Chapter 2

Coplanar Forces: Resolution and Composition of Forces

2.1 Introduction

In this chapter we will define the most basic concept in mechanics, the 'Force' and know its various characteristics and effects. We shall further learn to 'break', technically known as 'resolution' of a force and to 'combine' technically known as 'composition' of forces. We shall be studying the different 'systems of forces' and finally we shall learn about 'couples' and their properties.

2.2 Force

Force is defined as an agency which changes or tends to change the state of rest or of uniform motion of a body. For example a man trying to push a cupboard as shown in figure 2.1(a) exerts a force 'F'. If the force is sufficient to cause motion, the cupboard will slide to the right and if not sufficient, the force F only 'tends' to change the state of rest but not succeeding to cause motion.

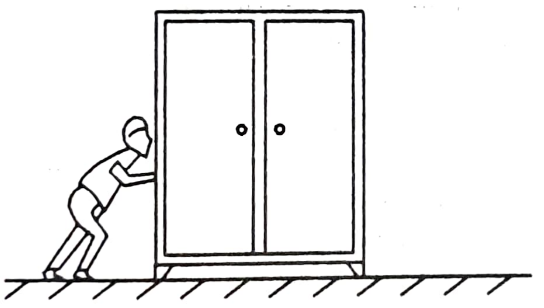


Fig. 2.1 (a)

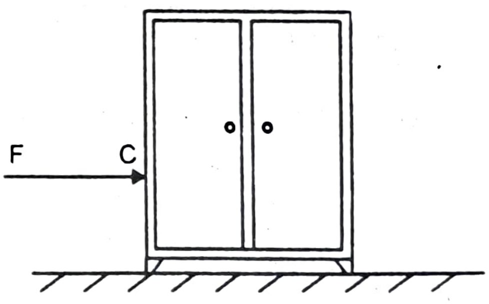


Fig. 2.1 (b)

Similarly Fig. 2.2 shows an arrangement to bring a rotating wheel to a stop by applying force P to press the brakes.

Force is a vector quantity.
It is characterised by
i) magnitude
ii) direction
iii) sense
iv) point of application

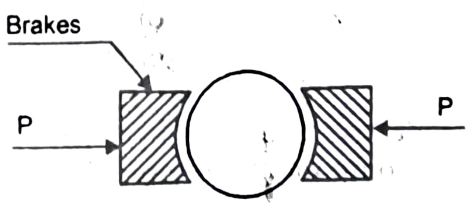


Fig. 2.2

For the person pushing the cupboard, refer Fig. 2.1 (a) and 2.1 (b) the man applies a force of magnitude F, whose direction is horizontal, sense is to the right and point of application is at C.

The S.I. unit of force is Newton (N) or its multiples i.e. kilo-Newton (kN), where 1 kN = 1000 N

The unit of force has been named after Sir Issac Newton and one Newton force is defined as a force required to produce an acceleration of 1 m/s^2 in a body of mass 1 kg.

2.3 Resolution of a Force

Resolution or resolving a force implies breaking the force into components, such that the components combined together would have the same effect as the original force. Fig. 2.3 (a) shows a force F acting at an angle 'θ'. This force can be resolved into components F_1 and F_2 as shown in Fig. 2.3 (b) or into components F_x and F_y as shown in Fig. 2.3 (c).

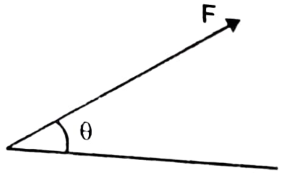


Fig. 2.3 (a)

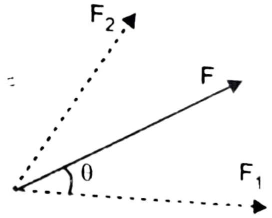


Fig. 2.3 (b)

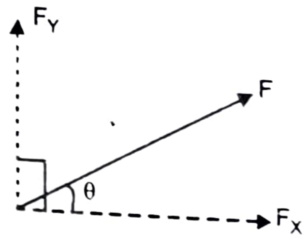


Fig. 2.3 (c)

The components F_x and F_y of the force F as shown in Fig. 2.3 (c) are known as the *rectangular* or *perpendicular* components of the force, since the two components are perpendicular to each other.

Usually we require rectangular components of force and hence let us learn how to find rectangular components of a force.

2.3.1. Resolution of a Force into Rectangular Components

Consider a force of magnitude F acting at an angle θ with the x-axis. Let the force be represented by a line OA drawn to scale. From A drop a perpendicular on the x-axis at E and on the y-axis at P.

Length (OE) represents the magnitude of the component along x-axis (F_x) with direction from O to E, while length (OP) represents the magnitude of the component along y-axis (F_y) with direction from O to P.

From Geometry $F_x = F \cos\theta \rightarrow$

$F_y = F \sin\theta \uparrow$

These are the two rectangular components of force F.

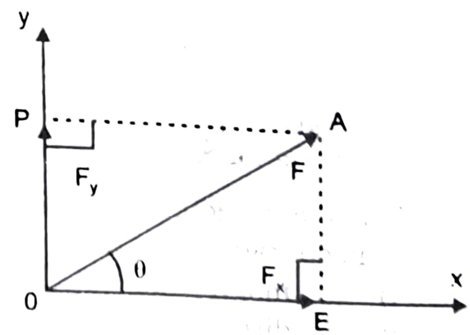
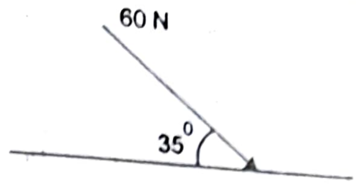


Fig. 2.4

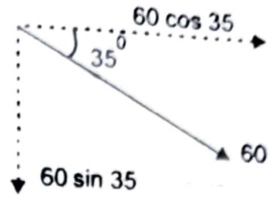
Ex. 2.1 Resolve the force P = 60 N into horizontal and vertical component.



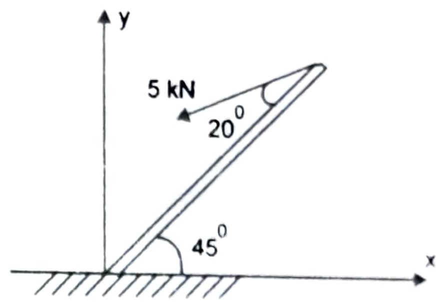
Solution:

F_x = 60 cos 35 = 49.15 N →

F_y = 60 sin 35 = 34.41 N↓



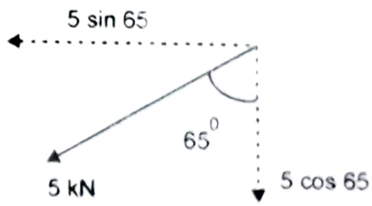
Ex. 2.2 A 5 kN force acts on one end of a rod as shown. Resolve the force along x and y direction.



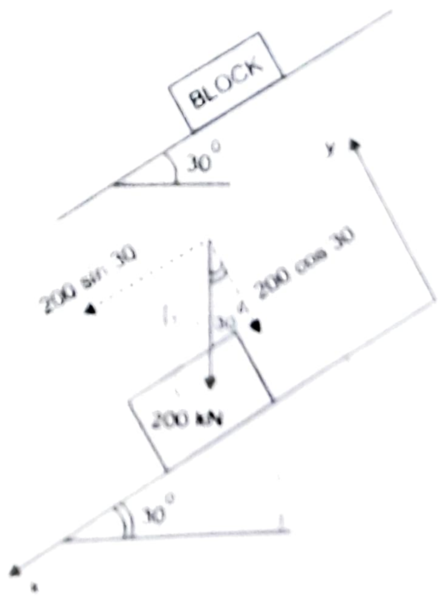
Solution: Total angle made by 5 kN force with the vertical is 65°

∴ F_x = 5 sin 65 = 4.53 kN ←

F_y = 5 cos 65 = 2.11 kN ↓



Ex. 2.3 A block of 200 kN weight is kept on a plane inclined at 30° with the horizontal. Find the components along and normal to the plane.



Solution :

The weight of the block shall act vertically down as shown. Let us choose the x-axis along the plane and y-axis normal to the plane.

Resolving we get

F_x = 200 sin 30 = 100 N

F_y = 200 cos 30 = 173.2 N

2.4 System of Forces

'System of Forces' tells us about how the forces are arranged. For example all the forces may lie on one plane or may lie on different planes. They may meet at a point, may be parallel to each other or may just neither be parallel, nor may meet at a point. 'System' defines this arrangement of forces and is classified as shown.

