



TÉCNICO
LISBOA

INTELLIGENT SYSTEMS

MECHANICAL ENGINEERING

Class Assignment 1

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1 Dataset 1: Diabetes Dataset (Regression)

Fuzzy C-Means Clustering was applied with a range of values for the number of clusters of 2 to 10 with an increment of 1 and the fuzziness coefficient was varied between 1.5 and 2.5 with an increment of 0.1. The results were evaluated using the Fuzzy Partition Coefficient (FPC) and culminated in a number of 2 clusters and a fuzziness coefficient of 1.5 achieving a FPC of 0.939724280208228.

The resulting TSK model achieved a MSE of 2534.134033203125.

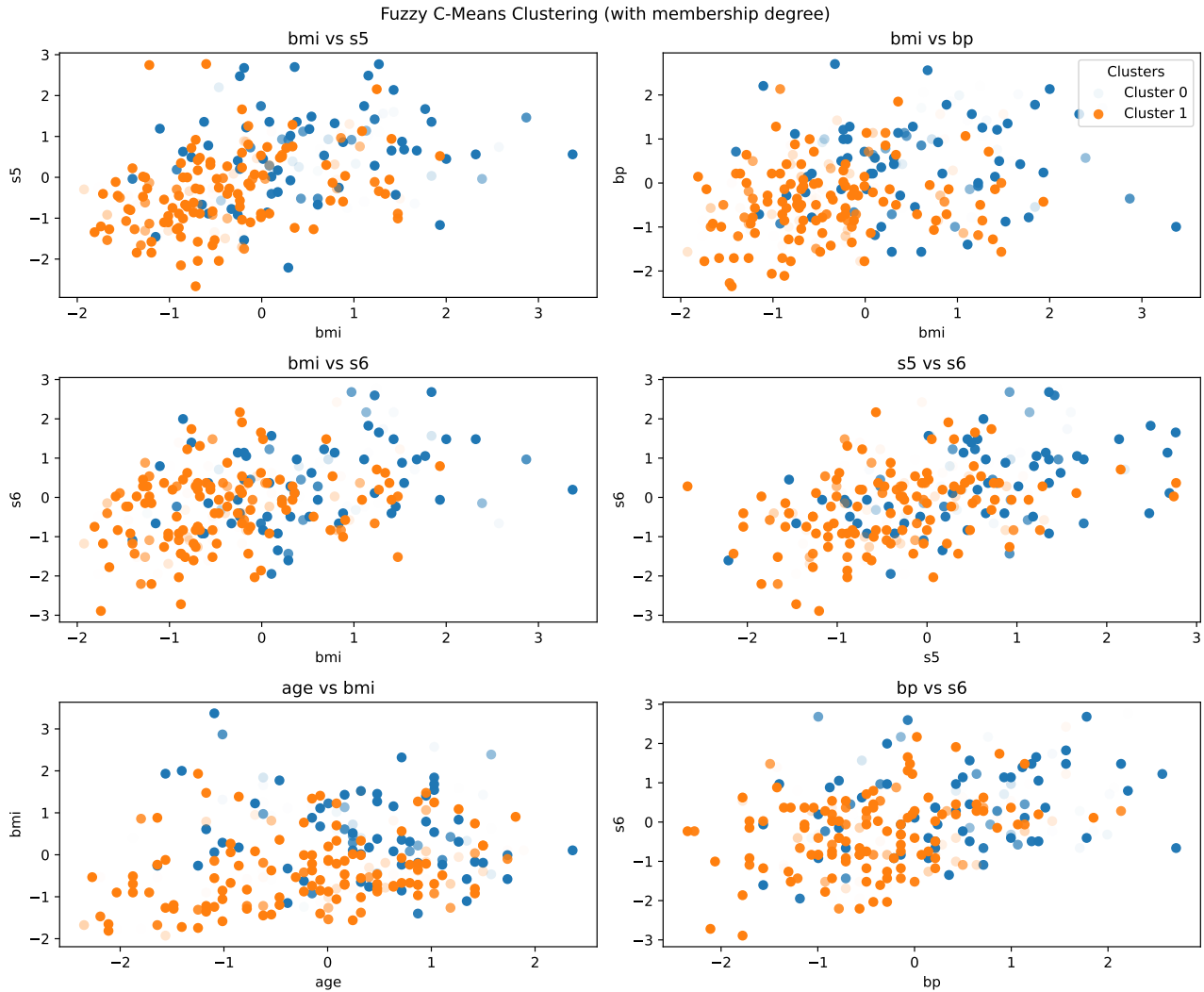


Figure 1: Fuzzy C-Means Clustering (with membership degree)

