOR write R first

R	S
$1 \rightarrow 1$	$3 \rightarrow 1$
$1 \rightarrow 2$	$4 \rightarrow 4$
$2 \rightarrow 3$	$2 \rightarrow 3$
$2 \rightarrow 4$	$2 \rightarrow 4$
$3 \rightarrow 4$	$1 \rightarrow 1$
$4 \rightarrow 1$	$1 \rightarrow 4$
$4 \rightarrow 2$	

$\{ (1,1)\ (1,4)\ (1,4)$
$(1,3)\ (2,1)\ (2,4)$
$(3,4)\ (4,1)\ (4,4)$
$(4,3)\ (4,4) \}$

$= \{ (1,1)\ (1,4)\ (1,3)$
$(2,1)\ (3,4)\ (4,1)$
$(4,4)\ (4,3) \}$

RoR

$1 \to 1$	$1 \to 1$
$1 \to 2$	$1 \to 2$
$2 \to 3$	$2 \to 3$
$2 \to 4$	$2 \to 4$
$3 \to 4$	$3 \to 4$
$4 \to 1$	$4 \to 1$
$4 \to 2$	$4 \to 2$

$(1,1)$ $(1,2)$ $(1,3)$ $(1,4)$
$(2,4)$ $(2,1)$ $(2,2)$
$(3,1)$ $(3,2)$ $(4,1)$ $(4,2)$
$(4,3)$ $(4,4)$ }

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1+2+\dots n)^2$$

1: Basis of Induction

$P(n) = P(1) \rightarrow$

$1^3 = 1^2$ LHS = RHS $\therefore$ true

2: Induction Step

$P(k)$ is assumed to be true

$$1^3 + 2^3 + 3^3 + \dots + k^3 = (1+2+3\dots+k)^2$$

$\hookrightarrow \left[\frac{k(k+1)}{2}\right]^2$

$P(k+1) \rightarrow$

$1^3 + 2^3 + 3^3 + \dots + \underline{\underline{(k+1)^3}} = (1 + 2 + 3 + \dots + (k+1))^2$

$= \left[\frac{(k+1)(k+2)}{2}\right]^2$

Hypothetical ind.

P(k+1) can also be written in terms of P(k)

$P(k) + (k+1)^3 = \left[k\frac{(k+1)}{2}\right]^2 + (k+1)^3$

$= \left[\frac{k(k+1)}{2}\right]^2 + \frac{4(k+1)^3}{4}$

$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$

$\Rightarrow (k+1)^2 \left[\frac{k^2 + 4(k+1)}{4}\right]$

$\Rightarrow \frac{(k+1)^2}{4}\left[k^2 + 4k + 4\right]$

$\Rightarrow \frac{(k+1)^2}{4}(k+2)^2$

$\Rightarrow \left[\frac{(k+1)(k+2)}{2}\right]^2$ //

Closure - the smallest relation R_1 on A such that it contains & possesses the property we desire.

Reflexive : A relation $R_1 = R \cup \Delta$ is the reflexive closure of R is $R \cup \Delta$ is the smallest relation that contain R & is reflexive.

