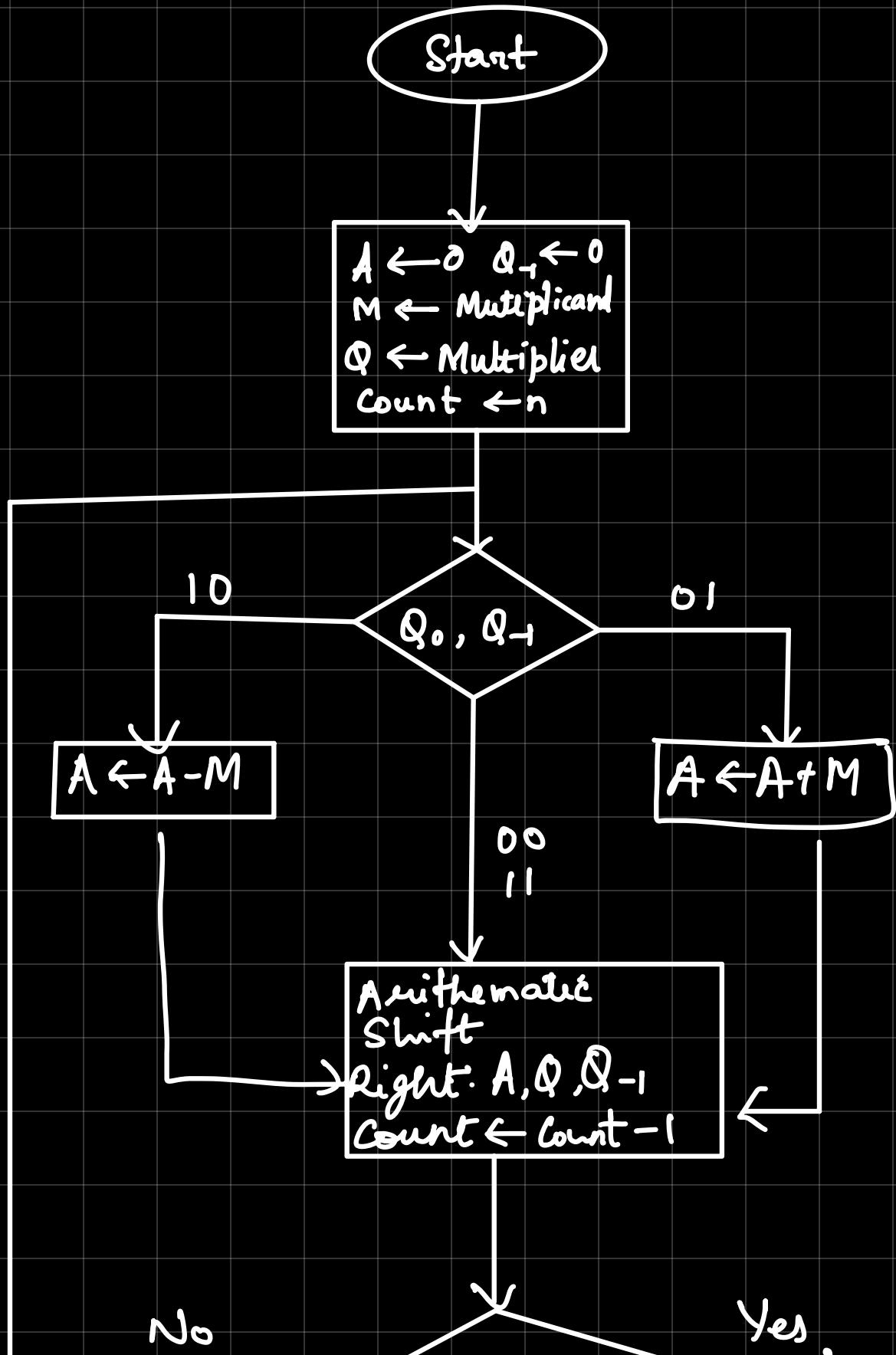


Booth's Algorithm

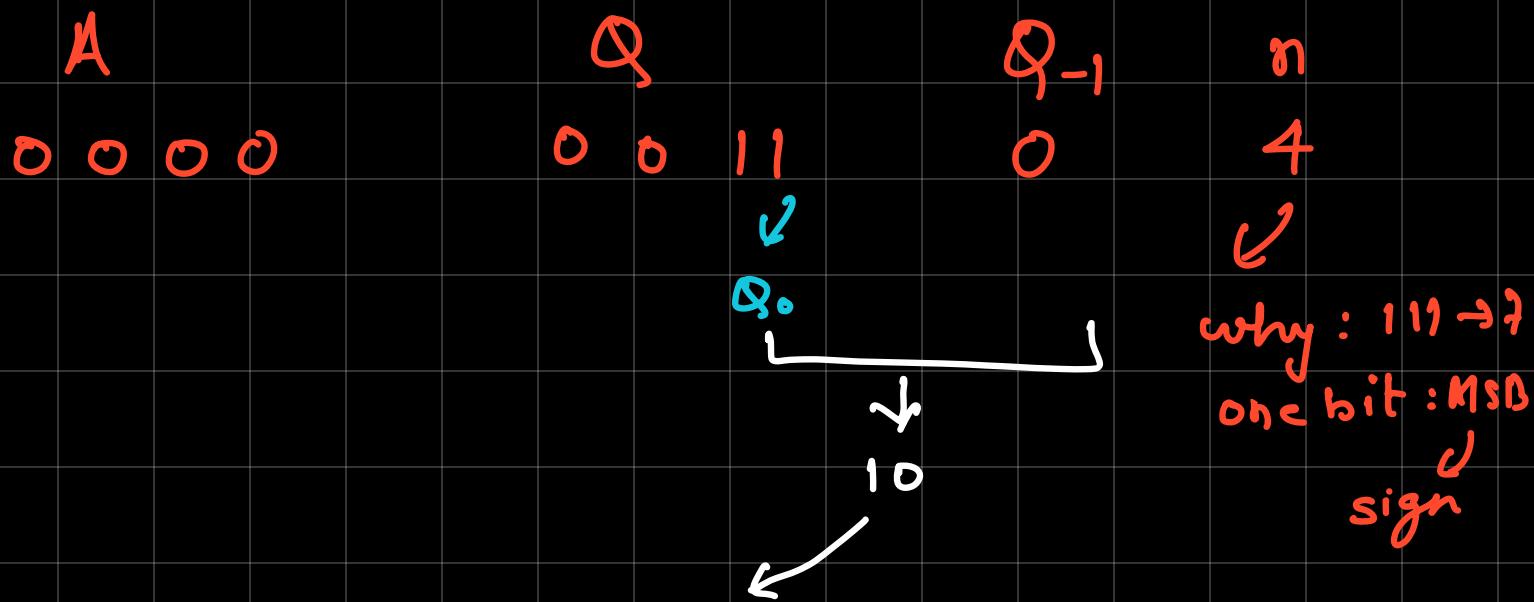


Count = 0 ?

M Q
7 3

$$M \Rightarrow 0111 \quad Q \Rightarrow 0011$$

$$-M \Rightarrow \begin{array}{r} 1000 \\ + 1 \\ \hline 1001 \end{array}$$



$$A \leftarrow A - M$$

$$\begin{array}{r} 0 0 0 0 \\ 1 0 0 1 \\ \hline \end{array}$$

1 0 0 1

1 0 0 1

0 0 11

0

Shift right

1 1 0 0

1 0 0 1

1 ↗ 3

11

1 1 1 0

0 1 0 0

1 ↗ 2

11

01

$A \leftarrow A + M$

1
1 1 1 0
0 1 1 1

1 0 1 0 1

0 0 1 0

1 0 1 0 0

0 1

11

0 0 0 1 0 1 0 1 0 0

Q. 7 * 3
 ↓ ↓
 M Q

$$\begin{array}{r}
 M \rightarrow 0111 \\
 -M \rightarrow 1000 \\
 + \quad \quad \quad | \\
 \hline
 1001
 \end{array}$$

A	00000	Q	0011	Q_1	0	n	$A \leftarrow A - M$
1100			1001		1	3	$\begin{array}{r} 00000 \\ 1001 \\ \hline 1001 \end{array}$
1110		0100				2	$A \leftarrow A + M$
0010		1010	0				$\begin{array}{r} 1100 \\ 0100 \\ \hline 1010 \end{array}$

0 0 0 | 0 1 0 | 0 0

Q. 12 * 13
↑ ↑
M S

$$\begin{array}{r}
 M \rightarrow 01100 \\
 -M \rightarrow 10011 \\
 + \\
 \hline
 10100
 \end{array}$$

Q → DISOI

A

Q

Q₋₁

n

0 0 0 0 0

0 1 1 0 1

0

5

$A \leftarrow A - M$

$$\begin{array}{r}
 0 0 0 0 0 \\
 1 0 1 0 0 \\
 \hline
 1 0 1 0 0
 \end{array}$$

1 0 1 0 0

1 1 0 1 0

0 0 1 1 0

1

4

$A \leftarrow A + M$

$$\begin{array}{r}
 1 1 0 1 0 \\
 0 1 1 0 0 \\
 \hline
 0 0 1 1 0
 \end{array}$$

0 0 1 1 0

0 0 0 1 1

0 0 0 1 1

0

3

$A \leftarrow A - M$

$$\begin{array}{r}
 0 0 0 1 1 \\