System of Forces

- COPLANAR
  - CONCURRENT
  - PARALLEL
  - GENERAL
- NON-COPLANAR (SPACE FORCES)

2.4.1 *Coplanar system*

In this arrangement all the forces lie in one plane. Fig. 2.5 shows a coplanar system consisting of four forces F_1, F_2, F_3 and F_4 all lying in the 'xoy' plane.

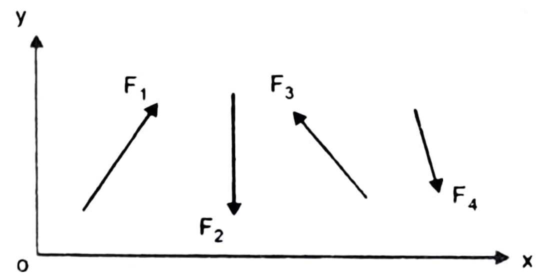


Fig. 2.5

A coplanar system is further sub-divided into

a) *Concurrent system* : In this system all the forces meet at a point. Examples of concurrent system are,

i) A lamp hanging from two strings. If T_1 and T_2 are the forces developed in the strings and W be the weight of the lamp, then we have a concurrent system at a point A

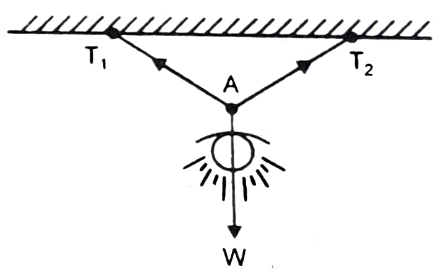


Fig. 2.6 (a)

ii) An electric pole supporting heavy electric cables. If F_1, F_2, F_3 are the forces in the cable, and W be the weight of the pole, then the forces form a concurrent system.

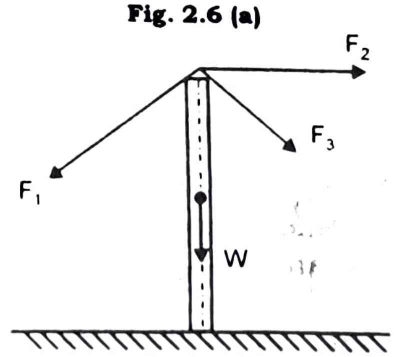


Fig. 2.6 (b)

...: In this system the lines of action of the forces are parallel.

For example;

i) A vegetable vendor weighing the vegetables, the weight W_1 of vegetables, the measured weight W_2, the force F applied by hand to hold the weighing scale, all three forces form a parallel system. Refer Fig. 2.7 (a).

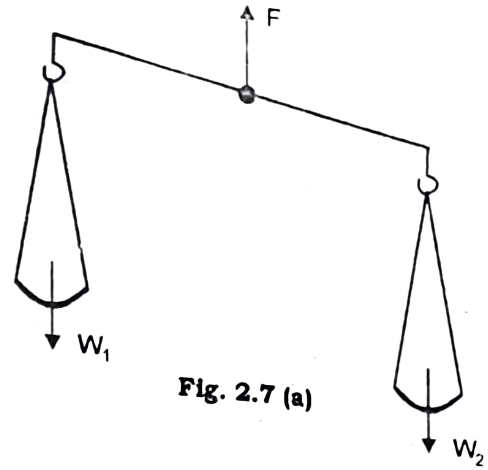


Fig. 2.7 (a)

ii) Person sitting on a bench. Fig. 2.7 (b) shows three persons of weights P_1, P_2 and P_3 sitting on a bench of self weight W. If R_1 and R_2 are the reactions offered by the ground then the six forces form a parallel system.

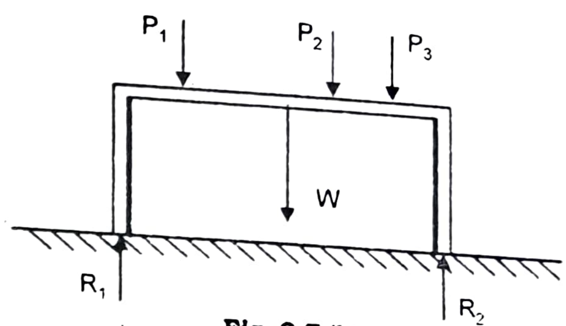


Fig. 2.7 (b)

c) *General system* : Also known as a 'non-concurrent and non-parallel' system has forces which do not meet at a single point, nor are parallel to each other. For example the forces acting on the rectangular plate form a general system. Refer Fig. 2.8.

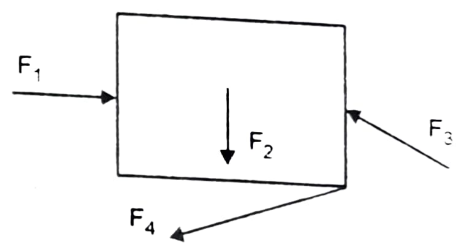


Fig. 2.8

2.4.2 *Non-coplanar System*

When the forces acting in a system do not lie in a single plane, they are termed as Non-coplanar forces or Space forces. Refer Fig. 2.9.

We shall learn to operate with Non-coplanar system in Chapter No. 9 on Space Force System.

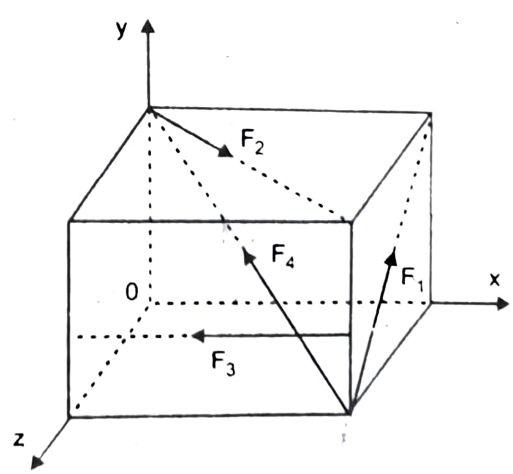


Fig. 2.9

2.5 Principle of Transmissibility of Force

It states "A force being a sliding vector continues to act along its line of action and therefore makes no change if it acts from a different point on its line of action on a rigid body". Consider a rigid body as shown in Fig. 2.10, acted upon by a force of magnitude F acting at A. The effect on the body would remain unchanged if it acted from point B, C or any other point on its line of action.

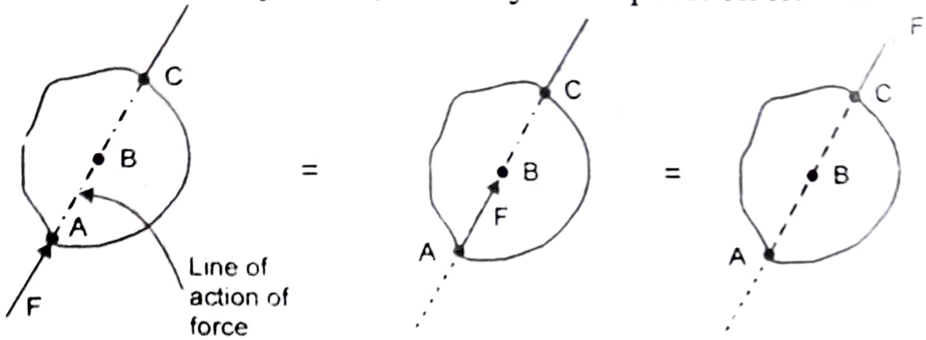


Fig. 2.10

2.6 Moment of a Force

We all are aware that a force can cause a body to slide, but at the same time it can cause a body to rotate also. *The rotational effect of a force is known as the moment of the force.* When we talk of the rotational effect, it has to be with respect to a point. The concerned point is known as the *moment centre*. The rotational effect of the same force will vary from one moment center to another and of course if the point (moment centre) lies on the line of action of the force, the moment of force about the point would be zero.

The rotational effect or moment is measured as the product of the force and the perpendicular distance from the moment centre to the force. This perpendicular distance is known as the *moment arm* 'd'.

$$\therefore \quad \mathbf{M = F \times d} \qquad \text{........... } \mathbf{2.1}$$

The tendency to rotate could be either clockwise or anti-clockwise. Fig. 2.11 shows a force of magnitude F acting on a rectangular plate ABCD as shown.