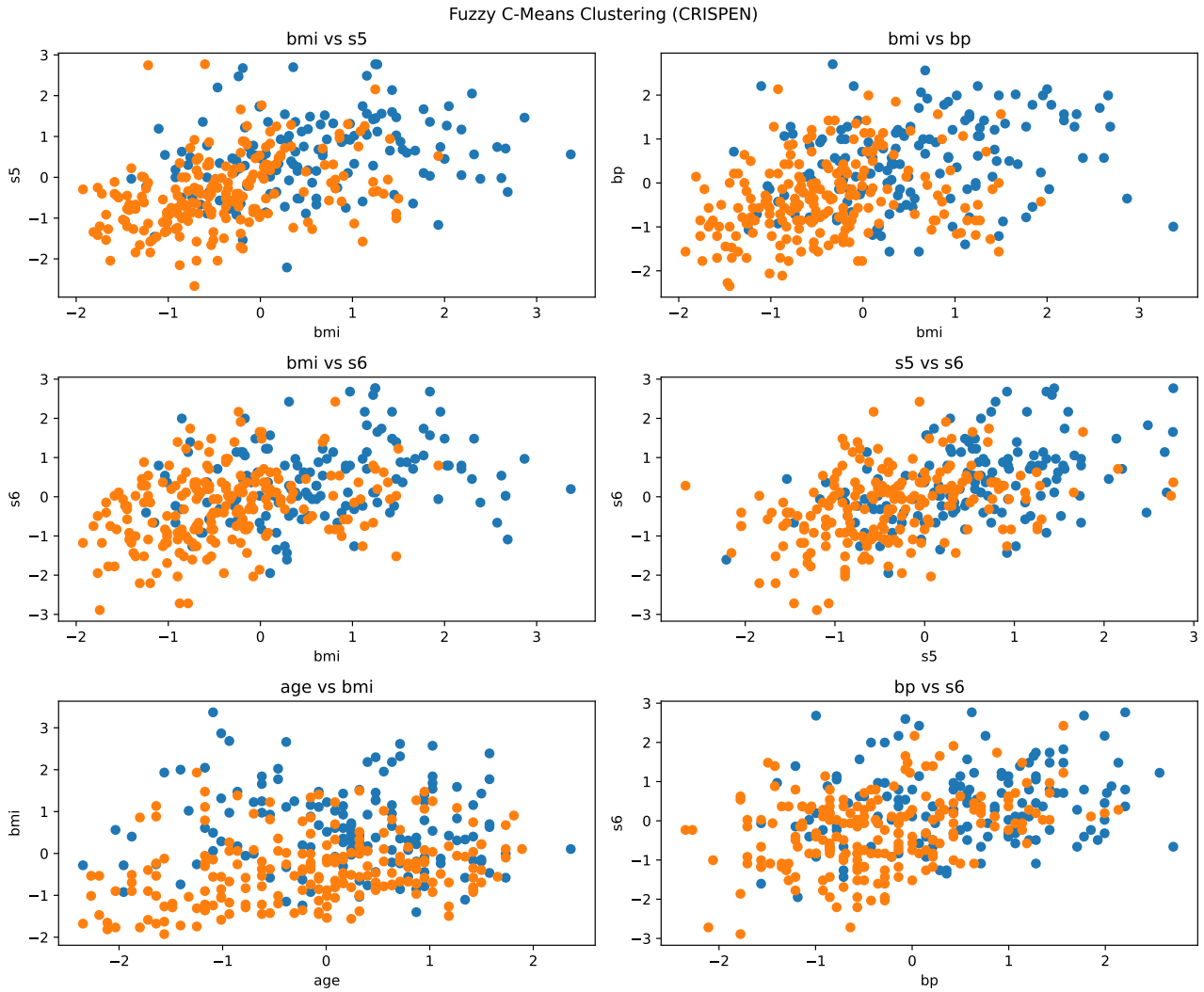


Figure 2: Fuzzy C-Means Clustering (CRISPEN)

2 Dataset 2: Pima Indians Diabetes Dataset (Classification)

Regarding data processing, several things were done to improve the dataset. Firstly, the target variable was converted from text strings to binary. The *Skin Thickness* and *Insulin* features lacked many rows of measurements; as such, these were discarded. Despite this, some measurements had 0 values that were equivalent to impossible; the rows containing these were discarded. The remaining data processing tasks were already implemented. The performance metrics improved after these measures were implemented.

Fuzzy C-Means Clustering was applied with a range of values for the number of clusters of 2 to 10 with an increment of 1 and the fuzziness coefficient was varied between 1.5 and 2.5 with an increment of 0.1. The results were evaluated using the Fuzzy Partition Coefficient (FPC) and culminated in a number of 2 clusters and a fuzziness coefficient of 1.5 achieving a FPC of 0.7231894414858948.

The resulting TSK model achieved an ACC of 0.8.

