Hashing

Searching

- ***To retrieve a record with a particular value.***

Searching

- *Searching is a very common operation in most computer applications.*

- ***If we browse the Internet there is virtually no page where we will not find a search button!***

Searching

- ***The Google search -search facility helps Internet users.***
- ***Windows operating systems also have search facility to find files and folders.***

Searching

Different types of Searching :-

- ***The different types of searching techniques are***
  - ***Linear (Sequential) Search***
  - ***Binary Search***

Linear Search

Linear (Sequential Search) :-

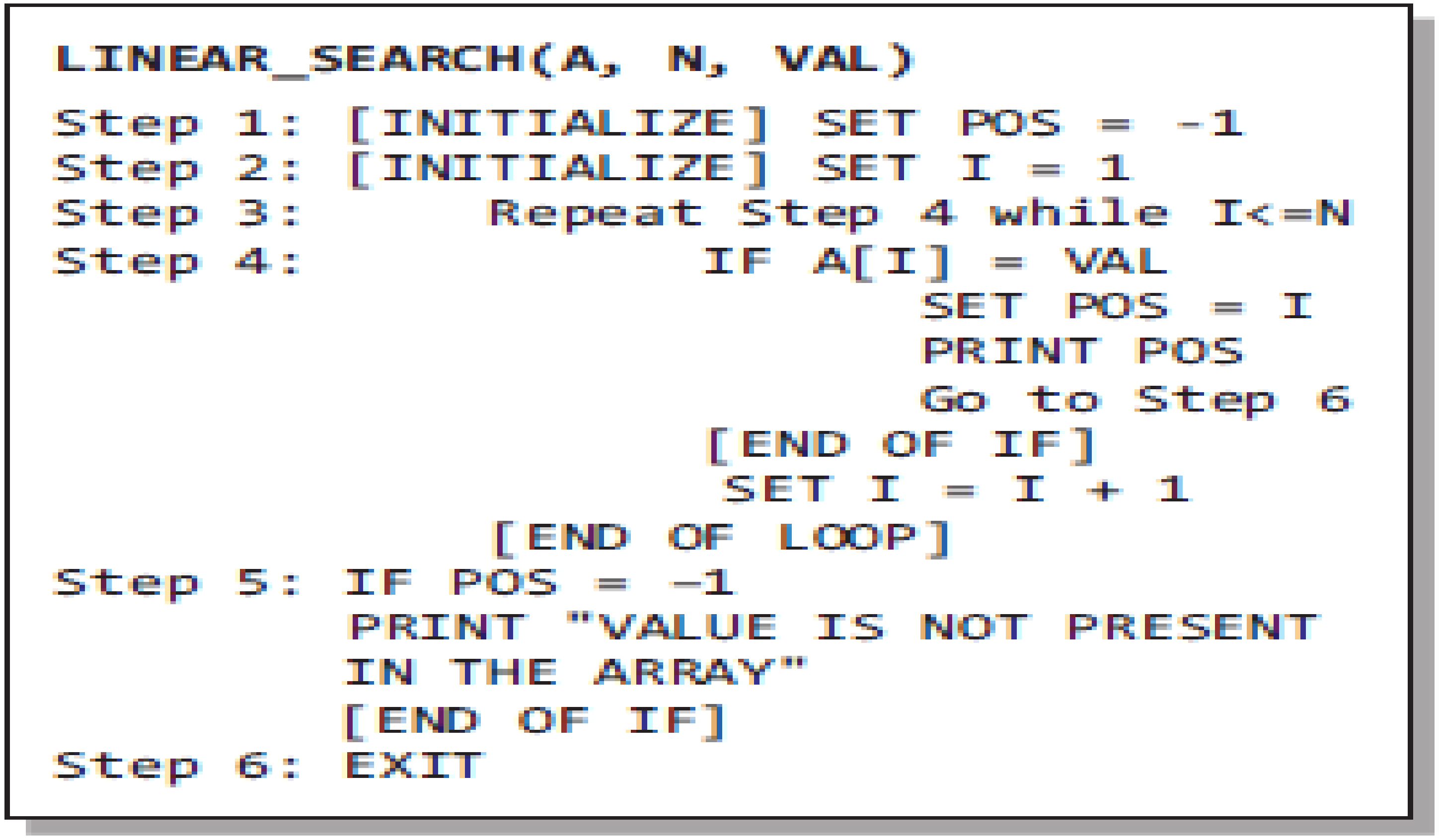
- Also called as *sequential search*,
- Simple method used for searching an array for a particular value.

Linear Search

Linear (Sequential Search) :-

- *The search begins at one end of the list,*
- *Scans the elements of the list , one by one*
- *until the desired record is found or the end of the array/list is reached.*
- ***If Search is successful then it will return the location of element***
- ***otherwise it will return the failure notification***

Linear Search



```
LINEAR_SEARCH(A, N, VAL)
Step 1: [INITIALIZE] SET POS = -1
Step 2: [INITIALIZE] SET I = 1
Step 3:       Repeat Step 4 while I<=N
Step 4:                 IF A[I] = VAL
                              SET POS = I
                              PRINT POS
                              Go to Step 6
                        [END OF IF]
                         SET I = I + 1
              [END OF LOOP]
Step 5: IF POS = -1
        PRINT "VALUE IS NOT PRESENT
        IN THE ARRAY"
        [END OF IF]
Step 6: EXIT
```

Linear Search

Linear (Sequential Search) :-

- Linear search is mostly used to search an unordered list of elements (array in which data elements are not sorted).

Binary Search :-

- Binary search is a searching algorithm that works efficiently with a sorted list.

Binary Search :-

- How do we find words in a dictionary?
- We first open the dictionary somewhere in the middle.
- Then, we compare the first word on that page with the desired word whose meaning we are looking for.
- If the desired word comes before the word on the page, we look in the first half of the dictionary, else we look in the second half.
- Again, we open a page in the first half of the dictionary and compare the first word on that page with the desired word and repeat the same procedure until we finally get the word.
- The same mechanism is applied in the binary search.

Binary Search :-

- *In Binary Search, the entire sorted list is divided into two parts.*
- *We first compare our input item with the mid element of the list and*
- ***Restrict our attention to only the first or second half of the list depending on whether the input item comes before or after the mid-element.***
- *In this way we reduce the length of the list at each step until we are getting a single element in a list.*

Binary Search :-

- Now, let us consider how this mechanism is applied to search for a value in a sorted array.

- Consider an array A[] that is declared and initialized as
- **int A[] = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10};**
- **The value to be searched is VAL = 9.**

Binary Search :-

- **int A[] = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10};**
- **The value to be searched is VAL = 9.**

- BEG = 0,
- END = 10,
- MID = (0 + 10)/2 = 5
- Now, VAL = 9 and A[MID] = A[5] = 5
- A[5] is less than VAL,
- therefore, we now search for the value in the second half of the array.