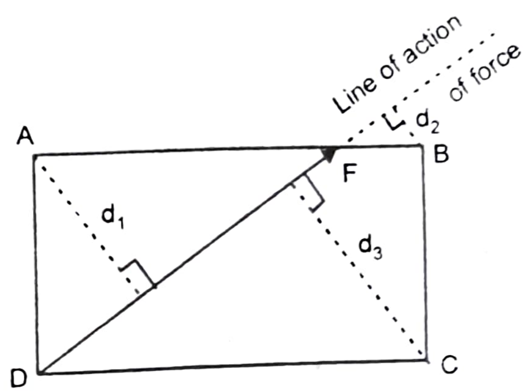


Fig. 2.11

The moment of F about A = $F \times d_1$ (anti-clockwise)
about B = $F \times d_2$ (clockwise)
about C = $F \times d_3$ (clockwise)
about D = 0

Units of moment are N-m or N-mm or kN-m

Sign convention: we shall take anti-clockwise moments as positive moments, and clockwise moments as negative. This shall be indicated as ↺ + ve

2.7 Varignon's Theorem

Varignon, a French mathematician (1654 – 1722) established that the sum of the moments of a concurrent system of forces about any point is equal to the moment of the resultant of the concurrent system about the same point. Though originally derived for a concurrent system of forces, this theorem can in fact be applied to any system of forces and is thus stated as "*the algebraic sum of the moments of a system of coplanar forces about any point in the plane is equal to the moment of the resultant force of the system about the same point*".

Mathematically it is written as

$$\Sigma M_A^F = M_A^R \qquad \text{............ 2.2}$$

Sum of moments of all forces about any point, say point A. = Moment of the resultant about the same point A.

Proof – Let P and Q be two concurrent forces at O, making angle θ_1 and θ_2 with the x-axis, let R be their resultant making an angle θ with x-axis.
Let A be a point on the y-axis about which we shall find the moments of P and Q and also of the resultant R. Let d_1, d_2 and d be the moment arm of P, Q and R from moment centre A.

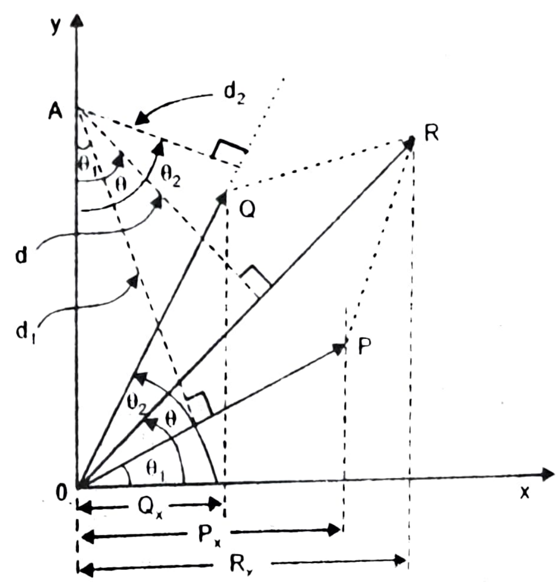


Fig. 2.12

Now, Moment of P about A = $M_A^P = P \times d_1$ --------(1)

Moment of Q about A = $M_A^Q = Q \times d_2$ --------(2)

Moment of R about A = $M_A^R = R \times d$

$= R\,(OA \cos\theta)$

$= OA\,(R_x)$ --------(3)

Adding equations (1) and (2) we have

$$M_A^P + M_A^Q = P\,d_1 + Q\,d_2$$

$$\Sigma M_A^F = P \times OA \cos\theta_1 + Q \times OA \cos\theta_2$$
$$= OA.\ P_x + OA.Q_x \qquad \text{since } P_x = P\cos\theta_1 \text{ and } Q_x = Q\cos\theta_2$$
$$= OA\,(P_x + Q_x)$$

$\therefore \quad \Sigma M_A^F = OA\,(R_x)$ ---------(4)

since the resultant of forces in the 'x' direction is the sum of components of forces in the 'x' direction

Comparing equation (4) with (3)

$$\Sigma M_A^F = M_A^R \qquad \text{----- Proved}$$

The above equation can similarly be extended for more than two forces in the system.

2.8 Composition of Forces

Composition means to combine the forces acting in a system into a single force, which has the same effect as the number of forces acting together. Such a single force is known as *resultant of the system*. Finding the 'resultant' helps to analyse the effect of the forces on the system and may form an important step in the solution of engineering problems. We shall learn to find the resultant of

(a) Concurrent system of two forces using law of parallelogram of forces.
(b) Concurrent system of more than two forces using method of resolution.
(c) Parallel system of forces.
(d) General system of forces.

2.9 Resultant of Concurrent System of Forces using Parallelogram Law of Forces

Parallelogram law of forces states "If two forces acting simultaneously on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces".

Let P & Q be the two forces acting at a point and making an angle 'α' with each other as shown in Fig.2.13 (a). The forces are drawn to scale in figure such that AB and AD represent forces P & Q. Completing the parallelogram ABCD, the diagonal AC gives the magnitude and direction of the resultant R. Refer Fig. 2.13 (b).

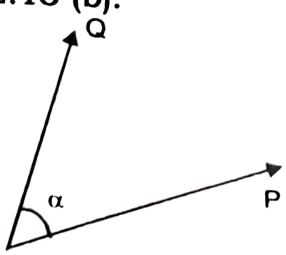


Fig. 2.13 (a)

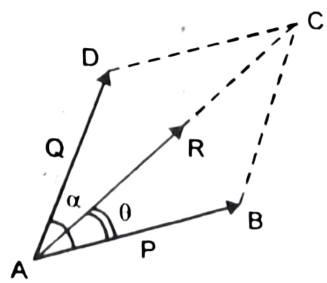


Fig. 2.13 (b)

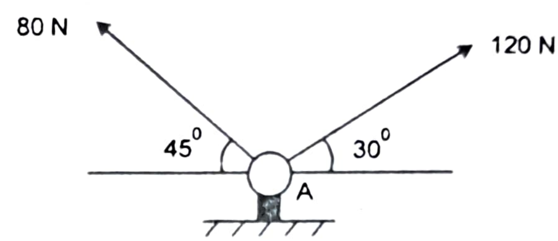
Mathematically $$\mathbf{R} = \sqrt{\mathbf{P^2 + Q^2 + 2PQ\cos\alpha}} \qquad \text{......... 2.3 (a)}$$

$$\tan\theta = \frac{\mathbf{Q\sin\alpha}}{\mathbf{P + Q\cos\alpha}} \qquad \text{......... 2.3 (b)}$$

Where θ is the angle made by resultant R with the force P

Though parallelogram law is for two forces, it can be used for more forces also but would require repeated use, taking two forces at a time.

Ex. 2.4 Two forces of 120 N and 80 N act on an eye-bolt at A as shown. Determine the resultant of the two forces.



Solution : Using parallelogram law of forces ,

Let P = 120 N, Q = 80 N

Angle between P and Q = α = 180 – 45 – 30 = 105°

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{120^2 + 80^2 + 2 \times 120 \times 80 \times \cos 105}$$

$$= 125.8 \text{ N}$$ .... **Ans.**

If 'θ' is the angle made by the resultant R with the force P then

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$$= \frac{80\sin 105}{120 + 80\cos 105}$$

$$= 0.778$$

or $\theta = 37.89°$ .... **Ans.**

Ex. 2.5 Find the magnitude of forces P & Q such that if they act at right angles their resultant is $\sqrt{34}$ N. If they act at an angle of 60°, their resultant is 7 N.

Solution :

Case 1: P and Q act at α = 90° and R = $\sqrt{34}$ N

Using $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\sqrt{34} = \sqrt{P^2 + Q^2 + 2PQ\cos 90}$$

squaring both sides, we have

$$34 = P^2 + Q^2 \quad \text{------------ (1)}$$

Case 2: P and Q act at $\alpha = 60°$ and R = 7 N

Using

$$R = \sqrt{P^2 + Q^2 + 2\,PQ\cos\alpha}$$

$$7 = \sqrt{P^2 + Q^2 + 2\,PQ\cos 60}$$

squaring both sides, we have

$$49 = P^2 + Q^2 + PQ \quad \text{------------ (2)}$$

substituting equation (1) in (2)

$$49 = 34 + PQ$$

$$15 = PQ$$

or

$$Q = 15/P \quad \text{------------ (3)}$$

substituting equation (3) in (1)

$$34 = P^2 + (15/P)^2$$

$$34P^2 = P^4 + 225$$

or

$$P^4 - 34P^2 + 225 = 0$$

This is a quadratic in P^2

$$\therefore \quad P^2 = \frac{34 \pm \sqrt{(34)^2 - 4 \times 1 \times 225}}{2 \times 1}$$

$$\therefore \quad P^2 = 25 \quad \text{or} \quad P^2 = 9$$

$$\therefore \quad P = 5\text{ N} \quad \text{or} \quad P = 3\text{ N}$$

If P = 5 N, Q = 3 N .... **Ans.**
If P = 3 N, Q = 5 N .... **Ans.**

2.10 Resultant of Concurrent System of Forces Using Method of Resolution

When more than two forces act at a point, the use of method of resolution is made to avoid tedious repetition of parallelogram law of forces to successive forces. The following steps are adopted in the solution.

