# * Z - Transforms

$$\boxed{Z(U_n) = \sum_{n=-\infty}^{\infty} U_n z^{-n} = \bar{U}(z)} \quad \text{———} \quad (1)$$

where $\bar{U}(z)$ is the Z-transform of $U_n$ & $z$ is a complex number.

Z-transform exists only when the infinite series in (1) is convergent

The sequence $U_n$ is called the inverse Z-transform of $\bar{U}(z)$ & is written as $Z^{-1}[\bar{U}(z)] = U_n$

Q] Show that $Z[a^n] = \dfrac{z}{z-a}$, $n \geq 0$

→

$$Z(U_n) = \sum_{n=0}^{\infty} U_n z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

$$= 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \cdots$$

Infinite series

$$\left[ 1 + r + r^2 + \ldots = \frac{1}{1-r} \right]$$

$$= \frac{1}{1 - \frac{a}{z}}$$

$$z[U_n] = \frac{z}{z-a}$$

Q) Show that z-transform of

$Z[n^p] = -z \frac{d}{dz}(z(n^{p-1}))$, $n \geq 0$ Also find $z(n)$ & $z(n^2)$

→

$$z(U_n) = \sum_{n=0}^{\infty} U_n z^{-n}$$

$$z(n^{p-1}) = \sum_{n=0}^{\infty} n^{p-1} z^{-n}$$

$$\frac{d}{dz}(z(n^{p-1})) = \sum_{n=0}^{\infty} n^{p-1} \cdot -n z^{-n-1}$$

$$= -\sum_{n=0}^{\infty} n^p \frac{z^{-n}}{z}$$

$$= \frac{-1}{z} \sum_{n=0}^{\infty} n^p z^{-n}$$

Inverse

$$= -\frac{1}{z} \sum_{n=0}^{\infty} n^p$$

$$\frac{d}{dz} z(n^{p-1}) = -\frac{1}{z} z[n^p]$$

$$\therefore \quad z(n^p) = -z \frac{d}{dz} z(n^{p-1})$$

$p = 1$

$$z(n) = -z \frac{d}{dz}(z(1))$$

$$= -z \frac{d}{dz}\left(\frac{z}{z-1}\right)$$

$$= -z \frac{((z-1)\cdot 1 - z)}{(z-1)^2}$$

$$z(n) = \frac{z}{(z-1)^2}$$

$z = 2$

$$z(n^2) = -z \frac{d}{dz}(z(n)$$

$$= \frac{z^2 + z}{(z-1)^3}$$

# * Find Z-Transform of the foll.

① Unit Impulse

$$f(n) = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

② Discrete unit step

$$f(n) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

Q] Find z-transform of foll. sequence

$$U(n) = \{\overset{-3}{3}, \overset{-2}{7}, \overset{-1}{5}, \underset{\uparrow}{\overset{0}{8}}, \overset{1}{0}, \overset{2}{1}, \overset{3}{2}, \overset{4}{3}\}$$

I

→

$$U(n) = \sum_{n=-3}^{4} U(n) z^{-n}$$

$$= U(-3)z^{3} + U(-2)z^{2} + U(-1)z + U(0) + U(1)z^{-1} + U(2)z^{-2} + U(3)z^{-3} + U(4)z^{-4}$$

$$= 3z^{3} + 7z^{2} + 5z + 8 + 0 + \frac{1}{z^2} + \frac{2}{z^3} + \frac{3}{z^4}$$

Q] Find z-transform of foll. sequence

II

$$U(n) = \{\overset{0}{3}, \overset{1}{7}, \overset{2}{5}, \overset{3}{8}, \overset{4}{0}, \overset{5}{1}, \overset{6}{2}, \overset{7}{3}\}$$

→ $$\bar{U}(z) = \sum_{n=0}^{7} U(n) z^{-n}$$

$$= U(0) + \frac{U(1)}{z} + \frac{U(2)}{z^2} + \frac{U(3)}{z^3} + \frac{U(4)}{z^4} + \frac{U(5)}{z^5} + \frac{U(6)}{z^6} + \frac{U(7)}{z^7}$$

$$= 3 + \frac{7}{z} + \frac{5}{z^2} + \frac{8}{z^3} + 0 + \frac{1}{z^5} + \frac{2}{z^6} + \frac{3}{z^7}$$