Binary Search :-

- So, we change the values of BEG and MID.
- Now, BEG = MID + 1 = 6,
- END = 10,
- MID = (6 + 10)/2 =16/2 = 8
- VAL = 9 and A[MID] = A[8] = 8

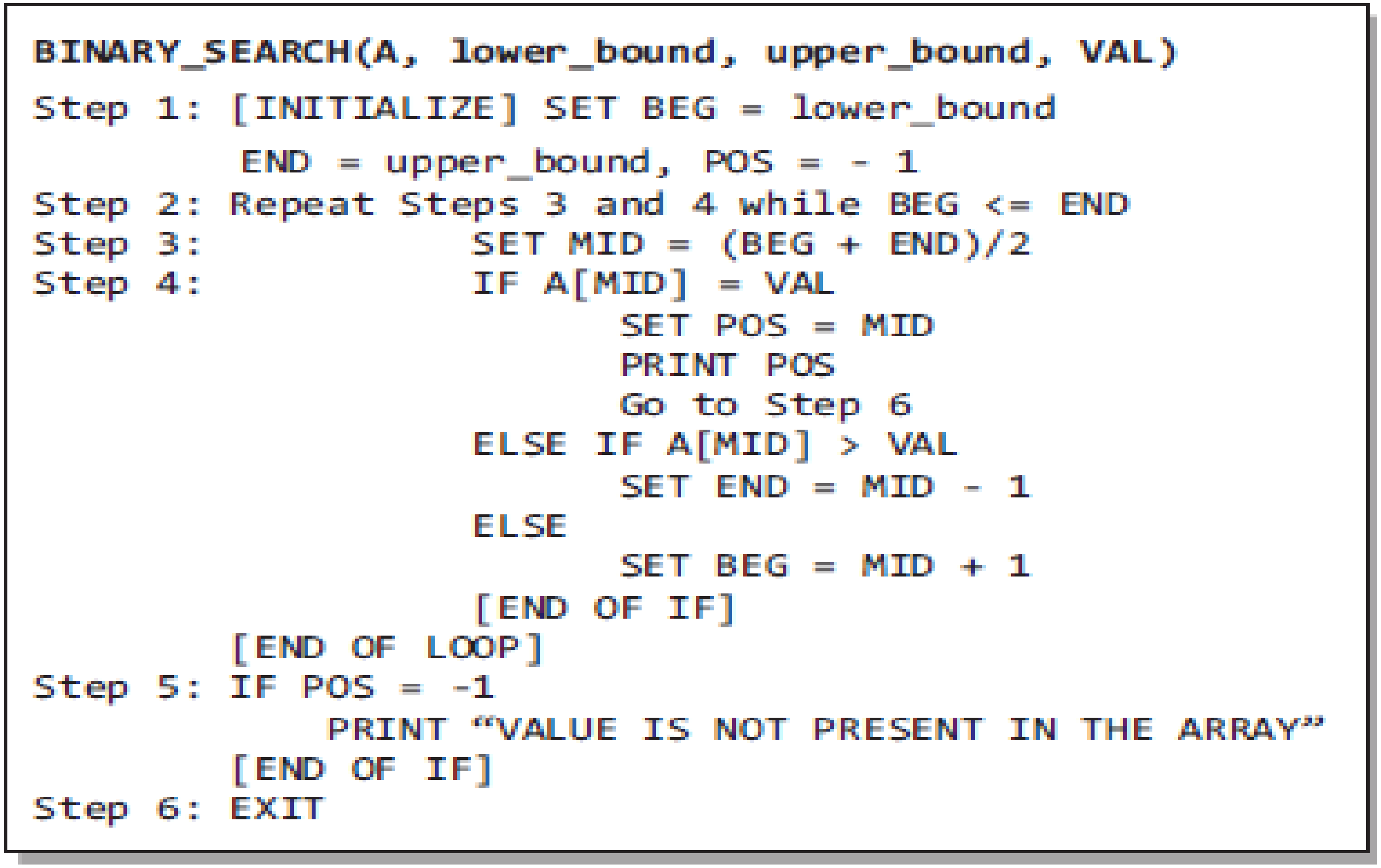
int A[] = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10};

- A[8] is less than VAL,
- therefore, we now search for the value in the second half of the segment.

- So, again we change the values of BEG and MID.
- Now, BEG = MID + 1 = 9,
- END = 10,
- MID = (9 + 10)/2 = 9
- Now, VAL = 9 and A[MID] = 9.

Binary Search :-

- **Algorithm-**



```
BINARY_SEARCH(A, lower_bound, upper_bound, VAL)
Step 1: [INITIALIZE] SET BEG = lower_bound
        END = upper_bound, POS = - 1
Step 2: Repeat Steps 3 and 4 while BEG <= END
Step 3:          SET MID = (BEG + END)/2
Step 4:          IF A[MID] = VAL
                       SET POS = MID
                       PRINT POS
                       Go to Step 6
                 ELSE IF A[MID] > VAL
                       SET END = MID - 1
                 ELSE
                       SET BEG = MID + 1
                 [END OF IF]
        [END OF LOOP]
Step 5: IF POS = -1
              PRINT "VALUE IS NOT PRESENT IN THE ARRAY"
        [END OF IF]
Step 6: EXIT
```

Hashing

- Various search algorithms allows us to search data in linear time complexity
  - **[O(n)] =>Linear Search**
  - **logarithmic time complexity [O(log n)]=>Binary Search**

- **To search a data in constant time complexity i.e. in O (1) time ??**

Hashing

- To search a data in constant time complexity i.e. in O (1) time ??
    - **Hash tables**
    - **Allows the storage and retrieval of data in an average time of O (1).**

Hashing

- Hash is an important Data Structure which is designed to use a **special function called the Hash function** which is used to map a given value with a particular key for faster access of elements.

Hashing

- **The efficiency of mapping depends of the efficiency of the hash function used.**

- It is a technique whereby items are placed into a structure based on a **key to-address transformation.**

Hashing

For storing record

$$\begin{tikzcd}
\textbf{Key} \arrow[d] \\
\textbf{Generate array index} \arrow[d] \\
\textbf{Store the record on that array index}
\end{tikzcd}$$

Hashing

For accessing record

$$\begin{tikzcd}
\text{Key} \arrow[d] \\
\text{Generate array index} \arrow[d] \\
\text{Get the record from that array index}
\end{tikzcd}$$

Hashing

Hash Table :-

- A hash table is a data structure that uses a **random access data structure, such as an array,** and a **mapping function, called a hash function,** to allow average constant time **O(1) searches**.

Hashing

Hash Function :-

- A hash function is a mapping between a set of input values and a set of integers, known as hash values.

- Denoted by H.

H(K)->A

Hashing

Good Hash Function :-

- Should have following properties

1) Efficiently computable.
2) Should uniformly distribute the keys (Each table position equally likely for each key)

3) Should generate unique addresses or addresses with minimum collision

Hashing

Hash Function :-

- For eg - A function that converts a given big phone number to a small practical integer value. The mapped integer value is used as an index in hash table.

- In simple terms, **a hash function maps a big number or string to a small integer that can be used as index in hash table.**

Hashing

Hash Function :-

Array Index	Keys
0	
1	
2	011 Delhi
3	
4	022 Kolkata
5	
6	033 Mumbai
7	
8	044 Chennai
9	

Hash Function :-

STD codes of the cities are keys

Hash function maps that into address by just adding the digits of the key

Array Index	Keys
0	
1	
2	011 Delhi
3	
4	022 Kolkata
5	
6	033 Mumbai
7	
8	044 Chennai
9	

Hashing

Hash of key :-

- Let h be a hash function & K is a key,
- H (K) is called hash-of-key.
- **The hash-of-key is the index at which a record with the key value K must be kept.**

Hashing

If each key is mapped on a unique hash table address then this situation is **ideal situation**

If the hash function is generating same hash table address for different keys, **the situation is called collision**

A good hash function should **generate minimum collision**

Hashing

2 things-

1) **Choosing a hash function which ensures minimum collision**
2) **Resolving collision**

Hashing

Some techniques for choosing hash functions are

Truncation Method

Hashing

Truncation Method

- Easiest method
- **A part of the key as address**
- **Can be rightmost or leftmost digit**

Eg-

82394561, 87139465, 83567271, 85943228

Suppose table size is 100 then take **the 2 rightmost digits for getting the addresses.**

Address will be 61, 65, 71 and 28

Hashing

Mid Square Method

- **Square** the key
- After getting the number, we take **some digits from the middle** of that no as address