Step 1: Resolve the forces along the horizontal x direction and the vertical y direction

Step 2: a) Add up the x components of forces using the sign convention

$\rightarrow$ + ve to get ΣF_x

b) Add up the y components of forces using the sign convention

$\uparrow$ + ve to get ΣF_y

c) The resultant force $R = \sqrt{\Sigma F_x^{\,2} + \Sigma F_y^{\,2}}$

Step 3: The direction of the resultant force is the angle θ made by it with the x axis.

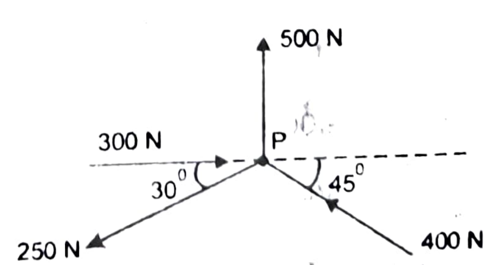
$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

Note: while using the above relation take positive values of ΣF_y and ΣF_x. The value of θ so obtained will always be less than 90°

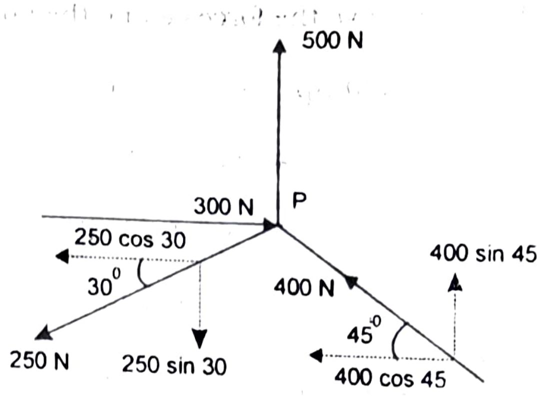
Step 4: Decide the quadrant of the resultant, depending on the signs of ΣF_x and ΣF_y

ΣF_x is + ve, ΣF_y is + ve	1 st Quadrent
ΣF_x is - ve, ΣF_y is + ve	2 nd Quadrent
ΣF_x is - ve, ΣF_y is – ve	3 rd Quadrent
ΣF_x is + ve, ΣF_y is – ve	4 th Quadrent

✓ **Ex. 2.6** Find the resultant of the four concurrent forces acting on a particle P.



Solution: This is a concurrent system of forces.



$\Sigma F_x \rightarrow$ + ve

$= 300 - 250 \cos 30 - 400 \cos 45$

$= -199.3$ N

$= 199.3$ N $\leftarrow$

$\Sigma F_y \uparrow$ + ve

$= 500 - 250 \sin 30 + 400 \sin 45$

$= 657.8$ N $\uparrow$

Using $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{199.3^2 + 657.8^2}$

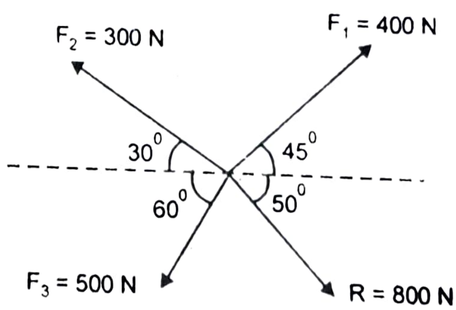
$\therefore \quad R = 687.4$ N

also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{657.8}{199.3}$

$\therefore \quad \theta = 73.1°$

Resultant force R = 687.4 N at θ = 73.1° acts at particle P. .... **Ans.**

Ex. 2.7: R = 800 N is the resultant of 4 concurrent forces. Find the fourth force F_4



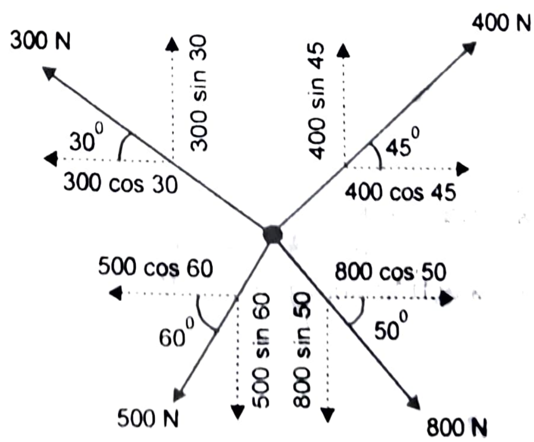
Solution: This is a concurrent system of four forces.

Let $(F_4)_x$ and $(F_4)_y$ be the perpendicular components of the fourth force

Since it is given R = 800 N at θ = 50°

$\therefore \quad \Sigma F_x = 800 \cos 50 \rightarrow$

and $\quad \Sigma F_y = 800 \sin 50 \downarrow$



$\Sigma F_x \rightarrow$ + ve

$800 \cos 50 = 400 \cos 45 - 300 \cos 30 - 500 \cos 60 + (F_4)_x$

$\therefore \quad (F_4)_x = 741.2 \text{ N} \rightarrow$

$\Sigma F_y \quad \uparrow \quad + \text{ve}$

$800 \sin 50 = 400 \sin 45 + 300 \sin 30 - 500 \sin 60 + (F_4)_y$

$\therefore \quad (F_4)_y = -612.6 \text{ N}$

$= 612.6 \text{ N} \downarrow$

Now $F_4 = \sqrt{(F_4)_x^{\,2} + (F_4)_y^{\,2}}$

$= \sqrt{741.2^2 + 612.6^2}$

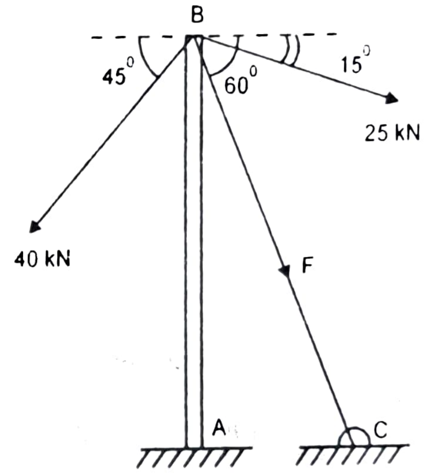
$= 961.6 \text{ N}$

also $\tan\theta = \dfrac{(F_4)_y}{(F_4)_x} = \dfrac{612.6}{741.2}$

$\therefore \quad \theta = 39.6°$

The fourth force $F_4 = 961.6$ N at $\theta = 39.6°$ ↘ ....**Ans.**

Ex. 2.8 Determine the force F in cable BC if the resultant of the 3 concurrent forces acting at B is vertical. Also determine the resultant.

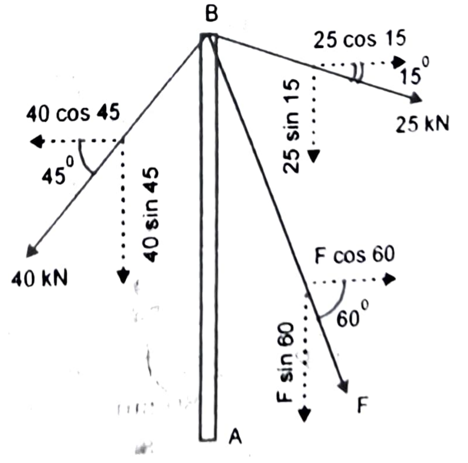


Solution: This is a concurrent system of three forces acting at B. Since the resultant force is vertical, it implies $\Sigma F_x = 0$

$\Sigma F_x \rightarrow + \text{ve}$

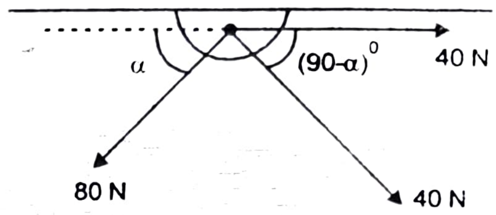
$25 \cos 15 - 40 \cos 45 + F \cos 60 = 0$

$\therefore \quad F = 8.27 \text{ kN}$ ....**Ans.**



Also resultant $R = \Sigma F_y$
$= -40 \sin 45 - 25 \sin 15 - 8.27 \sin 60$
$= -41.92$ kN
$\therefore\ R = 41.92$ kN $\downarrow$ ....**Ans.**

Ex. 2.9 Three coplanar forces act at a point on a bracket as shown. Determine the value of the angle α such that the resultant of the three forces will be vertical. Also find the magnitude of the resultant.