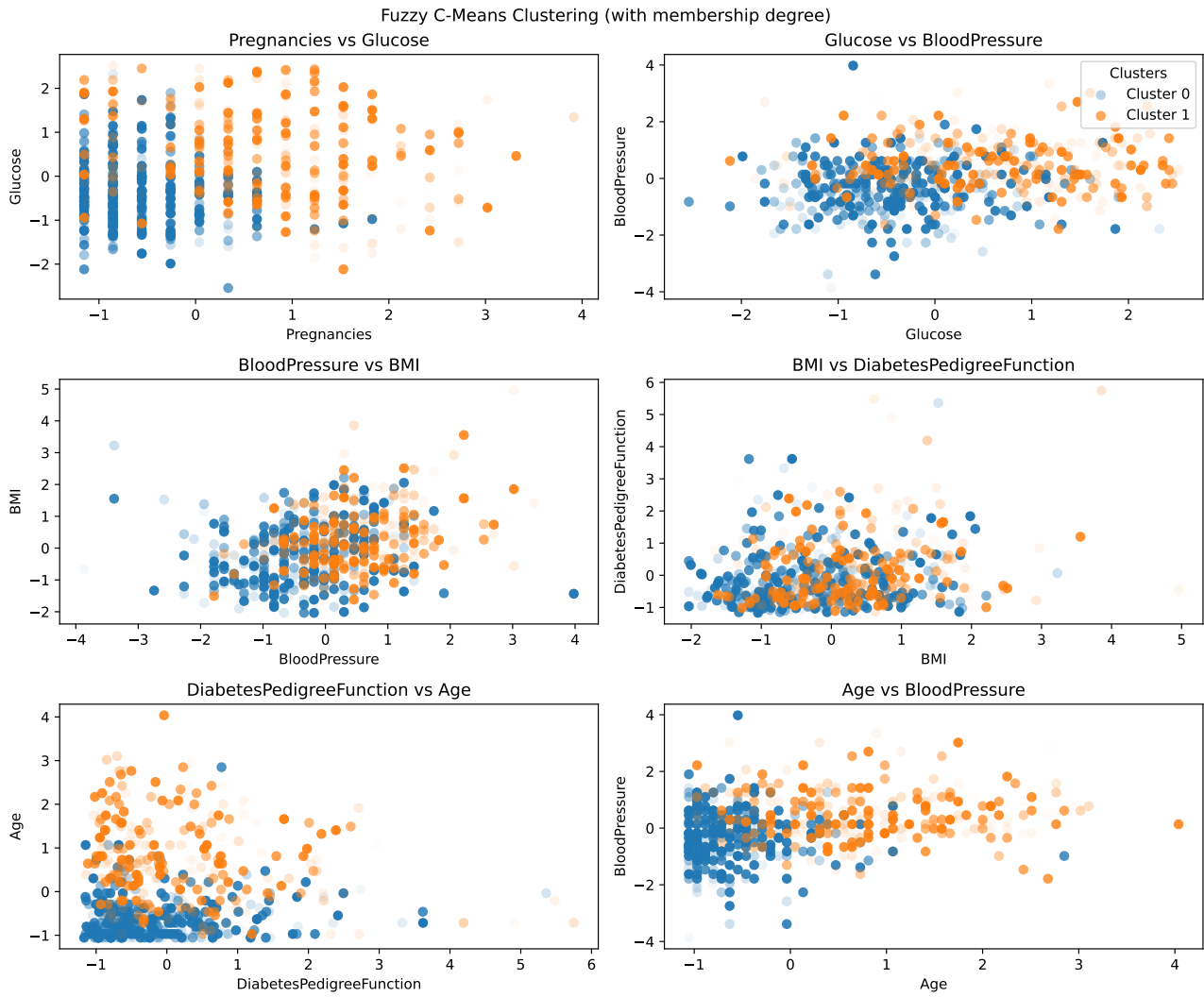


Figure 3: Fuzzy C-Means Clustering (with membership degree)

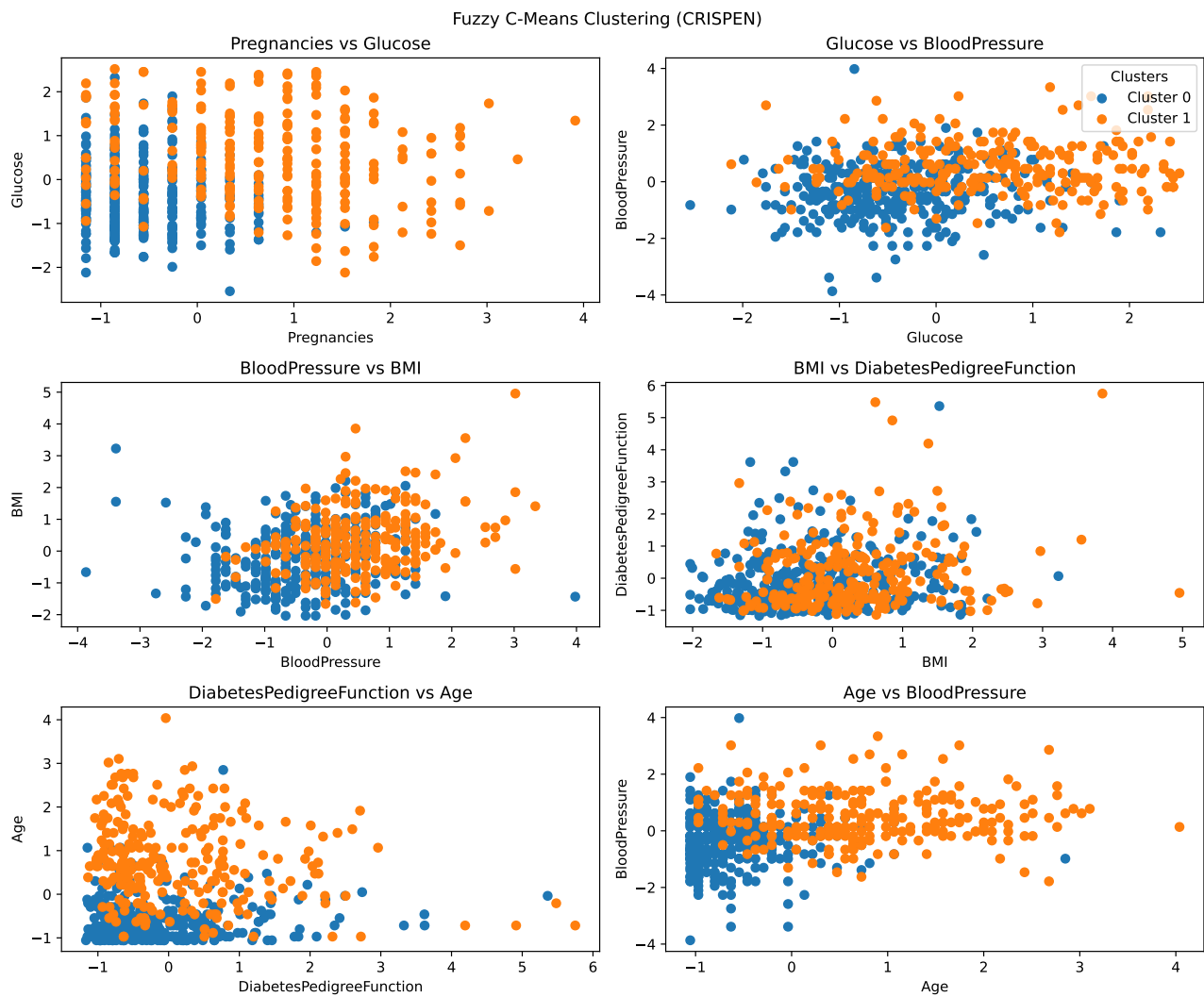


Figure 4: Fuzzy C-Means Clustering (CRISPEN)