Q) Find $Z(a^n \cos n\theta)$

$\rightarrow$ $$Z(u_n) = \sum_{n=0}^{\infty} a^n \cos n\theta \, z^{-n} \quad , \; n \geq 0$$

$$= \sum_{n=0}^{\infty} a^n \left[ \frac{e^{in\theta} + e^{-in\theta}}{2} \right] z^{-n}$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \left( \frac{ae^{i\theta}}{z} \right)^n + \frac{1}{2} \sum_{n=0}^{\infty} \left( \frac{ae^{-i\theta}}{z} \right)^n$$

$$= \frac{1}{2} \left( \frac{1}{1 - \frac{ae^{i\theta}}{z}} \right) + \frac{1}{2} \left( \frac{1}{1 - \frac{ae^{-i\theta}}{z}} \right)$$

$$= \frac{1}{2} \left[ \frac{z}{z - ae^{i\theta}} + \frac{z}{z - ae^{-i\theta}} \right]$$

$$= \frac{z}{2} \left[ \frac{z - ae^{-i\theta} + z - ae^{i\theta}}{z - ae^{i\theta} \cdot z - ae^{i\theta}} \right]$$

$$= \frac{z}{2} \left[ \frac{2z - a(e^{i\theta} + e^{-i\theta})}{z^2 - za(e^{i\theta} + e^{-i\theta}) + a^2} \right]$$

$$= \frac{z^2 - az\cos\theta}{z^2 - 2az\cos\theta + a^2}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\Rightarrow e^{i\theta} + e^{-i\theta} = 2\cos\theta$$

Q] Find $z(a^n \sin n\theta)$ , $n \geq 0$

$\rightarrow z(U_n) = \sum_{n=0}^{\infty} a^n \sin n\theta \, z^{-n}$

$\sin n\theta = \frac{e^{in\theta} - e^{-in\theta}}{2i}$

$$= \sum_{n=0}^{\infty} \frac{a^n}{z} \left[ \frac{e^{in\theta} - e^{-in\theta}}{2i} \right]$$

$$= \frac{1}{2i} \sum_{n=0}^{\infty} \left( \frac{ae^{i\theta}}{z} \right)^n - \frac{1}{2i} \sum_{n=0}^{\infty} \left( \frac{ae^{-i\theta}}{z} \right)^n$$

$$= \frac{1}{2i} \left( \frac{z}{z - ae^{i\theta}} - \frac{z}{z - ae^{-i\theta}} \right)$$

$$= \frac{z}{2i} \left[ \frac{\cancel{z} - ae^{-i\theta} - \cancel{z} + ae^{i\theta}}{(z - ae^{i\theta})(z - ae^{-i\theta})} \right]$$

$$= \frac{z}{2i} \left[ \frac{a(e^{i\theta} - e^{-i\theta})}{z^2 - za(e^{i\theta} + e^{-i\theta}) + a^2} \right]$$

$$= \frac{z}{\cancel{2i}} \left[ \frac{\cancel{2i} \sin\theta}{z^2 - z\,a\cos\theta + a^2} \right]$$

$e^{i\theta} - e^{-i\theta} = 2\sin\theta$

$$= \frac{z \sin\theta}{z^2 - za\cos\theta + a^2}$$

Q] Find Z-Transform of

$$U_n = \begin{cases} 5^n , & n < 0 \\ 3^n , & n > 0 \end{cases}$$

$\rightarrow z(U_n) = \sum_{n=-\infty}^{\infty} U_n z^{-n}$

$$Z(U_n) = \sum_{n=-\infty}^{-1} 5^n z^{-n} + \sum_{n=0}^{\infty} 3^n z^{-n}$$

$$= \left[ \left( \cdots + \frac{z^2}{5^2} + \frac{z}{5} \right) + \left( 1 + \frac{3}{z} + \frac{3^2}{z^2} + \cdots \right) \right]$$

