Menu

- Priority Queues
- Heaps
- Heapsort

Priority Queue

A data structure implementing a set S of elements, each associated with a key, supporting the following operations:

insert(S, x) : insert element x into set S

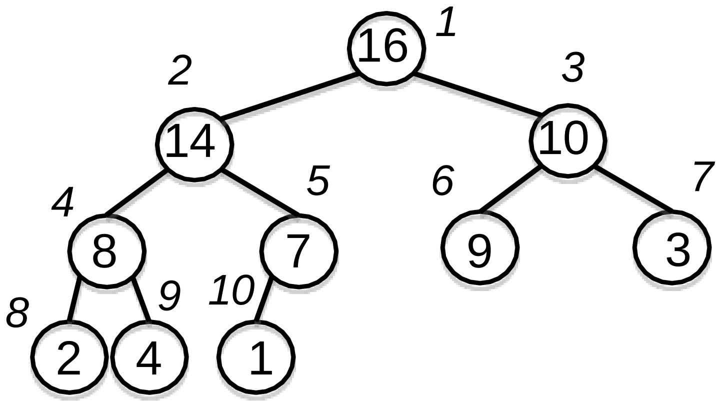
max(S) : return element of S with largest key return

extract_max(S) : element of S with largest key and remove it from S

increase_key(S, x, k) : increase the value of element x's key to new value k
(assumed to be as large as current value)

Heap

- Implementation of a priority queue
- An array, visualized as a nearly complete binary tree
- Max Heap Property: The key of a node is $\geq$ than the keys of its children (Min Heap defined analogously)



