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1. Note on 'De Broglie Hypothesis'.

→ If a light wave can act as a wave sometimes and as a particle at other times, then particles such as electrons should also act as waves at times. This is known as de Broglie hypothesis.

according to the hypothesis, any moving particle is associated with a wave. The waves associated with particles are known as de Broglie waves or matter waves.

The wavelength λ of matter waves associated with a particle moving with velocity v is inversely proportional to the magnitude of the momentum of the particle. Thus,

$$\nu = \lambda\nu = E/h \times h/p = E/p = mc^2/mv = c^2/v$$

$$\lambda = h/mv = h/p \quad (v < c \text{ always})$$

This is the de Broglie equation, and the wavelength λ is called de Broglie wavelength.

we may draw the following conclusions:

i. $\lambda \to \infty$ when the velocity of the particle is zero. it means matter waves are detectable only for moving particles.

ii. lighter the particle, smaller the value of mass m and hence longer is the wavelength λ associated with the matter wave, hence, λ of micro bodies will be more significant that of macrobodies.

iii. smaller the velocity, longer is the wavelength of the matter wave.

if a charged particle, say an electron is accelerated by a potential difference, of V volts, then its kinetic energy is given by KE = eV

$$mv^2/2 = eV$$

$$v = \sqrt{\frac{2eV}{m}}$$

Then the electron wavelength is given by,

$$\lambda = \frac{h}{mv} = \frac{h}{m}\sqrt{\frac{m}{2eV}}$$

$$\therefore \lambda = \frac{h}{\sqrt{2meV}}$$

$$\therefore \lambda = \frac{h}{\sqrt{2m(KE)}} \qquad \text{here, } K = \frac{p^2}{2m}$$

if particles are in thermal equilibrium at temperature T,

then, $KE = 3kT/2$

$$\therefore \lambda = \frac{h}{\sqrt{3mkT}}$$

for a charged particle of charge 'q' accelerated through a potential difference of V,

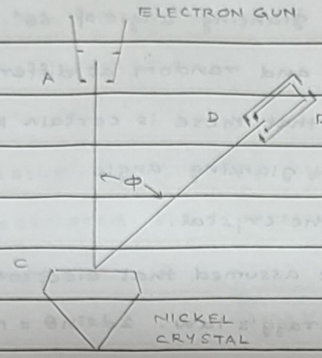
$$\lambda = \frac{h}{\sqrt{2mqV}}$$

-wave-particle duality

2. Note on 'Davison- Germer Experiment'.

→ APPARATUS:

The experimental arrangement of Davison- Germer Experiment is as follows:



The apparatus consisted of an electron gun, which produced collineated beam of electrons. An anode, A, connected to a variable voltage source accelerated the electrons. The energy can be computed from the accelerating potential. These electrons were scattered by a nickel crystal which can be rotated on its axis. A detector was placed in the scattering direction, which can also be ~~used~~ rotated in accordance to the crystal. The detector was attached with a scale to measure angle of scattering (ϕ) and hence, to deduce the glancing angle (θ).

EXPERIMENT:

Electrons emitted from a hot tungsten after getting accelerated and focused, fall on the nickel crystal. Electrons scattered from the crystal are collected at the detector, which produces the count in different directions with respect to the incident beam. The intensity of scattering (detector count) is plotted as a function of scattering angle for different values of accelerating voltage (V).

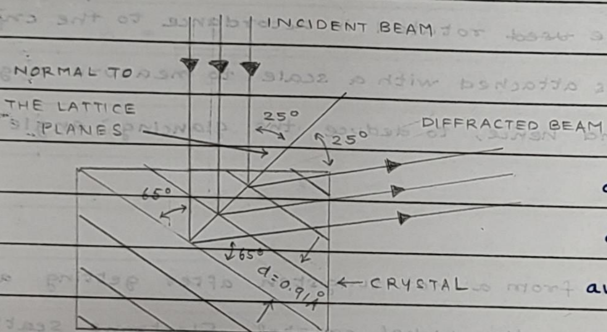
INVESTIGATIONS:

Davison and Germer found the detector current, which in turn depends upon intensity of scattered electrons increased significantly, at certain angle of scattering and values of accelerating voltage.