$\Delta \to (a, a) \quad a \in A$

$A = \{1, 2, 3\} \quad R = \{(1,1)\ (1,2)\ (2,3)\}$

$R_1 = R \cup \Delta \quad \Delta = \{(1,1)\ (2,2)\ (3,3)\}$

$R_1 = \{(1,1)\ (1,2)\ (2,2)\ (2,3)\ (3,3)\}$

Symmetric Closure

$R_1 = R \cup R^{-1}$

$R = \{ (a,b)\ (b,c)\ (a,c)\ (c,d) \}$

$R^{-1} = \{ (b,a)\ (c,b)\ (c,a)\ (d,c) \}$

$R \cup R^{-1} = \{ (a,b)\ (b,c)\ (a,c)\ (c,d)$
$(b,a)\ (c,b)\ (c,a)\ (d,c) \}$

Warshall

$A = \{1, 2, 3, 4\}$

$R = \{ (1,2) \ (2,3) \ (3,4) \ (2,1) \}$

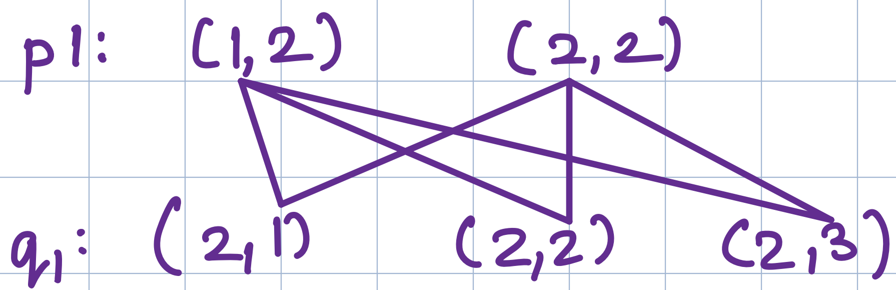
$$W_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

p1: (2,1)

q1: (1,2)

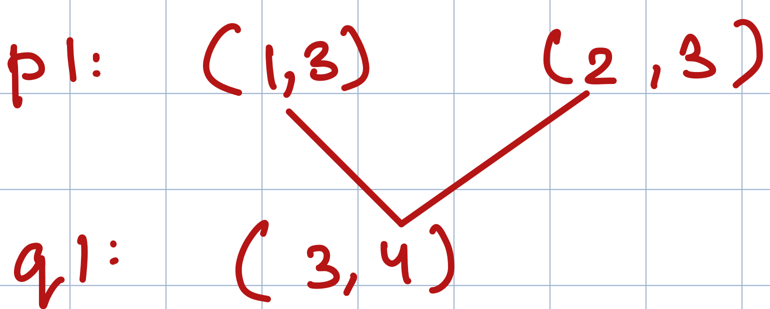
pair : (2,2)

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$



(1,1) (1,2) (1,3) (2,3) (2,1) (2,2)

$$W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & & & \\ 3 & & & & \\ 4 & & & & \end{array}\quad \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



(1,4) (2,4)

$$W_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & & & \\ 3 & & & & \\ 4 & & & & \end{array}\quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$A = \{1,2,3,4,5\}$

$R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,3)$
$(3,4)\ (4,3)\ (4,4)\ (5,5)\}$

$S = \{(1,1)\ (2,2)\ (3,3)\ (4,4)$
$(4,5)\ (5,4)\ (5,5)\}$

Find smallest eq relation cont. R & S

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$M_S = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & & & & & \\ 2 & & & & & \\ 3 & & & & & \\ 4 & & & & & \\ 5 & & & & & \end{array}$$

Sets

$A \subseteq B$ every element of A is in B

$A \subset B$ $A \subseteq B$ $A \neq B$

Power Set

$A = n$

$P(A) = 2^n$ (null → itself)

Difference

$A - B =$ in A but not in B

Symm. Difference

$A \oplus B = (A-B) \cup (B-A)$

$A \times B = \{(x,y) \ x \in A, y \in B\}$

Principle of mutual inclusion & exclusion

a) $|A \cup B| = |A| + |B| - |A \cap B|$

b) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

c) $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |B \cap C| - |A \cap C| - |A \cap D| - |B \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |B \cap C \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$

$|A - B| = |A| - |A \cap B|$

Q. 37 → fruits

33 → veg.

