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Tutorial-8

Laplace Transform

Q1. Find the laplace transform of the following function:

i) t^3 cos(t) ii)∫0^t sin(t)cos(t) dx iii) te^-2t H(t − 1)

In [121]:
```
# i

# Define the variable t,s
t,s = var('t,s')

f=(t^3) * cos(t)

L=f.laplace(t,s)

# Display the solution
show("Laplace Transform of t^3 cos(t) : ",L)
```

Laplace Transform of t^3 cos(t) : $\frac{48\,s^4}{(s^2+1)^4} - \frac{48\,s^2}{(s^2+1)^3} + \frac{6}{(s^2+1)^2}$

In [140]:
```
# ii
# Define variables
t, s= var('t s ')

# Assume t is positive
assume(t > 0)

# Compute the integral from 0 to t
F = integrate(sin(t) * cos(t), (t, 0, t))  # Use a different variable 'x' for integration

# Now compute the Laplace transform of the resulting function
L = laplace(F, t, s)

# Display the result
show("Laplace Transform of ∫0^t sin(x)cos(x) dx : ",L.full_simplify())
```

Laplace Transform of ∫0^t sin(x)cos(x) dx : $\frac{1}{s^3+4\,s}$

In [141]:
```
# iii

t, s = var('t s')

# Define the original function (before the shift by 1)
f_unshifted = t * exp(-2*t)

# Compute the Laplace transform of the unshifted function
laplace_f_unshifted = laplace(f_unshifted, t, s)

# Now apply the shifting property manually (shift by 1)
laplace_f_shifted = exp(-s) * laplace_f_unshifted

# Display the result
show("Laplace Transform of te^-2t H(t - 1) : ", laplace_f_shifted)
```

Laplace Transform of te^-2t H(t − 1) : $\frac{e^{(-s)}}{(s+2)^2}$

Q2. Find the inverse Laplace transform of the following function:

i) 1/(s^2+9)(s^2+1) ii) 11s^2-2s+5/2s^3-3s^2-3s+2

In [22]:
```
# i
t, s = var('t s')

F(s) = 1/((s^2+9)*(s^2+1))


print("Inverse Laplace Transform:")
show(inverse_laplace(F(s),s,t))
```

Inverse Laplace Transform:

$$-\frac{1}{24}\sin(3\,t) + \frac{1}{8}\sin(t)$$

In [23]:
```
# ii
t, s = var('t s')

F(s) = (11*s^2-2*s+5)/(2*s^3-3*s^2-3*s+2)


print("Inverse Laplace Transform:")
show(inverse_laplace(F(s),s,t))
```

Inverse Laplace Transform:

$$5\,e^{(2\,t)} - \frac{3}{2}\,e^{\left(\frac{1}{2}\,t\right)} + 2\,e^{(-t)}$$

Q3. Solve the follwing differential equation x'''(t) - 2x''(t) + 5x' = 0 x(0) = 0, x' (0) = 0, x''(0) = 1

In [109]:
```
# Define the variable t and the function x(t)
t = var('t')
x = function('x')(t)

# Define the differential equation
diff_eq = diff(x, t, 3) - 2*diff(x, t, 2) + 5*diff(x, t) == 0

# Solve the differential equation with initial conditions
solution = desolve_laplace(diff_eq, x, ics=[0,0, 0, 1])

# Display the solution
show("Solution of Differential Equation: ",solution)
```

Solution of Differential Equation: $-\frac{1}{10}\,(2\,\cos(2\,t) - \sin(2\,t))e^t + \frac{1}{5}$

Fourier Series

Q1.Find fourier Series of the follwoing function and also plot graph of function and fourier series. i) f(x) = e^(−x) in (0, 2π) ii) f(x) = xsinx in (0, 2π)

In [145]:
```
# i
var('x n')
L = pi  # Period of the function

# Define the function as piecewise over [0, 2*pi]
f(x) = exp(-x)

# Compute a0, an, and bn using the Fourier series formulas
a0 = (1/2L) * integrate(f(x), x, 0, 2*pi)

an = (1/L) * integrate(f(x) * cos(n * x), x, 0, 2*pi)

bn = (1/L) * integrate(f(x) * sin(n * x), x, 0, 2*pi)

# Create the Fourier series up to the 3rd term
s = a0+ sum(an * cos(n * x) + bn * sin(n *x / L), n, 1, 3)

# Show the results for a0, an, bn, and the Fourier series
show("a0 =",a0)
show("an =",an)
show("bn =",bn)
show("f(x) =",s.full_simplify())

plot(f,0,2*pi,legend_label="e^-x") + plot(s,0,2*pi,color = "red",legend_label="fourier series")
```