3 GitHub Repo

github.com/Pereira98/SI_Individual_87172.git

TSK_pytorch_REG

September 26, 2025

```
[1]: import numpy as np
from sklearn import datasets
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error, accuracy_score, classification_report
import skfuzzy as fuzz
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import torch.optim as optim
import pandas
```

```
[2]: # CHOOSE DATASET

# Regression dataset
data = datasets.load_diabetes(as_frame=True)

X = data.data.values
y = data.target.values
X.shape
```

[2]: (442, 10)

```
[3]: #train test splitting
test_size=0.2
Xtr, Xte, ytr, yte = train_test_split(X, y, test_size=test_size,
    random_state=42)
```

```
[4]: # Standardize features
scaler=StandardScaler()
Xtr= scaler.fit_transform(Xtr)
Xte= scaler.transform(Xte)
```

```
[5]: # Number of clusters
n_clusters = 2
m=2
```

```

# Concatenate target for clustering
Xexp=np.concatenate([Xtr, ytr.reshape(-1, 1)], axis=1)
#Xexp=Xtr

# Transpose data for skfuzzy (expects features x samples)
Xexp_T = Xexp.T

best_fpc = -1
best_params = None
for n_clusters in np.arange(2, 10, 1):      # number of clusters to test
    for m in np.arange(1.5, 2.5, 0.1):
        # Fuzzy C-means clustering
        centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
            Xexp_T, n_clusters, m, error=0.005, maxiter=1000, init=None,
        )

        if fpc > best_fpc:
            best_fpc = fpc
            best_params = (n_clusters, m)
            print(best_params)

print(best_fpc)
print(best_params[0])
print(best_params[1])

n_clusters=best_params[0]
m=best_params[1]

centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
    Xexp_T, n_clusters, m, error=0.005, maxiter=10000, init=None,
)

(np.int64(2), np.float64(1.5))
0.93972435685851
2
1.5

```

```
[6]: centers.shape
```

```
[6]: (2, 11)
```

```

[7]: # Compute sigma (spread) for each cluster
sigmas = []
for j in range(n_clusters):
    # membership weights for cluster j, raised to m
    u_j = u[j, :] ** m
    # weighted variance for each feature
    var_j = np.average((Xexp - centers[j])**2, axis=0, weights=u_j)

```



```

    sigma_j = np.sqrt(var_j)
    sigmas.append(sigma_j)
sigmas=np.array(sigmas)

```

```

[8]: # Hard clustering from fuzzy membership
cluster_labels = np.argmax(u, axis=0)
print("Fuzzy partition coefficient (FPC):", fpc)

# Diabetes feature names (sklearn's load_diabetes ordering)
feature_names = [
    "age", # 0
    "sex", # 1
    "bmi", # 2
    "bp", # 3
    "s1", # 4 (tc)
    "s2", # 5 (ldl)
    "s3", # 6 (hdl)
    "s4", # 7 (tch)
    "s5", # 8 (ltg)
    "s6" # 9 (glu)
]

# Choose 6 feature pairs (indices in Xexp)
pairs = [
    (2, 8), # bmi vs s5 (both strong predictors)
    (2, 3), # bmi vs bp
    (2, 9), # bmi vs glu
    (8, 9), # s5 vs glu
    (0, 2), # age vs bmi
    (3, 9) # bp vs glu
]

fig, axes = plt.subplots(3, 2, figsize=(12, 10))

for ax, (i, j) in zip(axes.ravel(), pairs):
    # Plot 2 features with fuzzy membership
    for k in range(n_clusters):
        ax.scatter(
            Xexp[cluster_labels == k, i], # Feature i
            Xexp[cluster_labels == k, j], # Feature j
            alpha=u[k, :], # transparency ~ membership degree
            label=f'Cluster {k}'
        )
    ax.set_xlabel(feature_names[i])
    ax.set_ylabel(feature_names[j])
    ax.set_title(f"{feature_names[i]} vs {feature_names[j]}")

