Chapter 4

Friction

4.1 Introduction

We have so far dealt with smooth surfaces, which offer a single reaction force R. In this chapter we deal with rough surfaces which offer an additional reaction known as *friction force*. Friction force is developed whenever there is a motion or tendency of motion of one body with respect to the other body involving **rubbing** of the surfaces of contact. We will understand the concept of friction and also present the laws of friction. Application of friction to the problems of ***blocks, ladder***, *wedges, square threaded screw and belt friction* will be dealt with in this chapter.

4.2 Frictional Force

Friction is of two types i) *Dry Friction* ii) *Fluid Friction*. Dry friction also known as Coulomb friction involves friction due to rubbing of rigid bodies, for example a block tending to move on table or a wheel rolling on the ground. Fluid friction is developed between layers of fluids as they move with different velocities inside a pipe, bodies moving over lubricated surface, etc. Our study would be limited to Dry Friction.

Frictional force is generated whenever a body moves or tends to move over another surface. This is best illustrated by the following discussion. Consider a block of weight W resting on a rough surface. Let the normal reaction be N. The block is in equilibrium under the action of two forces. Fig. 4.1 (a)

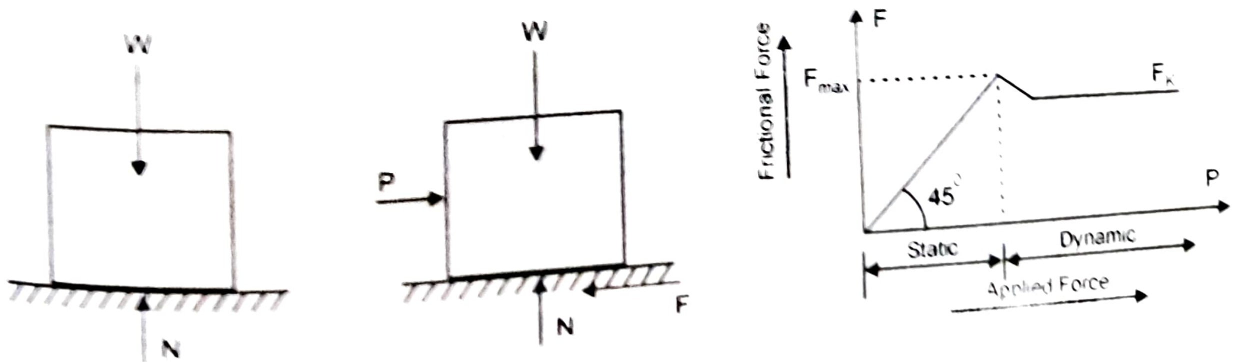


Fig. 4.1

Now if an attempt is made to disturb the equilibrium by applying a force P as shown in Fig. 4.1 (b), the rough surface generates a friction force F to maintain the equilibrium. The force F is equal to P and the block is maintained in equilibrium. If P is now increased the friction force F also increases {refer graph shown in Fig. 4.1 (c)}. However the surface can generate a maximum friction known as *limiting friction force* F_{max}. If P exceeds F_{max}, the body would be set into motion and the block is said to be in a *dynamic state*. In dynamic state, the same surfaces develop a lower friction force known as a *kinetic frictional force* F_k.

Friction force generated is generally due to presence of hills and valleys on any surface and to smaller extent due to molecular attraction. Fig. 4.2 shows a macroscopic view of two contacting surfaces enlarged 1000 times.

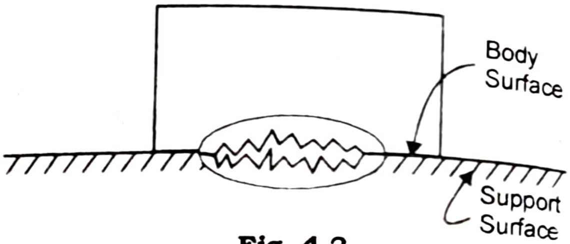


Fig. 4.2

When the body is in static condition, there is greater interlocking between the hills and the valleys of the two surfaces. As the body becomes dynamic, the interlocking reduces resulting in lowering the friction force to a new value F_k.

4.3 Coefficient of Friction

It has been experimentally found that the ratio of the limiting frictional force F_{max} and the normal reaction N is a constant. This constant is referred to as *coefficient of static friction*, denoted as μ_s.

$$\mu_s = \frac{F_{max}}{N} \qquad \text{......... 4.1}$$

Similarly the ratio of kinetic frictional force and the normal reaction is known as *coefficient of kinetic friction*, denoted as μ_k.

$$\mu_s = \frac{F_k}{N} \qquad \text{......... 4.2}$$

Since F_k is less than F_{max}, we have $\mu_k < \mu_s$

Coefficient of friction μ_s or μ_k depend on the nature of surfaces of contact and have value less than 1. For example the approximate value of coefficient of static friction (μ_s) between wood on glass ranges between 0.2 to 0.6. The corresponding coefficient of kinetic friction (μ_k) is around 25 % lower.

4.4 Laws of Friction

1. The frictional force is always tangential to the contact surface and acts opposite to the direction of impending motion.
2. The value of frictional force F increases as the applied disturbing force increases till it reaches the limiting value F_{max}. At this limiting stage the body is on the verge of motion.

3. The ratio of limiting frictional force F_{max} and the normal reaction N is a constant and it is referred to as coefficient of static friction (μ_s).
4. For bodies in motion, frictional force developed (F_k) is less than the limiting frictional force (F_{max}). The ratio of F_k and the normal reaction N is a constant and is referred to as coefficient of kinetic friction (μ_k).
5. The frictional force F generated between the two rubbing surfaces is independent of the area of contact.

4.5 Angle of Friction, Cone of Friction and Angle of Repose

Angle of Friction:

"It is the angle made by the resultant of the limiting frictional force F_{max} and the normal reaction N with the normal reaction". Fig. 4.3 shows a block of weight W under the action of the applied force P. Let N be the normal reaction. If the block is on the verge of impending motion, the frictional force F_{max} would be developed as shown.