 1 0 1 0 0 \\
 \hline
 1 0 1 1 1
 \end{array}$$

1 1 0 1 1

1 0 0 0 1

1

2

1 1 1 0 1

1 1 0 0 0

1

1

$A \leftarrow A + M$

$$\begin{array}{r}
 1 1 1 0 1 \\
 0 1 1 0 0 \\
 \hline
 0 0 1 0 1
 \end{array}$$

0 1 0 0 1

0 0 1 0 0 1 1 1 0 0 0 0

Q. -9 * 7
↑ ↑
M Q

$$\begin{array}{r} 8 4 2 1 \\ M \Rightarrow 0 1 0 0 1 \qquad Q \Rightarrow 0 0 1 1 1 \\ 1 0 1 1 0 \\ + \qquad \qquad \qquad \qquad \qquad \qquad | \\ -M \Rightarrow 1 0 1 1 1 \end{array}$$

In this case

$$\begin{array}{l} M \Rightarrow 1 0 1 1 1 \\ -M \Rightarrow 0 1 0 0 1 \end{array}$$

* one no. -ve \rightarrow

ans ka 2's compl

$$\begin{array}{c}
 A \\
 00000 \\
 \\
 \textcolor{red}{01001} \\
 \end{array}
 \quad
 \begin{array}{c}
 Q \\
 0111 \\
 \\
 \boxed{1}
 \end{array}
 \quad
 \begin{array}{c}
 Q^{-1} \\
 0 \\
 \\
 \boxed{1}
 \end{array}
 \quad
 \begin{array}{c}
 n \\
 5
 \end{array}
 \quad
 \begin{array}{c}
 A \rightarrow A - M \\
 00000 \\
 01001 \\
 \hline
 01001
 \end{array}$$

$$\begin{array}{ccc}
 00100 & 10011 & | \\
 & \boxed{1} &
 \end{array}
 \quad
 \begin{array}{c}
 4
 \end{array}$$

$$\begin{array}{ccc}
 00010 & 01001 & | \\
 & \boxed{1} &
 \end{array}
 \quad
 \begin{array}{c}
 3
 \end{array}$$

$$\begin{array}{ccc}
 00001 & 00100 & | \\
 & \boxed{1} &
 \end{array}
 \quad
 \begin{array}{c}
 2
 \end{array}$$

$$\begin{array}{c}
 \textcolor{red}{A \rightarrow A + M} \\
 \begin{array}{r}
 00001 \\
 10111 \\
 \hline
 11000
 \end{array}
 \end{array}$$

