Q] P.T. $(\bar{p}-\bar{q}).\left[(\bar{q}-\bar{r}) \times (\bar{r}-\bar{p})\right] = 0$

$\rightarrow$

$$L.H.S = (\bar{p}.\bar{q})\left[(\bar{q}-\bar{r}) \times (\bar{r}-\bar{p})\right]$$

$$= (\bar{p}.\bar{q}).\left[(\bar{q}\times\bar{r}) - (\bar{q}\times\bar{p}) - (\bar{r}\times\bar{r}) + (\bar{r}\times\bar{p})\right]$$

$$LHS = \bar{p}.(\bar{q}\times\bar{r}) - \cancelto{0}{\bar{p}.(\bar{q}\times\bar{p})} - \cancelto{0}{\bar{p}.(\bar{r}\times\bar{r})} + \cancelto{0}{\bar{p}.(\bar{r}\times\bar{p})} - \cancelto{0}{\bar{q}.(\bar{q}\times\bar{r})} + \cancelto{0}{\bar{q}.(\bar{q}\times\bar{p})} + \cancelto{0}{\bar{q}.(\bar{r}\times\bar{r})} - \bar{q}.(\bar{r}\times\bar{p})$$

$$= \bar{p}.(\bar{q}\times\bar{r}) - \bar{q}(\bar{r}\times\bar{p})$$

$$= [\bar{p}\,\bar{q}\,\bar{r}] - [\bar{q}\,\bar{r}\,\bar{p}]$$

$$= [p\,q\,r] - [p\,q\,r]$$

$$= 0$$

$$\therefore (\bar{p}-\bar{q}).\left[(\bar{q}-\bar{r}) \times (\bar{r}-\bar{p})\right] = 0$$

Q] $[\bar{p}+\bar{q} \quad \bar{q}+\bar{r} \quad \bar{r}+\bar{p}] = (\bar{p}+\bar{q})\left[(\bar{q}+\bar{r}) \times (\bar{r}+\bar{p})\right] = 2[\bar{p}\,\bar{q}\,\bar{r}]$

$\rightarrow$ LHS $= (\bar{p}+\bar{q}).\left[(\bar{q}+\bar{r}) \times (\bar{r}+\bar{p})\right]$

$$= (\bar{p}+\bar{q}).\left[\bar{q}\times\bar{r} + \bar{q}\times\bar{p} + \bar{r}\times\bar{r} + \bar{r}\times\bar{p}\right]$$

Simplify like above Q

$$\therefore \bar{p}.(\bar{q}\times\bar{r}) + \bar{q}(\bar{r}\times\bar{p})$$

$$= [p\,q\,r] + [q\,r\,p]$$

$$= [pqr] + [pqr] = 2[pqr]$$

Q] If $\bar{a}\bar{b}\bar{c}$ coplaner vectors P.T. $\bar{a}\times\bar{b}$, $\bar{b}\times\bar{c}$ & $\bar{c}\times\bar{a}$ coplaner as well

→ LHS = $[\bar{a}\ \bar{b}\ \bar{c}] = 0$

$[\bar{a}\times\bar{b}\ \ \bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}] = 0$

$= (\bar{a}\times\bar{b}).[(\bar{b}\times\bar{c})\times(\bar{c}\times\bar{a})]$

$= (\bar{a}\times\bar{b}).\{[\bar{b}\ \bar{c}\ \bar{a}]\bar{c} - [\bar{b}\ \bar{c}\ \bar{c}]\bar{a}\}$

$= (\bar{a}\times\bar{b})([\bar{b}\ \bar{c}\ \bar{a}]\bar{c})$

$= (\bar{a}\times\bar{b}\ .\bar{c})[\bar{b}\ \bar{c}\ \bar{a}]$

$= \cancel{[\bar{a}\ \bar{b}\ \bar{c}]}^{\,0}\ \cancel{[\bar{a}\ \bar{b}\ \bar{c}]}^{\,0}$ [As $\bar{a}\ \bar{b}\ \bar{c}$ are coplanar]

$= 0$

$\therefore\ \bar{a}\times\bar{b}$, $\bar{b}\times\bar{c}$ & $\bar{c}\times\bar{a}$ are coplanar as well

20. If the vector $\overline{x}$ and a scalar λ satisfy the equation $\overline{a}\times\overline{x} = \lambda\overline{a} + b \ and\ \overline{a}\,.\overline{x} = 1$, find the values of λ and $\overline{x}$ in terms of $\overline{a},\overline{b}$. Also determine them if $\overline{a} = i - 2j$ and $\overline{b} = 2i + j - 2k$.