1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
16	14	10	8	7	9	3	2	4	1

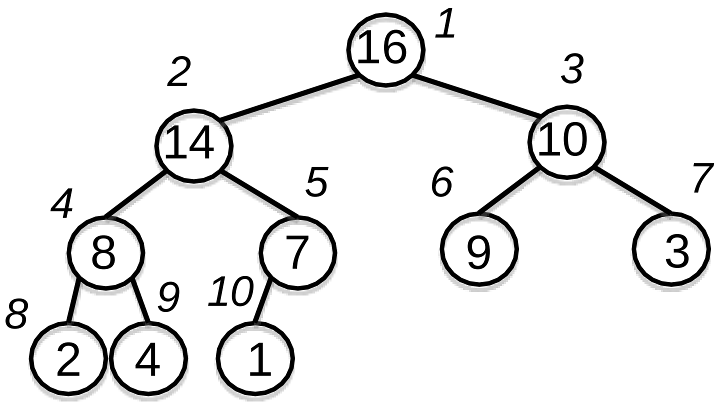
Heap as a Tree

root of tree.   : first element in the array, corresponding to i = 1

parent(i) =i/2: returns index of node's parent

left(i)=2i.   : returns index of node's left child

right(i)=2i+1: returns index of node's right child



1	2	3	4	5	6	7	8	9	10
16	14	10	8	7	9	3	2	4	1

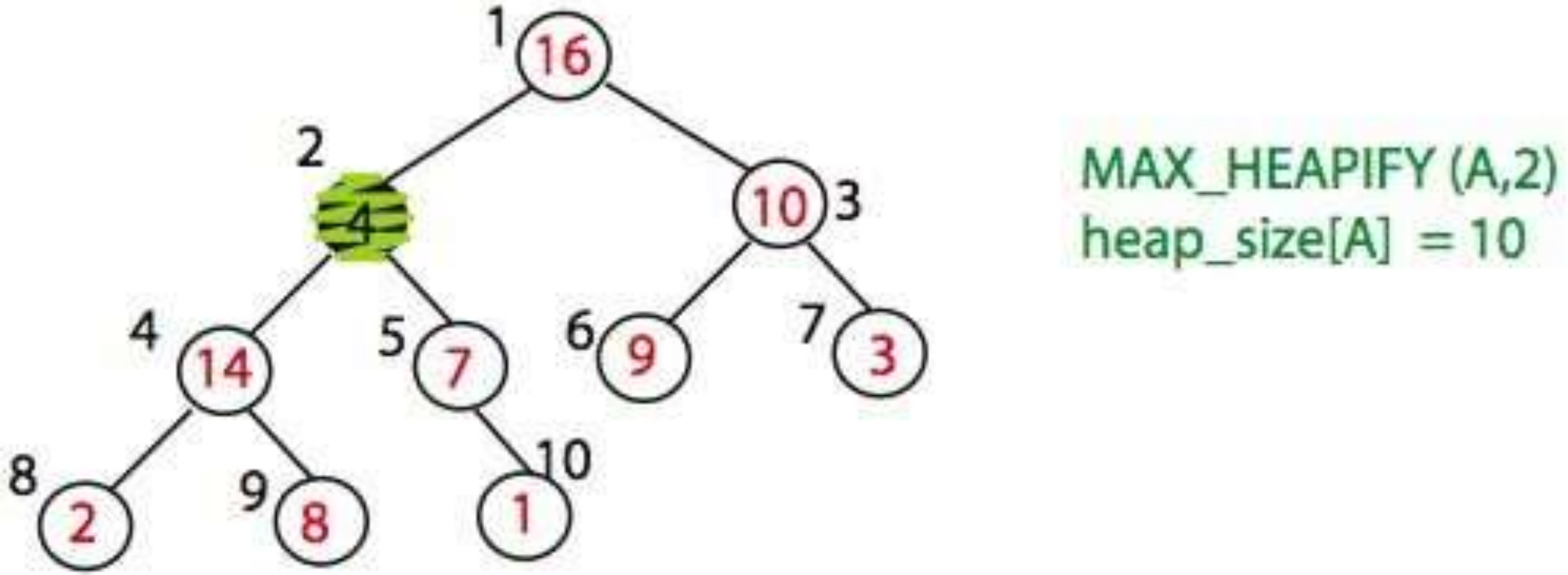
Heap Operations

build_max_heap : produce a max-heap from an unordered array

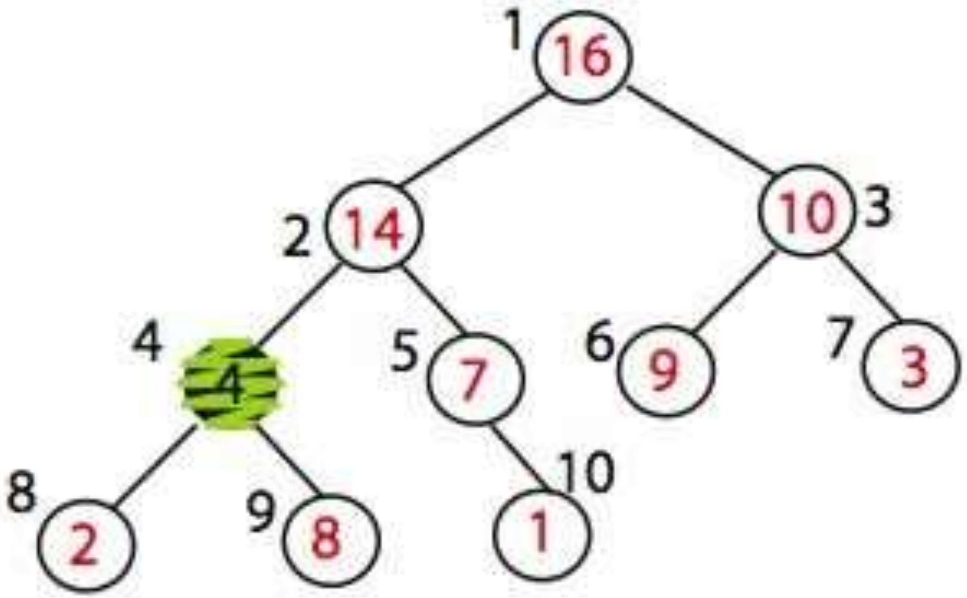
max_heapify : correct a single violation of the heap property in a subtree at its root

insert, extract_max, heapsort

Max_heapify (Example)

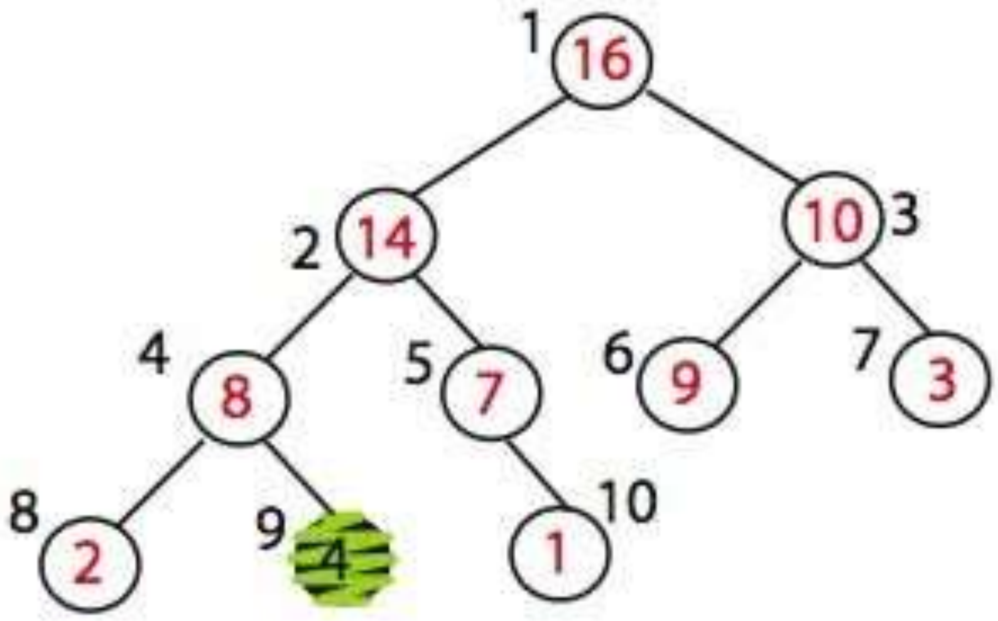


Max_heapify (Example)