Eg-

1337 , 1273, 1391, 1026

Square=1787569, 1620529, 1934881, 1052676

Lets take 3rd, 4th digit from each number as address

Let the table size be 100

Address=75, 05,48, 26

Hashing

Folding Method

- **Break the key into pieces, add them and get the hash address**

Eg-Lets take some 8 bit address

82394561, 87139465, 83567271, 85943228

Chop them in pieces 3,2 and 3 digits and them

Address will be->

82394561 = 823+94+561 =1478

87139465= 871+39+465 =1375

83567271= 835+67+271 = 1173

85943228 =859+43+228=1130

Hashing

Folding Method

Suppose the size of Hash table=1000,

Hash address will be from 0 to 999

Truncate the higher digits of the number

Address will be->

H(82394561) = 478 ✓

H(87139465) = 375 ✓

H(83567271) = 173 ✓

H(85943228) = 130 ✓

Hashing

Modular Method

- Best way
- **Perform Modulus operation, Remainder is address of hash table**
- Ensure address will be in range of hash table
- **Take table size as a prime number**
- **Table size should not be in Power of 2, else collisions increase**

Hashing

Modular Method

- Let us take some keys
- 82394561, 87139465, 83567271, 85943228
- Table size=97

Address=

82394561%97=45

87139465%%97=0

83567271%97=25

859432285%97=64

Hashing

- **Hashing**
    - **Closed Hashing**
    - **Open hashing**

Hashing

Closed Hashing(Open addressing)

- Uses array
- Techniques used in Closed Hashing are-

1) **Linear Probing**
2) **Quadratic Probing**
3) **Double Hashing**
4) **Rehashing**

Hashing

Open hashing:-

- Uses linked list for resolving collision resolution which has its own disadvantage

Linear Probing

1) Map the key on a particular address using Modular method

2) If 2 keys have the same hash address, then it will find the next empty position in the hash table

3) Hash table is used as circular array

4) If table size is N then after N-1 position, value is entered in 0th position

Hashing

Linear Probing

Elements=29,18,43,10,36,25,46

Table size=11

Hashing

Linear Probing

Elements=29,18,43,10,36,25,46

Table size=11

H(29)=29%11=7

H(18)=18%11=7

H(43)=43%11=10

H(10)=10%11=10

H(36)=36%11=3

H(25)=25%11=3

H(46)=46%11=2

Hashing

Linear Probing-Insertion

H(29)=29%11=7

H(18)=18%11=7

H(18)=7+1=8

H(43)=43%11=10

H(10)=10%11=10

H(10)=10+1=11%11=0

H(36)=36%11=3

H(25)=25%11=3

H(25)=3+1=4

H(46)=46%11=2

For 18, hash address 7 is already occupied, so next free position is 8th

43 and 10 has same hash address, so 10 will be inserted at 0th position in the table

0	10
1	
2	46
3	36
4	25
5	
6	
7	29
8	18
9	
10	43

Hashing

Linear Probing

For Searching the element

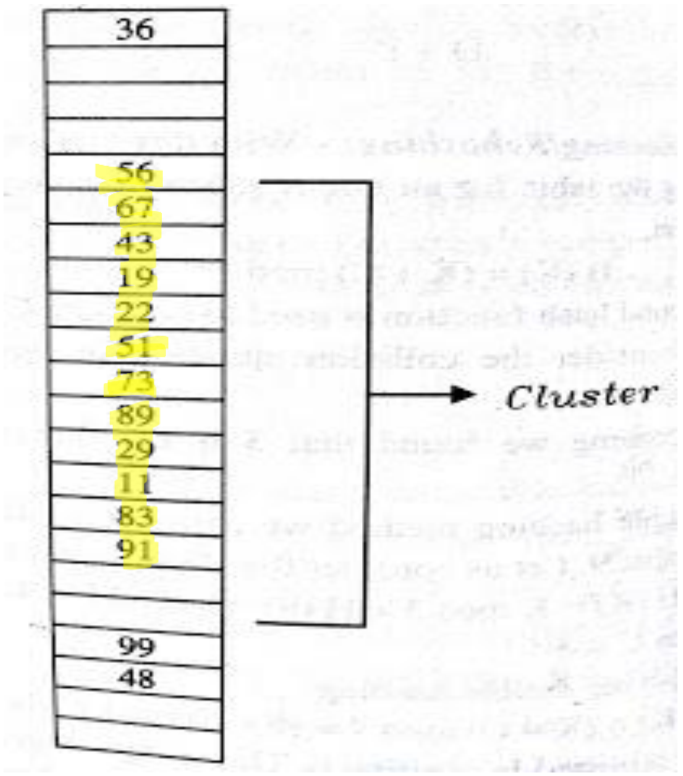
1) **First we check the hash address position in the table**
2) **If element is not available at that position,**
3) **Sequentially search the element after that hash address position**

0	10
1	
2	46
3	36
4	25
5	
6	
7	29
8	18
9	
10	43

Hashing

Disadvantage of Linear Probing

- Clustering Problem
- When filled sequence in a hash table becomes longer.
- It means that positions are occupied by elements and we have to search a longer period of time to get an empty cell.
- Searching also becomes slow



Hashing

Linear Probing

1) If Hash address is h
2) In case of collision, linear probing searches the location h,h+1,h+2……….(%SIZE)

Hashing

Linear Probing

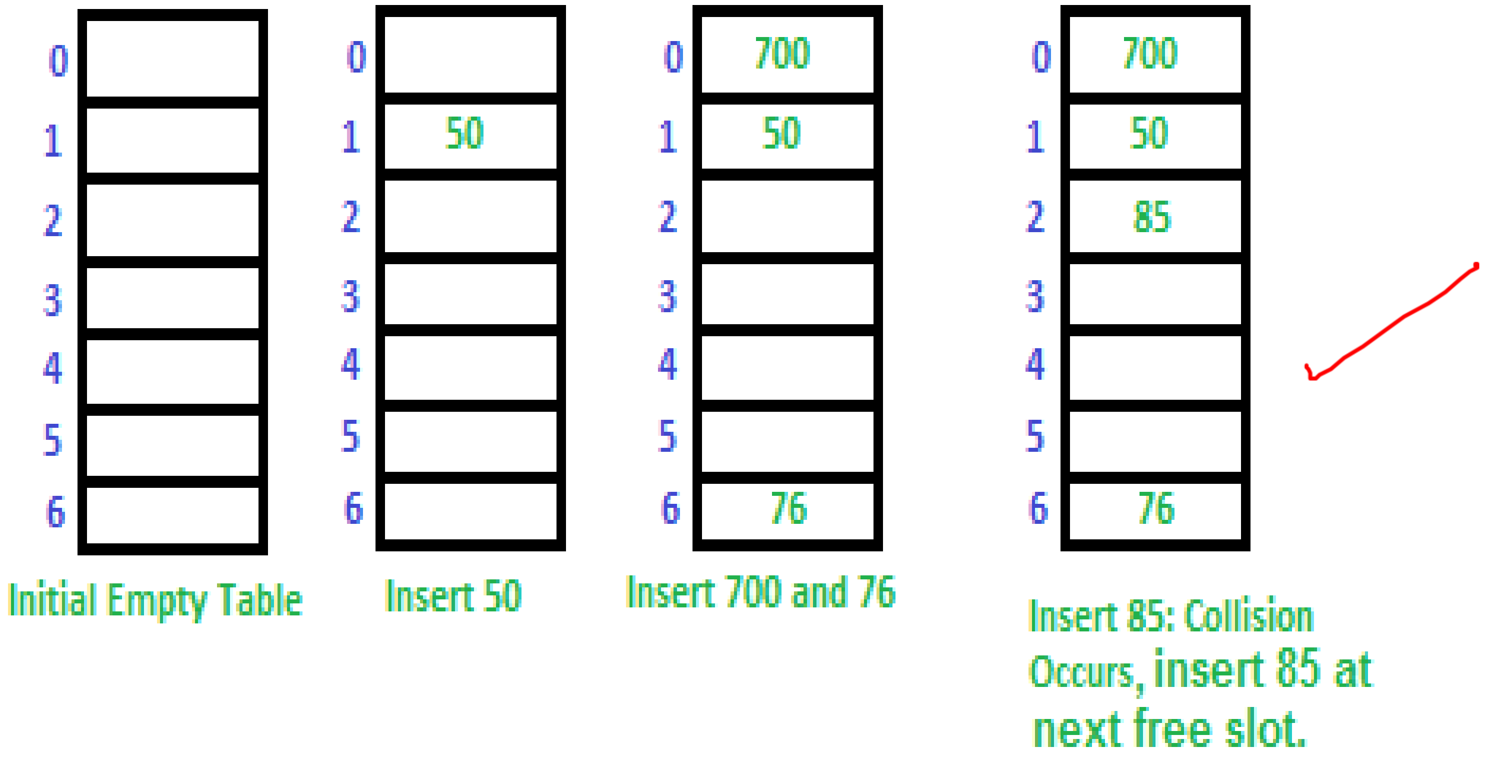
- If slot hash(x) % S is full, then we try (hash(x) + 1) % S
- If (hash(x) + 1) % S is also full, then we try (hash(x) + 2) % S
- If (hash(x) + 2) % S is also full, then we try (hash(x) + 3) % S
- S=size of hash table