Solution: It is given that the resultant of the three concurrent forces is vertical, also since ΣF_x and ΣF_y are the components of the resultant, we have

$\Sigma F_x = 0$ because resultant is vertical i.e. in the y direction

$\therefore\ \Sigma F_x \rightarrow +ve$

$$0 = -80 \cos\alpha + 40 \cos(90-\alpha) + 40$$
$$0 = -2\cos\alpha + \sin\alpha + 1$$

$$\frac{\cos\alpha}{1+\sin\alpha} = \frac{1}{2}$$

$$\frac{\sin(90-\alpha)}{1+\cos(90-\alpha)} = 0.5$$

$$\frac{\sin(90-\alpha)}{2\cos^2\frac{(90-\alpha)}{2}} = 0.5$$

$$\frac{2\sin\frac{(90-\alpha)}{2}\cos\frac{(90-\alpha)}{2}}{2\cos^2\frac{(90-\alpha)}{2}} = 0.5$$

$$\tan\frac{(90-\alpha)}{2} = 0.5$$

$$\frac{90-\alpha}{2} = 26.56$$

or $\alpha = 36.86°$ ....**Ans.**

Resultant $R = \Sigma F_y \uparrow +ve$
$= -80 \sin\alpha - 40 \sin(90-\alpha)$
$= -80 \sin 36.86 - 40 \sin 53.13$
$= -80$ N
$\therefore\ R = 80$ N $\downarrow$ ....**Ans.**

2.11 Resultant of Parallel System of Forces

To find the Resultant of Parallel Force System follow the given steps,

Step 1: Since in a parallel system, the forces are directed in one direction only, they can be simply added up using a sign convention for the sense of the force. i.e.

$$R = \Sigma F$$

Step 2: Location of the resultant force forms an important step. The point of application of the resultant force is found out using Varignon's theorem, discussed earlier in article 2.7. The resultant is initially assumed to act either to the right or left of the reference point at a $\perp$ distance d.

Varignon's theorem $\Sigma M_A{}^F = M_A{}^R$ is used. If a positive value of d is obtained then the assumption made earlier is true. If a negative value of d is obtained the resultant lies on the opposite side to what was assumed.

2.12 Couple

Couple is a special case of parallel forces. Two parallel forces of equal magnitude and opposite direction form a couple. The effect of a couple is to rotate the body on which it acts. Fig. 2.14 shows a couple formed by two forces of same magnitude F, separated by a $\perp$ distance d known as the arm of the couple.

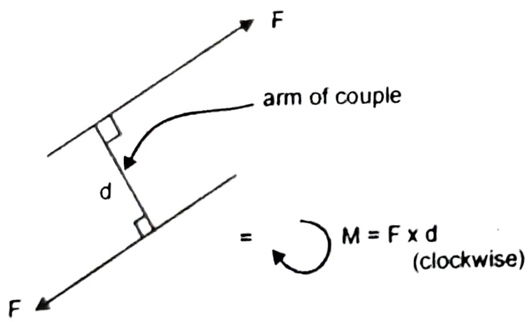


Fig. 2.14 (a)

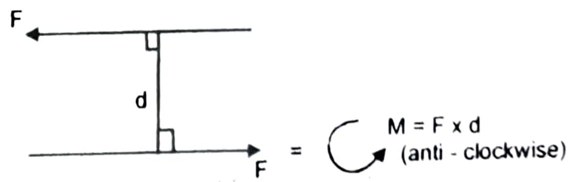


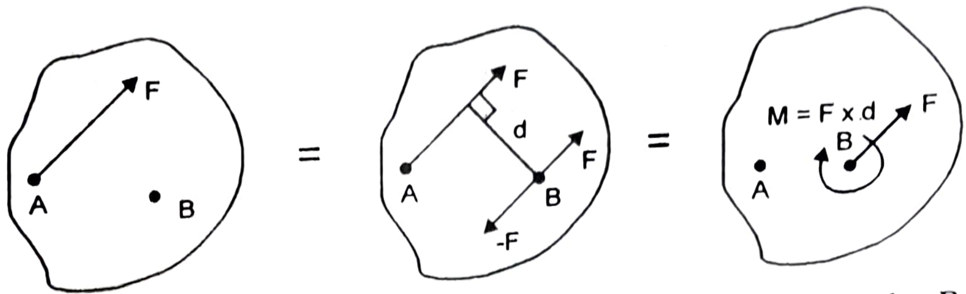
Fig. 2.14 (b)

The magnitude of rotation known as the moment of a couple is $M = F \times d$. Rotation of a couple could be clockwise Fig. 2.14 (a) or could be anti-clockwise Fig. 2.14(b). Couples are represented by curved arrows. Units are N.m, kN.m etc.

2.13 Properties of Couple

1. Couple tend to cause rotation of the body about an axis $\perp$ to the plane containing the two parallel forces.
2. The magnitude of rotation or moment of a couple is equal to the product of one of the forces and the arm of the couple.

3. Couple is a free vector because of which it can be moved anywhere on the body on which it acts without causing any change.
4. The resultant force of a couple system is zero.
5. To balance a system whose resultant is a couple, another couple of the same magnitude and opposite direction is required to be added.
6. To shift a force to a new parallel position, a couple is required to be added to the system. For example in Fig. 2.15 (a), force F is required to be shifted from its original position at A to a new parallel position B. This is done by adding two collinear forces of same magnitude F and – F at B Fig. 2.15 (b). The two parallel forces F at A and – F at B form a couple. Thus we have a single force F at B and a couple M = F × d in the system Fig. 2.15 (c).



Force originally at A

Fig. 2.15 (a)

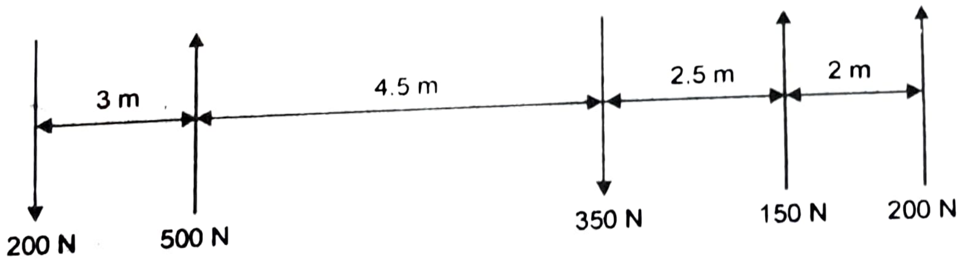
Two collinear forces added at B

Fig. 2.15 (b)

Force shifted to B with a couple added

Fig. 2.15 (c)

Ex. 2.10 Determine the resultant of the parallel system of forces.



Solution: This is a parallel system of five forces.

Resultant force $R = \Sigma F \quad \uparrow + ve$

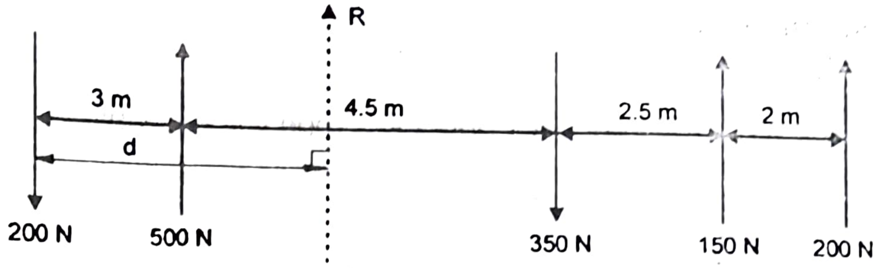
$= -200 + 500 - 350 + 150 + 200$

$= 300$ N

$\therefore \quad R = 300 \text{ N} \uparrow$

Location of the resultant force.

Let the resultant force be located at a ⊥ distance 'd' to the right of the 200 N force as shown. Let 'A' be a point on the line of action of the 200 N force.



Using Varignon's theorem

$\Sigma M_A{}^F = M_A{}^R$ ↺ + ve

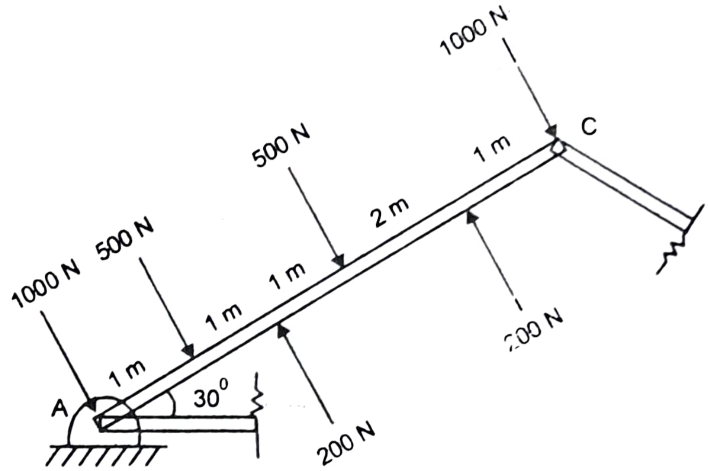
$= 500 \times 3 - 350 \times 7.5 + 150 \times 10 + 200 \times 12 = 300 \times d$

$\therefore \quad d = 9.25$ m

Hence, the resultant force R = 300 N ↑ lies at ⊥ distance d = 9.25 m to the right of A.