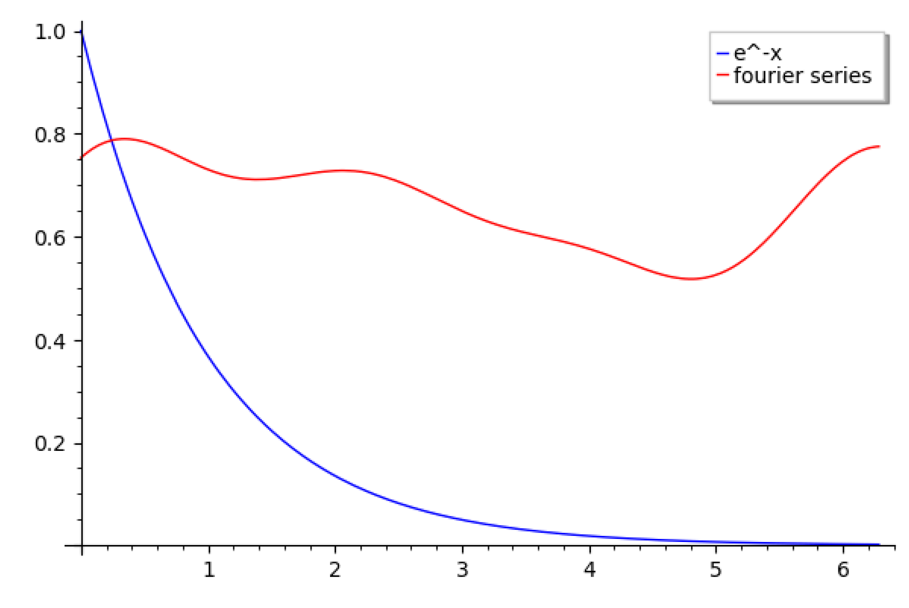
a0 $= -\frac{1}{2}\,e^{(-2\,\pi)} + \frac{1}{2}$

an $= -\frac{\frac{1}{n^2 e^{(2\,\pi)} + e^{(2\,\pi)}} - \frac{1}{n^2+1}}{\pi}$

bn $= -\frac{\frac{n}{n^2 e^{(2\,\pi)} + e^{(2\,\pi)}} - \frac{n}{n^2+1}}{\pi}$

f(x) $= -\frac{\left(4\left(\left(e^{(2\,\pi)}-1\right)\cos(x) + e^{(2\,\pi)} - 1\right)\sin(x)^2 - 5\,\pi\left(e^{(2\,\pi)}-1\right) - 6\left(e^{(2\,\pi)}-1\right)\cos(x) - 2\,e^{(-2\,\pi)}\left(6\left(e^{(2\,\pi)}-1\right)\cos\left(\frac{x}{\pi}\right)^2 + 4\left(e^{(2\,\pi)}-1\right)\cos\left(\frac{x}{\pi}\right) + e^{(2\,\pi)} - 1\right)\sin\left(\frac{x}{\pi}\right) - 2\,e^{(2\,\pi)} + 2\right)}{10\,\pi}$

Out[145]:



In [82]:
```
# ii
var('x n')
f(x) = x*sin(x)
assume(n, 'integer')
L = pi

# Fourier Coefficients
a0 = (1/(2*L)) * integrate(f(x), (x, 0, 2*pi))
an = (1/L) * integrate(f(x) * cos(n*pi*x/L), (x, 0, 2*pi))
bn = (1/L) * integrate(f(x) * sin(n*pi*x/L), (x, 0, 2*pi))

# Display results
show("a0 =", a0)
show("an =", an)
show("bn =", bn)

# Fourier Series Sum (first 5 terms)
g = a0

# Display the function f(x) and add terms in a loop
for i in range(2, 6):
    # Calculate an and bn for the current n
    an_i = an.subs(n=i)
    bn_i = bn.subs(n=i)

    # Add the current term to g
    g += an_i * cos(i * pi * x / L) + bn_i * sin(i * pi * x / L)

# Display f(x) and g
show("f(x) =", g)

plot(f,0,2*pi,legend_label="x * sin x") + plot(g,0,2*pi,color = "red",legend_label="fourier series")
```