Exchange A[2] with A[4]
Call MAX_HEAPIFY(A,4)
because max_heap property
is violated

Max_heapify (Example)



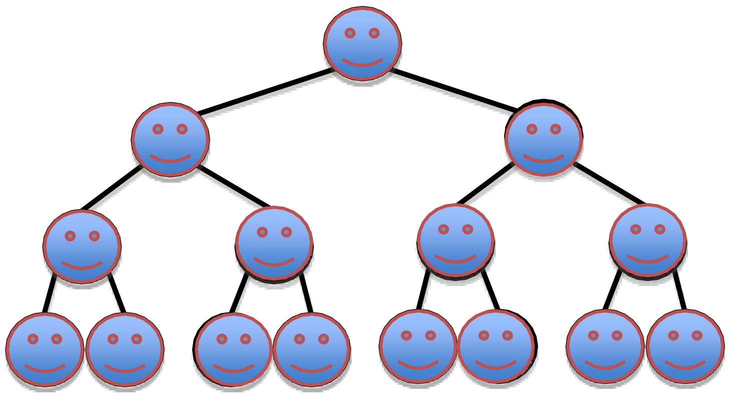
Exchange A[4] with A[9]
No more calls

Time=? O(log n)

Max_Heapify Pseudocode

```
Max_Heapify(A,i):
l = left(i)
r = right(i)
if (l <= heap-size(A) and A[l] > A[i])
   then largest = l
else largest = i
if (r <= heap-size(A) and A[r] > A[largest])
   then largest = r
if largest != i
   then exchange A[i] and A[largest]
        Max_Heapify(A, largest)
```

Build_Max_Heap(A)



- `Converts A[1…n] to a max heap`
- `Build_Max_Heap(A):`
- `heap-size(A) = length(A)`

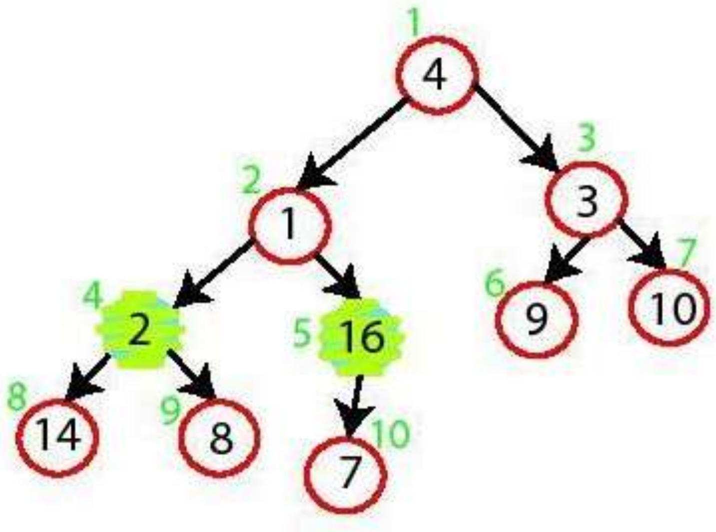
```
for i=n/2  downto 1
    do Max_Heapify(A, i)
```

- Why start at n/2?

- Because elements A[n/2 + 1 … n] are all leaves of the tree 2i > n, for i > n/2 + 1