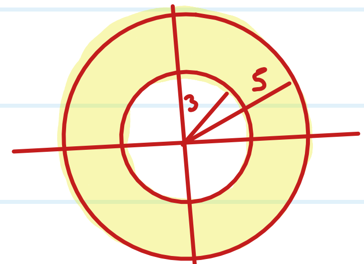
$$= \frac{z/5}{1 - z/5} + \frac{1}{1 - 3/z}$$

$$\left[ \left|\frac{z}{5}\right| < 1 \;\&\&\; \left|\frac{3}{z}\right| < 1 \right]$$

$$\boxed{Z(U_n) = \frac{z}{5 - z} + \frac{z}{z - 3}}$$

$|z| < 5$ && $|z| > 3$

$\therefore \quad 3 < |z| < 5$




ROC

* Properties of Z-Transform

① Change of scale or Domping Rule

if $Z[U_n] = \bar{U}(z)$

if $Z[a^{-n} U_n] = \bar{U}(az)$

if $Z[a^n U_n] = \bar{U}\left(\frac{z}{a}\right)$

Q] Find z-Transform of $na^n$ , $n \geq 0$

$z(na^n)$

$$z(n) = \frac{z}{(z-1)^2}$$

$$z(na^n) = \frac{a/z}{(a/z - 1)^2} = \frac{az}{(a-z)^2}$$

## * Shifting Property

$$Z(U_n) = \bar{U}(z)$$

$$z(U_{n-k}) = z^{-k}\left[\bar{U}(z) + \sum_{r=1}^{k} U_r z^r\right]$$

$$z(U_{n+k}) = z^{k}\left[\bar{U}(z) - U_0 - \frac{U_1}{z} + \dots - \frac{U_{k-1}}{z^{k-1}}\right]$$

If $z[f(k)] = F(z)$

then $z[f(k \pm n)] = z^{\pm n} F(z)$

Q] Find z-Transform of $(n-1)^2$, $n \geq 0$

$\rightarrow$ $$z(n^2) = \frac{z^2+z}{(z-1)^3} = U(z)$$

$$z(U_{n-1}) = z^{-1}\left[\bar{U}(z) + U_{-1} z^1\right]$$

$U(n) = n^2$
$U(-1) = (-1)^2 = 1$

$$= z^{-1}\left[\frac{z^2+z}{(z-1)^3} + 1 \cdot z\right] = \frac{z+1}{(z-1)^3} + 1 = \frac{z^3 - 3z^2 + 4z}{(z-1)^3}$$

## Properties of z-Transform

### Change of scale

$$Z[a^k f(k)] = F\left(\frac{z}{a}\right)$$

### Shifting

$$Z[f(k+n)] = z^n F(z)$$

$$Z[f(k-n)] = z^{-n} F(z).$$

### Multiplication by k

Imp $$Z[k \cdot f(k)] = -z \frac{d}{dz} F(z).$$

$$Z[k^n f(k)] = \left(-z \frac{d}{dz}\right)^2 F(z).$$

### Division by k

$$Z\left[\frac{f(k)}{k}\right] = \int_z^\infty \frac{1}{z} \cdot F(z)\, dz.$$

### Convolution

$$Z[f(k) \cdot g(k)] = Z[f(k)] \times Z[g(k)]$$

$$Z[U(k)] = \frac{z}{z-1}$$

$$Z[a^k] = \frac{z}{z-a}$$

$$Z[\sin\alpha k] = \frac{z \sin\alpha}{z^2 - 2z\cos\alpha + 1}$$

$$Z[\sinh\alpha k] = \frac{z \cdot \sinh\alpha}{z^2 - 2z\cosh\alpha + 1}$$

$$Z[\cos\alpha k] = \frac{z^2 - z\cos\alpha}{z^2 - 2z\cos\alpha + 1}$$

$$Z[\cosh\alpha k] = \frac{z^2 - z\cosh\alpha}{z^2 - 2z\cosh\alpha + 1}$$

Q) Find Z-Transform of $U_n = \frac{1}{n!}$

→ $U_n = \frac{1}{n!}$ , $U_{n+1} = \frac{1}{(n+1)!}$

$$z(U_n) = \sum_{n=0}^{\infty} U_n z^{-n} = \sum_{n=0}^{\infty} \frac{1}{n!} z^{-n}$$