Example

- A simple hash function as “key mod 7” and sequence of keys as 50, 700, 76, 85, 92, 73, 101

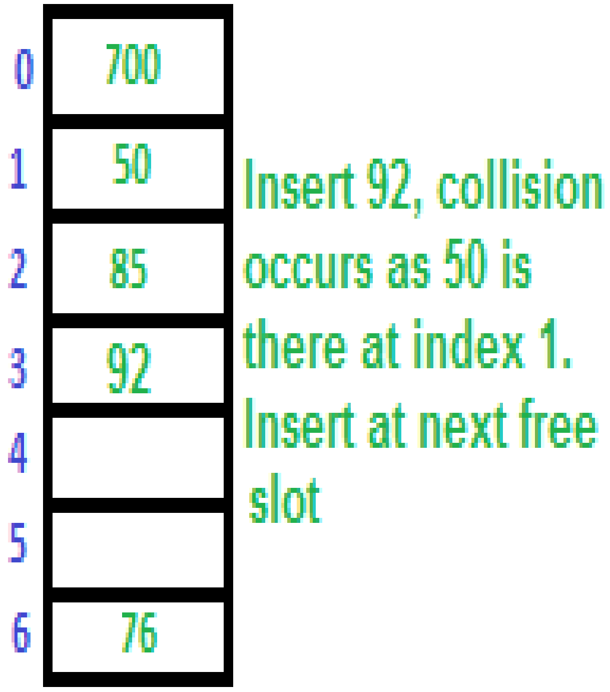
Example

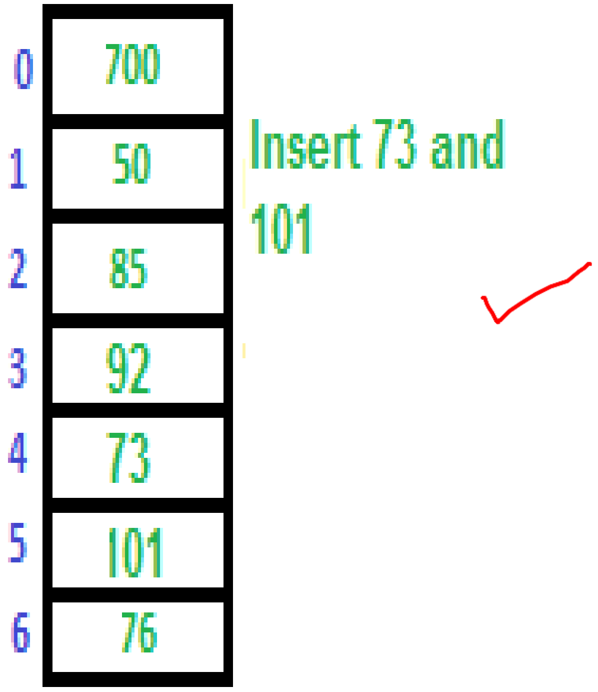
- A simple hash function as “key mod 7” and sequence of keys as 50, 700, 76, 85, 92, 73, 101



Example

- A simple hash function as "key mod 7" and sequence of keys as 50, 700, 76, 85, 92, 73, 101


0
700
1
50
2
85
3
92
4
5
6
76
Insert 92, collision occurs as 50 is there at index 1. Insert at next free slot



Hashing

Quadratic Probing

1) If Hash address is h
2) In case of collision, quadratic probing searches the location **$h+i^2\%SIZE$**
3) **For i=1,2,3….it searches for location h+1, h+4, h+9…..**
4) **Decreases the clustering problem**
5) **Cannot search all the locations**
6) **If hash table size is prime, it will search atleast half of the locations of the hash table**

Hashing

Quadratic Probing

- **How Quadratic Probing is done?**
  Let hash(x) be the slot index computed using the hash function.
- If the slot hash(x) % S is full, then we try (hash(x) + 1*1) % S.
- If (hash(x) + 1*1) % S is also full, then we try (hash(x) + 2*2) % S.
- If (hash(x) + 2*2) % S is also full, then we try (hash(x) + 3*3) % S.
- This process is repeated for all the values of i until an empty slot is found.
- S=size of the table