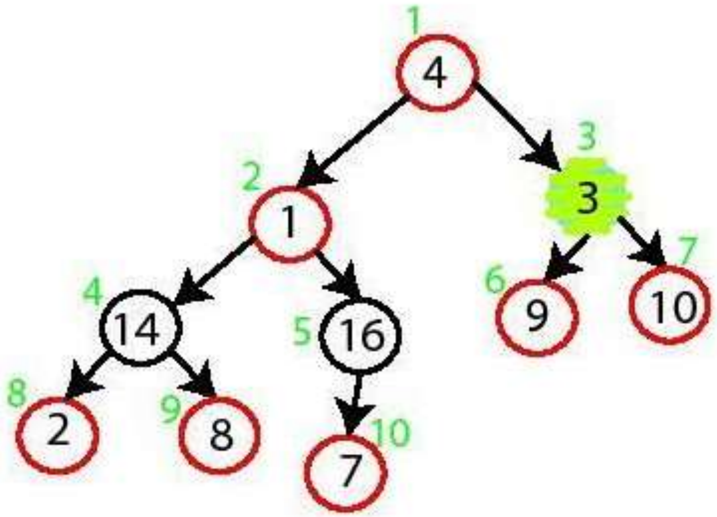
Time= O(n log n) via simple analysis

Build-Max-Heap Demo



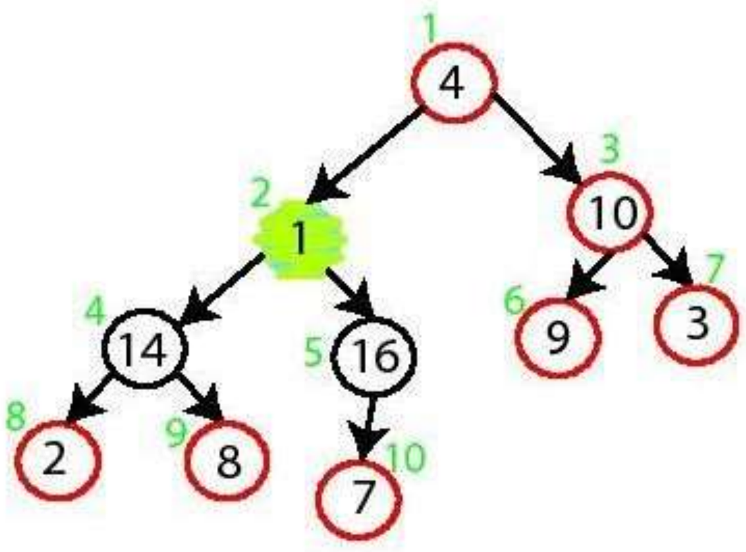
A									
4	1	3	2	16	9	10	14	8	7

MAX-HEAPIFY (A,5)
no change
MAX-HEAPIFY (A,4)
Swap A[4] and A[8]

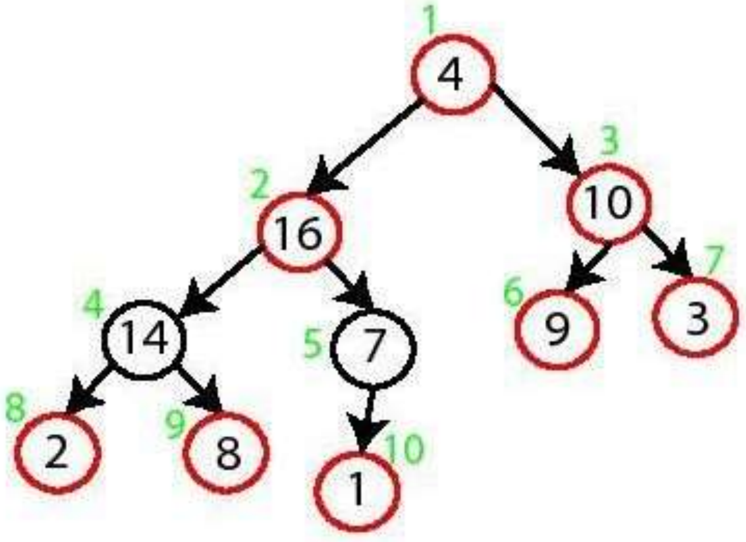


MAX-HEAPIFY (A,3)
Swap A[3] and A[7]

Build-Max-Heap Demo



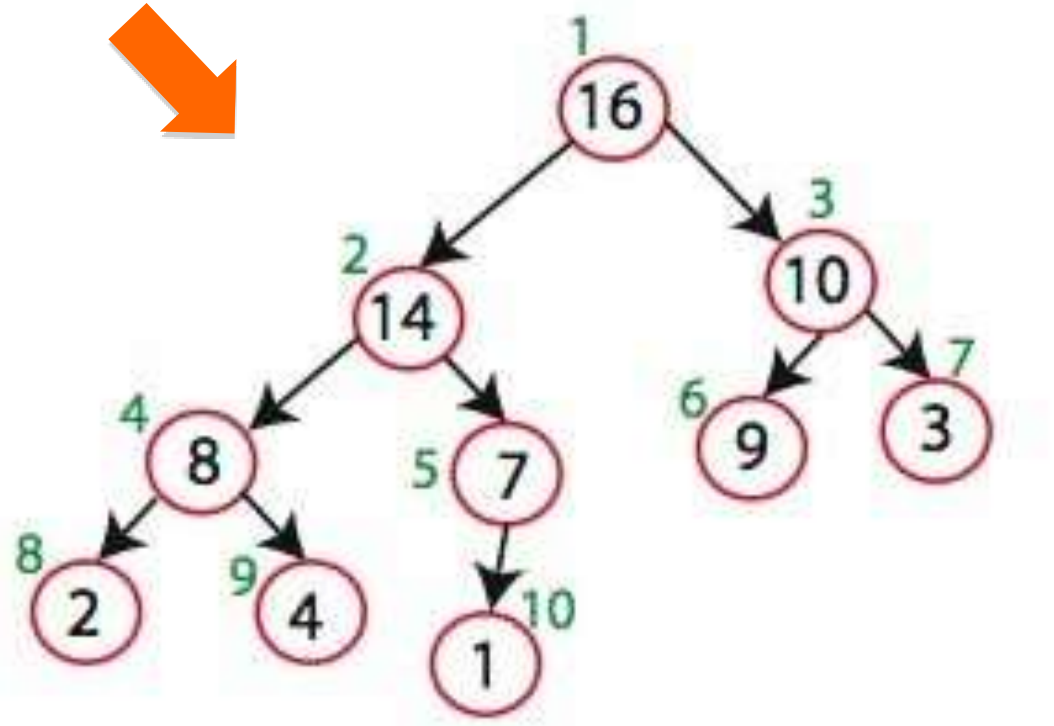
MAX-HEAPIFY (A,2)
Swap A[2] and A[5]
Swap A[5] and A[10]