9 → $C \cap F$

12 → $C \cap V$

10 → $F \cap V$

$\boxed{100}$ ↳ people

12 → only cheese

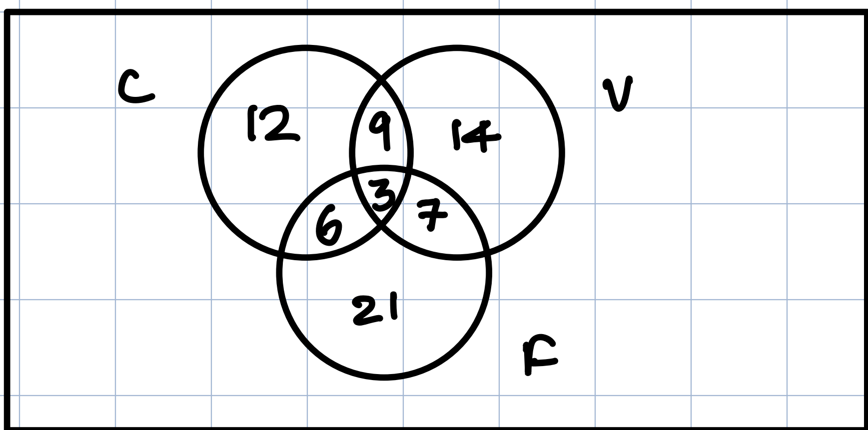
3 → eat all $(C \cap V \cap F)$

Solve any Two

The college catering service must decide if the mix of food that is supplied for receptions is appropriate. of 100 people questioned, 37 say they eat fruits, 33 say they eat vegetables, 9 say they eat cheese and fruits, 12 eat cheese and vegetables, 10 eat fruits and vegetables, 12 eat only cheese, and 3 report they eat all three offerings.

How many people surveyed eat cheese?

How many do not eat any of the offerings?



16

i) eat cheese $= 12 + 9 + 3 + 6 = 30$

ii) eat none ⇒ total − (eat all) $= 100 - 72 = \underline{\underline{28}}$

$$F \cup V \cup C = |F| + |V| + |C| - |F \cap V| - |V \cap C| - |F \cap C| + |F \cap V \cap C|$$

$$= 37 + 33 + 30 - 10 - 12 - 9 + 3$$

$$= 72$$

Q. 50 students

26 → A (first exam)

21 → A (2nd exam)

17 → did not get A in either

Scored A in both → $F \cap S$

scored in either

$F \cup S = |F| + |S| - |F \cap S|$

$50 - 17 = 33$

$33 = 26 + 21 - F \cap S$

$F \cap S = \underline{\underline{14}}$

Q. $1 \rightarrow 300$ divisible 3, 5, 7

divisible by neither

divisible by 3 but not 5 or 7

$|A|_3 \rightarrow \frac{300}{3} = 100$ $\qquad$ $|A \cap B| = \frac{300}{15} = 20$

$|B|_5 = \frac{300}{5} = 60$ $\qquad$ $|B \cap C| = \frac{300}{35} = 8$

$|C|_7 = \frac{800}{7} = 42$ $\qquad$ $|A \cap C| = \frac{300}{21} = 14$

↳ DO NOT ROUND OFF

$|A \cap B \cap C| = \frac{300}{105} = 2$

i) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C| = \underline{\underline{162}}$