Let R be the resultant of F_{max} and N, making an angle ϕ with the normal reaction. Here ϕ is known as the *angle of friction.*

Here $R = \sqrt{F_{max}{}^2 + N^2}$

$= \sqrt{(\mu_s N)^2 + N^2}$

also $\tan\phi = \dfrac{F_{max}}{N} = \dfrac{\mu_s N}{N}$

$\tan\phi = \mu_s$

$\therefore \quad \phi = \tan^{-1}\mu_s \quad \text{.........} \; \mathbf{4.3}$

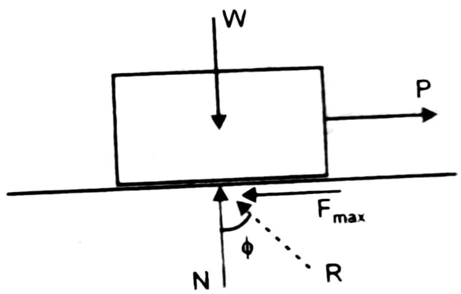


Fig. 4.3

Cone of Friction:

Fig. 4.4 shows a block of weight W on the verge of motion acted upon by force P. Let R be the resultant reaction at the contact surface acting at an angle of friction ϕ. If the direction of force is changed by rotating it through 360° in a plane parallel to the contact surface, the force R also rotates and generates a right circular cone of semi-central angle equal to ϕ. This right circular cone is known as the *cone of friction*. For a body to be stationary the reaction R should be within the cone of friction.

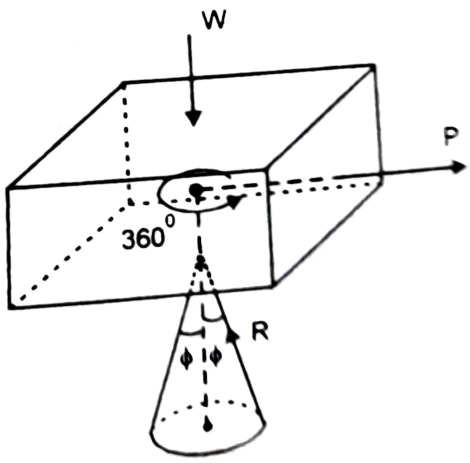


Fig. 4.4

Angle of Repose:

It is defined as the minimum angle of inclination of a plane with the horizontal for which a body kept on it will just slide on it without the application of any external force.

Consider a block of weight W resting on a rough horizontal plane. The plane is slowly tilted till the block is just on the verge of sliding down the plane, Fig. 4.5. The angle of inclination of the plane at this position is known as the *angle of repose*. It is denoted by letter α. Angle of repose is independent of the weight of the body and depends only on the coefficient of static friction. Let us derive the relation between angle of repose α and coefficient of static friction μ_s.

Applying COE

$\Sigma F_x = 0 \quad \nearrow +ve$

$F_{max} - W \sin\alpha = 0$

$\mu_s N - W \sin\alpha = 0 \quad$ --------- (1)

$\Sigma F_y = 0 \quad \nwarrow +ve$

$N - W\cos\alpha = 0$

$N = W\cos\alpha \quad$ --------- (2)

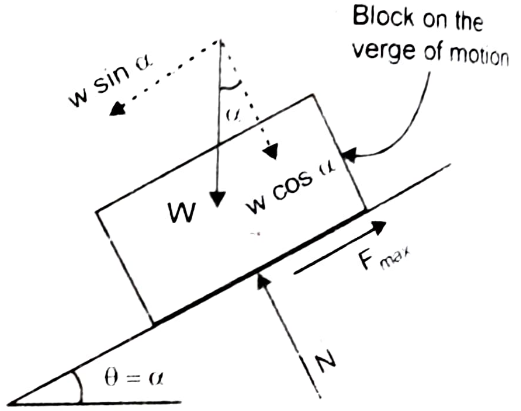


Fig. 4.5

Substituting (2) in (1)

$\mu_s(W\cos\alpha) - W\sin\alpha = 0$

$\tan\alpha = \mu$

or $\quad \alpha = \mathbf{tan^{-1}}\ \mu_s$ **4.4**

but we have seen that

$\phi = \tan^{-1}\mu_s$

$\therefore \quad \alpha = \phi$

i.e. Angle of Repose = Angle of Friction

Though magnitude wise both angle of repose and angle of friction have the same value, their meaning and application is different, as we have seen.

4.6 Problems on Blocks

We will encounter dimensionless blocks acted upon by forces tending to cause motion. The blocks can be treated as a particle forming a concurrent system of forces. The following steps are adopted in the solution of problems on blocks.

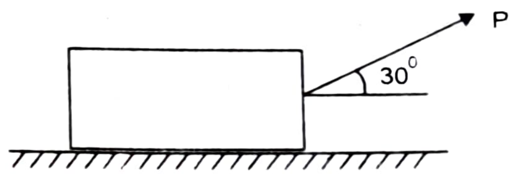
Step 1: Draw the FBD of the block showing the weight W, the normal reaction N, the friction force F and other applied forces. The direction of F is always directed opposite to the direction of impending motion. When the block is on verge of motion, $F = \mu_s N$ should be taken.

Step 2: Since we have a concurrent system of forces in equilibrium we apply two conditions of equilibrium viz.

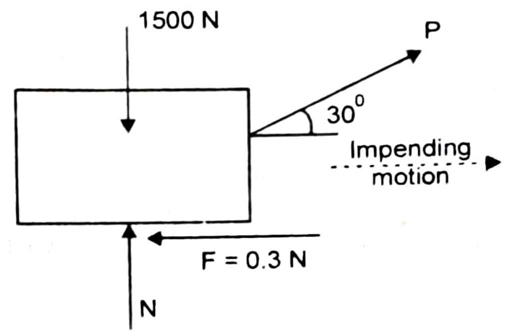
$\Sigma F_x = 0$

$\Sigma F_y = 0$

Ex. 4.1 A 1500 N block is kept on a rough horizontal surface having $\mu_s = 0.3$ and $\mu_k = 0.2$. Force P is applied as shown. Determine P for motion to just impend.



Solution: The FBD of the block is shown in the figure. Since the block is on the verge of motion, friction force $F = \mu_s N = 0.3\ N$.