$$\begin{array}{ccccc}
 11100 & 00010 & 0 & & | \\
 & \boxed{1} & & &
 \end{array}
 \quad
 \begin{array}{c}
 1
 \end{array}$$

$$\begin{array}{ccccc}
 11110 & 00001 & 0 & & | \\
 & & & &
 \end{array}
 \quad
 \begin{array}{c}
 0
 \end{array}$$

$$\begin{array}{ccccc}
 00001 & 11110 & & & | \\
 & \boxed{1} & & & \\
 & & & & \leftarrow 1\text{'s compliment} \\
 \hline
 00001 & 11111 & & & | \\
 & \boxed{1} & & & \\
 & & & & \leftarrow 2\text{'s compliment}
 \end{array}$$

$$= \boxed{-63}$$

$$Q \cdot -12 * -18$$

↑ ↑
 M Q

$$\begin{array}{r}
 M \rightarrow 001100 \\
 + 110011 \\
 \hline
 -M \rightarrow \overline{110100}
 \end{array}$$

$$\begin{array}{r}
 Q \rightarrow 010010 \\
 + 101101 \\
 \hline
 \overline{101110}
 \end{array}$$

So in this $M \rightarrow 110100$

$$-M \rightarrow 001100$$

A	Q	Q_{-1}	n
000000	101110	0	6
L			

000000	010111	0	S
L			
001100			

$A \rightarrow A - M$

$$\begin{array}{r}
 000000 \\
 001100 \\
 \hline
 \overline{001100}
 \end{array}$$

0 0 0 1 1 0 0 0 1 0 1 1 | 4

0 0 0 0 1 1 0 0 0 1 0 1 | 3

0 0 0 0 0 1 1 0 0 0 1 0 | 2 $A \rightarrow A + M$

1 1 0 1 0 1

1 1 1 0 1 0 1 1 0 0 0 1 0 1 $A \rightarrow A - M$

0 0 0 1 1 0

$\begin{array}{r} 000001 \\ 110100 \\ \hline 110101 \end{array}$

0 0 0 0 1 1 0 1 1 0 0 0 | 0

= 216

$$Q \cdot -15 * 15$$

\nearrow \nwarrow
 M Q

$$M \rightarrow 01111$$

$$\begin{array}{r} 10000 \\ + 1 \\ \hline \end{array}$$

$$\text{so } M \rightarrow 10001$$

$$-M \rightarrow 01111$$

$$-M \rightarrow 10001$$

$$Q \rightarrow 01111$$

A

$$00000$$

Q

$$01111$$

Q_1

$$0$$

n

$$5$$

$\boxed{}$

$$\begin{array}{r} A \rightarrow A - M \\ 00000 \\ 01111 \\ \hline 01111 \end{array}$$

$$01111$$

$$00111$$

$$10111$$

$$1$$

$$4$$

$\boxed{}$

$$00011$$

$$11011$$

$$1$$

$$3$$

$\boxed{}$

00001

11101

2

00000

11110

1

$A \rightarrow A + M$

$$\begin{array}{r} 00000 \\ 10001 \\ \hline 10001 \end{array}$$

(000)

