TSK_pytorch_REG

September 26, 2025

[1]:
```
import numpy as np
from sklearn import datasets
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.metrics import␣
↪mean_squared_error,accuracy_score,classification_report
import skfuzzy as fuzz
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import torch.optim as optim
import pandas
```

[2]:
```
# CHOOSE DATASET

# Regression dataset
data = datasets.load_diabetes(as_frame=True)

X = data.data.values
y = data.target.values
X.shape
```

[2]: (442, 10)

[3]:
```
#train test spliting
test_size=0.2
Xtr, Xte, ytr, yte = train_test_split(X, y, test_size=test_size,␣
↪random_state=42)
```

[4]:
```
# Standardize features
scaler=StandardScaler()
Xtr= scaler.fit_transform(Xtr)
Xte= scaler.transform(Xte)
```

[5]:
```
 # Number of clusters
n_clusters = 2
m=2
```

```
# Concatenate target for clustering
Xexp=np.concatenate([Xtr, ytr.reshape(-1, 1)], axis=1)
#Xexp=Xtr

# Transpose data for skfuzzy (expects features x samples)
Xexp_T = Xexp.T

best_fpc = -1
best_params = None
for n_clusters in np.arange(2, 10, 1):       # number of clusters to test
    for m in np.arange(1.5, 2.5, 0.1):
    # Fuzzy C-means clustering
        centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
            Xexp_T, n_clusters, m, error=0.005, maxiter=1000, init=None,
        )

        if fpc > best_fpc:
            best_fpc = fpc
            best_params = (n_clusters, m)
            print(best_params)

print(best_fpc)
print(best_params[0])
print(best_params[1])

n_clusters=best_params[0]
m=best_params[1]

centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
    Xexp_T, n_clusters, m, error=0.005, maxiter=10000, init=None,
)
```

```
(np.int64(2), np.float64(1.5))
0.93972435685851
2
1.5
```

[6]:
```
centers.shape
```

[6]: (2, 11)

[7]:
```
# Compute sigma (spread) for each cluster
sigmas = []
for j in range(n_clusters):
    # membership weights for cluster j, raised to m
    u_j = u[j, :] ** m
    # weighted variance for each feature
    var_j = np.average((Xexp - centers[j])**2, axis=0, weights=u_j)
```

```
    sigma_j = np.sqrt(var_j)
    sigmas.append(sigma_j)
sigmas=np.array(sigmas)
```

[8]:
```
# Hard clustering from fuzzy membership
cluster_labels = np.argmax(u, axis=0)
print("Fuzzy partition coefficient (FPC):", fpc)

# Diabetes feature names (sklearn's load_diabetes ordering)
feature_names = [
    "age",  # 0
    "sex",  # 1
    "bmi",  # 2
    "bp",   # 3
    "s1",   # 4 (tc)
    "s2",   # 5 (ldl)
    "s3",   # 6 (hdl)
    "s4",   # 7 (tch)
    "s5",   # 8 (ltg)
    "s6"    # 9 (glu)
]

# Choose 6 feature pairs (indices in Xexp)
pairs = [
    (2, 8),  # bmi vs s5 (both strong predictors)
    (2, 3),  # bmi vs bp
    (2, 9),  # bmi vs glu
    (8, 9),  # s5 vs glu
    (0, 2),  # age vs bmi
    (3, 9)   # bp vs glu
]

fig, axes = plt.subplots(3, 2, figsize=(12, 10))

for ax, (i, j) in zip(axes.ravel(), pairs):
    # Plot 2 features with fuzzy membership
    for k in range(n_clusters):
        ax.scatter(
            Xexp[cluster_labels == k, i],   # Feature i
            Xexp[cluster_labels == k, j],   # Feature j
            alpha=u[k, :],                  # transparency ~ membership degree
            label=f'Cluster {k}'
        )
    ax.set_xlabel(feature_names[i])
    ax.set_ylabel(feature_names[j])
    ax.set_title(f"{feature_names[i]} vs {feature_names[j]}")
```

