Application of Fourier series in image processing

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Description

The Fourier series decomposes periodic signals into sums of sine and cosine functions. In image processing, it breaks images into frequency components for tasks like filtering, noise reduction, and compression. By transforming to the frequency domain, key features are preserved, while noise can be removed. This technique is widely used in JPEG compression and edge detection, enabling efficient image storage and processing.
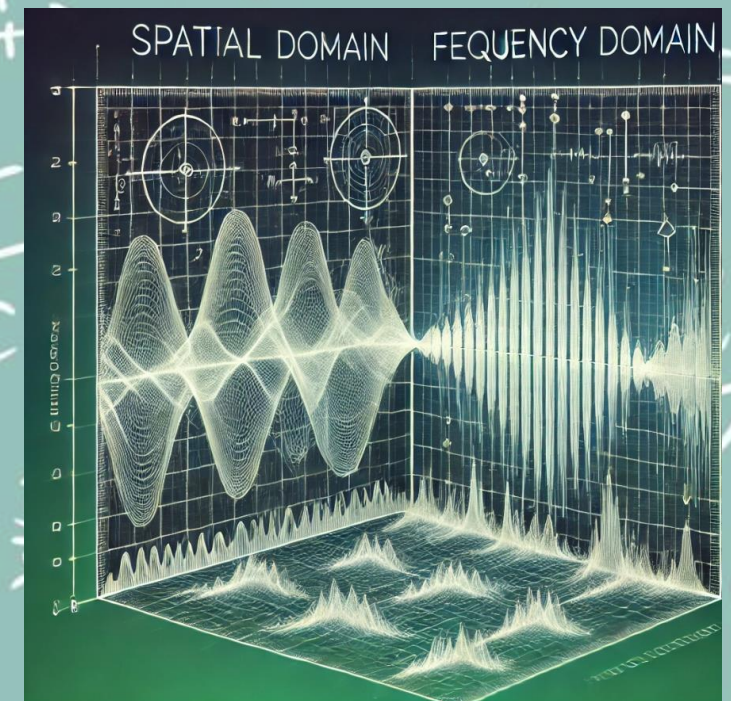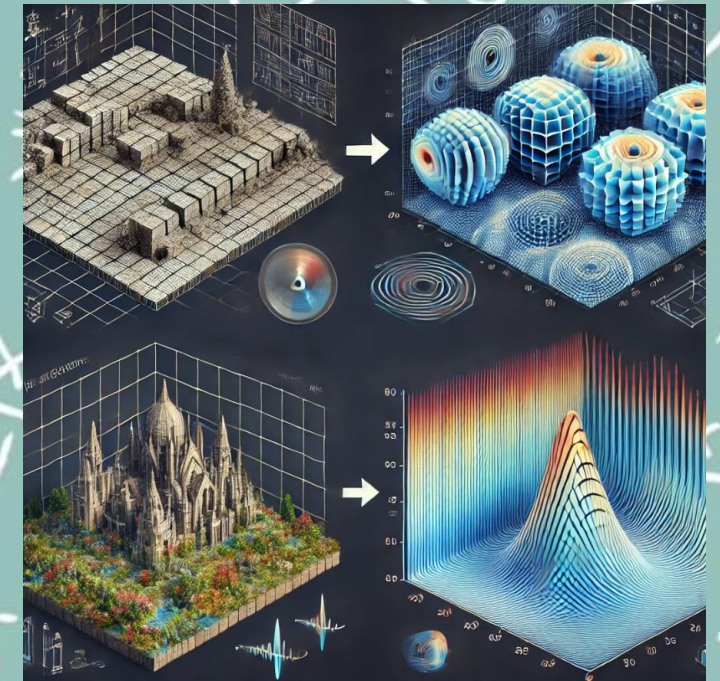

Equations

Fourier Series Decomposition

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(nx) + b_n \sin(nx))$$

Inverse Fourier Transform (2D)

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{2i\pi(\frac{ux}{M}+\frac{vy}{N})}$$

SPATIAL DOMAIN   FREQUENCY DOMAIN

Fourier series in image processing
Dynamics

Aspect	Description
Decomposition	Breaks down an image into sine and cosine frequency components.
Frequency Domain	Transforms spatial data into frequency data for analysis.
Noise Reduction	Filters out high-frequency noise while preserving image quality.
Image Compression	Reduces image size by discarding less significant frequencies.
Edge Detection	Highlights edges by focusing on high-frequency components.

Conclusion

- The Fourier series is a powerful tool for breaking down complex signals into frequency components.
- It plays a key role in noise reduction, filtering, and image compression.
- This method enhances the ability to process and manipulate images effectively.
- Despite its limitations, it remains foundational in frequency-based image analysis.
- Fourier series continues to drive advancements in digital signal and image processing fields.
- The Fourier series allows efficient transformation between the spatial and frequency domains for image analysis.
- It provides critical insights into patterns and textures within images, improving various image processing tasks.

Fourier series in image processing
Limitations

Limitation	Description
Non-periodic Signals	Fourier series struggles with non-periodic signals or finite images.
Edge Discontinuities	Gibbs phenomenon causes overshoot near sharp edges in images.
High Computational Cost	Transforming large images can be computationally expensive.
Limited Localization	Lacks spatial localization, making it hard to detect localized features.
Frequency Domain Artifacts	Can introduce artifacts when filtering or compressing images.
Difficulty with Colour Images	Processing RGB channels independently can lead to mismatches in quality.

With the interplay of frequencies and signals, Fourier series orchestrates a seamless transformation, revealing the hidden patterns that define the essence of image processing.

References

https://www.sciencedirect.com/topics/engineering/fourier-transform

https://homepages.inf.ed.ac.uk/rbf/HIPR2/fourier.htm