11000

11111

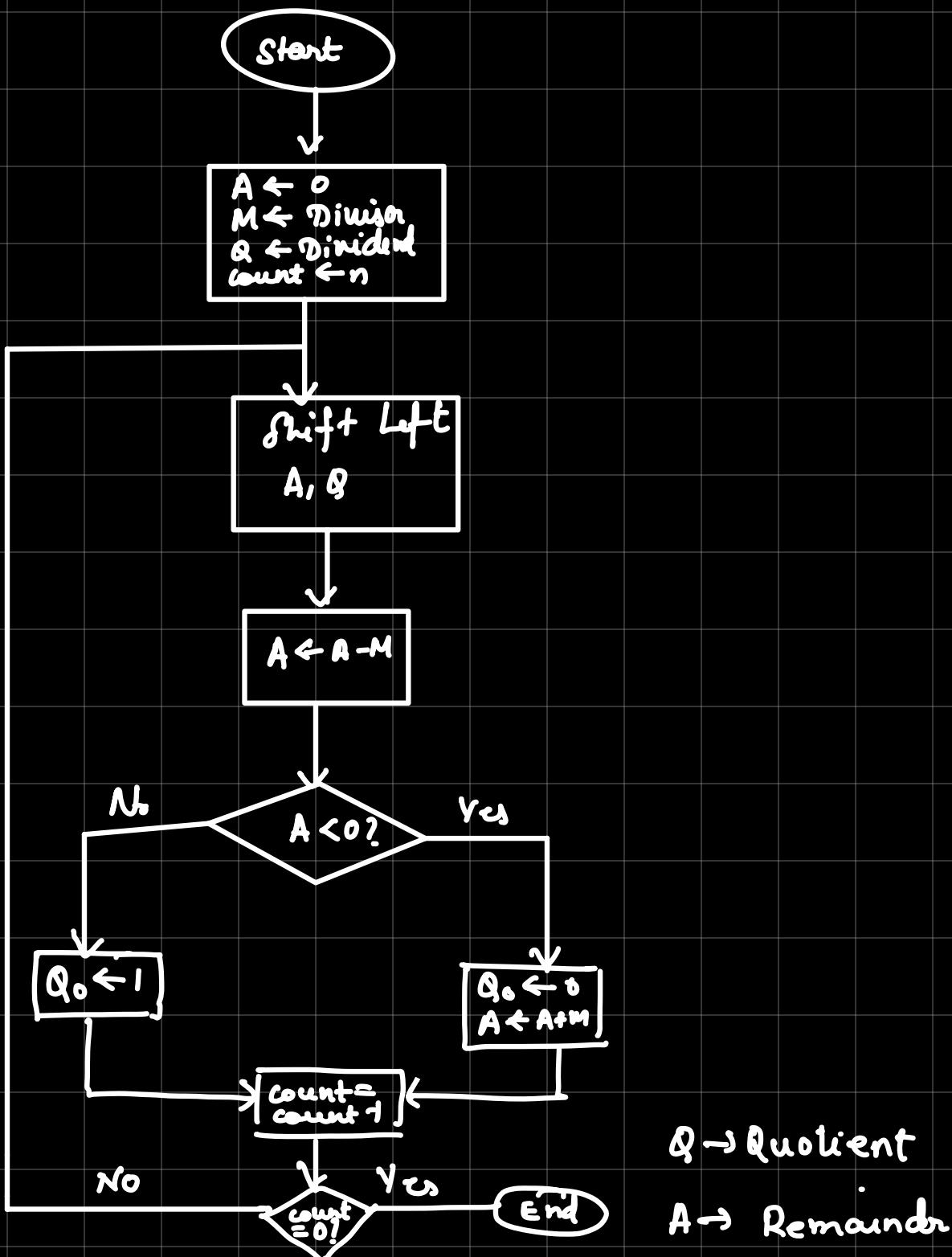
0

00111 00000

$$\begin{array}{r} + \\ 00111 \\ \hline 00001 \end{array}$$

$$= [-225]$$

Flowchart for Restoring Division [+ve only]



$Q \rightarrow \text{Quotient}$

$A \rightarrow \text{Remainder}$

$$7 \div 3$$

↗ ↙

Q M

$$Q \Rightarrow 0111$$

$$M \Rightarrow 0011$$

$$\begin{array}{r}
 1100 \\
 + \quad \quad \quad 1 \\
 \hline
 1101
 \end{array}$$

$(-M)$

$1 \rightarrow -ve$
 $0 \rightarrow +ve$

A	Q	n
0000	0111	4

↓
 -
 0000 111
 1101 111 0

$A \rightarrow A - M$
 $\begin{array}{r}
 0000 \\
 1101 \\
 \hline
 1101
 \end{array}$

$A \rightarrow A + M$
 $\begin{array}{r}
 1101 \\
 0011 \\
 \hline
 10000
 \end{array}$

$$\begin{array}{r}
 0001 \\
 -1110 \\
 \hline
 110\boxed{1}
 \end{array}$$

$$\begin{array}{r}
 A \rightarrow A-M \\
 \begin{array}{r}
 0001 \\
 1101 \\
 \hline
 1110
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 0001 \\
 110\boxed{0}
 \end{array} \quad 2$$

$$\begin{array}{r}
 A \rightarrow A+M \\
 \begin{array}{r}
 110 \\
 0011 \\
 \hline
 10001
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 0011 \\
 100\boxed{1}
 \end{array} \quad 1$$

$$\begin{array}{r}
 0000 \\
 100\boxed{11}
 \end{array} \quad 1$$

$$\begin{array}{r}
 A \rightarrow A-M \\
 \begin{array}{r}
 0011 \\
 1101 \\
 \hline
 10000
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 0001 \\
 -1110 \\
 \hline
 001\boxed{0}
 \end{array} \quad 0$$

$$\begin{array}{r}
 A \rightarrow A-M \\
 \begin{array}{r}
 0001 \\
 1101 \\
 \hline
 1110
 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \uparrow \\
 R=1
 \end{array} \quad
 \begin{array}{l}
 \uparrow \\
 Q=2
 \end{array}$$

$$\begin{array}{r}
 A \rightarrow A+M \\
 \begin{array}{r}
 110 \\
 0011 \\
 \hline
 0001
 \end{array}
 \end{array}$$

Restoring \rightarrow because regenerates the value
of A

$$Q \cdot 5 \div 5$$

\swarrow \uparrow
 Q M

$$Q \Rightarrow 0110$$

$$\begin{array}{r} M \Rightarrow 0110 \\ 1001 \\ + \quad \quad \quad | \\ \hline -M \Rightarrow 1010 \end{array}$$

A B n
 0 0 0 0 0 1 1 0 4

0 0 0 0 1 1 0 □

1 0 1 0 1 1 0 □

0 0 0 0 1 1 0 0 3

0 0 0 1 1 0 0 □

1 0 1 1 1 0 0 □

0 0 0 1 1 0 0 0 2

0 0 1 1 0 0 0 □

1 1 0 1 0 0 0 □

0 0 1 1 0 0 0 0 1

$A \rightarrow A - M$

$$\begin{array}{r} 0 0 0 0 \\ 1 0 1 0 \\ \hline 1 0 1 0 \end{array}$$

$A \rightarrow A + M$

$$\begin{array}{r} 1 0 1 0 \\ 0 1 1 0 \\ \hline 1 0 0 0 \end{array}$$

$A \rightarrow A - M$

$$\begin{array}{r} 0 0 0 1 \\ 1 0 1 0 \\ \hline 1 0 1 1 \end{array}$$

$A \rightarrow A + M$

$$\begin{array}{r} 1 0 1 1 \\ 0 1 1 0 \\ \hline 0 0 0 1 \end{array}$$

$A \rightarrow A - M$

$$\begin{array}{r} 0 0 1 1 \\ 1 0 1 0 \\ \hline 1 1 0 1 \end{array}$$

$A \rightarrow A + M$

$$\begin{array}{r} 1 1 0 1 \\ 0 1 1 0 \\ \hline 0 0 0 1 \end{array}$$

0110

000 □

0000

000 □

R = D

Q = I

D

A → A - M

1
0110

1010
—————
0000

0 01 1

$$Q \cdot 26 \div 12$$

\nearrow \nwarrow
 Q M

$$Q = 011010$$

$$M = 001100$$

$$\begin{array}{r}
 110011 \\
 + \quad \quad \quad 1 \\
 \hline
 110100
 \end{array}$$

$$\begin{array}{r}
 A \\
 00 \quad 000
 \end{array}$$

$$\begin{array}{r}
 Q \\
 011010
 \end{array}$$

$$\begin{array}{r}
 n \\
 6
 \end{array}$$

$$000000$$

$$11010 \boxed{\square}$$

$$A \rightarrow A - M$$

$$110100$$

$$11010 \boxed{0}$$

$$000000$$

$$110100$$

$$5$$

$$\begin{array}{r}
 000000 \\
 110100 \\
 110100 \\
 001100 \\
 \hline
 000000
 \end{array}$$