→ To determine λ, we multiply $\bar{a}\times\bar{x} = \lambda\bar{a} + b$ scalarly by $\bar{a}$

$\therefore\ \bar{a}(\bar{a}\times\bar{x}) = \bar{a}.\lambda\bar{a} + \bar{a}.\bar{b}$

$\therefore\ 0 = \bar{a}.\lambda\bar{a} + \bar{a}.\bar{b}$

$\therefore\ \lambda = -\dfrac{\bar{a}.\bar{b}}{a^2}$ —— ①

To find $\bar{x}$, we multiply $\bar{a}\times\bar{b} = \lambda\bar{a} + \bar{a}\bar{b}$ vectorially by $\bar{a}$

$\bar{a}\times(\bar{a}\times\bar{x}) = \bar{a} + \lambda\bar{a} + \bar{a}\times\bar{b}$

$= (\bar{a}.\bar{x}).\bar{a} - (\bar{a}.\bar{a})\bar{x} = \lambda\bar{a}\times\bar{a} + \bar{a}\times\bar{b}$

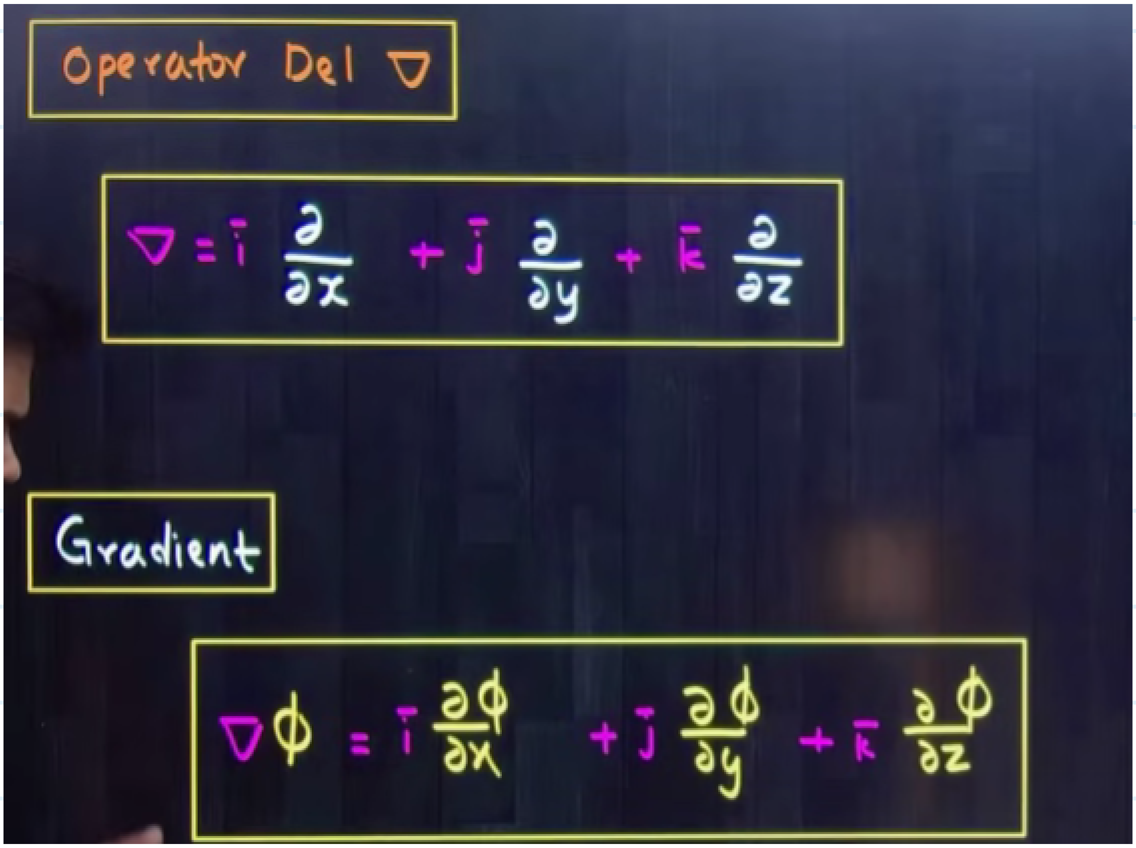
$(\bar{a} \times \bar{a} = 0)$ & $(\bar{a} \cdot \bar{x} = 1)$