Hashing

Quadratic Probing

Elements=29,18,43,10,46,54

Table size=11

Hashing

Quadratic Probing

Elements=29,18,43,10,46,54

Table size=11

H(29)=29%11=7

H(18)=18%11=7, collision h+1*1=8

0	
1	
2	
3	
4	
5	
6	
7	29
8	18
9	
10	

Hashing

Quadratic Probing

Elements=29,18,43,10,46,54

Table size=11

H(29)=29%11=7

H(18)=18%11=7, collision h+1*1=8

H(43)=43%11=10

H(10)=10%11=10, collision h+ 1*1=11%size=11%11=0

0	**10**
1	
2	
3	
4	
5	
6	
7	29
8	18
9	
10	**43**

Hashing

Quadratic Probing

Elements=29,18,43,10,46,54

Table size=11

H(29)=29%11=7

H(18)=18%11=7, collision h+1*1=8

H(43)=43%11=10

H(10)=10%11=10, collision h+ 1*1=11%size=11%11=0

H(46)=46%11=2

H(54)=54%11=10, collision h + 2*2=h+4=14%11=3

0	**10**
1	
2	**46**
3	**54**
4	
5	
6	
7	**29**
8	**18**
9	
10	**43**

Hashing

Quadratic Probing

Elements=29,18,43,10,46,54

Table size=11

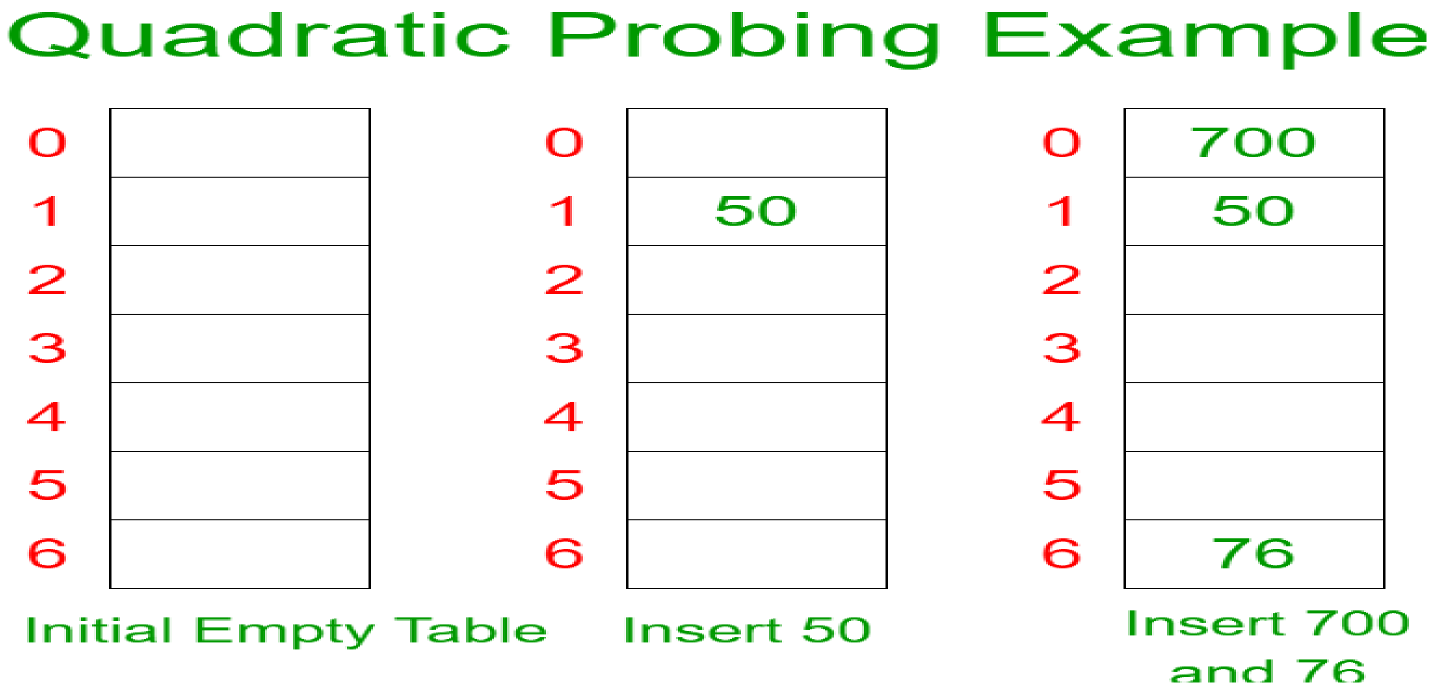
0	10
1	
2	46
3	54
4	
5	
6	
7	29
8	18
9	
10	43

Example

- Let us consider a simple hash function as “**key mod 7**” and sequence of keys as **50, 700, 76, 85, 92, 73, 101**.

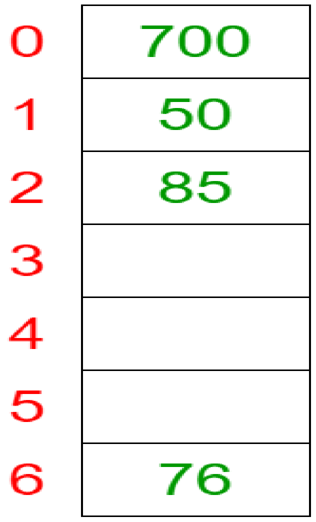
Example

- Let us consider a simple hash function as "**key mod 7**" and sequence of keys as **50, 700, 76, 85, 92, 73, 101**.
- H(50)=50%7=1
- H(700)=700%7=0
- H(76)=76%7=6



Example

- Let us consider a simple hash function as “**key mod 7**” and sequence of keys as **50, 700, 76, 85, 92, 73, 101**.
- H(50)=50%7=1   H(85)=85%7=1, Collision (h+1)=2
- H(700)=700%7=0   H(92)=92%7=1 , Collision again (h+4)=5
- H(76)=76%7=6

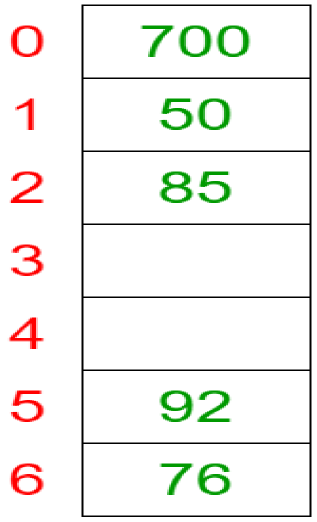


0	700
1	50
2	85
3	
4	
5	
6	76

Insert 85:

Collision occurs.

Insert at 1 + 1*1 position



0	700
1	50
2	85
3	
4	
5	92
6	76

Insert 92:

Collision occurs at 1.

Collision occurs at 1 + 1*1 position

Insert at 1 + 2*2 position.

Example

- Let us consider a simple hash function as “**key mod 7**” and sequence of keys as **50, 700, 76, 85, 92, 73, 101**.
- H(73)=73%7=3
- H(101)=101%7=3, collision (h+1)=4

0	700
1	50
2	85
3	73
4	101
5	92
6	76

Insert 73 and 101

Hashing

Double Hashing

- Choose 2nd Hash function
- Hashing 2nd time in case of Collision
- **h =hash value from 1st function**
- **h’=hash value from 2nd function**
- **In case of collision , Search the hash key location h, h+h’, h+2h’, h+3h’,………..**

Hashing

Double Hashing

- h=key%prime1
- h’=prime2-(key%prime2)
- In case of collision:-
- (h+h’)%prime1,(h+2h’)%prime1……..

Hashing

Double Hashing

- First hash function is typically
  - **hash1(key) = key % TABLE_SIZE**

- A popular second hash function is :
  - **hash2(key) = PRIME – (key % PRIME)**
  - **where PRIME is a prime smaller than the TABLE_SIZE.**

Hashing

Double Hashing

- 2 times calculation of hash function which creates it complex
- **Searching is slower than linear and quadratic probing**
- Very careful in choosing 2nd Hash function

Hashing

Double Hashing

h=key%13

h’=11-(key%11)

At the time collision, the next probe will be-

=(h+h’)mod13

=((key%13) + (11-(key%11)))mod13

Hashing

Double Hashing

Elements=8, 55, 48, 68

Table size=13

h=key%13

h'=11-(key%11)

0	
1	
2	
3	55
4	
5	
6	
7	
8	8
9	48
10	
11	
12	68

Hashing

Double Hashing

Elements=8, 55, 48, 68

Table size=13

H(8)=8%13=8

H(55)=55%13=3

H(43)=48%13=9

No collision till now

0	
1	
2	
3	**55**
4	
5	
6	
7	
8	**8**
9	**48**
10	
11	
12	

Hashing

Double Hashing

Elements=8, 55, 48, 68

Table size=13

H(8)=8%13=8

H(55)=55%13=3

H(43)=48%13=9

H(10)=68%13=3, collision

Next address=

=(68%13)+(11-(68%11))%13

=(3+(11-2))%13

=(3+9)%13

=12%13

=12

Position for 68=12

0	
1	
2	
3	**55**
4	
5	
6	
7	
8	**8**
9	**48**
10	
11	
12	**68**

Let us take elements 19, 27,36,10
Perform Double Hashing using the following functions

Lets say, Hash1 (key) = key % 13

Hash2 (key) = 7 – (key % 7)

Lets say, Hash1 (key) = key % 13

Hash2 (key) = 7 – (key % 7)

	Hash1(19) = 19 % 13 = 6
	Hash1(27) = 27 % 13 = 1
	Hash1(36) = 36 % 13 = 10
h	Hash1(10) = 10 % 13 = 10
h’	Hash2(10) = 7 – (10%7) = 4
(h+h’)mod 13	(Hash1(10) + 1*Hash2(10))%13= 1
(h+2h’)mod 13	(Hash1(10) + 2*Hash2(10))%13= 5

Collision

Hashing

ReHashing

- Chances of Insertion Failure when table is full
- **Soln=>**
- **Create a new hash table with double size of previous hash table**
- Use the new hash function and Insert all the elements of the previous hash table in the new table

Hashing

ReHashing

- Scan the elements of the previous hash table one by one
- Calculate the hash key with new hash function
- Insert them in new hash table