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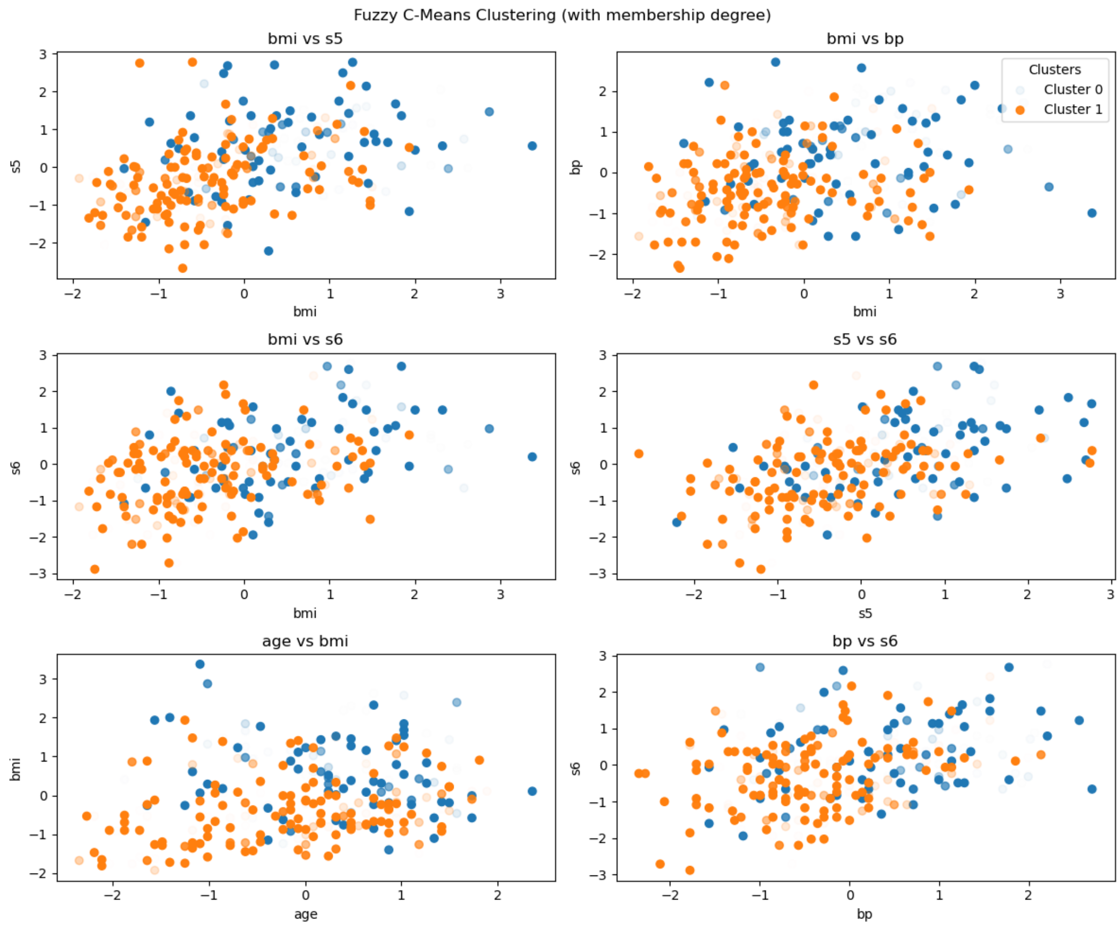
fig.suptitle("Fuzzy C-Means Clustering (with membership degree)")
axes[0, 1].legend(title="Clusters", loc="upper right")
plt.tight_layout()
plt.show()