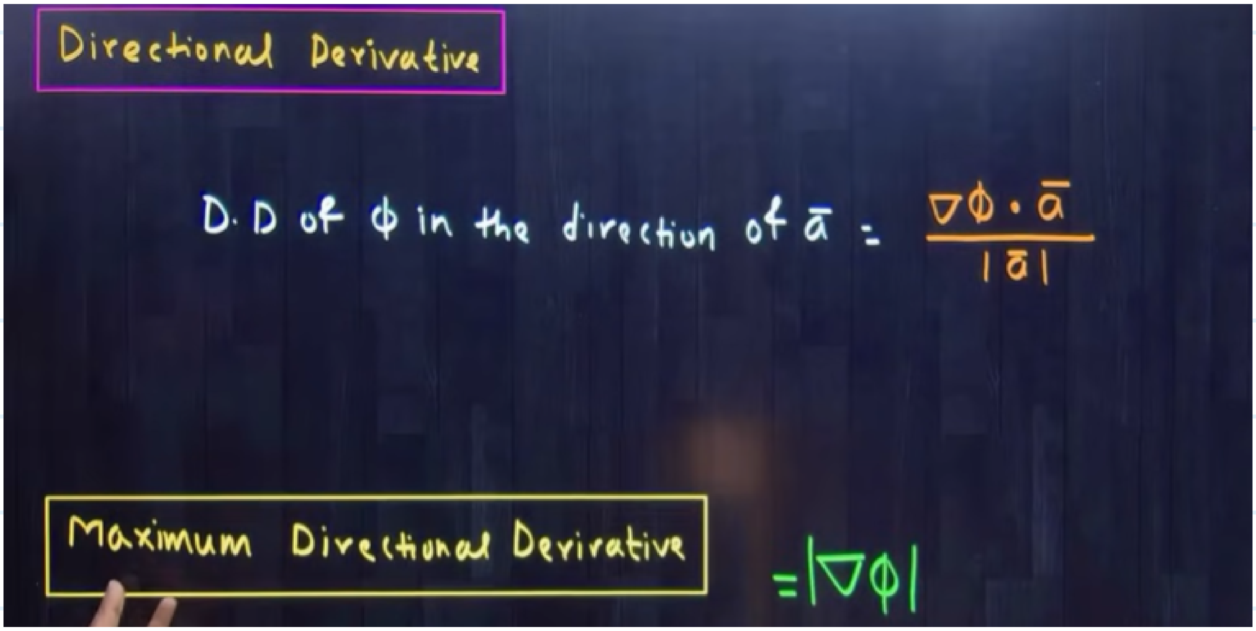
$\therefore \quad \bar{a} - \bar{a}^2 \bar{x} = \bar{a} \times \bar{b}$

$\therefore \quad \bar{x} = \dfrac{\bar{a} - \bar{a} \times \bar{b}}{a^2}$ — ②

from ① by subs, $\lambda = 0$

② " " , $x = -\dfrac{(3\hat{i} + 4\hat{j} + 5\vec{k})}{5}$


Operator Del ∇
$\nabla = \bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}$
Gradient
$\nabla\phi = \bar{i}\frac{\partial\phi}{\partial x} + \bar{j}\frac{\partial\phi}{\partial y} + \bar{k}\frac{\partial\phi}{\partial z}$


Directional Derivative
D. D of ϕ in the direction of $\bar{a} = \frac{\nabla\phi\cdot\bar{a}}{|\bar{a}|}$
Maximum Directional Derivative
$= |\nabla\phi|$

Let $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$|\bar{r}| = \sqrt{x^2+y^2+z^2}$