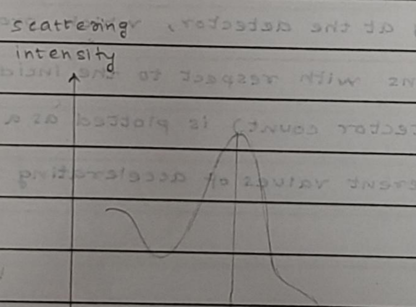
In a particular observation, they found maximum detector current corresponding to scattering angle of 50° for accelerating voltage of 54 V. With reference to the selected orientation of the nickel crystal (lattice points), this scattering angle corresponded to a glancing angle of 65°. The detector current was very low and random at different angle of scattering. This suggested that there is certain kind of preference to a particular ~~angle~~ glancing angle when the electrons scatter from the crystal.

This could only be explained if it is assumed that electrons have a wave nature and obey the Bragg's law: $2d\sin\theta = n\lambda$

~~ANALY~~ ANALYSIS:



The interplanar spacing (d) for the selected orientation of nickel crystal obtained from X-ray analysis is d = 0.91Å.



The glancing angle for which maximum count ~~was~~ obtained is 65°.

Using Bragg's law, it gives wavelength of possible waves as

$$\lambda = \frac{2d\sin\theta}{n}$$

corresponding to the first order of diffraction,

$$\lambda = 2 \times 0.91 \times \sin 65$$

$$\therefore \lambda = 1.65 \text{ Å}$$

Assuming de-broglie hypothesis to be correct, Davison and Germer estimated the wavelength of electron waves using de Broglie equation, $\lambda = \frac{h}{\sqrt{2mqV}}$

$$\therefore \lambda = \frac{6.63 \times 10^{-14}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}} = 1.67 \text{ Å}$$

Due to the excellent closeness of the answers, de Broglie equation indeed has significance and wave nature of matter was established experimentally ~~fr~~ for the first time.

- shor's algo
- quantum hardware

- quantum computing
- quantum algo

3. Write a short note on 'wave Packets'.

→ A wave packet consists of a group of harmonic waves. Each wave has slightly different wavelength. The superposition of a very large no. of harmonic waves differing infinitesimally in frequency will produce a single ~~wac~~ wave packet.

The velocity with which the wave packet propagates is called the group velocity, v_g. Individual waves forming the wave packet propagate at a velocity known as the phase velocity, v_p.

$$v_{phase} = \frac{E}{p} = \frac{c^2}{v_{par}}$$

$$v_g = \frac{d\omega}{dk}$$

4. Electromagnetic and matter waves:

→ Electromagnetic waves:

i. Electromagnetic waves are associated with photon, which has zero rest mass.

ii. A single de' Broglie wave can be associated with the particle (photon).

iii. The quantities those vary periodically with time and space are the electric and magnetic fields.

iv. Electric and magnetic fields are real physical quantities and can be measured experimentally.

v. Square of field amplitude gives intensity of electromagnetic waves.

Matter Waves:

i. Matter waves are associated with all moving particles having non-zero rest mass.

ii. A single de' Broglie wave ~~can~~ cannot be associated with the material particle.

iii. The quantity that varies periodically with time and space is called the wave function.

iv. The wave function is ~~as~~ an abstract mathematical quantity and has no direct physical interpretation.

v. Square of wave function gives probability of locating the particle in a given interval.

Their wavelength is inversely proportional to momentum of the particle.

They are indepent of the charge of the particle.

A particle cannot be ~~associated by~~ represented by a single wave, instead, it is associated with a large number of waves ~~&~~ forming a wave packet.

5. State Heisenberg's uncertainity principle and illustrate ~~&~~ it by diffraction of beam of electrons by a narrow slit.