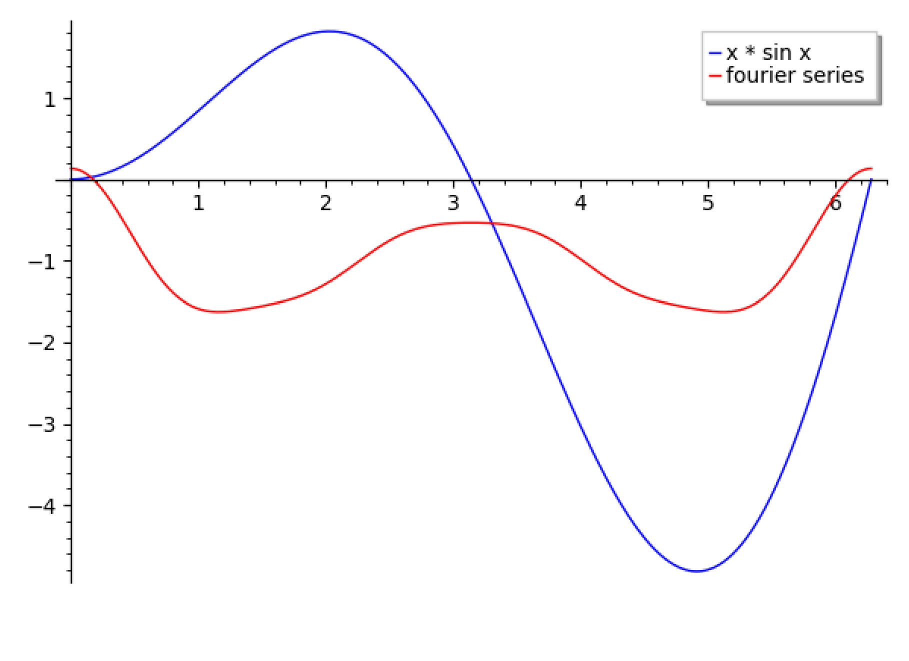
a0 $= -1$

an $= \frac{2}{n^2-1}$

bn $= 0$

f(x) $= \frac{1}{12}\cos(5\,x) + \frac{2}{15}\cos(4\,x) + \frac{1}{4}\cos(3\,x) + \frac{2}{3}\cos(2\,x) - 1$

Out[82]:



Q2. Obtain half range sine series in (0,pi) for cosx

In [80]:
```
# Define variables and function
var('x n')
f(x) = cos(x)
L = pi

# Calculate the Fourier sine coefficients
b_n = (2 / L) * integrate(f(x) * sin(n * pi * x / L), (x, 0, L))
b_n = b_n.full_simplify()

# Display the result for b_n
show("b_n =", b_n)

# Initialize the series sum
g = 0

# Partial sum of the series (first 5 terms)
for i in range(2, 10):
    bn_i = b_n.subs(n=i)

    # Add the current term to g
    g += bn_i * sin(i * pi * x / L)

# Display f(x) and g
show("Half-range sine series (first 5 terms) =", g)
```

b_n $= -\frac{2\left((-1)^n n + n\right)}{\pi - \pi n^2}$

Half-range sine series (first 5 terms) $= \frac{32\,\sin(8\,x)}{63\,\pi} + \frac{24\,\sin(6\,x)}{35\,\pi} + \frac{16\,\sin(4\,x)}{15\,\pi} + \frac{8\,\sin(2\,x)}{3\,\pi}$

Q3. Obtain half range cosine series in (0,π) for f(x) = x(π - x)

In [86]:
```
# Define variables and function
var('x n')
f(x) =  x*(pi- x)
L = pi

# Calculate the Fourier sine coefficients

a0 = (1/L) * integrate(f(x) , (x, 0, L))
a_n = (2 / L) * integrate(f(x) * cos(n * pi * x / L), (x, 0, L))


# Display the result for b_n
show("a0 =", a0)
show("a_n =", a_n.full_simplify())

# Initialize the series sum
g = a0

# Partial sum of the series (first 5 terms)
for i in range(2, 10):
    an_i = a_n.subs(n=i)

    # Add the current term to g
    g += an_i * cos(i * pi * x / L)

# Display f(x) and g
show("Half-range sine series (first 5 terms) =", g)
```

a0 $= \frac{1}{6}\,\pi^2$

a_n $= -\frac{2\left((-1)^n + 1\right)}{n^2}$

Half-range sine series (first 5 terms) $= \frac{1}{6}\,\pi^2 - \frac{1}{16}\cos(8\,x) - \frac{1}{9}\cos(6\,x) - \frac{1}{4}\cos(4\,x) - \cos(2\,x)$

Q4. Find the complex form of the Fourier series for f(x) = 2x in (0,2π)

In [100]:
```
# Define variables
var('x n')

# Define the function
f(x) = 2*x

# Define the period length
L = 2*pi

# Calculate the Fourier coefficients C_n
C_n = (1/(2*L)) * integrate(f(x) * exp(-I*n*pi*x/L), (x, 0, L))
C_n = C_n.full_simplify()

# Display the result for C_n
show("C_n =", C_n)

# Initialize the Fourier series
F_series = 0


for i in range(1, 6):
    Cn_i = C_n.subs(n=i)  # Substitute i for n

    # Add the current term to F_series
    F_series += Cn_i * exp(I * i * pi * x / L)


show("Complex Fourier series for f(x) =", F_series)
```

C_n $= -\frac{2\left((-i\,\pi n - 1)(-1)^n + 1\right)}{\pi n^2}$

Complex Fourier series for f(x) $= -\frac{2\,(5i\,\pi+2)e^{\left(\frac{5}{2}i\,x\right)}}{25\,\pi} - \frac{2\,(3i\,\pi+2)e^{\left(\frac{3}{2}i\,x\right)}}{9\,\pi} - \frac{2\,(i\,\pi+2)e^{\left(\frac{1}{2}i\,x\right)}}{\pi} + \frac{1}{2}i\,e^{(2i\,x)} + i\,e^{(i\,x)}$

END