$r = \sqrt{x^2+y^2+z^2}$

$r^2 = x^2+y^2+z^2$

partially diff w.r.t. x

$\cancel{2} r \frac{\partial r}{\partial x} = \cancel{2} x$

$\boxed{\frac{\partial r}{\partial x} = \frac{x}{r}}$ Similarly $\boxed{\frac{\partial r}{\partial y} = \frac{y}{r}}$ $\boxed{\frac{\partial r}{\partial z} = \frac{z}{r}}$

$$\nabla f(r) = \bar{i}\,\frac{\partial f(r)}{\partial x} + \bar{j}\,\frac{\partial f(r)}{\partial y} + \bar{k}\,\frac{\partial f(r)}{\partial z}$$

$$\nabla f(r) = f'(r)\frac{\partial r}{\partial x}\bar{i} + f'(r)\frac{\partial r}{\partial y}\bar{j} + f'(r)\frac{\partial r}{\partial z}\bar{k}$$

$$\nabla f(r) = f'(r)\left[\frac{\partial r}{\partial x}\bar{i} + \frac{\partial r}{\partial y}\bar{j} + \frac{\partial r}{\partial z}\bar{k}\right]$$

$$= f'(r)\left[\frac{x}{r}\bar{i} + \frac{y}{r}\bar{j} + \frac{z}{r}\bar{k}\right]$$