.... **Ans.**

Ex. 2.11 A roof truss is acted upon by wind and other forces as shown. All the forces are parallel and normal to the AC portion of the truss. Find the resultant of the forces and its location w.r.t. hinge A.



Solution: This is a parallel system of six forces acting on the roof truss.

Resultant R = ΣF ↖ + ve

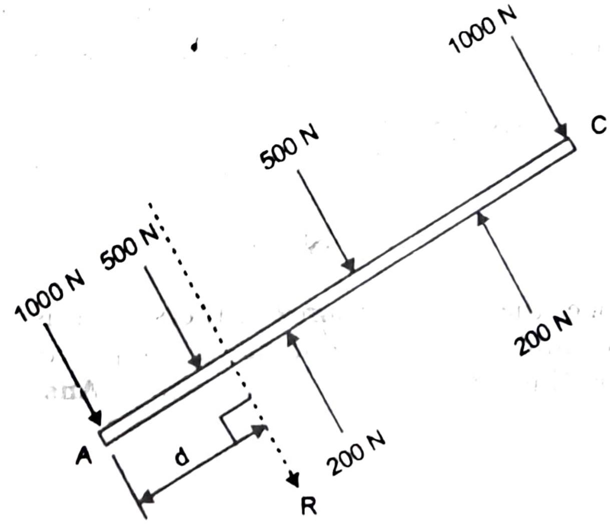
= – 1000 – 500 + 200 – 500 + 200 – 1000

= – 2600 N

= 2600 N ↘ 60°

Location of the resultant force

Let the resultant force be located at a perpendicular distance d to the right of A.

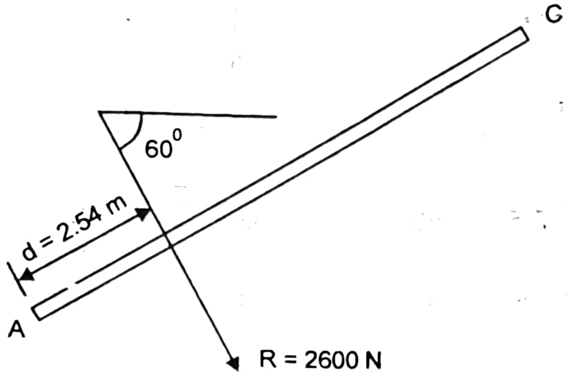


Using Varignon's theorem

$\Sigma M_A^F = M_A^R$ ↺ + ve

$$-500 \times 1 + 200 \times 2 - 500 \times 3 + 200 \times 5 - 1000 \times 6 = -2600 \times d$$

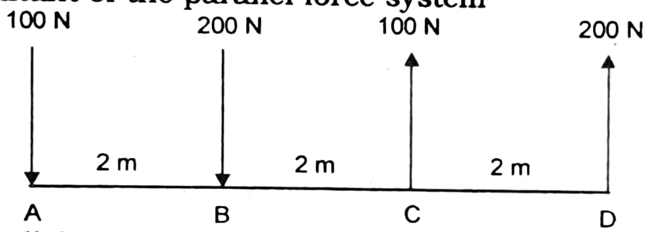
$\therefore \quad d = 2.54$ m



Hence the resultant R = 2600 N at θ = 60° ↘ lies at a ⊥ distance d = 2.54 m, to the right of A

.... **Ans.**

Ex. 2.12 Find the resultant of the parallel force system



Solution: This is a parallel system of four forces

$R = \Sigma F \uparrow + ve$

$= -100 - 200 + 100 + 200$

$= 0$

$\therefore$ the resultant force $R = 0$

Since the resultant force of the parallel system is zero, the resultant could be a couple. To find the value of the couple, taking moments of all forces about any point say A.

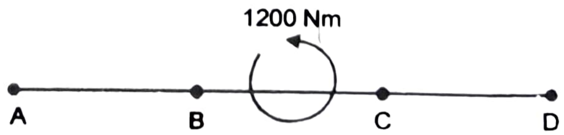
ΣM_A ↺ + ve

$= -200 \times 2 + 100 \times 4 + 200 \times 6$

$=$ 1200 Nm

$=$ 1200 Nm ↺

Hence, the resultant force of the system is zero and the resultant is a couple of 1200 Nm ↺ .... **Ans.**

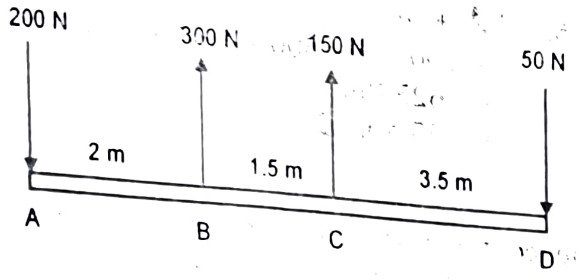


Ex. 2.13 Figure shows four parallel forces acting on a beam ABCD.

i) Determine the resultant of the system and its location from A.

ii) Replace the system by a single force and a couple acting at point B.

iii) Replace the system by a single force and couple acting at point D.



Solution: This is a parallel system of four forces acting on the beam ABCD.

i) Resultant $R = \Sigma F \uparrow +ve$

$= -200 + 300 + 150 - 50$

$= 200$ N

$= 200$ N $\uparrow$

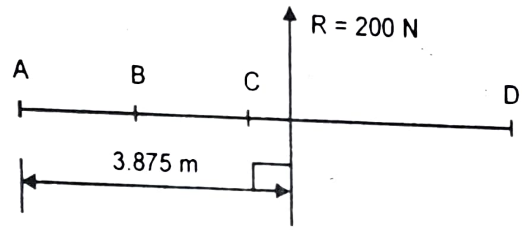
Location of resultant force from A

Let the resultant force be located at a perpendicular distance 'd' to the right of A.
Using Varignon's theorem

$\Sigma M_A^F = M_A^R \circlearrowleft +ve$

$-300 \times 2 - 150 \times 3.5 + 50 \times 7 = -200 \times d$

$\therefore \quad d = 3.875$ m



$\therefore$ The resultant force is $R = 200$ N $\uparrow$ at a perpendicular distance $d = 3.875$ m to the right of A. .... **Ans.**

(ii) The single force acting at B would be the resultant $R = 200$ N $\uparrow$

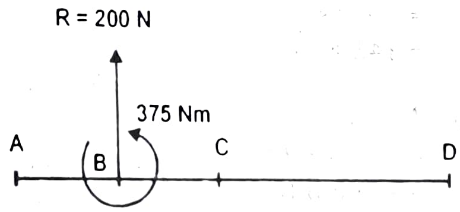
To find the couple at B, take moments of all forces about B

$\Sigma M_B \circlearrowleft +ve$

$= 200 \times 2 + 150 \times 1.5 - 50 \times 5$

$= 375$ Nm

$= 375$ Nm $\circlearrowleft$



Hence, the system of four forces can be replaced by a single force $R = 200$ N $\uparrow$ and a couple of 375 Nm $\circlearrowleft$ at B. This is referred to as a force couple system at B. .... **Ans.**

(iii) The single force acting at D would be the resultant $R = 200$ N $\uparrow$

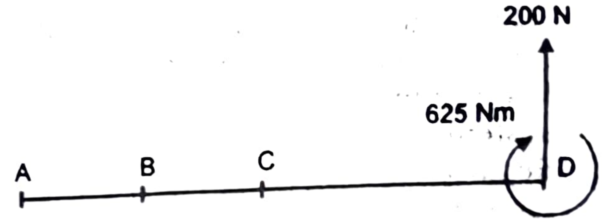
To find the couple at D, take moments of all forces about D

ΣM_D ↺ + ve

$= 200 \times 7 - 300 \times 5 - 150 \times 3.5$

$= -625$ Nm

$= 625$ Nm ↻



Hence, the system of four forces can be replaced by a single force $R = 200$ N ↑ and a couple of 625 Nm ↻ at D. This is referred to as a force couple system at D. .... **Ans.**