Applying COE

$$\Sigma F_x = 0 \rightarrow +ve$$

$$P \cos 30 - 0.3\ N = 0 \quad \text{--------} \ (1)$$

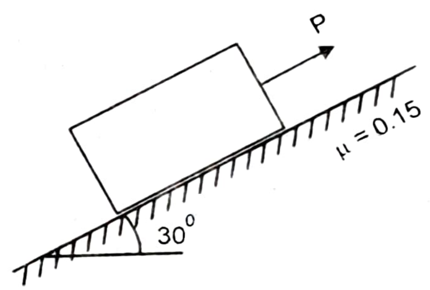
$$\Sigma F_y = 0 \uparrow +ve$$

$$N - 1500 + P \sin 30 = 0 \quad \text{--------} \ (2)$$

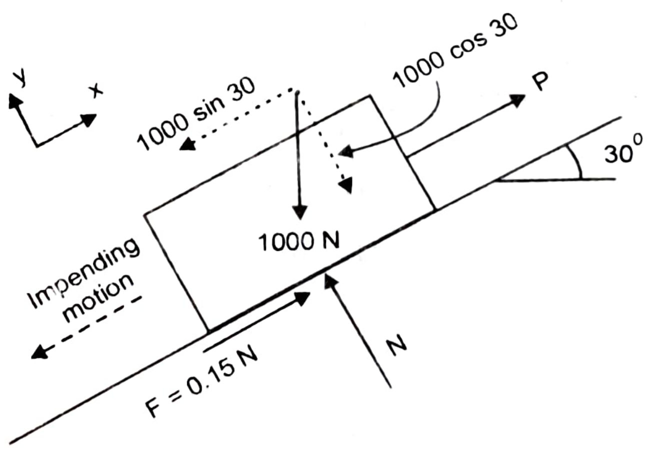
Solving equations (1) and (2)

$$P = 442.9\ N \quad \text{--------} \ \textbf{Ans.}$$

Ex. 4.2 A block of weight 1000 N is kept on a rough inclined surface. A force P is applied parallel to plane to keep the block in equilibrium. Determine range of values of P for which the block will be in equilibrium.



Solution: Since we have to find range of values of P for equilibrium, let us first find P_{min}, which would be just sufficient to prevent the block from moving down the plane. The friction force therefore acts up the plane.
Taking the axis as shown



Applying COE

$$\Sigma F_y = 0$$

$$N - 1000 \cos 30 = 0$$

$$N = 866\ N$$

$$\Sigma F_x = 0$$

$$P - 1000 \sin 30 + 0.15\ N = 0$$

$$P - 1000 \sin 30 + 0.15\,(866) = 0$$

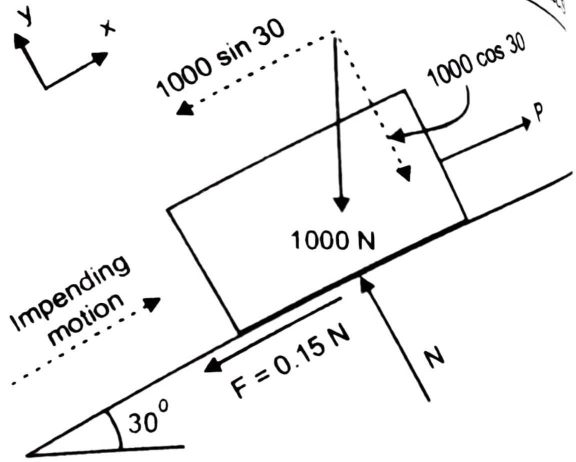
$$P_{min} = 370.1\ N$$

When P is maximum for equilibrium of the block, it tends to just cause the block to move up the plane, thereby the friction force acts down the plane.

Applying COE

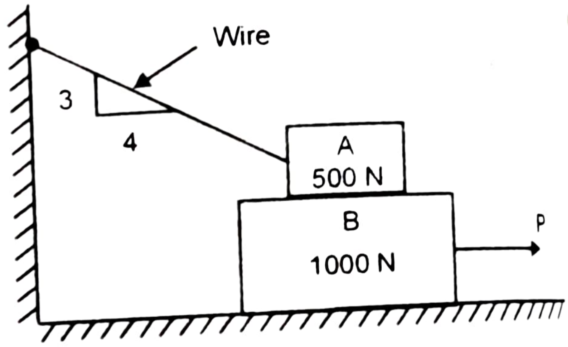
$$\Sigma F_y = 0$$
$$N - 1000 \cos 30 = 0$$
$$N = 866 \text{ N}$$

$$\Sigma F_x = 0$$
$$P - 1000 \sin 30 - 0.15 N = 0$$
$$P - 1000 \sin 30 - 0.15 (866) = 0$$
$$P_{max} = 629.9 \text{ N}$$



The block is in equilibrium within the range $370.1 \text{ N} \le P \le 629.9 \text{ N}$ .... **Ans.**

Ex. 4.3 The upper block is tied to a vertical wall by a wire. Determine the horizontal force P required to just pull the lower block. Coefficient of friction for all surfaces is 0.3



Solution: Figure shows the FBD of the entire system. We find there are three unknowns viz. P, N_1 and T, and we have only two equations of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ for dimensionless blocks. We will therefore have to isolate the two blocks.

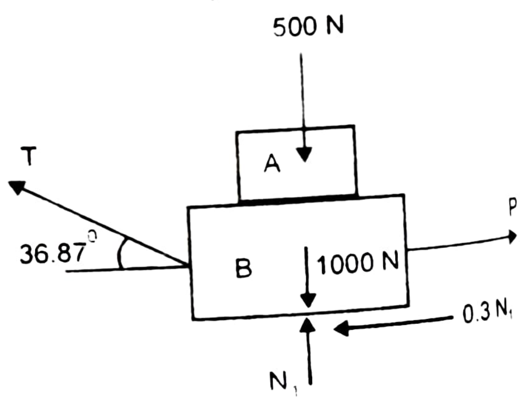


Figure shows the blocks isolated. Since block B tends to move to the right the friction force acts to the left. Hence for the block A, friction acts to the right.