MAX-HEAPIFY (A,1)
Swap A[1] with A[2]
Swap A[2] with A[4]
Swap A[4] with A[9]

Build-Max-Heap

A	4	1	3	2	16	9	10	14	8	7



Heap-Sort

Sorting Strategy:

1. Build Max Heap from unordered array;

2. Find maximum element A[1];

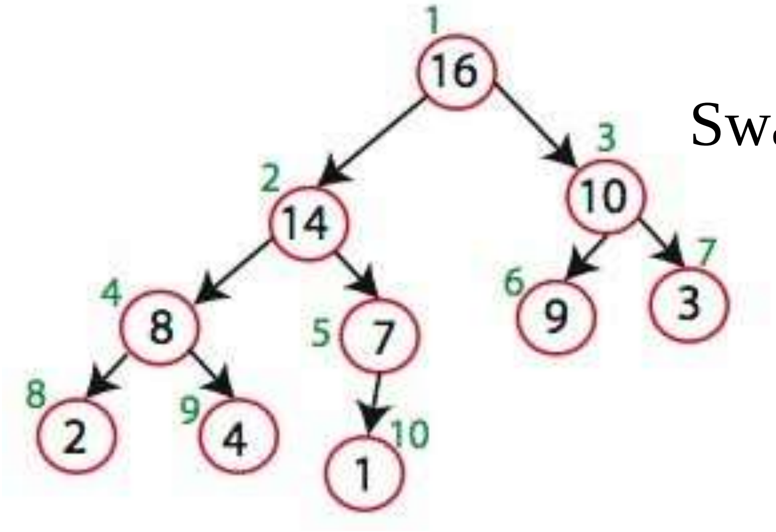
3. Swap elements A[n] and A[1]:
   now max element is at the end of the array!

4. Discard node n from heap
   (by decrementing heap-size variable)

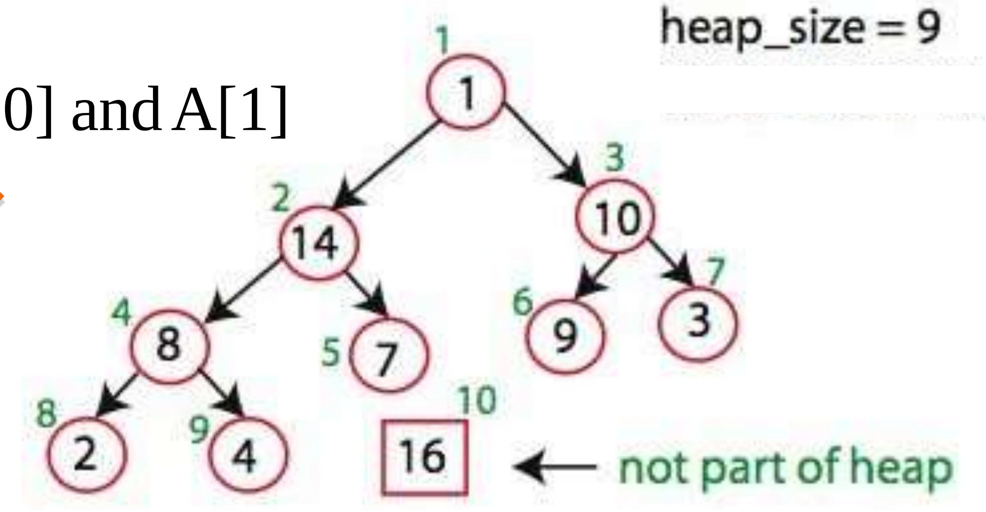
5. New root may violate max heap property, but its children are max heaps. Run max_heapify to fix this.