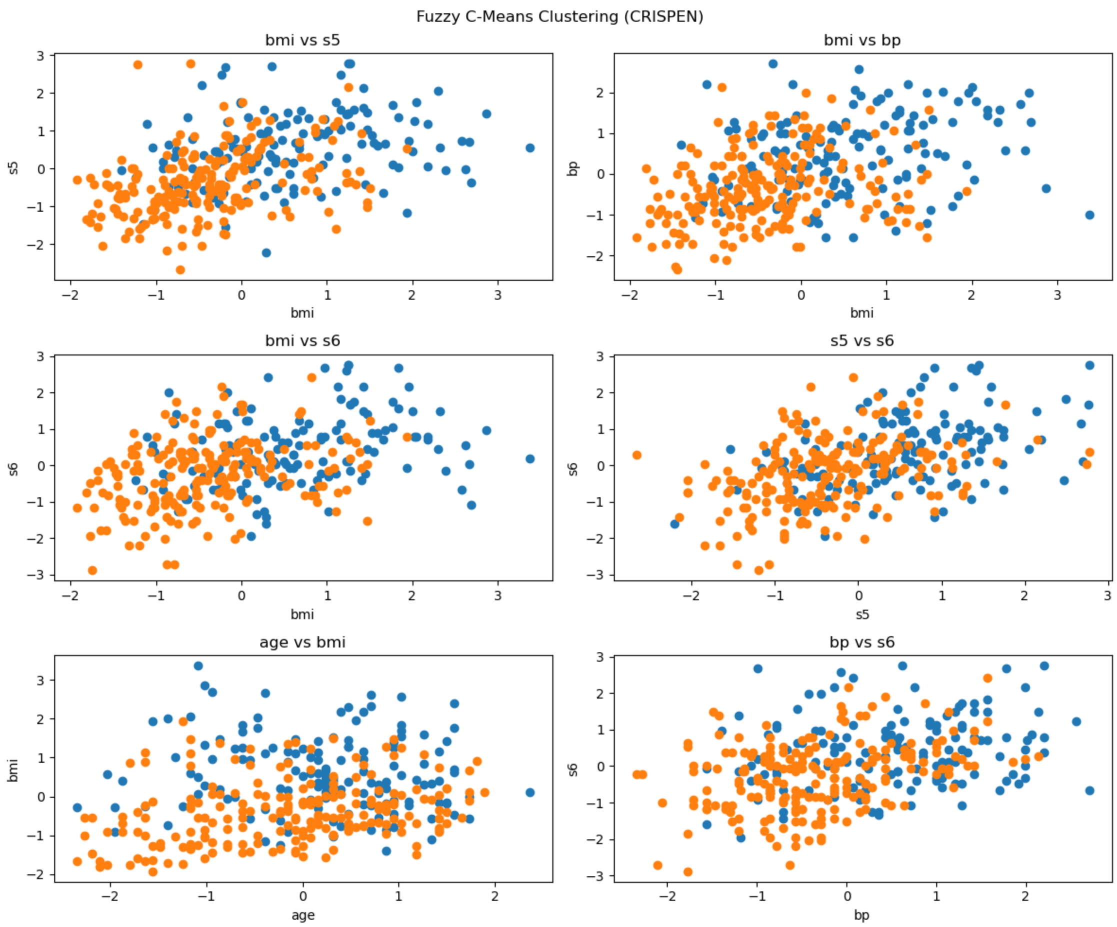
fig.savefig("Fuzzy C-Means Clustering (with membership degree) REG.pdf",␣
↪format="pdf", bbox_inches="tight")
```

```
Fuzzy partition coefficient (FPC): 0.9397242309334268
```



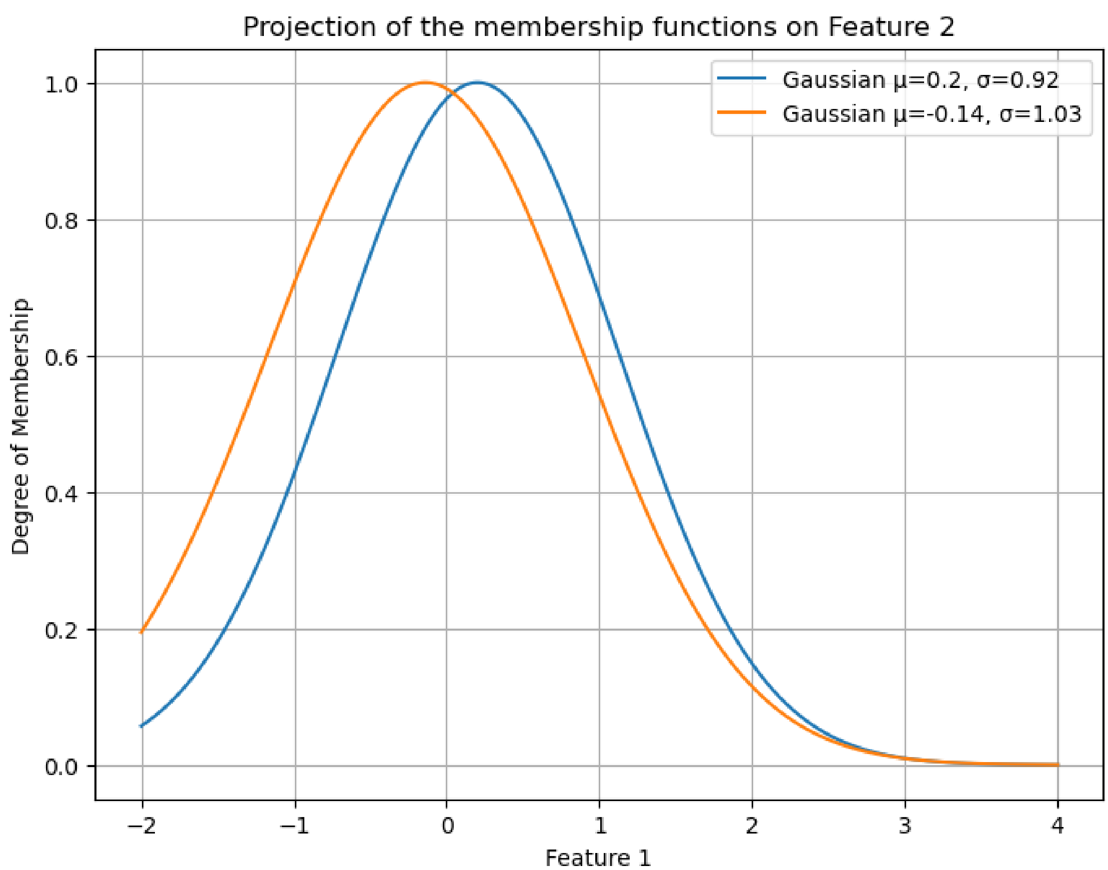