Applying COE to block A

$\Sigma F_x = 0$

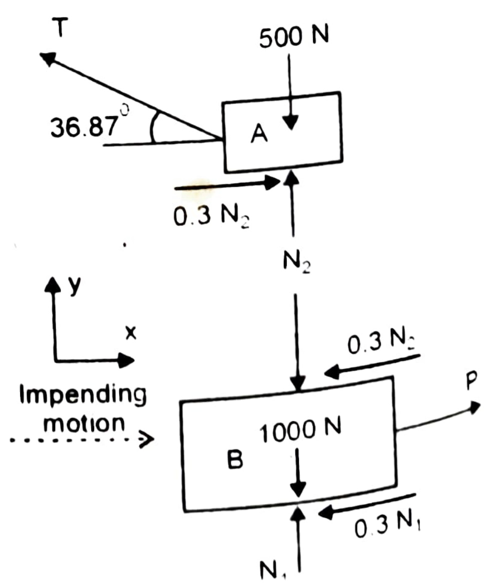
$-T \cos 36.87 + 0.3 N_2 = 0$ -------- (1)

$\Sigma F_y = 0$

$-500 + N_2 + T \sin 36.87 = 0$ -------- (2)

Solving equations (1) and (2), we get

$N_2 = 408.2 \text{ N}$



Applying COE to block B

$\Sigma F_y = 0$

$N_1 - N_2 - 1000 = 0$

$N_1 - 408.2 - 1000 = 0$

$N_1 = 1408.2 \text{ N}$

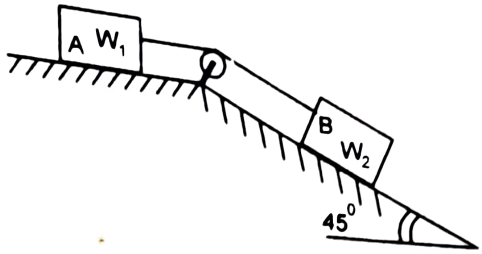
$\Sigma F_x = 0$

$P - 0.3 N_2 - 0.3 N_1 = 0$

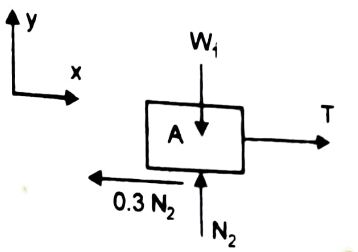
$P - 0.3 (408.2) - 0.3 (1408.2) = 0$

$P = 544.9 \text{ N}$ -------- **Ans.**

Ex. 4.4 Two blocks weighing W_1 and W_2 are connected by a string passing over a small smooth pulley as shown. If $\mu = 0.3$ for both the planes, find the minimum ratio W_1/W_2 required to maintain equilibrium.



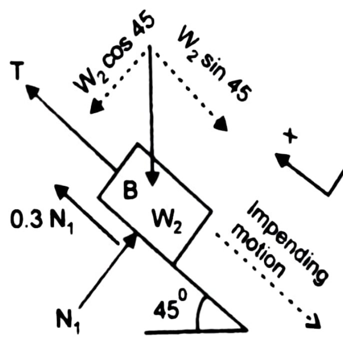
Solution: The component of weight of block B is responsible for causing motion of the system to impend down the plane.



Isolating the two blocks

Taking different axes for A and B as shown

Applying COE to block B



$$\Sigma F_y = 0$$
$$N_1 - W_2 \cos 45 = 0$$
$$N_1 = 0.707\ W_2 \quad \text{--------(1)}$$

$$\Sigma F_x = 0$$
$$T + 0.3\ N_1 - W_2 \sin 45 = 0$$
$$T + 0.3\ (0.707\ W_2) - W_2 \sin 45 = 0$$
$$T = 0.4949\ W_2 \quad \text{--------(2)}$$

Applying COE to block A

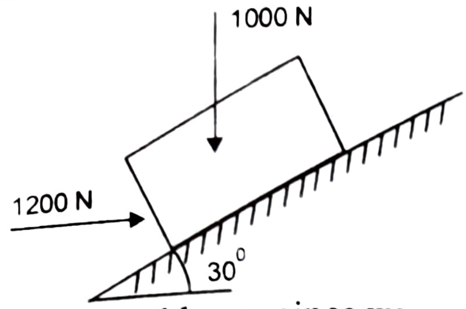
$$\Sigma F_y = 0$$
$$N_2 - W_1 = 0$$
$$\therefore \quad N_2 = W_1 \quad \text{--------(3)}$$

$$\Sigma F_x = 0$$
$$T - 0.3\ N_2 = 0$$

Substituting values of T and N_2

$$0.4949\ W_2 - 0.3\ W_1 = 0$$
$$\therefore \quad \frac{W_1}{W_2} = 1.65 \quad \text{----------- } \textbf{Ans.}$$

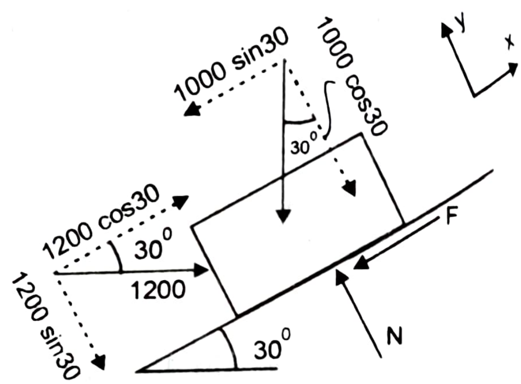
Ex. 4.5 If a horizontal force of 1200 N is applied to block of 1000 N, then block will be held in equilibrium or slide down or move up? Take $\mu = 0.3$.



Solution: This problem is different from the previous problems, since we are required to find out the state of the block i.e. whether it is in equilibrium or not. If not, in which direction it is moving.

Let F be the friction force acting down the plane be required to keep the block in equilibrium. Here we cannot take $F = \mu N$ because the block may not be on the verge of motion.

Taking the axes as shown



$$\Sigma F_y = 0$$

$$N - 1000 \cos 30 - 1200 \sin 30 = 0$$

$$N = 1466 \text{ N}$$

$$\Sigma F_x = 0$$

$$1200 \cos 30 - 1000 \sin 30 - F = 0$$

$F = 539.2$ .... $F_{required}$

$\therefore$ $F = 539.2$ N force is required to act down the plane to keep the block in equilibrium.

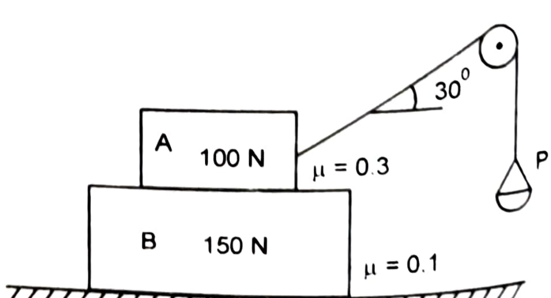
Now the maximum friction force the contact surface can produce

$= \mu N = 0.3 \times 1466$

$= 439.8$ N .... $F_{available}$

Since $F_{required} > F_{available}$ the block is not in equilibrium, but is moving up since F is directed down.

Ex. 4.6 Blocks A and B are resting on ground as shown. μ between ground and block is 0.1 and that between A and B is 0.3. Find the minimum value of P in the pan so that motion starts.