$$= f'(r)\left[\frac{x\bar{i} + y\bar{j} + z\bar{k}}{r}\right]$$

$$= \frac{f'(r)}{r}\left[x\bar{i} + y\bar{j} + z\bar{k}\right]$$

$$= \frac{f'(r)}{r}\,\bar{r}.$$

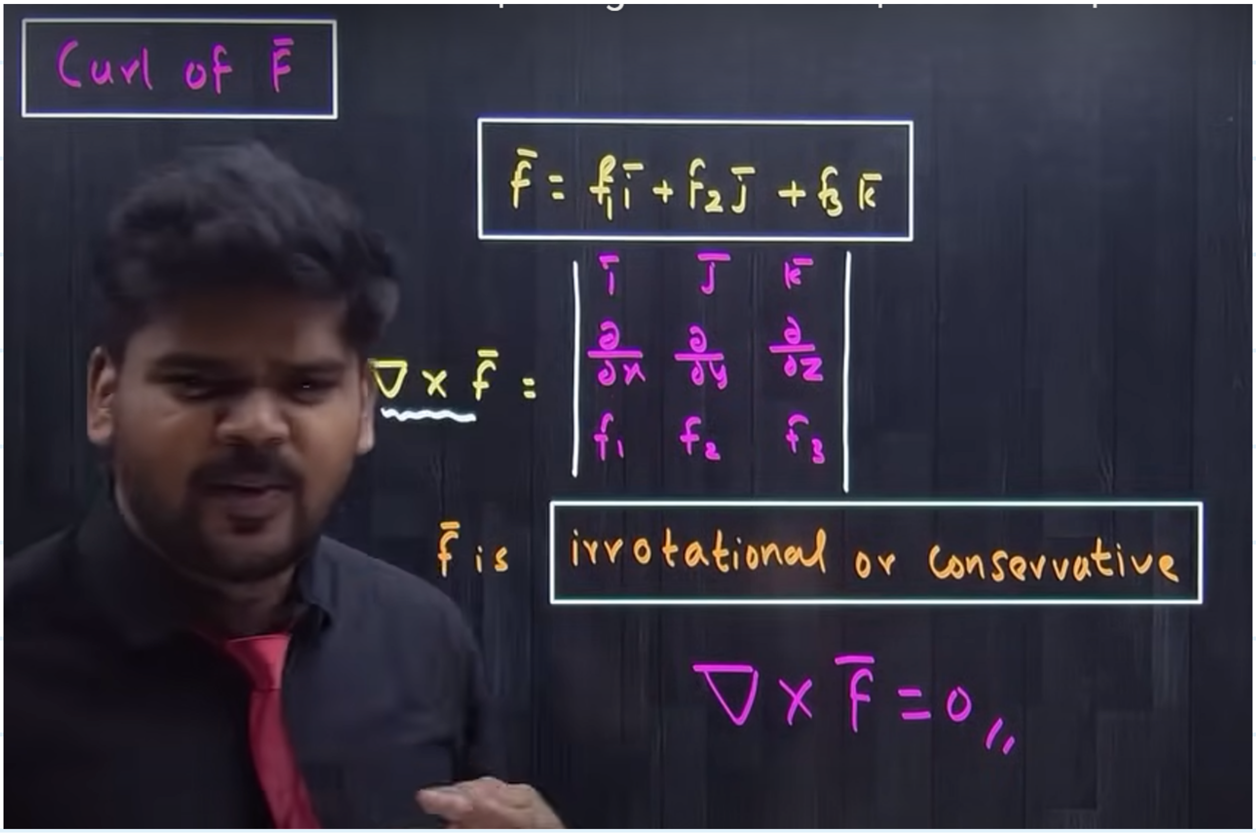
$\boxed{\text{Divergence}}$

$\bar{f} = f_1\bar{i} + f_2\bar{j} + f_3\bar{k}$

$$\boxed{\operatorname{div} \bar{f} = \nabla \cdot \bar{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}}$$

$\boxed{\bar{f} \text{ is solenoidal}}$

$\nabla \cdot \bar{f} = 0$


Curl of $\bar{F}$
$\bar{F} = f_1 \bar{i} + f_2 \bar{j} + f_3 \bar{k}$
$\nabla \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$
$\bar{F}$ is irrotational or conservative
$\nabla \times \bar{F} = 0$

Vector Differentiation

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$\vec{F}$

$\vec{F} = if_1 + jf_2 + kf_3$

Gradient

$$\text{grad } F = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\vec{F}$$

$$i\frac{\partial F}{\partial x} + j\frac{\partial F}{\partial y} + k\frac{\partial F}{\partial z}$$

Normal vector $\vec{N} = \text{grad } f$

Unit Normal vector $\hat{N} = \frac{\text{grad } f}{|\text{grad } f|}$

Directional derivative in $\vec{A}$ direc

$$= (\text{grad } f) \cdot \hat{A}$$

$$\text{div } \vec{f} = \nabla \cdot f = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$\text{curl } \vec{f} = \nabla \times f = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix}$$

Type 1: f & g (a,b,c) angle b/w f & g

$(\text{grad } f)_{abc}$

$(\text{grad } g)_{abc}$

$$\cos\theta = \frac{(\text{grad} f)_{abc} (\text{grad } g)_{abc}}{|\text{grad} f| \cdot |\text{grad } g|}$$

STANDARD RESULTS:

If $\bar{a}, \bar{b}, \bar{c}$ are differentiable vector function of a scalar variable t and Φ is a scalar function of t then

(i) $\frac{d}{dt}(\bar{a} \pm \bar{b}) = \frac{d\bar{a}}{dt} \pm \frac{d\bar{b}}{dt}$

(ii) $\frac{d}{dt}(\bar{a} \cdot \bar{b}) = \bar{a} \cdot \frac{d\bar{b}}{dt} + \bar{b} \cdot \frac{d\bar{a}}{dt}$

(iii) $\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{a} \times \frac{d\bar{b}}{dt} + \frac{d\bar{a}}{dt} \times \bar{b}$

(iv) $\frac{d}{dt}(\Phi\bar{a}) = \Phi\frac{d\bar{a}}{dt} + \bar{a}\frac{d\Phi}{dt}$

(v) $\frac{d}{dt}[\bar{a} \ \bar{b} \ \bar{c}] = \left[\frac{d\bar{a}}{dt} \ \bar{b} \ \bar{c}\right] + \left[\bar{a} \ \frac{d\bar{b}}{dt} \ \bar{c}\right] + \left[\bar{a} \ \bar{b} \ \frac{d\bar{c}}{dt}\right]$

(vi) $\frac{d}{dt}[\bar{a} \times (\bar{b} \times \bar{c})] = \frac{d\bar{a}}{dt} \times (\bar{b} \times \bar{c}) + \bar{a} \times \left(\frac{d\bar{b}}{dt} \times \bar{c}\right) + \bar{a} \times \left(\bar{b} \times \frac{d\bar{c}}{dt}\right)$