```python
[9]: fig, axes = plt.subplots(3, 2, figsize=(12, 10))

for ax, (i, j) in zip(axes.ravel(), pairs):
    # Plot 2 features with fuzzy membership
    for k in range(n_clusters):
        ax.scatter(
            Xexp[cluster_labels == k, i],
            Xexp[cluster_labels == k, j],
```

```
                label=f'Cluster {k}'
            )

        ax.set_xlabel(feature_names[i])
        ax.set_ylabel(feature_names[j])
        ax.set_title(f"{feature_names[i]} vs {feature_names[j]}")

fig.suptitle("Fuzzy C-Means Clustering (CRISPEN)")
fig.tight_layout()
fig.show()

fig.savefig("Fuzzy C-Means Clustering (CRISPEN) REG.pdf", format="pdf",␣
 ↪bbox_inches="tight")
```



[10]:
```
# Gaussian formula
def gaussian(x, mu, sigma):
    return np.exp(-0.5 * ((x - mu)/sigma)**2)
```

```
lin=np.linspace(-2, 4, 500)
plt.figure(figsize=(8,6))

y_aux=[]
feature=0
for j in range(n_clusters):
# Compute curves
    y_aux.append(gaussian(lin, centers[j,feature], sigmas[j,feature]))

# Plot
    plt.plot(lin, y_aux[j], label=f"Gaussian  ={np.
 ↪round(centers[j,feature],2)},  ={np.round(sigmas[j,feature],2)}")

plt.title("Projection of the membership functions on Feature 2")
plt.xlabel("Feature 1")
plt.ylabel("Degree of Membership")
plt.legend()
plt.grid(True)
plt.show()
```



[11]:

```
# ---------------------------
# Gaussian Membership Function
# ---------------------------
class GaussianMF(nn.Module):
    def __init__(self, centers, sigmas, agg_prob):
        super().__init__()
        self.centers = nn.Parameter(torch.tensor(centers, dtype=torch.float32))
        self.sigmas = nn.Parameter(torch.tensor(sigmas, dtype=torch.float32))
        self.agg_prob=agg_prob

    def forward(self, x):
        # Expand for broadcasting
        # x: (batch, 1, n_dims), centers: (1, n_rules, n_dims), sigmas: (1,␣
 ↪n_rules, n_dims)
        diff = abs((x.unsqueeze(1) - self.centers.unsqueeze(0))/self.sigmas.
 ↪unsqueeze(0)) #(batch, n_rules, n_dims)

        # Aggregation
        if self.agg_prob:
            dist = torch.norm(diff, dim=-1)  # (batch, n_rules) # probablistic␣
 ↪intersection
        else:
            dist = torch.max(diff, dim=-1).values  # (batch, n_rules) # min␣
 ↪intersection (min instersection of normal funtion is the same as the max on␣
 ↪dist)

        return torch.exp(-0.5 * dist ** 2)