6. Go to Step 2 unless heap is empty.

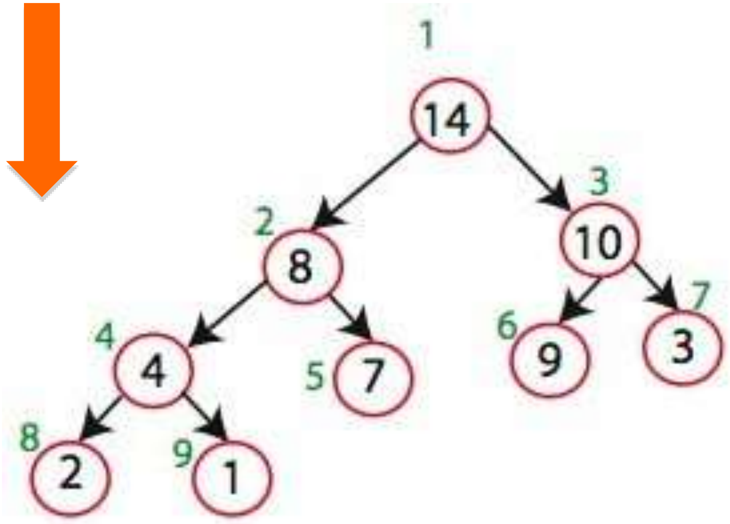
Heap-Sort Demo



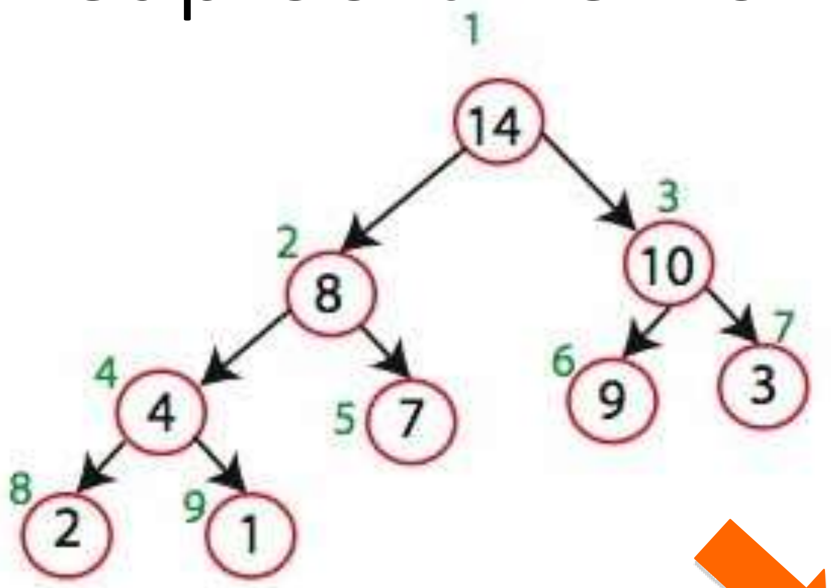
Swap A[10] and A[1]



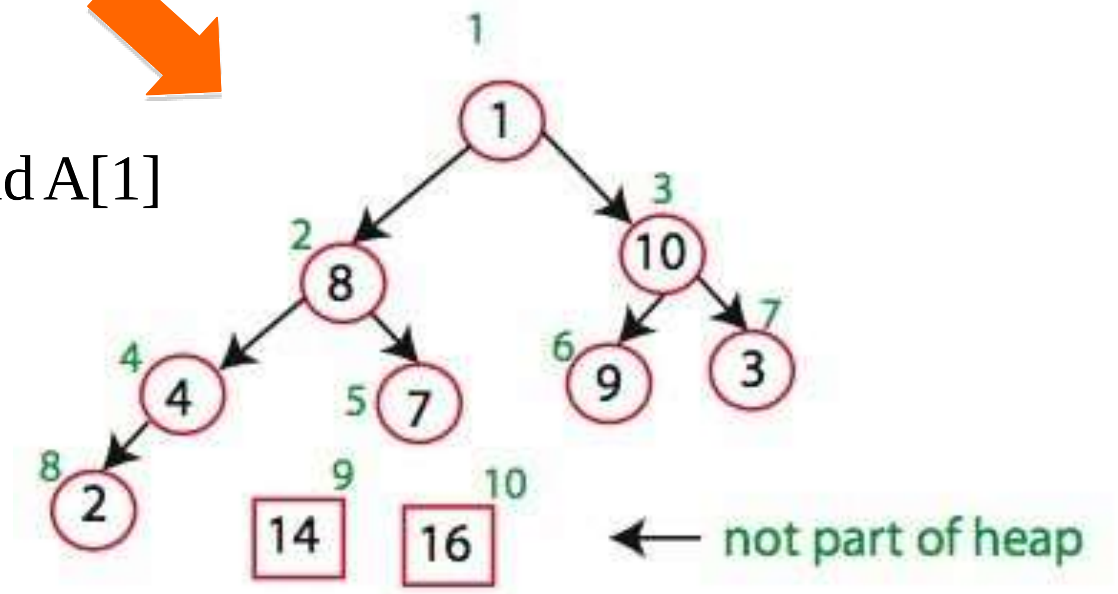
Max_heapify(A,1)