Hashing

Rehashing

Elements=7,18,43,10,36,25

Table size=11

H(7)=7%11=7

H(18)=18%7=7, collision

H(43)=43%11=10

H(36)=36%11=3

H(10)=10%11=10, collision

H(25)=25%11=3

0	10
1	
2	
3	36
4	25
5	
6	
7	7
8	18
9	
10	43

Hashing

Rehashing

Elements=7,18,43,10,36,25

Table size=11

Now insert 46

H(46)=46%11=2

0	10
1	
2	46
3	36
4	25
5	
6	
7	7
8	18
9	
10	43

Hashing

Rehashing

- Now to perform Rehashing
- Create a new table
- **For Size of New Hash table, We choose the nearest bigger prime number to double the size of the original hash table**

0	10
1	
2	46
3	36
4	25
5	
6	
7	7
8	18
9	
10	43

Hashing

Rehashing

- Original Size=11
- Double Size=22
- Nearest Bigger prime no=23
- New Hash function:-
- **H(K)=K mod 23**

0	10
1	
2	46
3	36
4	25
5	
6	
7	7
8	18
9	
10	43

Hashing

Rehashing

- New Hash Table-
- **H(K)=K mod 23**

H(7)=7%23=7

H(18)=18%23=18

H(43)=43%23=20

H(36)=36%23=13

H(25)=25%23=2

Now insert 46

H(46)=46%23=23%SIZE=23%23=0

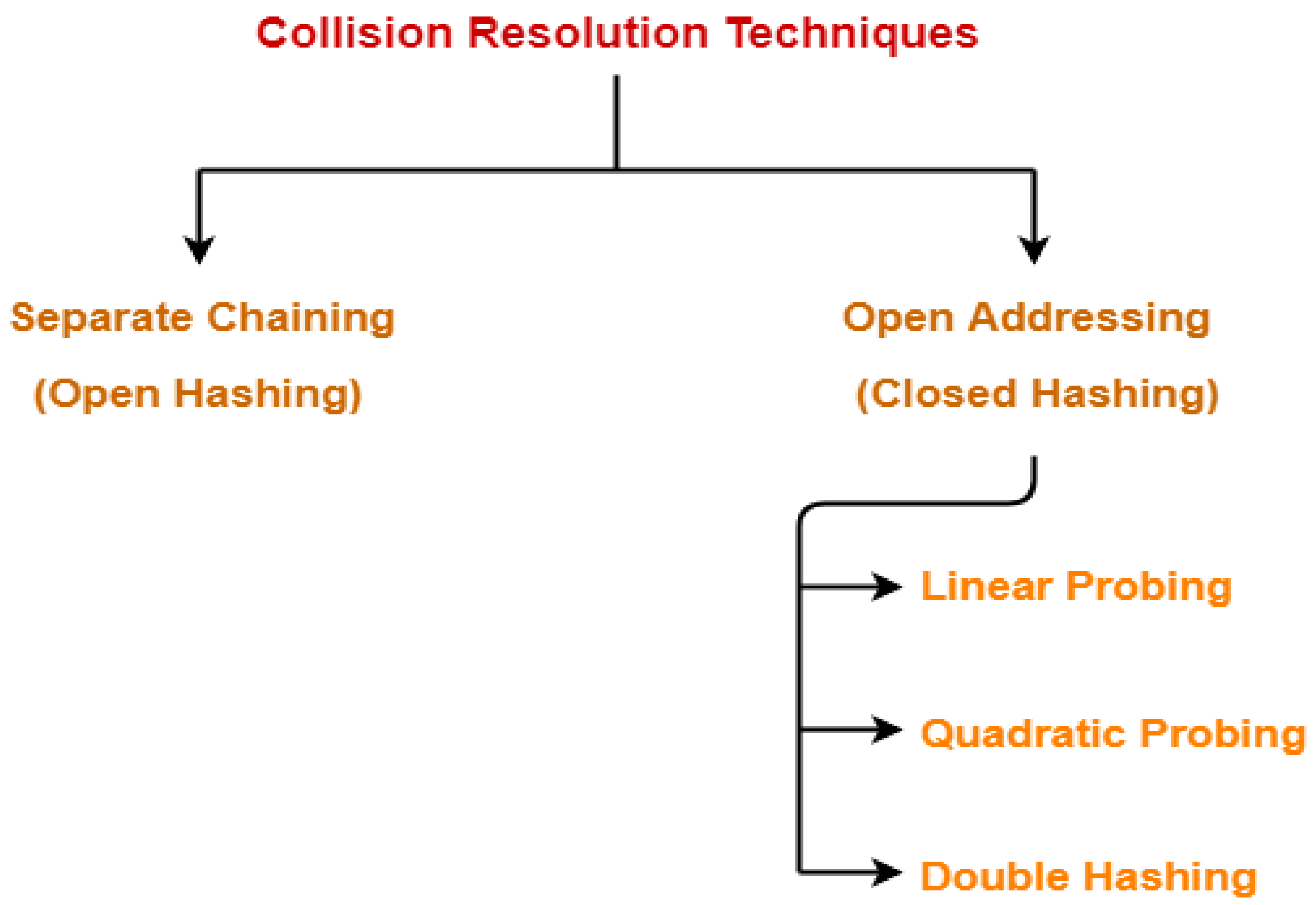
0	46
1	
2	25
3	
4	
5	
6	
7	7
8	
9	
10	10
11	
12	
13	36
14	
15	
16	
17	
18	18
19	
20	43
21	
22	

Hashing

ReHashing

- Bit more expensive technique but it happens very few times
- Decision of rehashing can be taken on different conditions like
  - **Table is occupied more than half,**
  - **Insertion of new element failure or on any given case**

Hashing


Collision Resolution Techniques
Separate Chaining
(Open Hashing)
Open Addressing
(Closed Hashing)
Linear Probing
Quadratic Probing
Double Hashing

Open Hashing

Separate Chaining

- **Maintain chain of elements which have same hash address**

Open Hashing

Separate Chaining

- **Hash table is an array of pointers which point to the linked list**
- **Maintain Linked list in sorted order and the elements which have same hash address will be in this linked list**