```

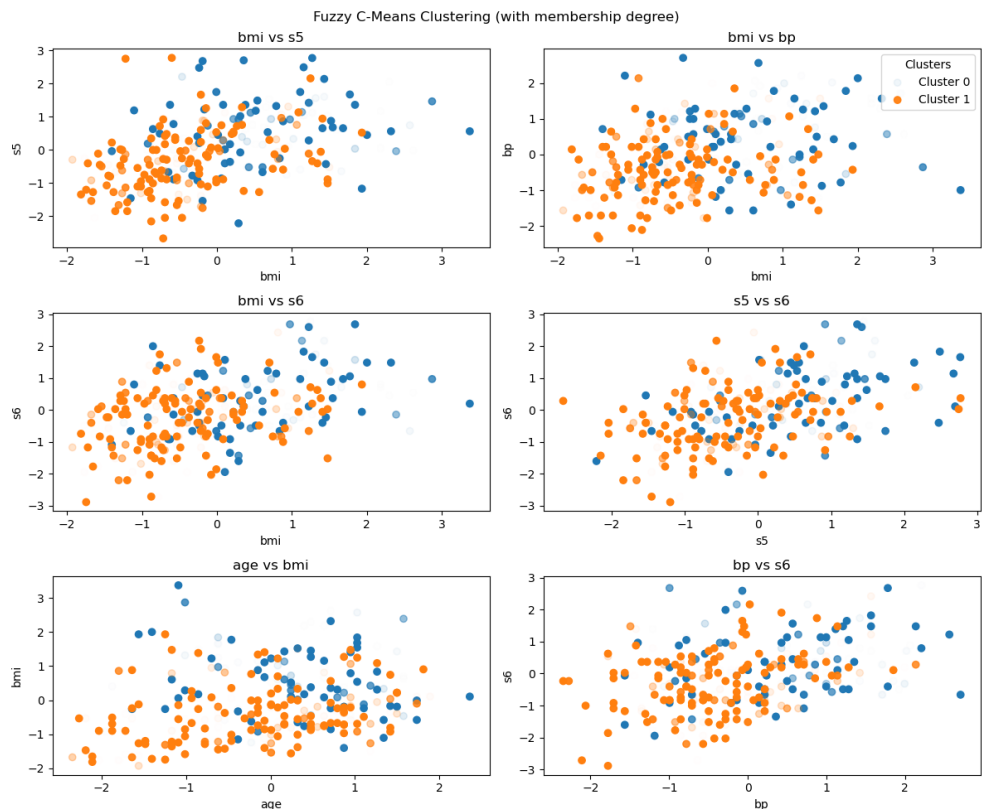
```

fig.suptitle("Fuzzy C-Means Clustering (with membership degree)")
axes[0, 1].legend(title="Clusters", loc="upper right")
plt.tight_layout()
plt.show()

fig.savefig("Fuzzy C-Means Clustering (with membership degree) REG.pdf",
            format="pdf", bbox_inches="tight")

```

Fuzzy partition coefficient (FPC): 0.9397242309334268



```

[9]: fig, axes = plt.subplots(3, 2, figsize=(12, 10))

for ax, (i, j) in zip(axes.ravel(), pairs):
    # Plot 2 features with fuzzy membership
    for k in range(n_clusters):
        ax.scatter(
            Xexp[cluster_labels == k, i],
            Xexp[cluster_labels == k, j],

```

```

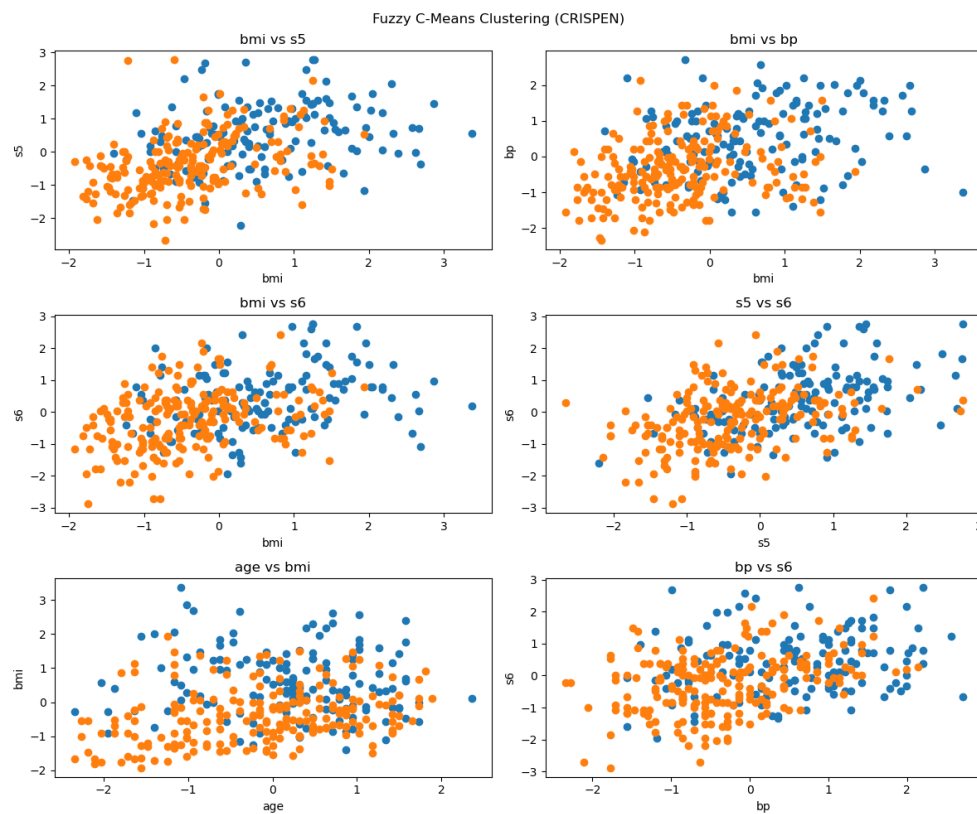
        label=f'Cluster {k}'
    )

    ax.set_xlabel(feature_names[i])
    ax.set_ylabel(feature_names[j])
    ax.set_title(f"{feature_names[i]} vs {feature_names[j]}")

fig.suptitle("Fuzzy C-Means Clustering (CRISPEN)")
fig.tight_layout()
fig.show()

fig.savefig("Fuzzy C-Means Clustering (CRISPEN) REG.pdf", format="pdf",
            bbox_inches="tight")