Heap-Sort Demo

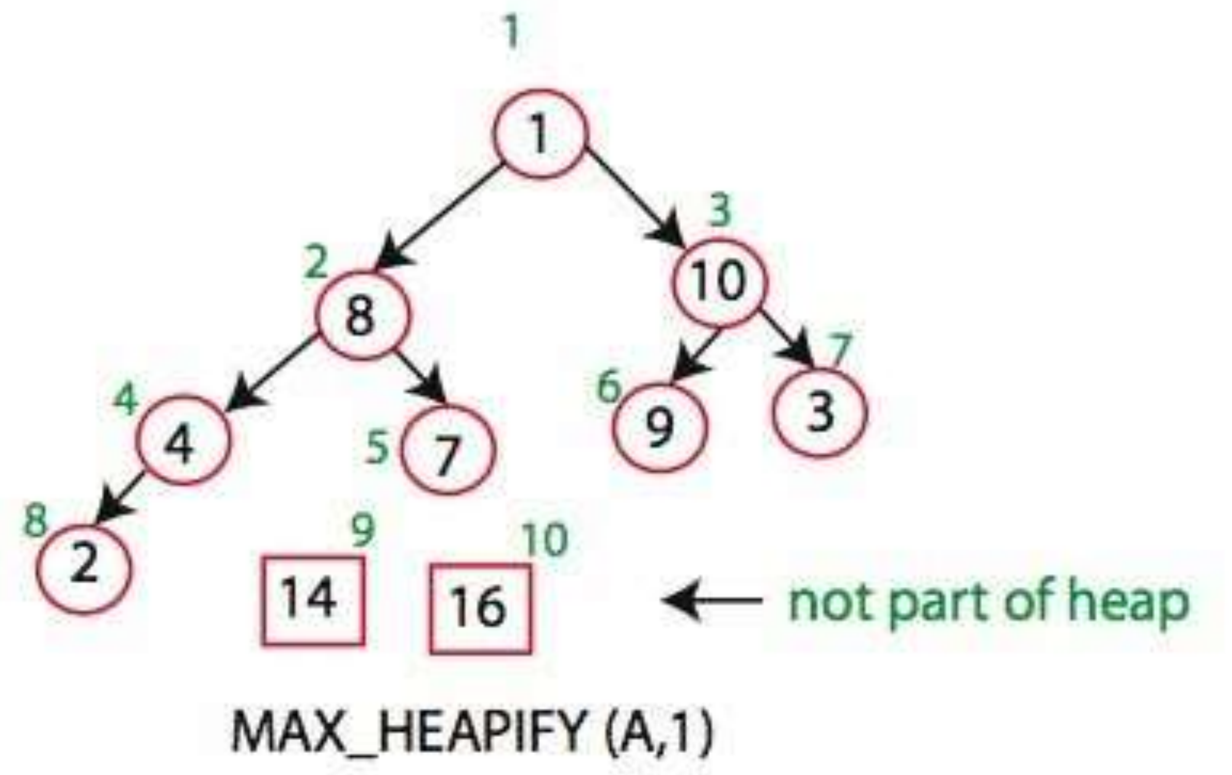


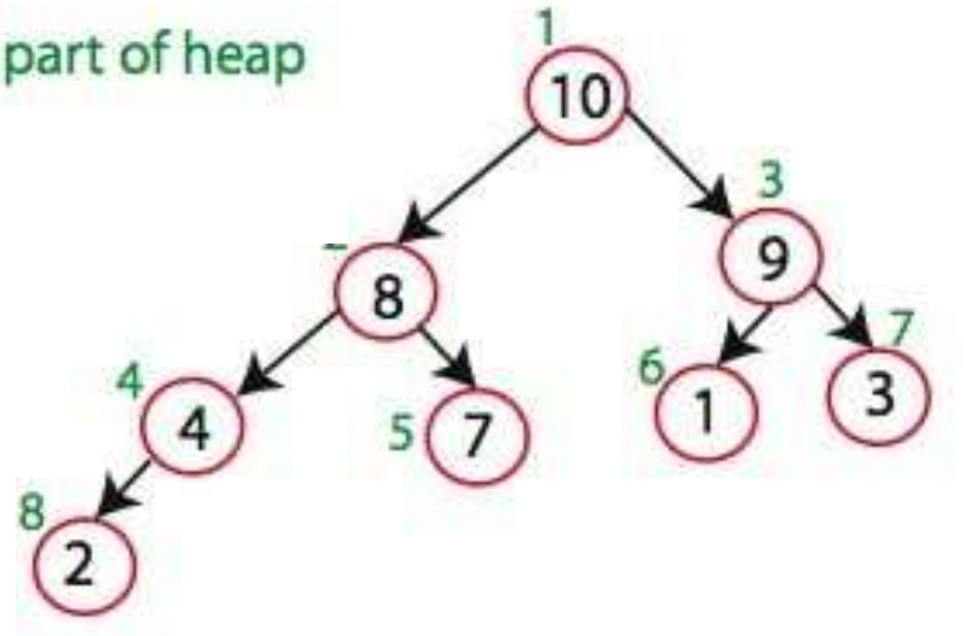
Swap A[9] and A[1]