$$= 1 + \frac{1}{1!\,z} + \frac{1}{2!\,z^2} + \frac{1}{3!\,z^3} + \cdots$$

$$z\left(\frac{1}{n!}\right) = e^{1/z} = \bar{U}(z)$$

$$z(U_{n+1}) = z^1(\bar{U}(z) - U_0)$$

$$= z^1\left[e^{1/z} - 1\right]$$

$U_n = \frac{1}{(n+1)!}$

$U_0 = \frac{1}{(0+1)!} = 1$

Q] If $\{f(k)\} = 4^k$ , $k < 0$ Find $z\{f(k)\}$

$3^k$ , $k \geq 0$

→ $$z(f(k)) = \sum_{-\infty}^{\infty} f(k) z^{-k}$$

$$= \sum_{-\infty}^{-1} 4^k z^{-k} + \sum_{0}^{\infty} 3^k z^{-k}$$

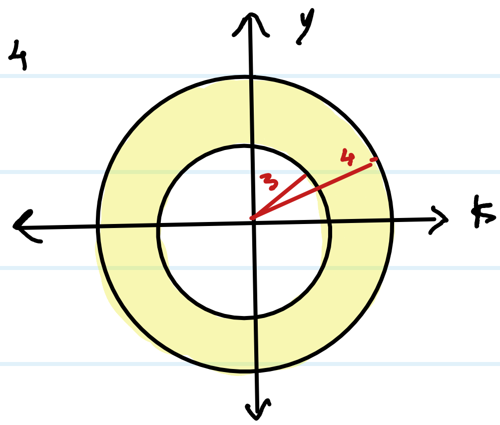
$$= \left[ \frac{z}{4} + \left(\frac{z}{4}\right)^2 + \left(\frac{z}{4}\right)^3 + \cdots \right] + \left[ 1 + \frac{3}{z} + \left(\frac{3}{z}\right)^2 + \left(\frac{3}{z}\right)^3 + \cdots \right]$$

$$a + ar + ar^2 + ar^3 + \ldots\ldots = \frac{a}{1-r} \quad \text{if } |r| < 1$$

$$= \frac{z/4}{1 - z/4} + \frac{1}{1 - \frac{3}{z}} \quad \text{if } \left|\frac{z}{4}\right| < 1 \ \&\& \ \left|\frac{3}{z}\right| < 1$$

$$= \frac{z}{4 - z} + \frac{z}{z - 3} \quad \text{if } |z| < 4 \ \&\& \ |z| > 3$$

$$3 < |z| < 4$$




$$= \frac{z^2 - 3z + 4z - z^2}{(4-z)(z-3)}$$

$$= \frac{z}{(z-4)(z-3)}$$

Q] $f(k) = ka^k, \quad k \geq 0$

$$\rightarrow Zf(k) = \sum_{-\infty}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=0}^{\infty} ka^k z^{-k}$$

$$= 0 + 1az^{-1} + 2a^2z^{-2} + 3a^3z^{-3} + \ldots\ldots$$

$$= \frac{a}{z} + 2\left(\frac{a}{z}\right)^2 + 3\left(\frac{a}{z}\right)^3 + \cdots$$

$$= \frac{a}{z}\left[1 + 2\left(\frac{a}{z}\right) + 3\left(\frac{a}{z}\right)^2 + \dots\right]$$

$$1 + nx + \frac{n(n-1)}{2!}x^2 + \dots = (1+x)^{-2}$$

$$1 + 2x + 3x^2 + \dots\dots = (1-x)^{-2}$$

$$\frac{a}{z}\left[1 - \frac{a}{z}\right]^{-2} = \frac{a}{z}\cdot\frac{1}{\left(1-\frac{a}{z}\right)^2} = \frac{az}{(z-a)^2}, \quad \text{if } \left|\frac{a}{z}\right| < 1$$

i.e $|a| < |z|$

$\therefore$ ROC is $|z| > |a|$

Q] $z\, f(k) = {}^nC_k$, $\quad 0 \le k \le n$

$$\rightarrow z[f(k)] = \sum_{-\infty}^{\infty} f(k)\, z^{-k} = \sum_{k=0}^{n} {}^nC_k\, z^{-k}$$

$$= {}^nC_0 z^0 + {}^nC_1 z^{-1} + {}^nC_2 z^{-2} + {}^nC_3 z^{-3} + \dots\dots$$

$$= 1 + {}^nC_1\left(\frac{1}{z}\right) + {}^nC_2\left(\frac{1}{z^2}\right) + {}^nC_3\left(\frac{1}{z^3}\right) + \dots + {}^nC_n\left(\frac{1}{z}\right)^n$$

$$1 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \dots + {}^nC_n x^n = (1+x)^n$$