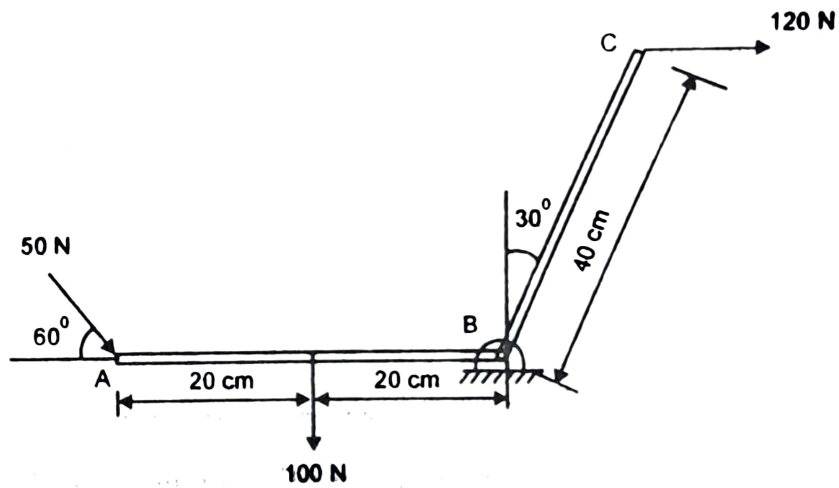
2.14. Resultant of General Force System

To find the Resultant of General Force System follow the given steps.

Step 1 : Follow the same procedure as discussed in article 2.10 and get the resultant force R of the system using Method of Resolution.

Step 2 : To locate the position of the resultant follow step 2 of article 2.11.

Ex. 2.14 Find the resultant of the forces acting on the bell crank lever shown. Also locate its position w.r.t hinge B.



Solution: This is a general system of three forces acting on the bell crank.

$\Sigma F_x \rightarrow +$ ve

$= 50 \cos 60 + 120$

$= 145$ N →

$\Sigma F_y \uparrow +$ ve

$= -50 \sin 60 - 100$

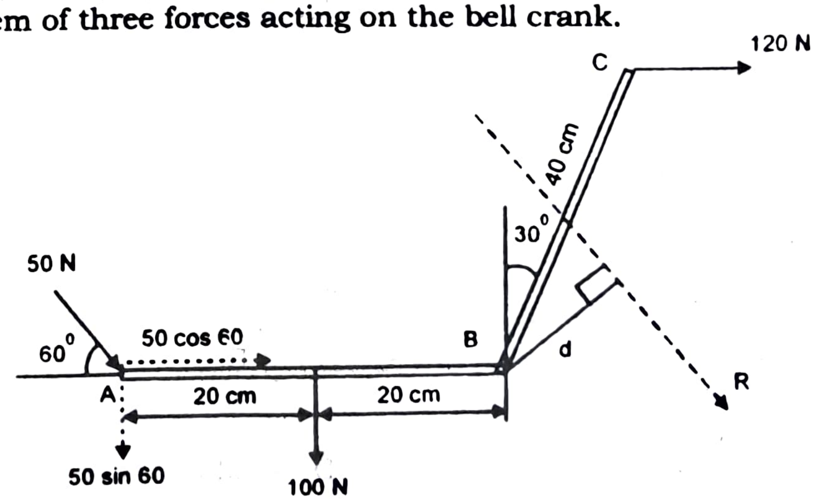
$= -143.3$ N

$= 143.3$ N ↓

Using $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{145^2 + 143.3^2}$

$= 203.8$ N



also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{143.3}{145}$

$\therefore \quad \theta = 44.65°$

$\therefore \quad R = 203.8$ N at $\theta = 44.66°$ ↘ .... **Ans.**

Location of resultant force

Let the resultant be located at a perpendicular distance 'd' to the right of B as shown by dotted line.

Using Varignon's theorem

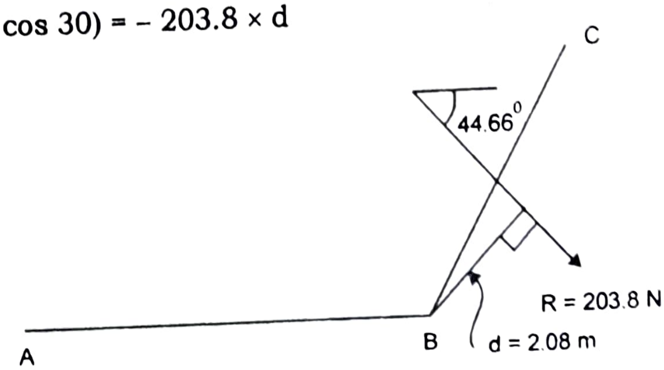
$\Sigma M_B^F = M_B^R$ ↺ + ve

$50 \sin 60 \times 40 + 100 \times 20 - 120 \times (40 \cos 30) = -203.8 \times d$

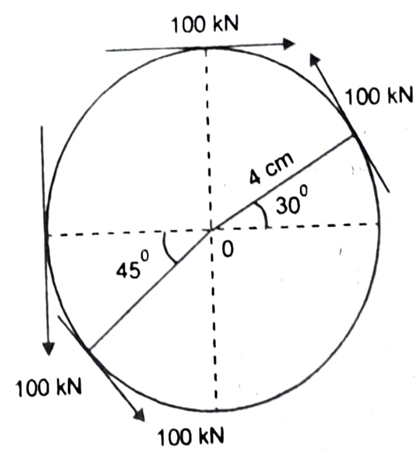
$424.8 = 203.8 \times d$

$\therefore \quad d = 2.08$ cm

Hence, the resultant R = 203.8 N at θ = 44.66° ↘ is located at a ⊥ distance d = 2.08 cm to the right of B as shown. .... **Ans.**



Ex. 2.15 Determine the resultant of four forces tangential to the circle of radius 4 cm as shown. What will be the location of the resultant with respect to the center of the circle?



Solution: This is a general system of four forces acting on a circle.

$\Sigma F_x \rightarrow$ + ve

$= 100 + 100 \cos 45 - 100 \sin 30$

$= 120.7$ kN →

$\Sigma F_y \quad \uparrow$ + ve

$= -100 - 100 \sin 45 + 100 \cos 30$

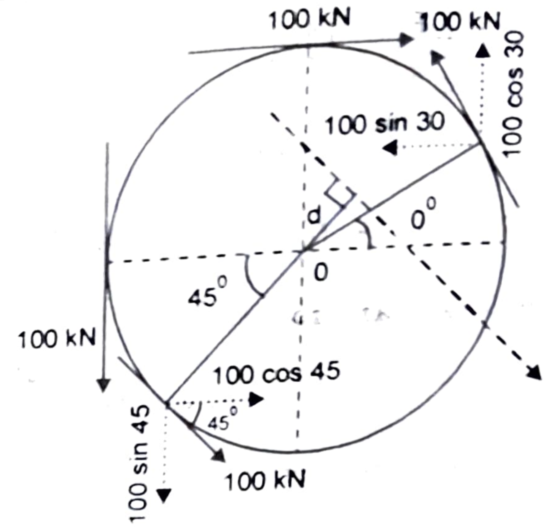
$= -84.1$ kN

$= 84.1$ kN ↓

Using $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{120.7^2 + 84.1^2}$

$= 147.1 \text{ N}$



also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{84.1}{120.7}$

$\therefore \theta = 34.86°$

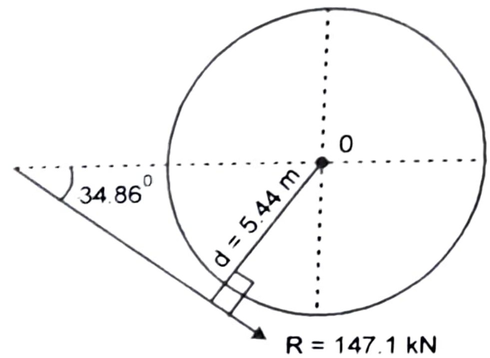
$R = 147.1$ kN at $\theta = 34.86°$

Location of resultant force

Let the resultant force be located at a perpendicular distance d to the right of O as shown.