Solution: There are two possibilities. One is that block A moves over block B, while the other possibility is that both blocks A and B move together over the ground.

1st Possibility: Let block A move over block B

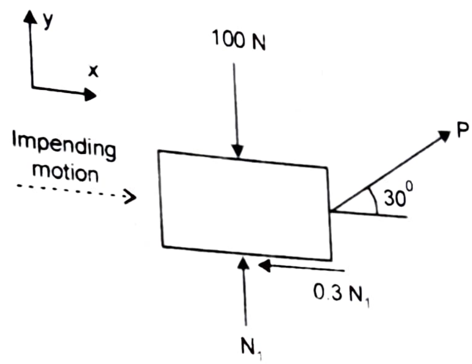
Applying COE

$\Sigma F_x = 0$

$P \cos 30 - 0.3\ N_1 = 0$ --------- (1)

$\Sigma F_y = 0$

$N_1 - 100 + P \sin 30 = 0$ --------- (2)



Solving equations (1) and (2)

$P = 29.53$ N

2nd Possibility: Both blocks A and B move together over the ground

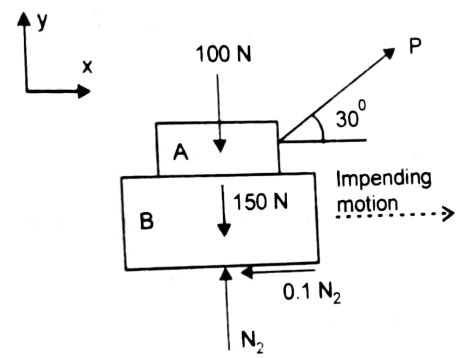
Applying COE

$\Sigma F_x = 0$

$P \cos 30 - 0.1\ N_2 = 0$ ----(3)

$\Sigma F_y = 0$

$N_2 - 100 - 150 + P \sin 30 = 0$ ----(4)

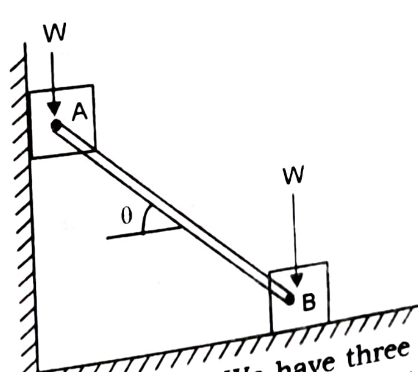


Solving equations (3) and (4)

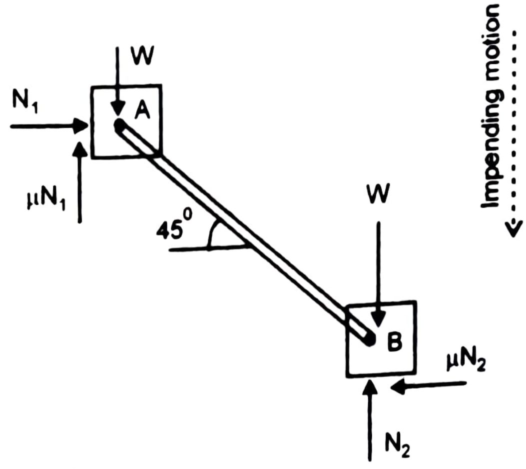
$P = 27.29$ N

Since P required to move A and B together over the ground is less than P required for A to move over B, the system is set in motion at P = 27.29 N with both blocks moving together. .... **Ans.**

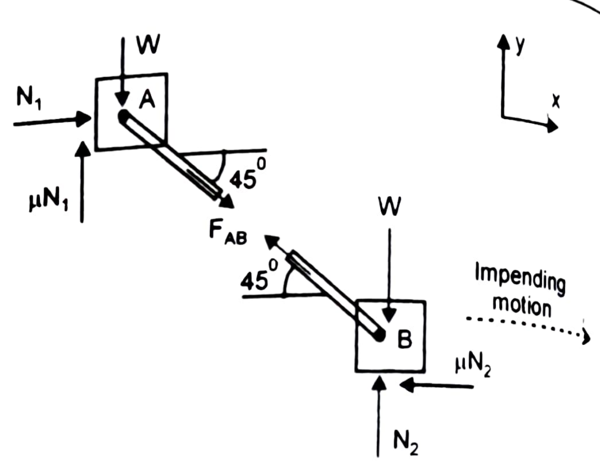
Ex.4.7 Two identical blocks A and B are pin-connected by rod as shown. If sliding impends when θ = 45°, determine μ , assuming it to be same at both floor and wall.



Solution: Figure (a) shows the FBD of the entire system. We have three unknowns N_1, N_2 and μ and two COE. We will therefore have to isolate the blocks by cutting the rod as shown in figure (b). Let F_{AB} be the force in the rod, assuming it to be of tensile nature.



(a)



(b)

Applying COE to block A

$$\Sigma F_x = 0$$

$$N_1 + F_{AB} \cos 45 = 0 \qquad \text{-------- (1)}$$

$$\Sigma F_y = 0$$

$$\mu N_1 - W - F_{AB} \sin 45 = 0 \qquad \text{-------- (2)}$$

Eliminating N_1 from equations (1) and (2) we get,

$$W = - F_{AB} \cos 45 \, (1 + \mu) \qquad \text{--------- (A)}$$

Applying COE to the block B

$$\Sigma F_x = 0$$

$$- F_{AB} \cos 45 - \mu N_2 = 0 \quad \text{---------- (3)}$$

$$\Sigma F_y = 0$$

$$N_2 - W + F_{AB} \sin 45 = 0 \quad \text{---------- (4)}$$

Eliminating N_2 from (3) and (4) we get,

$$W = - F_{AB} \cos 45 \left(\frac{1}{\mu} - 1\right) \qquad \text{--------- (B)}$$

Equating (A) and (B)