0 0 0 0 0 1 1 0 1 0 0 1 []

A → A-M

$$\begin{array}{r} 0 0 0 0 0 1 \\ 1 1 0 1 0 0 \\ \hline 1 1 0 1 0 1 \end{array}$$

1 1 0 1 0 1 1 0 1 0 0 1 [0]

4

0 0 0 0 0 1 1 0 1 0 0 0

A → A+M

$$\begin{array}{r} 1 1 0 1 0 1 \\ 0 0 1 1 0 0 \\ \hline 0 0 0 0 0 1 \end{array}$$

0 0 0 0 1 1 0 1 0 0 0 []

A → A-M

$$\begin{array}{r} 0 0 0 0 1 1 \\ 1 1 0 1 0 0 \\ \hline 1 1 0 1 1 1 \end{array}$$

0 0 0 0 1 1 0 1 0 0 0 0

3

0 0 0 1 1 0 1 0 0 0 0 []

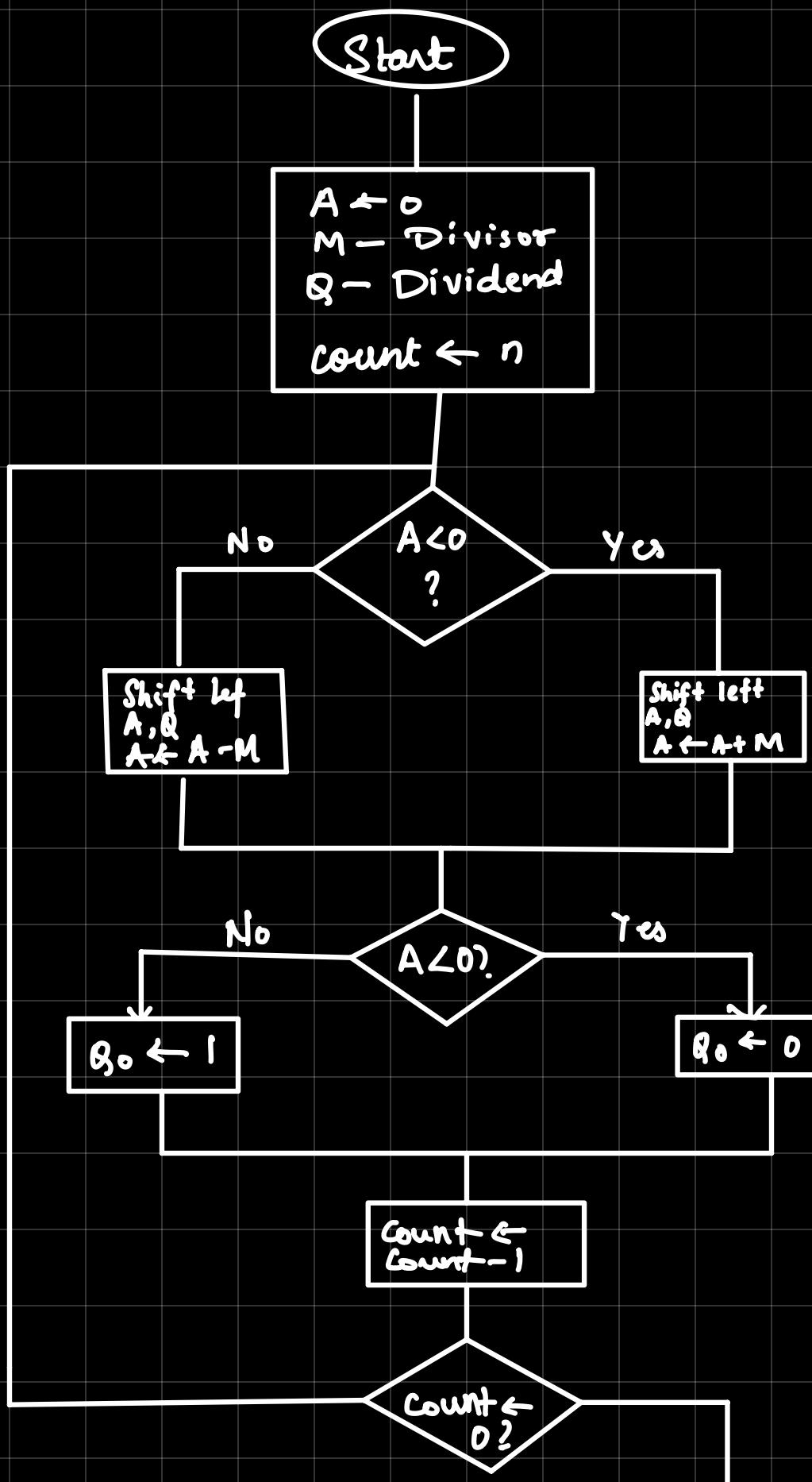
A → A+fM

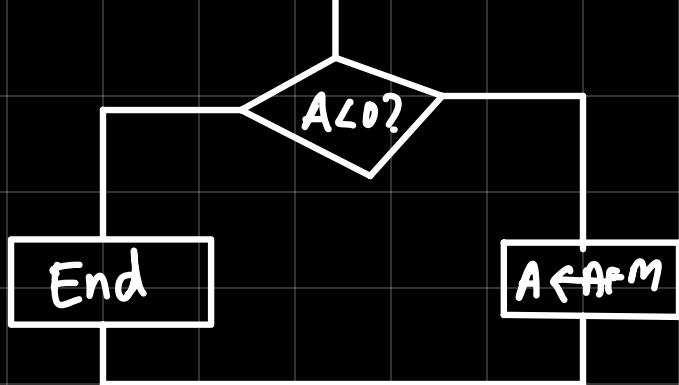
$$\begin{array}{r} 1 1 0 1 1 1 \\ f 0 0 1 1 0 0 \\ \hline 0 0 0 1 1 1 \end{array}$$