```



```

[10]: # Gaussian formula
def gaussian(x, mu, sigma):
    return np.exp(-0.5 * ((x - mu)/sigma)**2)

```

```

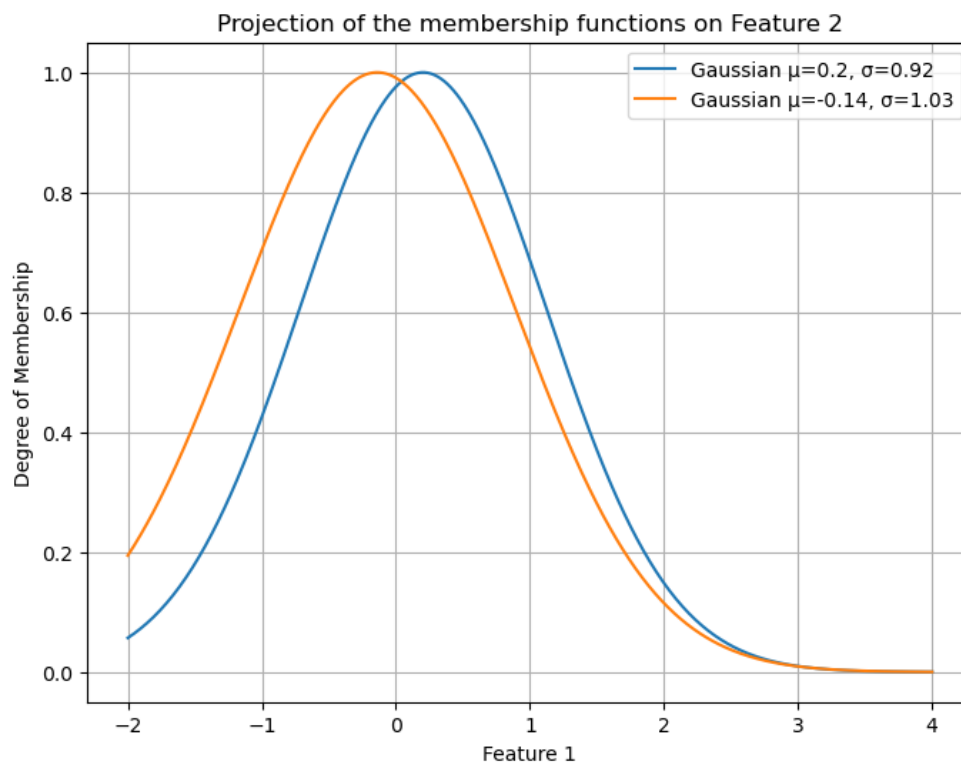
lin=np.linspace(-2, 4, 500)
plt.figure(figsize=(8,6))

y_aux=[]
feature=0
for j in range(n_clusters):
    # Compute curves
    y_aux.append(gaussian(lin, centers[j,feature], sigmas[j,feature]))

# Plot
plt.plot(lin, y_aux[j], label=f"Gaussian  $\mu$ ={np.
round(centers[j,feature],2)},  $\sigma$ ={np.round(sigmas[j,feature],2)}")

plt.title("Projection of the membership functions on Feature 2")
plt.xlabel("Feature 1")
plt.ylabel("Degree of Membership")
plt.legend()
plt.grid(True)
plt.show()

```



```
[11]: # -----
# Gaussian Membership Function
# -----
class GaussianMF(nn.Module):
    def __init__(self, centers, sigmas, agg_prob):
        super().__init__()
        self.centers = nn.Parameter(torch.tensor(centers, dtype=torch.float32))
        self.sigmas = nn.Parameter(torch.tensor(sigmas, dtype=torch.float32))
        self.agg_prob=agg_prob

    def forward(self, x):
        # Expand for broadcasting
        # x: (batch, 1, n_dims), centers: (1, n_rules, n_dims), sigmas: (1,
↵n_rules, n_dims)
        diff = abs((x.unsqueeze(1) - self.centers.unsqueeze(0))/self.sigmas.
↵unsqueeze(0)) #(batch, n_rules, n_dims)

        # Aggregation
        if self.agg_prob:
            dist = torch.norm(diff, dim=-1) # (batch, n_rules) # probabilistic
↵intersection
        else:
            dist = torch.max(diff, dim=-1).values # (batch, n_rules) # min
↵intersection (min instersection of normal funtion is the same as the max on
↵dist)

        return torch.exp(-0.5 * dist ** 2)

# -----
# TSK Model
# -----
class TSK(nn.Module):
    def __init__(self, n_inputs, n_rules, centers, sigmas,agg_prob=False):
        super().__init__()
        self.n_inputs = n_inputs
        self.n_rules = n_rules

        # Antecedents (Gaussian MFs)

        self.mfs=GaussianMF(centers, sigmas,agg_prob)

        # Consequents (linear functions of inputs)
        # Each rule has coeffs for each input + bias
        self.consequents = nn.Parameter(
            torch.randn(n_inputs + 1,n_rules)
        )
```

```

def forward(self, x):
    # x: (batch, n_inputs)
    batch_size = x.shape[0]

    # Compute membership values for each input feature
    # firing_strengths: (batch, n_rules)
    firing_strengths = self.mfs(x)

    # Normalize memberships
    # norm_fs: (batch, n_rules)
    norm_fs = firing_strengths / (firing_strengths.sum(dim=1, keepdim=True) + 1e-9)

    # Consequent output (linear model per rule)
    x_aug = torch.cat([x, torch.ones(batch_size, 1)], dim=1) # add bias

    rule_outputs = torch.einsum("br,rk->bk", x_aug, self.consequents) # (batch, rules)
    # Weighted sum
    output = torch.sum(norm_fs * rule_outputs, dim=1, keepdim=True)

    return output, norm_fs, rule_outputs