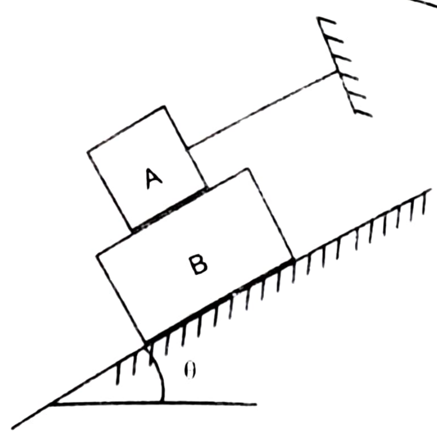
$$- F_{AB} \cos 45 \, (1 + \mu) = - F_{AB} \cos 45 \left(\frac{1}{\mu} - 1\right)$$

$$1 + \mu = \frac{1}{\mu} - 1$$

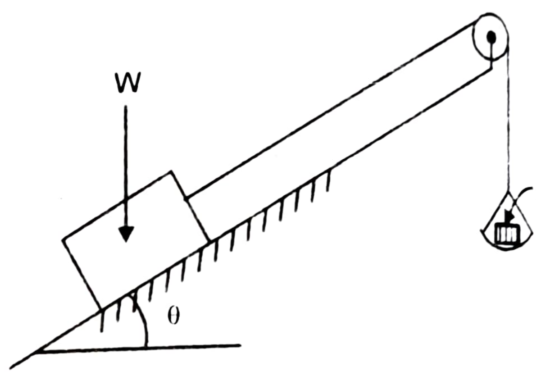
$$\mu^2 + 2\mu - 1 = 0$$

$$\mu = 0.414 \qquad \text{-------- Ans.}$$

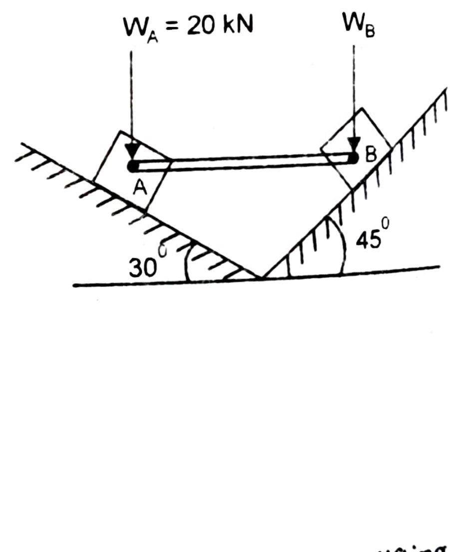
P14. What should be the value of angle θ for the motion of block B weighing 90 N to impend down the plane. The coefficient of friction for all surfaces of contact is 1/3. Block A weighs 30 N.



P15. Figure shows a weight W resting on a rough inclined plane having an angle of friction ϕ ($\theta > \phi$). It is connected to a pan of negligible weight by a string passing over a smooth pulley. Find the minimum value of weight P in the pan for equilibrium.



P16. Find the maximum value of W_B for the rod AB to remain horizontal. Also find the corresponding axial force in the rod.
Take μ = 0.2 for all contact surfaces.



4.7 Wedges

Wedges are tapering shaped devices which are used for lifting or shiftin[g] to little extent heavy blocks, machinery, pre-cast beams and columns et[c]. Usually two or more wedges are used in combination. Fig. 4.6 shows tw[o] arrangements of wedges. In Fig. 4.6 (a) two wedges are used for imparting small vertical movement to heavy machinery of weight W. Effort P is applied t[o] the wedge B which is driven inside causing the vertical movement. In Fig. 4.6 (b) two wedges are used in combination to cause a small horizontal movement t[o] the heavy block of weight W. Effort P is applied to wedge B, driving it down thereby causing horizontal movement of the heavy block.

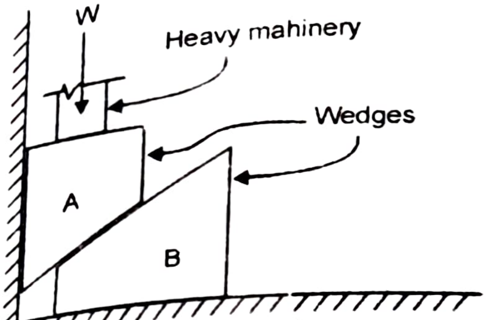


Fig. 4.6 (a)

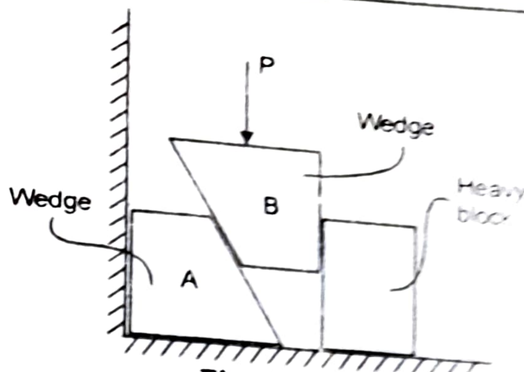
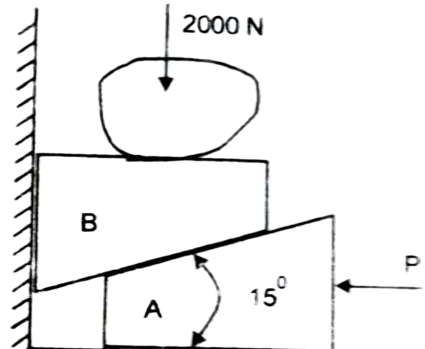


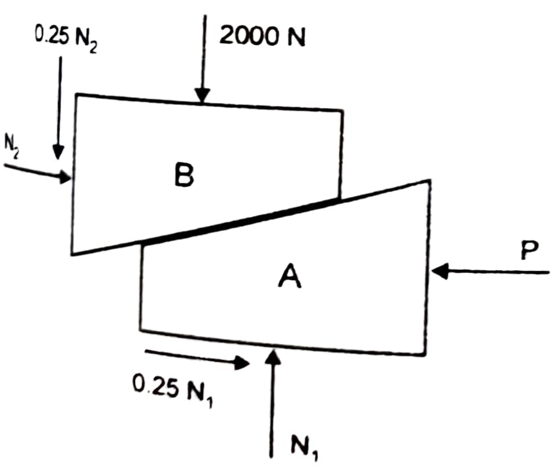
Fig. 4.6 (b)

For solving problems on wedges, we require to isolate the wedges and then apply COE to each wedge separately. Since the dimensions are neglected, wedges are treated as particles forming a concurrent force system. Only two COE viz. $\Sigma F_x = 0$ and $\Sigma F_y = 0$ are therefore useful.