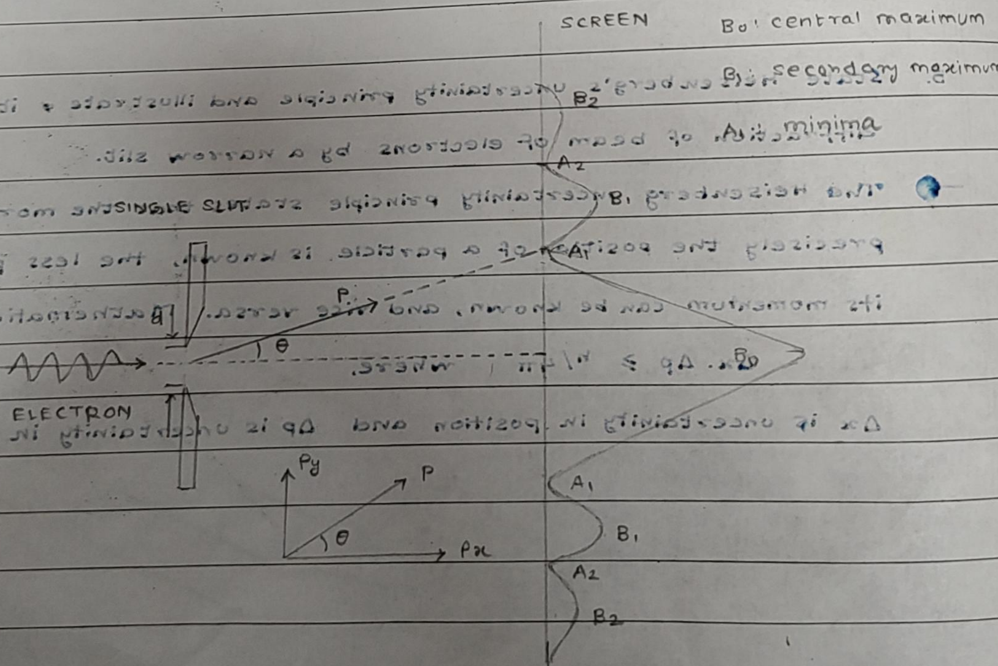
→ The Heisenberg uncertainity principle states that the more precisely the position of a particle is known, the less precisely its momentum can be known, and vice versa. Mathematically,

$$\Delta x . \Delta p \geq h/4\pi$$ where,

Δx is uncertainity in position and Δp is uncertainity in momentum

Single slit diffraction of electrons:

Here, we consider that electron is a wave of wavelength λ which is incident on a narrow slit of width d. ~~The narrow slit~~ Let p be the corresponding initial momentum as per de' Broglie equation. The single narrow slit has the same effect as it would have on a monochromatic beam of light. As an electron passes through the slit, its position becomes uncertain by an amount, Δy. Since it can pass anywhere from the slit, we can take $\Delta y \approx d$. Further, the electron gains y-component of momentum as it can reach anywhere on the screen on either sides of central maximum B_0. Let p be the momentum of electron after passing through the slit. The y-component of momentum is given by $p_y = p\sin\theta$. For convenience, let the electron reach at point A_1 on the screen. As this point corresponds to first minimum in the diffraction pattern, it is given by an equation, $\sin\theta = \lambda/d$ where θ is the angle of diffraction.



Now, letting ~~th~~ the uncertainity in the y-component of momentum to be at the most equal to the y-component of momentum itself,

$\Delta p_y \approx p_y \approx p\sin\theta$

$\Rightarrow p\sin\theta \approx \frac{h}{\lambda}\cdot\frac{\lambda}{d} \approx \frac{h}{\Delta y}$

$\therefore \quad \Delta p_y \approx \frac{h}{\Delta y}$ ... by using de broglie's equation, expression for the first minimum in the diffraction pattern and ~~substituting~~ substituting $\Delta y \approx d$.

Thus, $\Delta y \Delta p_y \approx h$, as required.

P.T.O.

6. Write a short note on 'qubits'.

→ Qubit is a quantum bit and it is called so as it need not be ON and OFF state of a transistor. It can be represented by multiple types: spin up, spin down, ground, excited states, left and right circular polarization, state of magnetization, etc.

There are two base states on which quantum computers are dependent on 0 and 1

Advantage of quantum computers is the superimposed state in which operations can be carried out on both the bits simultaneously.