Using Varignon's theorem

$\Sigma M_O^F = M_O^R$ ↺ + ve

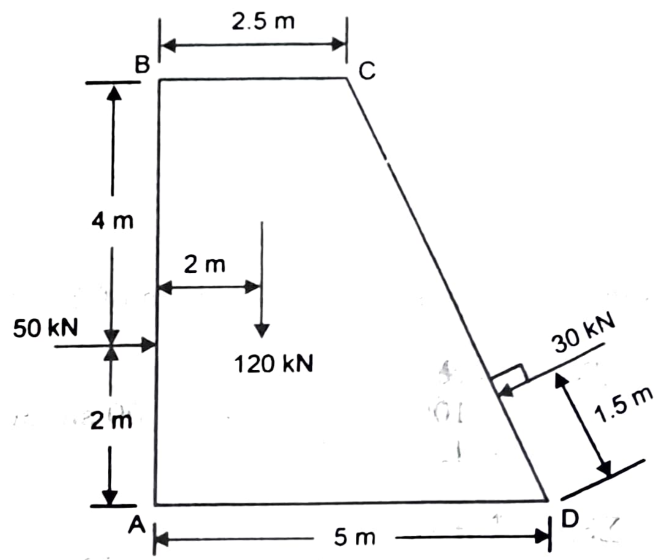


$-100 \times 4 + 100 \times 4 + 100 \times 4$
$+ 100 \times 4 = -147.1 \times d$
$d = -5.44$ m

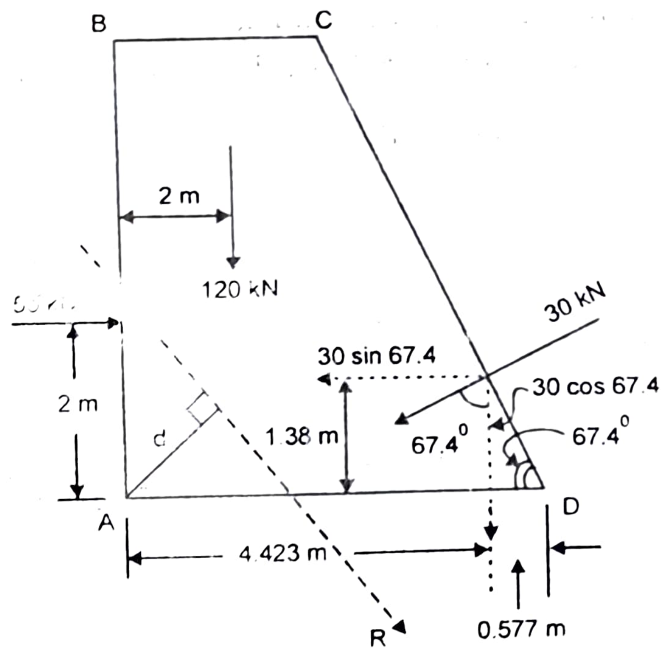
or $d = 5.44$ m (left of O)

Hence, the resultant force $R = 147.1$ kN at $\theta = 34.86°$ is located at a $\perp$ distance $d = 5.44$m, left of O. .... **Ans.**

Ex. 2.16 A dam is subjected to three forces, 50 kN on the upstream face AB, 30 kN force on the down stream inclined face and its own weight of 120 kN as shown. Determine the single force and locate its point of intersection with the base AD assuming all the forces to lie in a single plane.



Solution: This is a general system of three coplanar forces acting on the dam.



$\Sigma F_x \rightarrow +ve$

$= 50 - 30 \sin 67.4$

$= 22.3 \text{ kN} \rightarrow$

$\Sigma F_y \uparrow +ve$

$= -120 - 30 \cos 67.4$

$= -131.5 \text{ kN}$

$= 131.5 \text{ kN} \downarrow$

Using $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{22.3^2 + 131.5^2}$

$= 133.4 \text{ kN}$

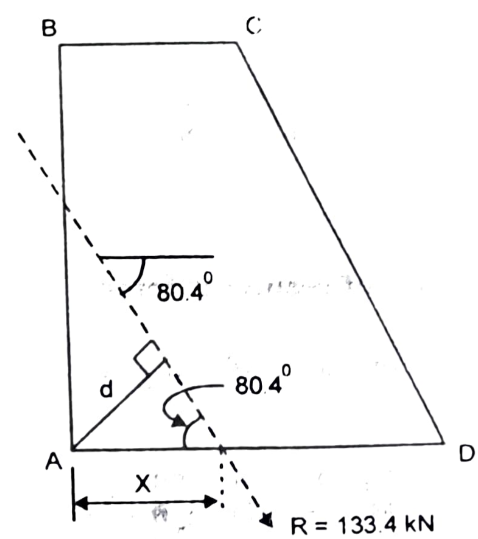
also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{131.5}{22.3}$

$\therefore \quad \theta = 80.4°$

$\therefore \quad R = 133.4 \text{ kN at } \theta = 80.4°$ .... **Ans.**

Point of intersection of the resultant force with base AD

Let the resultant force lie at a perpendicular distance 'd' to the right of A, cutting the base at a distance **x** from end A, as shown.



Using Varignon's theorem

$\Sigma M_A^F = M_A^R$ ↺ + ve

$-50 \times 2 - 120 \times 2 - (30 \cos 67.4) \times 4.423 + (30 \sin 67.4) \times 1.38 = -133.4 \times d$

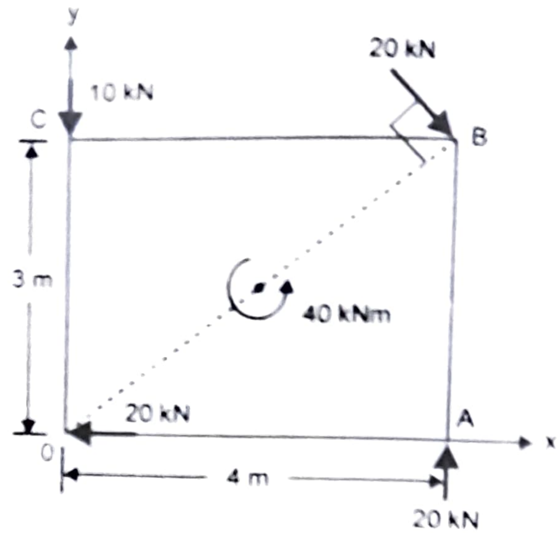
$\therefore \quad d = 2.64 \text{ m}$

From geometry $\sin 80.4 = \dfrac{d}{x} = \dfrac{2.64}{x}$

$\therefore \quad x = 2.68 \text{ m}$

Hence, Resultant force $R = 133.4$ kN at $\theta = 80.4°$ lies at $\perp$ distance $d = 2.64$ m right of A and cuts the base AD at $x = 2.68$ m. .... **Ans.**

Ex. 2.17 Find the resultant of the force system acting on a body OABC shown in figure.
Find the distance of the resultant from O. Also find the points where the resultant will cut the x and y axis.



Solution: This is a general force system of four forces and a couple of 40 kNm acting on the body OABC.

$\Sigma F_x \quad \rightarrow +ve$

$= 20 \cos 53.13 - 20$

$= -8$

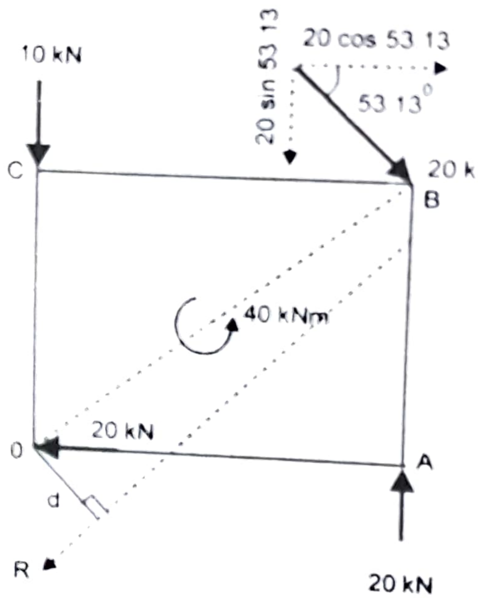
$= 8 \text{ kN} \leftarrow$

$\Sigma F_y \quad \uparrow +ve$

$= -10 - 20 \sin 53.13 + 20$

$= -6$

$= 6 \text{ kN} \downarrow$



Using $\quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{8^2 + 6^2}$

$= 10 \text{ kN}$

also $\quad \tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{6}{8} = 0.75$

$\theta = 36.86°$

$\therefore \quad R = 10 \text{ kN at } \theta = 36.86° \swarrow$

Location of resultant force

Let the resultant force be located at a $\perp$ distance 'd' to the right of O as shown in the figure.

Using Varignon's theorem

$\Sigma M_O^F = M_O^R \quad \circlearrowleft +ve$

$40 - (20 \sin 53.13) \times 4 - (20 \cos 53.13) \times 3 + 20 \times 4 = -10 \times d$

$d = -2 \text{ m}$

$= 2 \text{ m} \quad (\text{left of O})$

$\therefore$ The resultant $R = 10$ kN at $\theta = 36.86° \swarrow$ lies at $\perp$ distance $d = 2$ m to the left of O.

.... **Ans.**