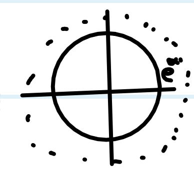
$= \left(1 + \frac{1}{z}\right)^n$ ROC is z-plane except z –

Q] $z[e^{k\alpha}] = \sum_{k=0}^{\infty} e^{\alpha k} z^{-k}$ (for $k \geq 0$)

$$= 1 + \frac{e^{\alpha k}}{z} + \frac{e^{2\alpha k}}{z^2} + \frac{e^{3\alpha k}}{z^3} + \dots \quad \text{(G.P.)}$$

$$= \frac{1}{1 - \frac{e^{\alpha}}{z}} \qquad \left|\frac{e^{\alpha}}{z}\right| < 1 \qquad \therefore |z| > e^{\alpha}$$

$$= \frac{z}{z - e^{\alpha}} \quad \text{for } |z| > e^{\alpha}$$

[ 

all points outside circle of radius $e^{\alpha}$ $(z = x + iy)$ ]

Q] $z[a^{|k|}]$

$\rightarrow$ $z[a^{|k|}] = \sum_{k=0}^{\infty} a^{|k|} z^{-k} = \sum_{k=-\infty}^{-1} a^{-k} z^{-k} + \sum_{k=0}^{\infty} a^{k} z^{-k}$

$\therefore$ $z[a^{|k|}] = \left(az + (az)^2 + (az)^3 + \dots\right] + \left[1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \left(\frac{a}{z}\right)^3 + \dots\right]$

$$= \frac{az}{1 - az} + \frac{1}{1 - \frac{a}{z}} = \frac{az}{1 - \frac{a}{z}} + \frac{z}{z - a}$$

[for $|az| < 1$ & $\left|\frac{a}{z}\right| < 1$]

$$= \frac{-a^2 z + z}{(1 - az)(z - a)}$$

[for $a < |z| < 1/a$ & only possible when $0 < a < 1$]

Q] Find $Z(f(k))$ if $f(k) = \frac{\sin ak}{k}$ $k>0$

$\rightarrow$ $Z[\sin ak] = \frac{z \sin a}{z^2 - 2z\cos a + 1}$

$$Z\left[\frac{\sin ak}{k}\right] = \int_z^\infty \frac{1}{\cancel{z}} \cdot \frac{\cancel{z} \sin a}{z^2 - 2z\cos a + 1} dz$$

$$= \int_z^\infty \frac{\sin a}{z^2 - 2z\cos a + \cos^2 a + \sin^2 a} dz$$

$\sin^2\theta + \cos^2\theta = 1$

$$= \sin a \int_z^\infty \frac{1}{\underbrace{(z - \cos a)^2}_{x} + \underbrace{\sin^2 a}_{a}} dz$$

$$\boxed{\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)}$$

$$= \cancel{\sin a}\left[\frac{1}{\cancel{\sin a}} \tan^{-1}\left(\frac{z - \cos a}{\sin a}\right)\right]_z^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{z - \cos a}{\sin a}\right)$$

$$= \cot^{-1}\left(\frac{z - \cos a}{\sin a}\right)$$

Q] Find $Z\{f(k)\}$ if $F(k) = 2^k \cos(3k+2)$ $k \geq 0$

$$\cos(3k+2) = \cos 3k \cdot \cos 2 - \sin 3k \cdot \sin 2$$

$$Z[\cos(3k+2)] = Z[\cos 3k \cos 2] - Z[\sin 3k \cdot \sin 2]$$

$$= \cos 2\, Z[\cos 3k] - \sin 2\, Z[\sin 3k]$$

$$= \cos 2 \cdot \frac{z^2 - z\cos 3}{z^2 - 2z\cos 3 + 1} - \sin 2 \cdot \frac{z \sin 3}{z^2 - 2z\cos 3 + 1}$$

$$= \frac{z^2 \cos 2 - z\cos 2 \cos 3}{z^2 - 2z\cos 3 + 1} - \frac{z \sin 3 \cdot \sin 2}{z^2 - 2z\cos 3 + 1}$$

$$= \frac{z^2 \cos 2 - z\cos 2\cos 3 - z\sin 3 \cdot \sin 2}{z^2 - 2z\cos 3 + 1}$$