1 1 1 0 1 0 1 0 0 0 0 [0]

$$\begin{array}{r} 0 0 0 1 1 0 \\ 1 1 0 1 0 0 \\ \hline 1 1 1 0 1 0 \end{array}$$

Non-Restoring





$$Q \cdot 4 \div 2$$

↗ ↑
Q M

$$\begin{array}{r}
 4 - 0100 \\
 2 - 0010
 \end{array}$$

$$\begin{array}{r}
 0010 \\
 1101 \\
 + \\
 \hline
 1110 (-M)
 \end{array}$$

A	Q	n	
0000	0100	4	0000
0000	100		1110

$$\begin{array}{r}
 0000 \\
 1110 \\
 \hline
 1110
 \end{array}$$

111 0	1 00	<u>0</u>	3	1101
1101	0 00	<u> </u>		<u>0010</u>

$$\begin{array}{r} \\ \hline 1111 \end{array}$$

111 1	0 00	<u>10</u>	2
1110	0 00	<u> </u>	

0 000	0 00	<u>1</u>	1
0 000	0 01	<u> </u>	

0 000	0 01	<u> </u>	$A \leftarrow A - M$
1110	0 01	<u>10</u>	$\begin{array}{r} 0000 \\ 1110 \\ \hline 1110 \end{array}$

0 000	0 010	
-------	-------	--

R

$Q = 2$

$$\begin{array}{r} 11 \\ | \\ 1110 \\ 0010 \\ \hline 10000 \end{array}$$

$$Q. M = 12$$

$$Q = 26$$

$$26 \div 12$$

↑
M

~~16~~ 8 4 2 1
 $26 \rightarrow 011010 Q$

$$12 \rightarrow 001100 M$$

$$\begin{array}{r} 11011 \\ + 1 \\ \hline 110100 (-M) \end{array}$$

A- Q n

000000 011010 6

000000 11010 []

110100 11010 [0] 5

101001 10100 []

000000 10100 [1] 4

$$\begin{array}{r} 110100 \\ - 001100 \\ \hline 000000 \end{array}$$

000001	01001	□	000001 110100 ----- 110101
110101	01001	10□	3
101010	10010	1□	101010 001100 ----- 110100
110110	10010	10□	2
101101	00100	□	101101 001100 ----- 110001
111101	00100	10□	1 11101
111010	01000	□	110010 001100 ----- 111110
00010	01000	1□	

↗

$R=2$

$$\begin{array}{r} 111110 \\ 001100 \\ \hline 001010 \end{array}$$

$$\begin{array}{r} 1111010 \\ 001100 \\ \hline 001010 \end{array}$$

$$\overline{12 \sqrt{26}}$$

Division of signed numbers

1. Load the divisor into M register and the dividend into A, Q registers.

The dividend must be expressed as a $2n$ -bit two's compliment number. This

e.g. \rightarrow 4-bit.: 0111 becomes 00000111

and 1001 becomes 11111001

2. Shift A, Q left by 1 position

3. If M & A have the same signs

$A \leftarrow A - M$ otherwise $A \leftarrow A + M$.

4. The preceding operation is successful if the sign of A is the same before & after the operation.

a. If operation is successful or $A=0$
then set $Q_0 \leftarrow 1$

b. If operation is unsuccessful & $A \neq 0$
set $Q_0 \leftarrow 0$ & restore the prev.
value of A

5. Repeat steps 2 to 4 as many times as there are bit positions in Q
6. The remainder is in A. If the signs of divisor & dividend were same the quotient is in Q. Otherwise the correct quotient is two's complement of Q.

$$Q. \quad 7 \div -3$$

\nearrow \nearrow
 Q M

$$\begin{array}{r} -M \rightarrow 0011 \\ - \\ (-M) \rightarrow \begin{array}{r} 1100 \\ + \\ \hline 1101 \end{array} \\ Q \rightarrow 0111 \end{array}$$

$\Leftarrow M$

$$\left(\begin{array}{c} A \\ 0000 \\ \hline 0000 \\ 1101 \\ 0000 \end{array} \quad \begin{array}{c} Q \\ 0111 \\ \hline 111 \\ 111 \\ 111 \end{array} \quad \begin{array}{c} n \\ 4 \\ \hline 3 \end{array} \right)$$

$$A \leftarrow A + M$$

$$\begin{array}{r} 0000 \\ 1101 \end{array}$$

$$\left(\begin{array}{c} 0001 \\ 1110 \\ 0001 \end{array} \quad \begin{array}{c} 110 \\ 110 \\ 110 \end{array} \quad \begin{array}{c} \square \\ \square \\ \square \end{array} \end{array} \right)$$

2

$$A \leftarrow A + M$$

$$\begin{array}{r} 0001 \\ 1101 \\ \hline 1110 \end{array}$$

0011 100

0000 100 1

0001 001

1110 001

0001 0010 0

↑
R=1
1101
+ 1

1110

A & A+M

0011
1101

10000

0001
1101

1110

$Q \cdot -7 \div 3$

↑ ↑
Q M

$$Q \rightarrow 0111 \text{ (Q)} \quad M \rightarrow 0011$$

$$\begin{array}{r} 1000 \\ + 1 \\ \hline 1001 \end{array} \text{ (Q)}$$

A	Q	n
1111	1001	4
1111	0011	
0010	0010	
1111	0010	3

$$\begin{array}{r} 1111 \\ 1111 \\ - 0011 \\ \hline 1001 \end{array}$$

1110	0101	
0001	0100	
1110	0100	2

$$\begin{array}{r} 11 \\ 1110 \\ - 0011 \\ \hline 0001 \end{array}$$

1100	1001	
1111	1001	1

$$\begin{array}{r} 1100 \\ - 0011 \\ \hline 1111 \end{array}$$

1.