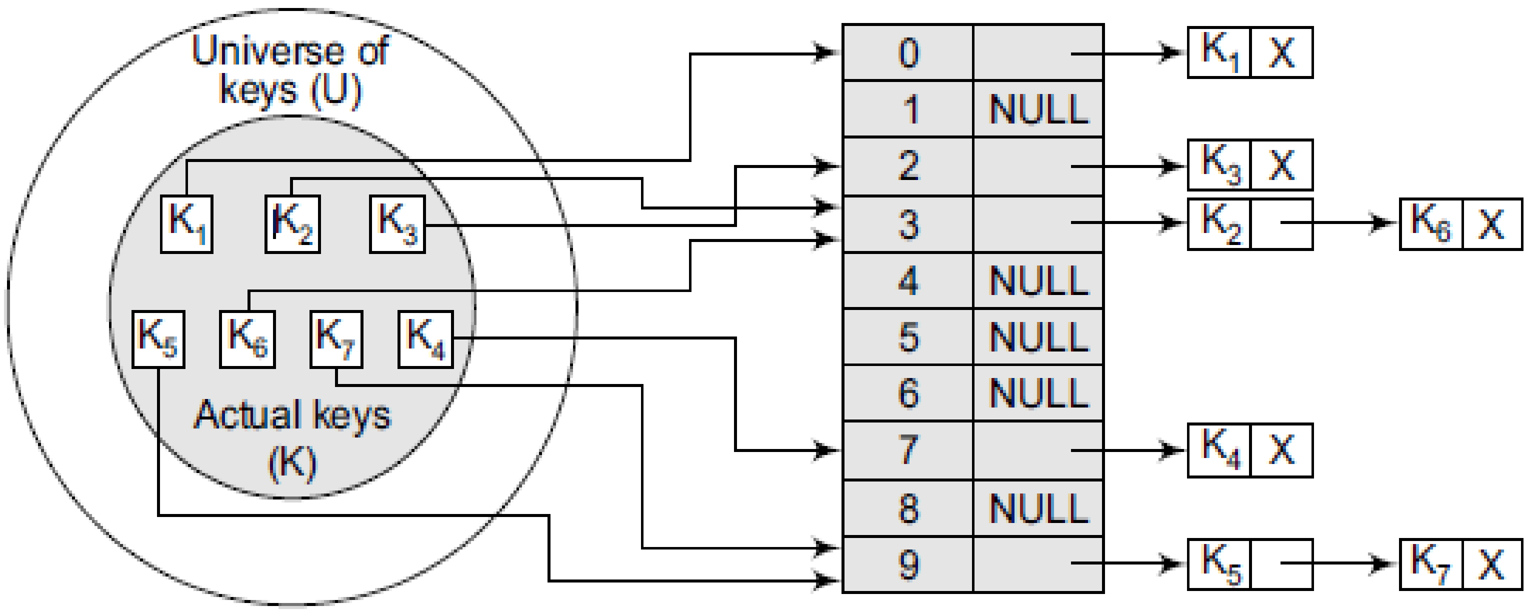
Hashing

Separate Chaining

- For inserting one element, we have to get the hash value through the hash function
- Hash value will be mapped in the hash table position then that element will be inserted in the Linked list
- Searching is also same

Hashing

Separate Chaining

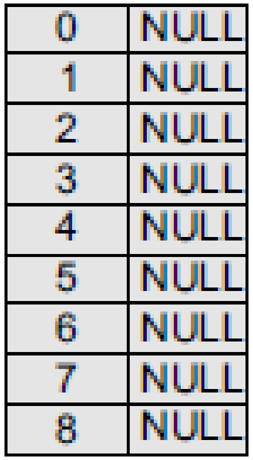


Separate Chaining

- Insert the keys 7, 24, 18, 52, 36, 54, 11, and 23 in a chained hash table of 9 memory locations. Use h(k) = k mod m.
- In this case, m=9

Separate Chaining

- Insert the keys 7, 24, 18, 52, 36, 54, 11, and 23 in a chained hash table of 9 memory locations. Use h(k) = k mod m.
- In this case, m=9
- Initially, the hash table can be given as



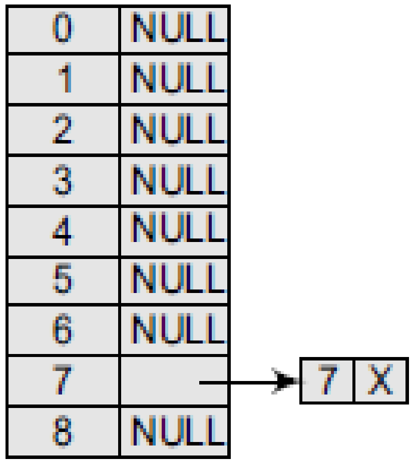
0	NULL
1	NULL
2	NULL
3	NULL
4	NULL
5	NULL
6	NULL
7	NULL
8	NULL

Separate Chaining

Step 1

```
Key = 7
h(k) = 7 mod 9
     = 7
```

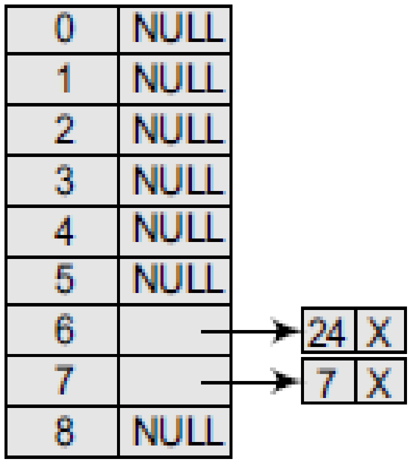
Create a linked list for location 7 and store the key value 7 in it as its only node.



Step 2

```
Key = 24
h(k) = 24 mod 9
     = 6
```

Create a linked list for location 6 and store the key value 24 in it as its only node.

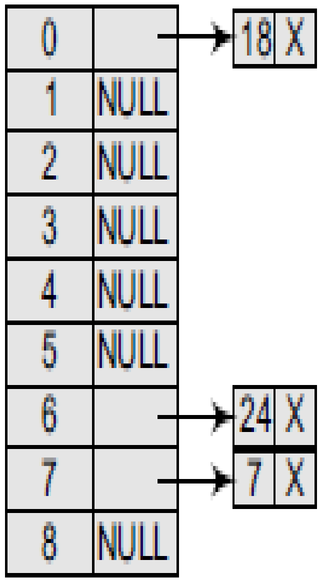


Separate Chaining

Step 3 Key = 18

h(k)= 18 mod 9 = 0

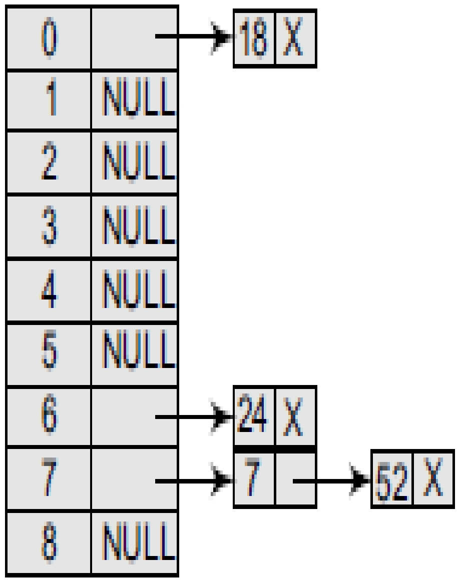
Create a linked list for location 0 and store the key value 18 in it as its only node.



Step 4 Key = 52

h(k) = 52 mod 9 = 7

Insert 52 at the end of the linked list of location 7.

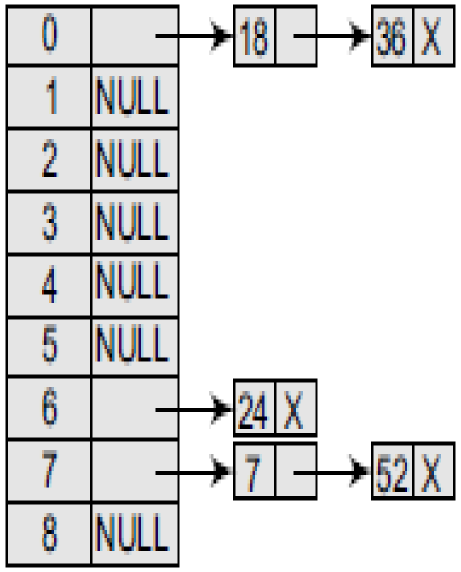


Separate Chaining

Step 5: Key = 36

h(k)= 36 mod 9 = 0

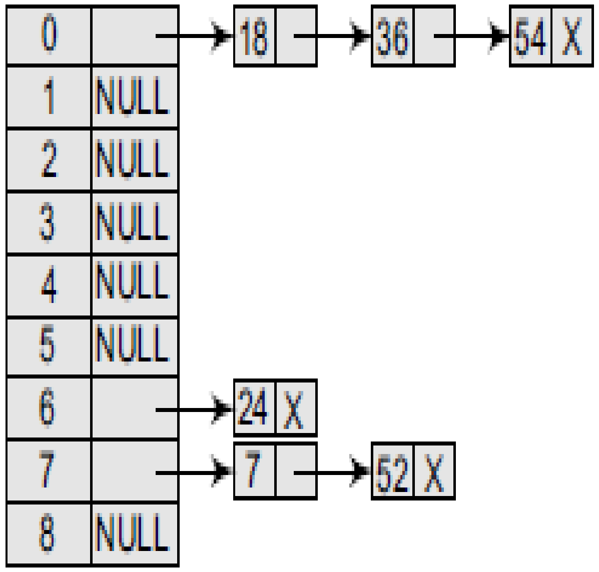
Insert 36 at the end of the linked list of location 0.