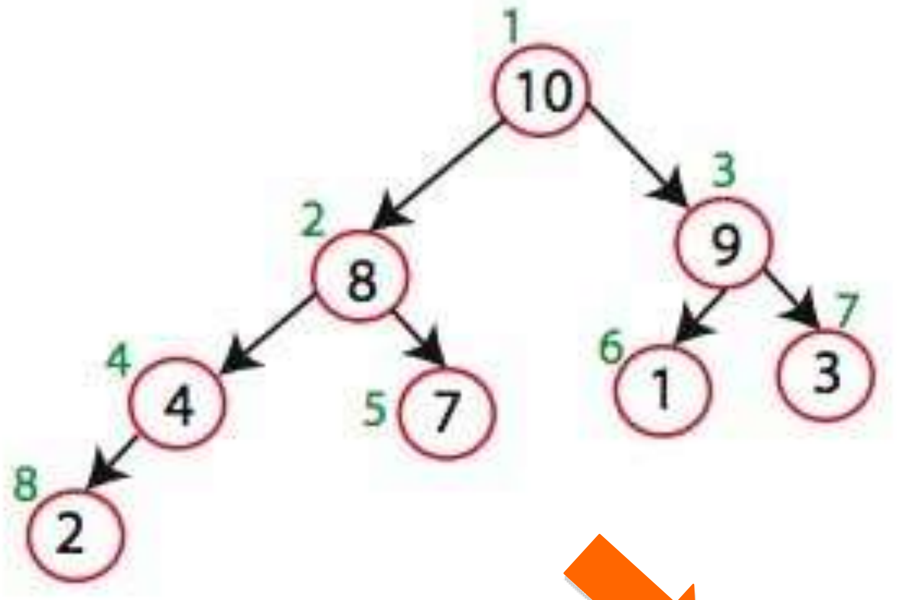
MAX_HEAPIFY (A,1)

Heap-Sort Demo

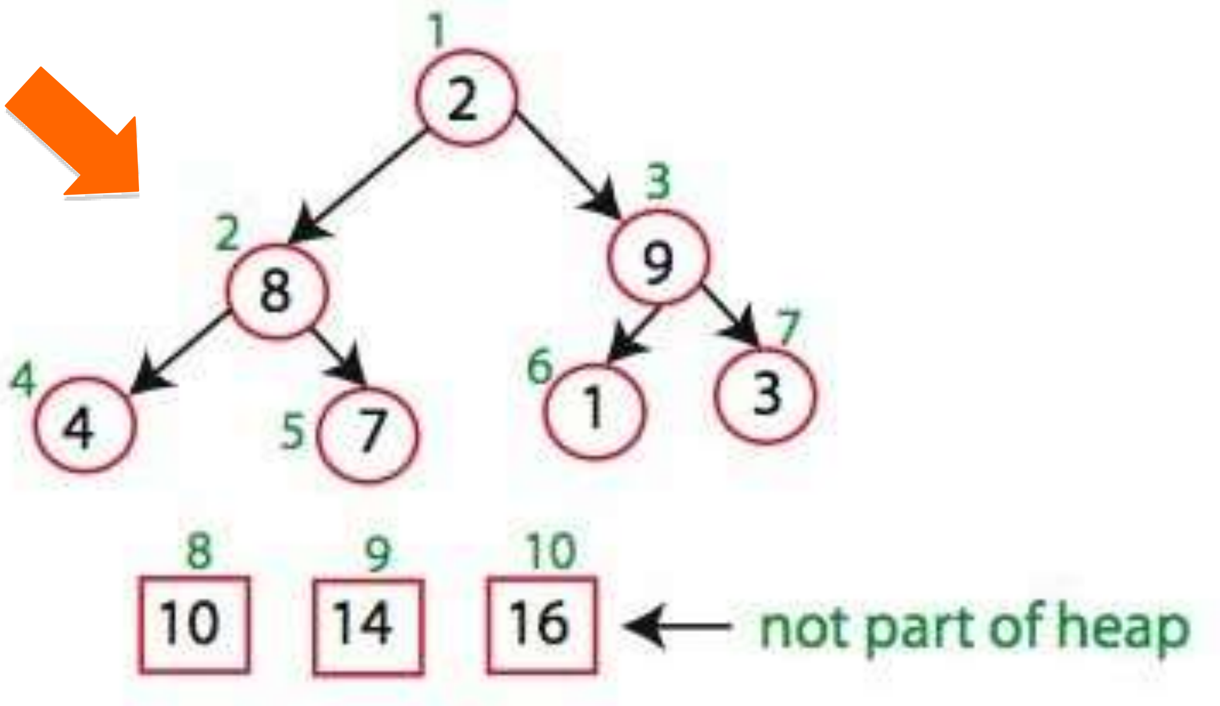




Heap-Sort Demo



Swap A[8] and A[1]



Heap-Sort

- Running time:
  - after n iterations the Heap is empty
  - every iteration involves a swap and a max_heapify operation; hence it takes O(log n) time

Overall O(n log n)