# ---------------------------
# TSK Model
# ---------------------------
class TSK(nn.Module):
    def __init__(self, n_inputs, n_rules, centers, sigmas,agg_prob=False):
        super().__init__()
        self.n_inputs = n_inputs
        self.n_rules = n_rules

        # Antecedents (Gaussian MFs)

        self.mfs=GaussianMF(centers, sigmas,agg_prob)

        # Consequents (linear functions of inputs)
        # Each rule has coeffs for each input + bias
        self.consequents = nn.Parameter(
            torch.randn(n_inputs + 1,n_rules)
        )
```

```
    def forward(self, x):
        # x: (batch, n_inputs)
        batch_size = x.shape[0]

        # Compute membership values for each input feature
        # firing_strengths: (batch, n_rules)
        firing_strengths = self.mfs(x)

        # Normalize memberships
        # norm_fs: (batch, n_rules)
        norm_fs = firing_strengths / (firing_strengths.sum(dim=1, keepdim=True)␣
↪+ 1e-9)

        # Consequent output (linear model per rule)
        x_aug = torch.cat([x, torch.ones(batch_size, 1)], dim=1)  # add bias

        rule_outputs = torch.einsum("br,rk->bk", x_aug, self.consequents)  #␣
↪(batch, rules)
        # Weighted sum
        output = torch.sum(norm_fs * rule_outputs, dim=1, keepdim=True)

        return output, norm_fs, rule_outputs
```

[12]:
```
# ---------------------------
# Least Squares Solver for Consequents (TSK)
# ---------------------------
def train_ls(model, X, y):
    with torch.no_grad():
        _, norm_fs, _ = model(X)

        # Design matrix for LS: combine normalized firing strengths with input
        X_aug = torch.cat([X, torch.ones(X.shape[0], 1)], dim=1)

        Phi = torch.einsum("br,bi->bri", X_aug, norm_fs).reshape(X.shape[0], -1)

        # Solve LS: consequents = (Phi^T Phi)^-1 Phi^T y

        theta= torch.linalg.lstsq(Phi, y).solution


        model.consequents.data = theta.reshape(model.consequents.shape)
```

[13]:
```
# ---------------------------
# Gradient Descent Training
```

```python
# ---------------------------
def train_gd(model, X, y, epochs=100, lr=1e-3):
    optimizer = optim.Adam(model.parameters(), lr=lr)
    criterion = nn.MSELoss()
    for _ in range(epochs):
        optimizer.zero_grad()
        y_pred, _, _ = model(X)
        loss = criterion(y_pred, y)
        print(loss)
        loss.backward()
        optimizer.step()
```

[14]:
```python
# ---------------------------
# Hybrid Training (Classic ANFIS)
# ---------------------------
def train_hybrid_anfis(model, X, y, max_iters=10, gd_epochs=20, lr=1e-3):
    train_ls(model, X, y)
    for _ in range(max_iters):
        # Step A: GD on antecedents (freeze consequents)
        model.consequents.requires_grad = False
        train_gd(model, X, y, epochs=gd_epochs, lr=lr)

        # Step B: LS on consequents (freeze antecedents)
        model.consequents.requires_grad = True
        model.mfs.requires_grad = False
        train_ls(model, X, y)

        # Re-enable antecedents
        model.mfs.requires_grad = True
```

[15]:
```python
# ---------------------------
# Alternative Hybrid Training (LS+ gradient descent on all)
# ---------------------------
def train_hybrid(model, X, y, epochs=100, lr=1e-4):
    # Step 1: LS for consequents
    train_ls(model, X, y)
    # Step 2: GD fine-tuning
    train_gd(model, X, y, epochs=epochs, lr=lr)
```

[16]:
```python
# Build model
model = TSK(n_inputs=Xtr.shape[1], n_rules=n_clusters, centers=centers[:,:-1],␣
 ↪sigmas=sigmas[:,:-1])

Xtr = torch.tensor(Xtr, dtype=torch.float32)
ytr = torch.tensor(ytr, dtype=torch.float32)
Xte = torch.tensor(Xte, dtype=torch.float32)
yte = torch.tensor(yte, dtype=torch.float32)
```

[17]:
```
# Training with LS:
train_ls(model, Xtr, ytr.reshape(-1,1))
```

[18]:
```
y_pred, _, _=model(Xte)
#performance metric for regression
print(f'MSE:{mean_squared_error(yte.detach().numpy(),y_pred.detach().
 ↪numpy())}') #regression
```

```
MSE:2534.13330078125
```