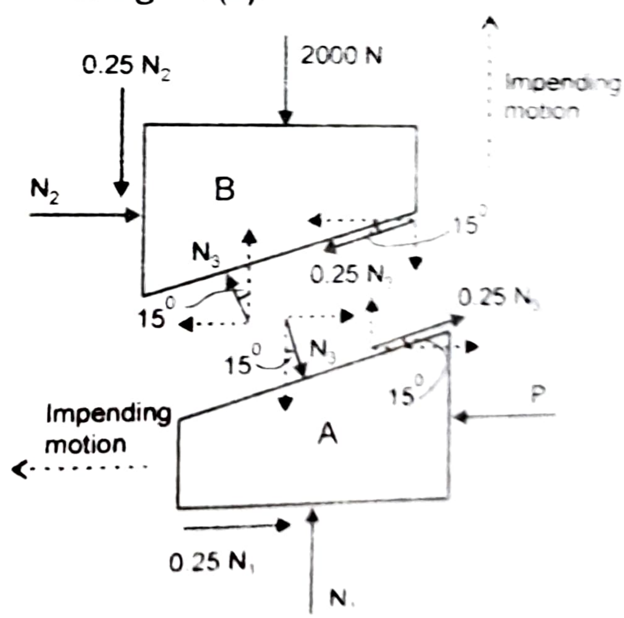
Ex. 4.8 To raise a heavy stone block weighing 2000 N, the arrangement shown is used. What horizontal force P is necessary to be applied to the wedge in order to raise the block. μ = 0.25. Neglect the weight of the wedges.



Solution: Figure (a) shows the FBD of the entire system. We find there are three unknowns N_1, N_2 and P and we have two COE viz. $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Hence we will have to isolate the wedges as shown in figure (b).



(a)



(b)

Applying COE to wedge B

$\Sigma F_x = 0$

$N_2 - N_3 \sin 15 - 0.25\ N_3 \cos 15 = 0$ ------- (1)

$\Sigma F_y = 0$

$N_3 \cos 15 - 0.25\, N_3 \sin 15 - 0.25\, N_2 - 2000 = 0$ -------- (2)

Solving equations (1) and (2)

$N_3 = 2576.6$ N

Applying COE to wedge A

$\Sigma F_y = 0$

$N_1 - N_3 \cos 15 + 0.25\, N_3 \sin 15 = 0$

$N_1 - 2576.6 \cos 15 + 0.25\,(2576.6) \sin 15 = 0$

$\therefore$ $N_1 = 2322$ N

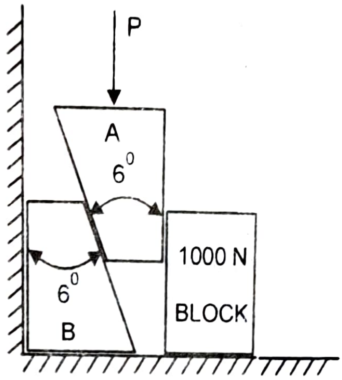
$\Sigma F_x = 0$

$-P + 0.25\, N_1 + N_3 \sin 15 + 0.25\, N_3 \cos 15 = 0$

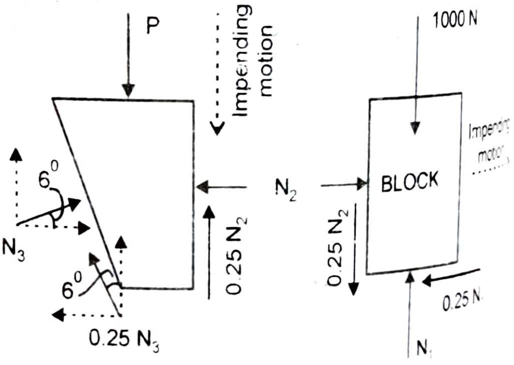
$-P + 0.25\,(2322) + 2576.6 \sin 15 + 0.25\,(2576.6) \cos 15 = 0$

$\therefore$ $P = 1869.6$ N .... **Ans.**

Ex. 4.9 Two 6° wedges are used to push the block horizontally as shown. Calculate the minimum force P required to push the block of weight 10000 N. $\mu = 0.25$ for all surfaces.



Solution: Figure below shows the FBD of the isolated wedge A and the block.



Applying COE to block

$\Sigma F_x = 0$

$N_2 - 0.25\, N_1 = 0$ -------- (1)

$\Sigma F_y = 0$

$N_1 - 10000 - 0.25\, N_2 = 0$ -------- (2)

Solving equations (1) and (2)

$N_2 = 2666.7$ N

Applying COE to wedge A

$\Sigma F_x = 0$

$N_3 \cos 6 - 0.25\, N_3 \sin 6 - N_2 = 0$

$N_3 \cos 6 - 0.25\, N_3 \sin 6 - 2666.7 = 0$

$N_3 = 2753.7$ N

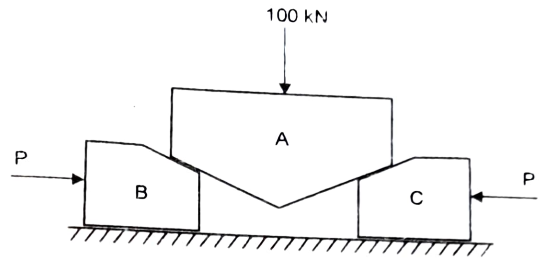
$\Sigma F_y = 0$

$-P + N_3 \sin 6 + 0.25\, N_3 \cos 6 + 0.25\, N_2 = 0$

$-P + 2753.7 \sin 6 + 0.25\,(2753.7) \cos 6 + 0.25\,(2666.7) = 0$

$P = 1639.2$ N.......... **Ans.**

Ex. 4.10 Calculate the magnitude of the horizontal force P acting on the wedges B and C to raise a load of 100 kN resting on A. μ between wedge and ground is 0.25 and between wedges and A is 0.2. Also assume symmetry of loading and neglect the weights of A, B, and C. Slope of wedges are 1:10.



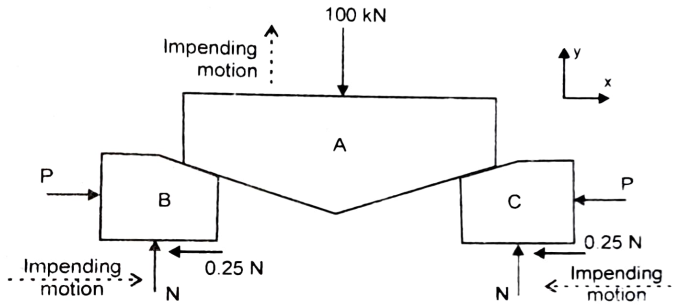
Solution: Figure (a) shows the FBD of the entire system. Because of symmetry of loading, the normal reaction offered by the ground on both the wedges is same.