$$\begin{array}{r}
 1111 \quad 001 \boxed{\square} \\
 0010 \quad 001 \boxed{0} \\
 \hline
 1111 \quad 0010 \\
 \end{array}$$

↗ ↘ 1101
 $R = -1$ + $\overline{1}$
 $\overline{1110}$
 ↗ Q =

$$\begin{array}{r}
 1111 \\
 0011 \\
 \hline
 0010
 \end{array}$$

γ

$$Q \cdot -7 \div -3$$

\nearrow

τ_M

$$Q \rightarrow 0111$$
$$-Q (Q) \rightarrow 1000$$

|

1001

$$-M \rightarrow 0011$$
$$M \rightarrow 1100$$

|

1101

A

1111

Q

Booth's Recoding / Bit-pair Recoding

↳ Derived from booth's algorithm

- ① Find table of M
- ② Reduce the value of Q
- ③ Perform $M * Q$
 $(S * 4)$

$$M = S \quad Q = 4$$
$$\begin{array}{r} 0101 \\ \times \quad 0100 \\ \hline \end{array}$$

- ① Find table of M

Operations	value		
0	A	0000	N
+1 (M)	<u>0000</u>	<u>0000</u>	<u>0101</u>
-1 ($-M$)	<u>1111</u>	<u>1011</u>	<u>1010</u>
+2 (leftshift M)	0000	1010	=
-2 (leftshift $-M$)	1111	0110	=

② Reduced value of Q

$$\begin{array}{c} Q \quad Q_0 \\ \begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} & \end{array} \\ \begin{array}{ccccc} 1 & -1 & 0 & 0 & \\ \boxed{} & \boxed{} & & & \end{array} \\ 2(1) + (-1) \quad 2(0) + 0 \\ 1 \quad 0 \end{array}$$

if $100 \rightarrow 0$
 $01 \rightarrow 1$
 $10 \rightarrow -1$

③ Multiply $M * Q$

$$\begin{array}{r} 0 \ 1 \ 0 \ 1 \\ * \quad \quad \quad 1 \ 0 \\ \hline 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \mid 0 \ 1 \ + \ f \end{array}$$

$$-5 + 4$$

① find table of M

Operation	Value	
	A	M
0	0000	0000
+1 (M)	1111	1011
-1 (-M)	0000	0101
+2 (M LS)	1111	011 <u>0</u>
-2 (M LS)	0001	10 <u>1</u> 0

② Reduce Q

$$\begin{matrix} Q & Q_0 \\ \begin{matrix} 0 & 1 & 0 & 0 \\ \sqcup & \sqcup & \sqcup & \sqcup \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$

$$\begin{matrix} | & -1 & 0 & 0 \\ \sqcup & \sqcup & \sqcup \end{matrix}$$

$$\begin{matrix} 2(1) + (-1) & 2(0) + 0 \\ 1 & 0 \end{matrix}$$

$$\begin{array}{r}
 & 1 & 0 & 1 & 1 \\
 * & & & 1 & 0 \\
 \hline
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 1 & 1 & + & + \\
 \hline
 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
 & 0 & 0 & 0 & 1 & 0 & 0 & ' & 1 & 1 \\
 \hline
 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0
 \end{array}$$

Q q * -6

8421

01001

① Operation

Value

A M

0	00000	00000	
+1(M)	00001	<u>01001</u>	01001
-1(M)	11111	10111	<u>01010</u>
+2(LSM)	00000	10010	=
-2(LSM)	11111	01110	

② Reduce value Q

01001
10111
1
0

00110
11001
1
11010

Q
 $\begin{array}{c} 1 \ 1 \ 0 \ 1 \ 0 \\ \sqcup \sqcup \sqcup \sqcup \sqcup \end{array}$
 Q_0
 $\begin{array}{c} 0 \ -1 \ 1 \ -1 \ 0 \\ \sqcup \ \sqcup \end{array}$

$11 / 00 \rightarrow 0$
 $01 \rightarrow 1$
 $10 \rightarrow -1$

$$\begin{array}{r}
 2(-1) + 1 \quad 2(-1) + 0 \\
 -1 \qquad \qquad \qquad -2
 \end{array}$$

$$\begin{array}{r}
 0 \ 1 \ 0 \ 0 \ 1 \\
 * \qquad \qquad \qquad -1 \ -2 \\
 \hline
 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \\
 \qquad \qquad \qquad + \ +
 \end{array}$$

$$Q. \quad 15 * -10$$

$$15 \rightarrow 01111(M)$$

$$-10 \rightarrow 01010$$

$$\begin{array}{r} 10101 \\ + 1 \\ \hline \end{array}$$

$$\begin{array}{r} \\ \\ \hline 10110 \end{array}$$

operation

Value

A M

0

00000 00000

+1(M)

00001 01111

-1(M)

11111 10001

+2(LSM)

00000 1110

-2 (LSM)

11111 00010

8

8.

1 0 1 1 0

0

U U U U L

-1 1 0 -1 0

U L L

2(1)10 2(-1) -2

-1 2 -2

0 1 1 1 1

* -1 2 -2

1 1 1 1 1 0 0 0 1 0

0 0 0 1 1 1 0 + +

1 1 0 0 0 1 + + + +

1 1 0 1 1 0 1 0 1 0

0 0 1 0 0 1 0 1 0 1

1

0 0 1 0 0 1 0 1 1 0

12 67 32 16 8 4 2 1

Q. - 13 * - 20

$$\begin{array}{r} 13 \rightarrow 001\ 1\ 0\ 1 \\ -13 \rightarrow 1\ 1\ 0\ 0\ 1\ 0 \\ \hline 1\ 1\ 0\ 0\ 1\ 1 \end{array}$$

$$\begin{array}{r} 20 \rightarrow 0\ 1\ 0\ 1\ 0\ 0 \\ -20 \rightarrow 1\ 0\ 1\ 0\ 1\ 1 \\ \hline 1\ 0\ 1\ 1\ 0\ 0 \end{array}$$