ii) neither $= 300 - 162 = 138$

iii) $|A - (B \cup C)| = |A| - |A \cap (B \cup C)|$

$= 100 - [A \cap B + B \cap C - A \cap B \cap C]$

$$= 100 - [20 + 14 - 2]$$
$$= 100 - 32 = \underline{68}$$

Q. 100 / 120 → allcand one lang.

$\therefore$ $F \cup G \cup R = 100$

$F = 65$ $\quad$ $F \cap G = 20$

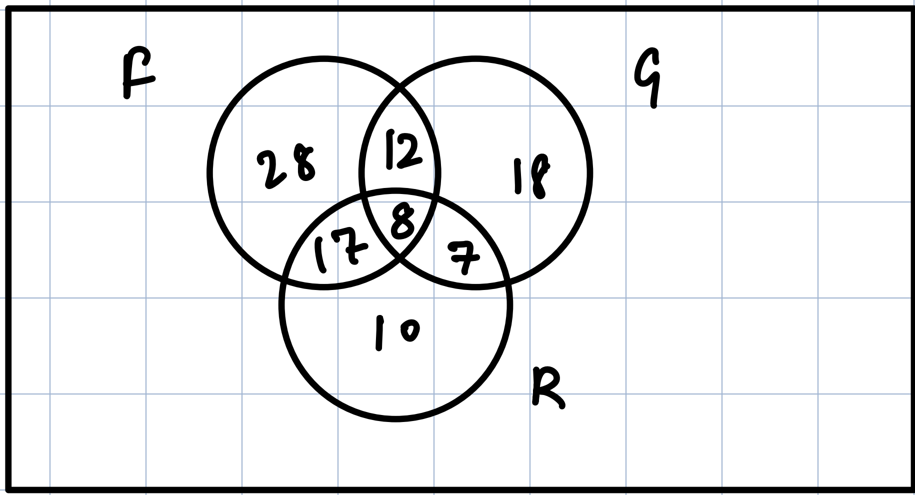
$G = 45$ $\quad$ $F \cap R = 25$

$R = 42$ $\quad$ $G \cap R = 15$

$$|F \cup G \cup R| = F + G + R - |F \cap G| - |F \cap R| - |G \cap R| + |F \cap G \cap R|$$

$$100 = 65 + 45 + 42 - 20 - 25 - 15 + |F \cap G \cap R|$$

$$F \cap G \cap R = 8$$



exactly 1 lang $= 28 + 18 + 10 = 56$

exact 2 $= 17 + 7 + 12 = 36$

Question

In a class of 100 students, 12 students drink only milk and 5 students drink only coffee and 8 students drink only tea. Other report says 30 students take both coffee and tea, 25 students take milk and tea and 20 students take only milk and coffee. 10 students drink all the three. Find the number of students who do not drink anything.

Q. 100 students

$M \cap T \cap C = 10$

$M \cap C = 20$

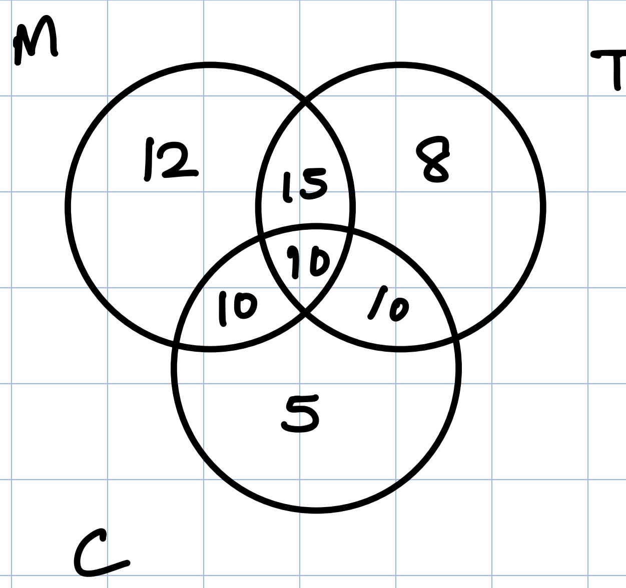
$M \cap T = 25$

$C \cap T = 20$

12 = only M

8 = ~~t~~ " T

5 = " C



$|M| = 47$ $|C| = 35$ $|T| = 43$

$M \cup T \cup C = 47 + 35 + 43 - 20 - 25 - 20 + 10$

$= 70$

$\therefore$ none $= 100 - 70 = 30$

Partition $\Rightarrow A = \{A_1, A_2, A_3 \dots\}$

such that $A = A_1 \cup A_2 \cup A_3 \dots$

$\phi = A_1 \cap A_2 \cap A_3 \dots$

Q. $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\{\{1,3,5\}\ \{2,6\}\ \{4,8,9\}\}$ × 7 miss

$\{\{1,3,5\}\ \{2,4,6,8\}\ \{5,7,9\}\}$ × 5 common

$\{\{1,3,5\}\ \{2,4,6,8\}\ \{7,9\}\}$ ✓

Q. $A \cup (A^c \cap B) = A \cup B$

Distr.

$(A \cup A^c) \cap (A \cup B)$

Complem.

$U \cap (A \cup B)$

Identity

$= (A \cup B)$