Step 6: Key = 54

h(k) = 54 mod 9 = 0

Insert 54 at the end of the linked list of location 0.

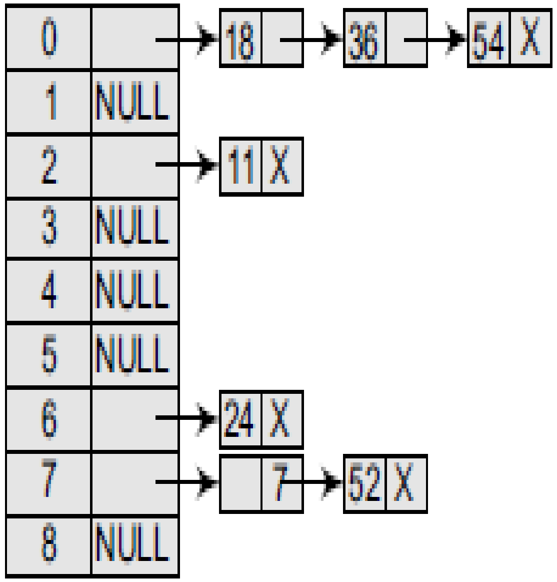


Separate Chaining

Step 7: Key = 11

$h(k) = 11 \bmod 9 = 2$

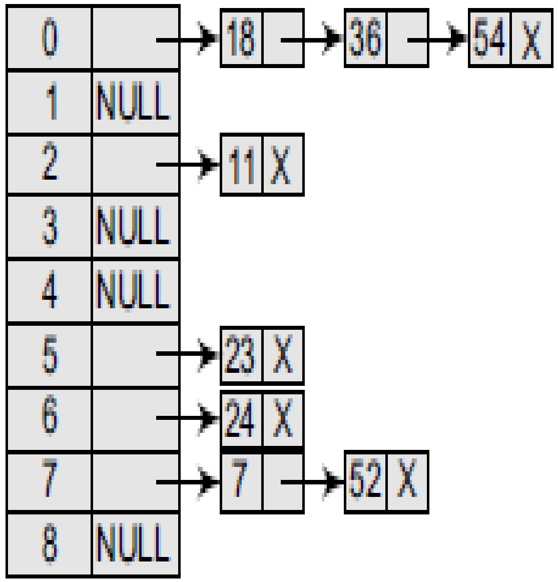
Create a linked list for location 2 and store the key value 11 in it as its only node.



Step 8: Key = 23

$h(k) = 23 \bmod 9 = 5$

Create a linked list for location 5 and store the key value 23 in it as its only node.



Hashing

Advantage of Separate Chaining

1) Simple to implement.

2) Hash table never fills up, we can always add more elements to the chain.

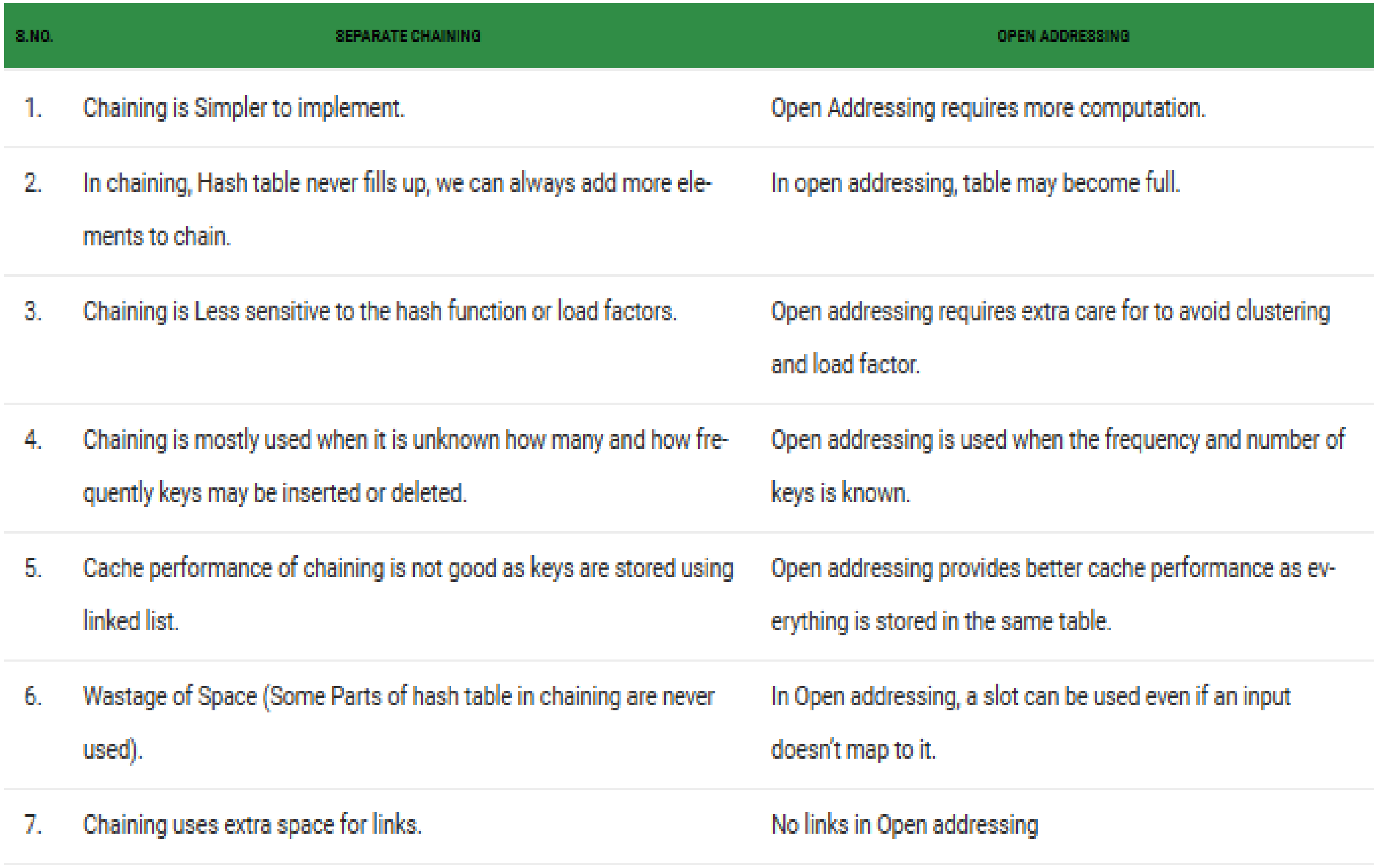
3) Less sensitive to the hash function

4) It is mostly used when it is unknown how many and how frequently keys may be inserted or deleted.

Hashing

Disadvantages of Separate Chaining

1) **Cache performance of chaining is not good as keys are stored using a linked list. Open addressing provides better cache performance as everything is stored in the same table.**
2) Wastage of Space (Some Parts of hash table are never used)
3) If the chain becomes long, then search time can become O(n) in the worst case.
4) Uses extra space for links.



S.NO.	SEPARATE CHAINING	OPEN ADDRESSING
1.	Chaining is Simpler to implement.	Open Addressing requires more computation.
2.	In chaining, Hash table never fills up, we can always add more elements to chain.	In open addressing, table may become full.
3.	Chaining is Less sensitive to the hash function or load factors.	Open addressing requires extra care for to avoid clustering and load factor.
4.	Chaining is mostly used when it is unknown how many and how frequently keys may be inserted or deleted.	Open addressing is used when the frequency and number of keys is known.
5.	Cache performance of chaining is not good as keys are stored using linked list.	Open addressing provides better cache performance as everything is stored in the same table.
6.	Wastage of Space (Some Parts of hash table in chaining are never used).	In Open addressing, a slot can be used even if an input doesn't map to it.
7.	Chaining uses extra space for links.	No links in Open addressing