Operation

Value

A M

0

0000000 0000000

+1 (M)

1111111 1100111

-1 (-M)

0000000 0011011

+2 (LSM)

1111111 00110

-2 (LSM)

0000000 011010

Q

Q₀

1 0 1 1 0 0 0
U U U U U U

-1 1 0 -1 0 0
U U U

2(-1) + 1 2(0) - 1 2(0) + 0

-1 -1 0

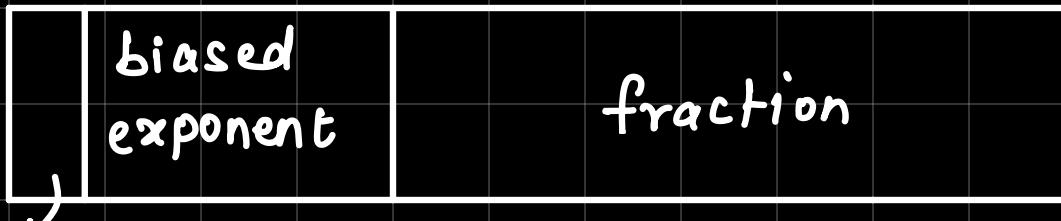
$$\begin{array}{r} 110011 \\ -1-10 \\ \hline 00000100 \quad 0000000 \\ 0000000 \quad 1101++ \\ 000011 \quad 01+++ \\ \hline 000100 \quad 000100 \\ 2^8 \quad + 2^2 \end{array}$$

$$= 260$$

Inst. of Elec. & Electronic Engineers

IEEE 754 formats

← 8 bits → ← 23 bits →



sign
bit

$$(1 \cdot N)^2 E-127$$

32-BIT.



$$(1 \cdot N)^2 E-1023$$

64-BIT.

→ most common representation today for
real no. on computers .

Draw IEEE 754 Floating

1. Convert Decimal \rightarrow Binary

2. Normalization

$$\Rightarrow 111.011 = 1.11011 \times 2^2 \quad \text{exponent}$$

$$\Rightarrow 0.00010 = 00001.0 \times 2^{-4}$$

3. Biasing

Single Precision : E-127

Double Precision : E - 1023

$$Q \cdot 12 \cdot 25 \quad (\underline{1-N})$$



$$\begin{array}{r} 12 \\ 2 \Big| \quad 6 - 0 \\ 2 \Big| \quad 3 - 0 \\ 2 \Big| \quad 1 - 1 \end{array}$$

100001

$$\begin{array}{r} 0.25 \\ \times 2 \\ \hline 0.50 \\ \times 2 \\ \hline 1.00 \end{array}$$

2. Normalization

$$\frac{1.10001 \times 2^3}{\text{mantissa}}$$

↑ exponent

3. Biasing

$$Z = E - 127 : \text{Single Precision}$$

$$E = 130$$

$3 = E - 1023$

: Double Precision

$E = 1026$

$$\begin{array}{r|l} 2 & 130 \\ 2 & 65 - 0 \\ 2 & 32 - 1 \\ 2 & 16 - 0 \\ 2 & 8 - 0 \\ 2 & 4 - 0 \\ 2 & 2 - 0 \\ 2 & 1 - 0 \end{array} \Rightarrow 10000010$$

$$\begin{array}{r|l} 2 & 1026 \\ 2 & 514 - 0 \\ 2 & 257 - 0 \\ 2 & 128 - 1 \\ 2 & 64 - 0 \\ 2 & 32 - 0 \\ 2 & 16 - 0 \end{array}$$

2	8 - 0	
2	4 - 0	
2	2 - 0	
	1 - 0	$\Rightarrow 10000000010$

Single Precision

0	10000010	10001
---	----------	-------

sign bit 8 bits 23 bits

$\Rightarrow 32 \text{ BIT}$

Double Precision

0	100000000010	10001
---	--------------	-------

↑
sign bit 11 bits 52 bits

$\Rightarrow 64 \text{ bit}$

Q. 25.44

$$\begin{array}{r|rr} 2 & 25 \\ 2 & 12 - 1 \\ 2 & 6 - 0 \\ 2 & 3 - 0 \\ 1 & - 1 \end{array} \Rightarrow 11001$$

$$\begin{array}{r} 0.44 \\ \times \quad 2 \\ \hline 0.88 \end{array}$$

$$\Rightarrow 01110000101000111101$$

0111

$$\begin{array}{r} x \quad 2 \\ \hline 1.52 \end{array}$$

$$\begin{array}{r} x \quad 2 \\ \hline 1.04 \end{array}$$

$$\begin{array}{r} x \quad 2 \\ \hline 0.08 \end{array}$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$0.16$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$0.32$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$0.64$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$1.28$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$0.56$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$1.12$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$0.24$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$0.48$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$0.96$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$1.92$$

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

$$1.84$$

$$\begin{array}{r} \times 2 \\ \hline 1.68 \end{array}$$

$$\begin{array}{r} \times 2 \\ \hline 1.36 \end{array}$$

$$\begin{array}{r} \times 2 \\ \hline 0.72 \end{array}$$

$$\begin{array}{r} \times 2 \\ \hline 1.44 \end{array}$$

$$\begin{array}{r} \times 2 \\ \hline 0.88 \end{array}$$

$$\begin{array}{r} \times 2 \\ \hline 1.76 \end{array}$$

$$\begin{array}{r} \times 2 \\ \hline 1.52 \end{array}$$

$$\begin{array}{r} \times 2 \\ \hline 1.04 \end{array}$$

11001.01110000101000111010111

FP Arithmetic +/-

- Check for zeroes
- Align significands (exponent)
- Add / subtract significands
- Normalize Result

$$8.70 \times 10^{-1} \text{ with } 9.95 \times 10^1$$

$$\hookrightarrow 0.087 \times 10^1 \text{ with } 9.95 \times 10^1$$

$$9.95 + 0.087 = 10.037$$

$$10.037 \times 10^1$$

↓

$$1.0037 \times 10^2$$