```

```

[12]: # -----
# Least Squares Solver for Consequents (TSK)
# -----
def train_ls(model, X, y):
    with torch.no_grad():
        _, norm_fs, _ = model(X)

        # Design matrix for LS: combine normalized firing strengths with input
        X_aug = torch.cat([X, torch.ones(X.shape[0], 1)], dim=1)

        Phi = torch.einsum("br,bi->bri", X_aug, norm_fs).reshape(X.shape[0], -1)

        # Solve LS: consequents = (Phi^T Phi)^-1 Phi^T y

        theta = torch.linalg.lstsq(Phi, y).solution

        model.consequents.data = theta.reshape(model.consequents.shape)

```

```

[13]: # -----
# Gradient Descent Training

```

```
# -----
def train_gd(model, X, y, epochs=100, lr=1e-3):
    optimizer = optim.Adam(model.parameters(), lr=lr)
    criterion = nn.MSELoss()
    for _ in range(epochs):
        optimizer.zero_grad()
        y_pred, _, _ = model(X)
        loss = criterion(y_pred, y)
        print(loss)
        loss.backward()
        optimizer.step()
```

```
[14]: # -----
# Hybrid Training (Classic ANFIS)
# -----
def train_hybrid_anfis(model, X, y, max_iters=10, gd_epochs=20, lr=1e-3):
    train_ls(model, X, y)
    for _ in range(max_iters):
        # Step A: GD on antecedents (freeze consequents)
        model.consequents.requires_grad = False
        train_gd(model, X, y, epochs=gd_epochs, lr=lr)

        # Step B: LS on consequents (freeze antecedents)
        model.consequents.requires_grad = True
        model.mfs.requires_grad = False
        train_ls(model, X, y)

        # Re-enable antecedents
        model.mfs.requires_grad = True
```

```
[15]: # -----
# Alternative Hybrid Training (LS+ gradient descent on all)
# -----
def train_hybrid(model, X, y, epochs=100, lr=1e-4):
    # Step 1: LS for consequents
    train_ls(model, X, y)
    # Step 2: GD fine-tuning
    train_gd(model, X, y, epochs=epochs, lr=lr)
```

```
[16]: # Build model
model = TSK(n_inputs=Xtr.shape[1], n_rules=n_clusters, centers=centers[:, :-1],
           sigmas=sigmas[:, :-1])

Xtr = torch.tensor(Xtr, dtype=torch.float32)
ytr = torch.tensor(ytr, dtype=torch.float32)
Xte = torch.tensor(Xte, dtype=torch.float32)
yte = torch.tensor(yte, dtype=torch.float32)
```

```
[17]: # Training with LS:  
train_ls(model, Xtr, ytr.reshape(-1,1))  
  
[18]: y_pred, _, _=model(Xte)  
      #performance metric for regression  
      print(f'MSE:{mean_squared_error(yte.detach().numpy(),y_pred.detach()  
      ↪numpy())}') #regression
```

MSE:2534.13330078125

TSK_pytorch-CLF

September 26, 2025

```
[1]: import numpy as np
      from sklearn import datasets
      from sklearn.preprocessing import StandardScaler
      from sklearn.model_selection import train_test_split
      from sklearn.metrics import \
          mean_squared_error, accuracy_score, classification_report
      import skfuzzy as fuzz
      import matplotlib.pyplot as plt
      import torch
      import torch.nn as nn
      import torch.optim as optim
      import pandas
      import copy

[2]: # CHOOSE DATASET

      # Binary classification dataset
      data = datasets.fetch_openml(name="diabetes", version=1, as_frame=True)

      X = data.data.values
      y = data.target.values
      X.shape

      # Keep as DataFrame for named-column ops
      df = data.data.copy()
      y = np.where(data.target.values == "tested_positive", 1, 0).astype(np.float32)

      # indices of features with invalid zeros
      invalid_idx = [1, 2, 3, 4, 5, 7]

      # count zeros per feature
      zero_counts = (X[:, invalid_idx] == 0).sum(axis=0)
      rows_with_zero = (X[:, invalid_idx] == 0).any(axis=1).sum()

      print("Zeros per feature:\n", zero_counts)
      print(f"Rows with 1 zero: {rows_with_zero} / {len(df)}")
```

```

(len(X), zero_counts, rows_with_zero)

# Drop columns 3 and 4 (0-based indexing)
X = np.delete(X, [3, 4], axis=1)

# Keep only rows where Glucose, BloodPressure, BMI are non-zero
mask = (X[:, [1, 2, 3, 4]] != 0).all(axis=1)
X = X[mask]
y = y[mask]

```

Zeros per feature:

```
[ 5  35 227 374  11   0]
```

Rows with 1 zero: 376 / 768

```

[3]: #train test splitting
test_size=0.2
Xtr, Xte, ytr, yte = train_test_split(X, y, test_size=test_size,
    random_state=42)

```

```

[4]: # Standardize features
scaler=StandardScaler()
Xtr= scaler.fit_transform(Xtr)
Xte= scaler.transform(Xte)