$$= z\left[\frac{z\cos 2 - (\cos 3 \cdot \cos 2 + \sin 3 \sin 2)}{z^2 - 2z\cos 3 + 1}\right]$$

$$= \frac{z[z\cos 2 - \cos(3-2)]}{z^2 - 2z\cos 3 + 1}$$

$$Z[\cos(3k+2)] = \frac{z[z\cos 2 - \cos 1]}{z^2 - 2z\cos 3 + 1}$$

$$Z[2^k \cdot \cos(3k+2)] = \frac{\frac{z}{2}\left[\frac{z}{2}\cos 2 - \cos 1\right]}{\left(\frac{z}{2}\right)^2 - \cancel{2} \cdot \frac{z}{\cancel{2}} \cos 3 + 1}$$

$$= \frac{z[z\cos 2 - 2\cos 1]}{z^2 - 4z\cos 3 + 4}$$

Find $Z\{f(k)\}$ if $F(k) = k5^k, k \geq 0$

$\rightarrow$ $Z[5^k] = \dfrac{z}{z-5}$

$$Z[k.5^k] = -z \cdot \frac{d}{dz}\left(\frac{z}{z-5}\right)$$

$$= -z \cdot \left[\frac{(z-5).(1) - z(1)}{(z-5)^2}\right]$$

$$= -z\left[\frac{-5}{(z-5)^2}\right]$$

$$= \frac{5z}{(z-5)^2}$$

Q] Find $Z(X_k)$ if $X_k = \dfrac{1}{1^k} \cdot \dfrac{1}{2^k} \cdot \dfrac{1}{3^k}$ $\quad k \geq 0$

$\rightarrow$

$A(k) = \dfrac{1}{1^k}$ , $B(k) = \dfrac{1}{2^k}$ , $C(k) = \dfrac{1}{3^k}$

$$Z[A(k)] = \sum_{k=0}^{\infty} \frac{1}{1^k} z^{-k}$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3}$$

$$= \frac{1}{1 - \frac{1}{z}} = \frac{z}{z-1}$$

$$z[B(k)] = \frac{2z}{2z-1}$$

$$z[C(k)] = \frac{3z}{3z-1}$$

$$F(X_k) = z\left[\frac{1}{1^k} \cdot \frac{1}{2^k} \cdot \frac{1}{3^k}\right]$$

By Convolution Property

$$= z\left(\frac{1}{1^k}\right) \cdot z\left(\frac{1}{2^k}\right) \cdot z\left(\frac{1}{3^k}\right)$$

$$= \left(\frac{z}{z-1}\right) \cdot \left(\frac{2z}{2z-1}\right) \cdot \left(\frac{3z}{3z-1}\right)$$

# Inverse

8] $Z^{-1}\left[\dfrac{z}{(z-1)(z-2)}\right]$, if $|z| \geq 2$

$$F(z) = \frac{z}{(z-1)(z-2)}$$

$$\frac{F(z)}{z} = \frac{1}{(z-1)(z-2)}$$

Partial Fraction

$$\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$\frac{1}{(z-1)(z-2)} = (z-2)A + (z-1)B$$

$B = 1$, $A = -1$

$$\frac{1}{(z-1)(z-2)} = \frac{-1}{z-1} + \frac{1}{z-2}$$

$$\frac{F(z)}{z} = \frac{-1}{z-1} + \frac{1}{z-2}$$

$$F(z) = \frac{-z}{z-1} + \frac{z}{z-2}$$

$$Z^{-1}[F(z)] = Z^{-1}\left[\frac{-z}{z-1}\right] + Z^{-1}\left[\frac{z}{z-2}\right]$$