Applying COE to the entire system

$\Sigma F_y = 0$

$N + N - 100 = 0$

$N = 50$ kN



(a)

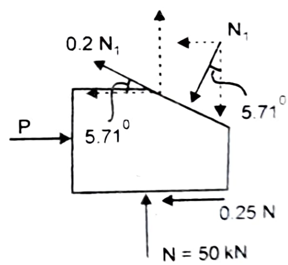
Isolating wedge B as shown in figure (b)

Slope : 1:10

$\therefore$ $\tan\theta = 1/10$

$\therefore$ $\theta = 5.71°$

Applying COE.



(b)

$\Sigma F_y = 0$

$50 + 0.2\, N_1 \sin 5.71 - N_1 \cos 5.71 = 0$

$\therefore$ $N_1 = 51.27$ N

$\Sigma F_x = 0$

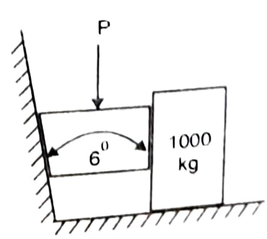
$P - 0.2\, N_1 \cos 5.71 - N_1 \sin 5.71 - 0.25\, N = 0$

$P - 0.2\, (51.27) \cos 5.71 - (51.27) \sin 5.71 - 0.25\, (50) = 0$

$\therefore$ $P = 27.8$ kN .... **Ans.**

Excercise 4.2

P1. The horizontal position of the 1000 kg block is adjusted by 6° wedge. If coefficient of friction for all surfaces is 0.6, determine the least value of force P required to move the block.

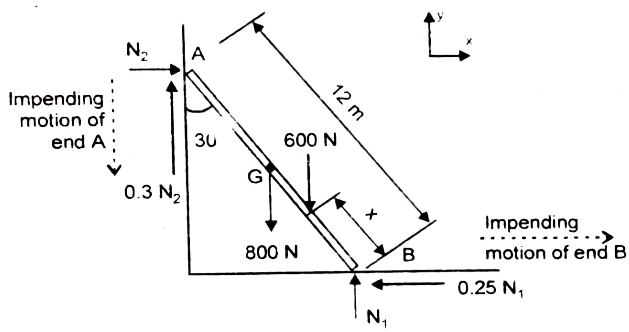


4.8. Problems on Ladders

Forces acting on ladder viz. the normal reactions and friction force at the floor and wall contact points, the weight of the ladder, the weight of the person climbing the ladder and any other force, form a non-concurrent force system. It will therefore require all the three COE viz. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$ for analysing the equilibrium of a ladder.

Ex. 4.11 A 12 m ladder is resting against a vertical wall making 30° angle with the wall. Static friction between wall and ladder is 0.3 and that between ground and ladder is 0.25. A 600 N man ascends the ladder. How high will he be able to go before the ladder slips? Assume the weight of the ladder to be 800 N.

Solution: Figure shows the FBD of ladder AB resting against the wall and floor. Let the person climb the distance x on the ladder when the ladder is on the verge of slipping. The weight of the ladder acts through its C.G.



Applying COE to the ladder

$$\Sigma F_x = 0$$
$$N_2 - 0.25\, N_1 = 0 \qquad \text{------ (1)}$$

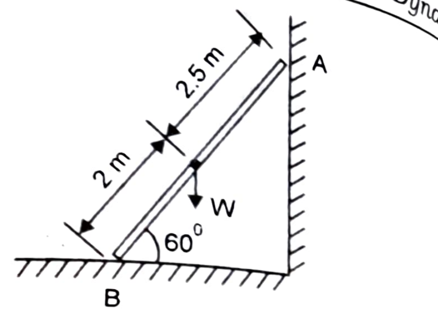
$$\Sigma F_y = 0$$
$$0.3\, N_2 + N_1 - 800 - 600 = 0 \qquad \text{------ (2)}$$

Solving equations (1) and (2)

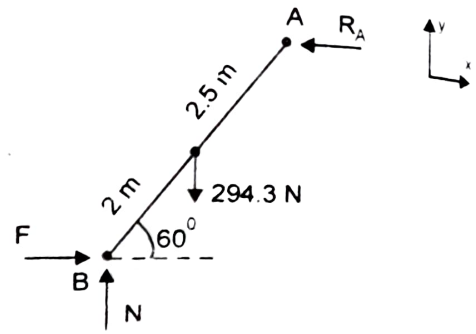
$$N_1 = 1302.4 \text{ N},$$
$$N_2 = 325.6 \text{ N}$$

$$\Sigma M_B = 0 \quad \circlearrowleft +\text{ve}$$
$$-N_2 \times 12 \cos 30 - 0.3\, N_2 \times 12 \sin 30 + 800 \times 6 \sin 30 + 600 \times x \sin 30 = 0$$
$$\therefore \quad x = 5.23 \text{ m} \qquad \text{............ } \textbf{Ans.}$$

Ex. 4.12 A non-homogeneous ladder shown rests against a smooth wall at A and a rough horizontal floor at B. The mass of the ladder is 30 kg and is concentrated at 2 m from the bottom. μ_s between ladder and floor is 0.35. Will the ladder stand in 60° position as shown?



Solution: In this problem the state of the ladder is unknown. Hence we cannot take $F = \mu_s N$. Let F be the friction force required at the ground to prevent the ladder from slipping. Since the wall at A is smooth it offers a single reaction force R_A.



Applying COE to the ladder

$$\Sigma F_x = 0$$
$$F - R_A = 0 \quad \text{-----------} \ (1)$$

$$\Sigma F_y = 0$$
$$N - 294.3 = 0$$
$$N = 294.3 \text{ N}$$

$$\Sigma M_B = 0 \quad +ve$$
$$-294.3 \times 2 \cos 60 + R_A \times 4.5 \sin 60 = 0$$
$$\therefore \quad R_A = 75.51 \text{ N}$$

Substituting in equation (1)

$$F = 75.51 \text{ N} \text{ ------------- } F_{required}$$

The maximum friction force the ground can produce

$$F = \mu_s N$$
$$= 0.35 \times 294.3$$
$$F = 103 \text{ N} \text{ ----------- } F_{available}$$

Since $F_{required} < F_{available}$ the ladder is in equilibrium and will stand in the 60° position.