In quantum mechanics, these qubits are represented by matrix notations,

$0 \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ or $|0\rangle$ – a single-qubit state

→ This is Dirac Notation

$1 \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ or $|1\rangle$ → called as Ket vector in Dirac Notation

Dagger operation in matrices

= complex conjugate + transpose

if a state is real, complex conjugate is not needed, then,

dagger = transpose

$|0\rangle \longrightarrow |0\rangle^{+}$

$|i\rangle = \begin{bmatrix} i \\ 0 \end{bmatrix}$ ⇒ on applying dagger operation, we get,

$|i\rangle^{+} = \begin{bmatrix} -i & 0 \end{bmatrix}$

similarly, $|0\rangle^{+} = |0\rangle^{*T} = \langle 0|$ – row matrix

$\langle 0| = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Multiplication of qubits :

(i) Inner or scalar product:

eg: $\langle 0|0\rangle = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = 1$ - scalar / number

$\langle 0|1\rangle = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = 0$

(ii) vector product: non-commutative (order dependent)

eg: $|0\rangle\langle 0| = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$

$|0\rangle\langle 1| = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$

(iii) Tensor product:

eg: $|0\rangle|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$

denoted by, $\oplus \Rightarrow |0\rangle \oplus |0\rangle$

it increases the rank of a matrix i.e. higher rank vector

similarly, $|0\rangle \oplus |1\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$

in Dirac notation, $|0\rangle \oplus |0\rangle = |00\rangle$

this is called as a two-qubit state.

n-qubit state : $|0\rangle^{\oplus n}$

Superposted state :

Any state, say, $|\psi\rangle$ is said to be in superposed state if it is expressed as a linear combination of the basic qubits where α and β are arbitary constants.

condition: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

probability that $|\psi\rangle$ can be found in state $|0\rangle$ after operation:

$$P = |\langle 0|\psi\rangle|^2$$

$$= |\langle 0|(\alpha|0\rangle + \beta|1\rangle)|^2$$

$$= |\langle 0|\alpha|0\rangle + \langle 0|\beta|1\rangle|^2$$

$$= |\alpha\langle 0|0\rangle + \beta\langle 0|1\rangle|^2$$

we know that, $\langle 0|0\rangle = 1$ and $\langle 0|1\rangle = 0$

$$\therefore P = |\alpha|^2$$

similarly, when, $P = |\langle 1|\psi\rangle|^2$

$$\therefore P = |\langle 1|\alpha|0\rangle + \langle 1|\beta|1\rangle|^2$$

$$\therefore P = |\alpha\langle 1|0\rangle + \beta\langle 1|1\rangle|^2$$

$$\therefore P = |\beta|^2$$

superposed state can be found in two ways: $\alpha = \beta = 1/\sqrt{2}$

(i) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\Rightarrow |+\rangle$

(ii) $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ $\Rightarrow |-\rangle$

Dirac Notations

7. Quantum Logic.

→ Quantum NOT Gate is called as $\boxed{\text{Pauli-}x\text{ gate}}$ and is denoted by, σ_x

σ_x is defined as,

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

operating σ_x on state zero,

$$\sigma_x|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$\sigma_x|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

similarly,

~~αx~~ $\cancel{\sigma_x|+\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}$ $\sigma_x|+\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} (|1\rangle + |0\rangle)$$

$$\Rightarrow \sigma_x|+\rangle = |+\rangle$$

$\boxed{\text{Pauli - z gate}}$ denoted by σ_z is given as,

$$\sigma_z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

on multiplying σ_z with $|+\rangle$, we get,

$$\sigma_z|+\rangle = \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

$$\therefore \sigma_z|+\rangle = |-\rangle$$

Hadamard gate : produces a superposed state

It is defined as, $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$

SOLVE PROPERLY IN EXAM

similarly, $H|1\rangle = |-\rangle$

vice versa, $H|+\rangle = |0\rangle$ and $H|-\rangle = |1\rangle$

Hence, we can say, Hadamard Gate is reversible.

8. Short note on 'Quantum Circuits'.

→ Quantum circuits are a combination of several logic gates

for example,

$|0\rangle$ — $\boxed{\sigma_x}$ — $\boxed{H}$ → ?

$|0\rangle$ σ_x will give 1

then, $H|1\rangle = |-\rangle$

* controlled NOT Gate: similar to XOR gate

it is a 2-bit gate. denoted by, for example,

$|00\rangle$

↓ control bit

→ data bit

if control bit is 1, flip the data bit; if control bit is 0, do nothing.

⇒ CNOT $|1,1\rangle \longrightarrow |10\rangle$

CNOT $|0,1\rangle \longrightarrow |01\rangle$

no commas