$|z| \geq 2$

$\hookrightarrow |z| \geq 1$

$$= -1^k + 2^k$$

$$z^{-1}[F(z)] = 2^k - 1$$

$$z^{-1}\left[\frac{z}{z-a}\right] = a^k$$

Q] $z^{-1}\left[\dfrac{z^2}{(z-\frac{1}{2})(z-\frac{1}{3})}\right]$ $|z| > \frac{1}{2}$

$$\rightarrow \quad F(z) = \frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$\frac{F(z)}{z} = \frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$\frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{(z-\frac{1}{2})} + \frac{B}{(z-\frac{1}{3})}$$

$$z = A\left(z-\frac{1}{3}\right) + B\left(z-\frac{1}{2}\right)$$

$\therefore$ $B = -2$ & $A = 3$

$$\frac{F(z)}{z} = \frac{3}{(z-\frac{1}{2})} + \frac{-2}{(z-\frac{1}{3})}$$

$$F(z) = 3 \cdot \frac{z}{(z-\frac{1}{2})} - 2 \cdot \frac{z}{(z-\frac{1}{3})}$$

$$z^{-1}[F(z)] = 3\, z^{-1}\left[\frac{z}{z-\frac{1}{2}}\right] - 2\, z^{-1}\left[\frac{z}{z-\frac{1}{3}}\right]$$

$$F(K) = 3 \cdot \left(\frac{1}{2}\right)^K - 2\left(\frac{1}{3}\right)^K \qquad |z| > \frac{1}{2}$$

$$|z| > \frac{1}{3}$$

8] $Z^{-1}\left[\dfrac{3z^2 + 2z}{z^2 - 3z + 2}\right] \qquad 1 < |z| < 2$

$$\frac{F(z)}{z} = \frac{3z + 2}{(z-2)(z-1)}$$

$$\frac{3z+2}{(z-2)(z-1)} = \frac{A}{z-2} + \frac{B}{z-1}$$

$$3z + 2 = (z-1)A + B(z-2)$$

$B = -5$ & $A = 8$

$$\frac{F(z)}{z} = \frac{8}{z-2} + \frac{-5}{z-1}$$

$$z^{-1}[F(z)] = 8z^{-1}\left[\frac{z}{z-2}\right] - 5z^{-1}\left[\frac{z}{z-1}\right]$$

$$= -8 \cdot 2^K - 5 \, 1^K \qquad 1 < |z| < 2$$

$$F(K) = -8 \cdot 2^K - 5$$

Q] $Z^{-1}\left(\frac{1}{z-a}\right)$ when $|z|<|a|$ if $|z|>|a|$

$|z|<|a|$

$\left|\frac{z}{a}\right|<1$

$$f(z)=\frac{1}{z-a}$$

$$=\frac{1}{a\left(\frac{z}{a}-1\right)}$$

$$=-\frac{1}{a\left(1-\frac{z}{a}\right)}$$

$$=-\frac{1}{a}\left[1+\frac{z}{a}+\left(\frac{z}{a}\right)^2+\left(\frac{z}{a}\right)^3+\dots\right]$$

$$=-\left[\frac{1}{a}+\frac{z}{a^2}+\frac{z^2}{a^3}+\dots\right]$$

$$\frac{z^k}{a^{k+1}}=\underbrace{a^{-k-1}}_{\text{coefficient}}z^k$$

coefficient $z^{-k}=-a^{k-1}$

$$Z^{-1}\left(\frac{1}{z-a}\right)=-a^{k-1} \quad |z|<|a|$$

$|z|<2$

$$Z^{-1}\left(\frac{1}{z-2}\right)$$

$|z|<2$

$\left|\frac{z}{2}\right|<1$

$$\frac{1}{z-2}$$

$$\frac{1}{2\left(\frac{z}{2}-1\right)}$$

$$-\frac{1}{2}\frac{1}{\left(1-\frac{z}{2}\right)}$$

$$-\frac{1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3+\dots\right]$$

$$-\left[\frac{1}{2}+\frac{z}{2^2}+\frac{z^2}{2^3}+\dots\right]$$

$$=\boxed{2^{k-1}}$$

$|z| > |a|$

$|a| < |z|$

$\left|\frac{a}{z}\right| < 1$

$$F(z) = \frac{1}{z-a}$$

$$= \frac{1}{z\left(1-\frac{a}{z}\right)}$$

$$= \frac{1}{z}\left[1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \left(\frac{a}{z}\right)^3 + \cdots\right]$$

$$= \left[\frac{1}{z} + \frac{a}{z^2} + \frac{a^2}{z^3} + \cdots\right]$$

$$\frac{a^{k-1}}{z^k} = a^{k-1} z^{-k}$$

coefficient of $z^{-k}$ is $a^{k-1}$

$$Z^{-1}\left(\frac{1}{z-a}\right) = a^{k-1} \qquad |z| > |a|$$