```

```

[5]: # Concatenate target for clustering
Xexp=np.concatenate([Xtr, ytr.reshape(-1, 1)], axis=1)
#Xexp=Xtr

# Transpose data for skfuzzy (expects features x samples)
Xexp_T = Xexp.T

best_fpc = -1
best_params = None

for n_clusters in np.arange(2, 10, 1):      # number of clusters to test
    for m in np.arange(1.5, 2.5, 0.01):
        # Fuzzy C-means clustering
        centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
            Xexp_T, n_clusters, m, error=0.005, maxiter=1000, init=None,
        )

        if fpc > best_fpc:
            best_fpc = fpc
            best_params = (n_clusters, m)

print(best_fpc)
print(best_params[0])

```

```

print(best_params[1])

n_clusters=best_params[0]
m=best_params[1]

centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
    Xexp_T, n_clusters, m, error=0.005, maxiter=10000, init=None,
)

```

0.7231811657391913

2

1.5

[6]: centers.shape

[6]: (2, 7)

```

[7]: # Compute sigma (spread) for each cluster
sigmas = []
for j in range(n_clusters):
    # membership weights for cluster j, raised to m
    u_j = u[j, :] ** m
    # weighted variance for each feature
    var_j = np.average((Xexp - centers[j])**2, axis=0, weights=u_j)
    sigma_j = np.sqrt(var_j)
    sigmas.append(sigma_j)
sigmas=np.array(sigmas)

```

```

[8]: # Hard clustering from fuzzy membership
cluster_labels = np.argmax(u, axis=0)
print("Fuzzy partition coefficient (FPC):", fpc)

feature_names = [
    "Pregnancies", "Glucose", "BloodPressure",
    "BMI", "DiabetesPedigreeFunction", "Age"
]

# Choose 4 feature pairs (indices in Xexp)
pairs = [(0, 1), # Pregnancies vs Glucose
         (1, 2), # Glucose vs BloodPressure
         (2, 3), # BloodPressure vs BMI
         (3, 4), # BMI vs Pedigree
         (4, 5), # Pedigree vs Age
         (5, 2)] # Age vs Glucose

fig, axes = plt.subplots(3, 2, figsize=(12, 10))

for ax, (i, j) in zip(axes.ravel(), pairs):

```

```

# Plot 2 features with fuzzy membership
for k in range(n_clusters):
    ax.scatter(
        Xexp[cluster_labels == k, i],          # Feature 1
        Xexp[cluster_labels == k, j],          # Feature 2
        alpha=u[k, :],                          # transparency ~ membership
        label=f'Cluster {k}'
    )

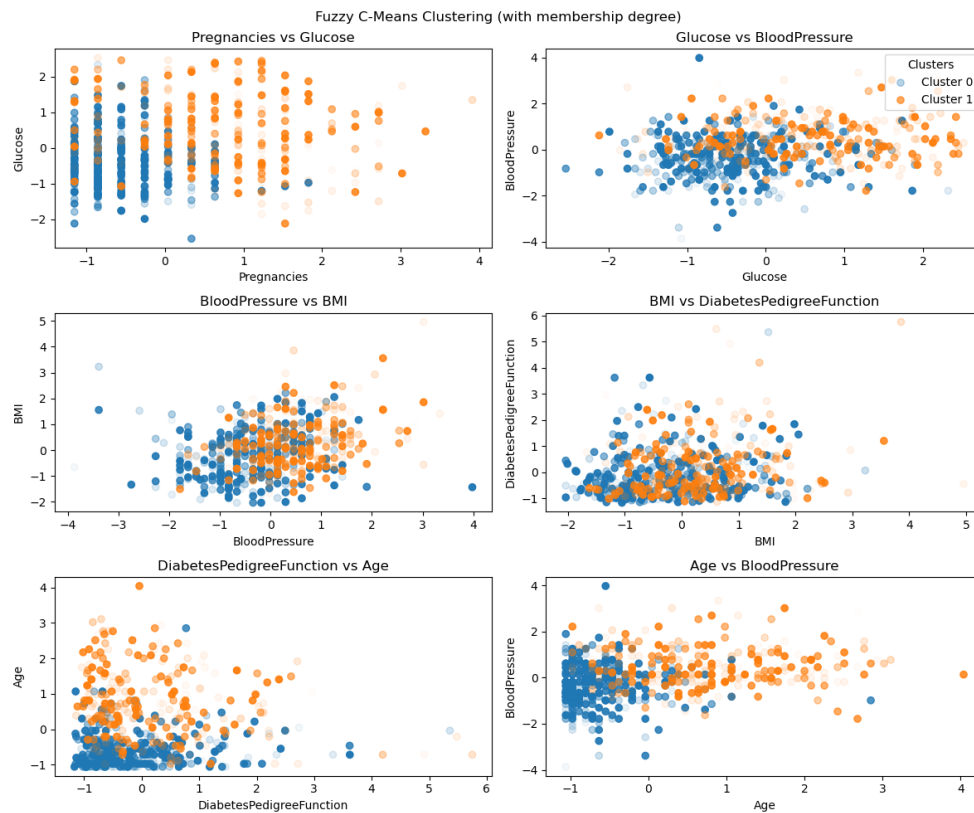
    ax.set_xlabel(feature_names[i])
    ax.set_ylabel(feature_names[j])
    ax.set_title(f"{feature_names[i]} vs {feature_names[j]}")

fig.suptitle("Fuzzy C-Means Clustering (with membership degree)")
axes[0,1].legend(title="Clusters", loc="upper right")
plt.tight_layout()
plt.show()

fig.savefig("Fuzzy C-Means Clustering (with membership degree) CLF.pdf",
            format="pdf", bbox_inches="tight")

```

Fuzzy partition coefficient (FPC): 0.723188876981547



```
[9]: fig, axes = plt.subplots(3, 2, figsize=(12, 10))

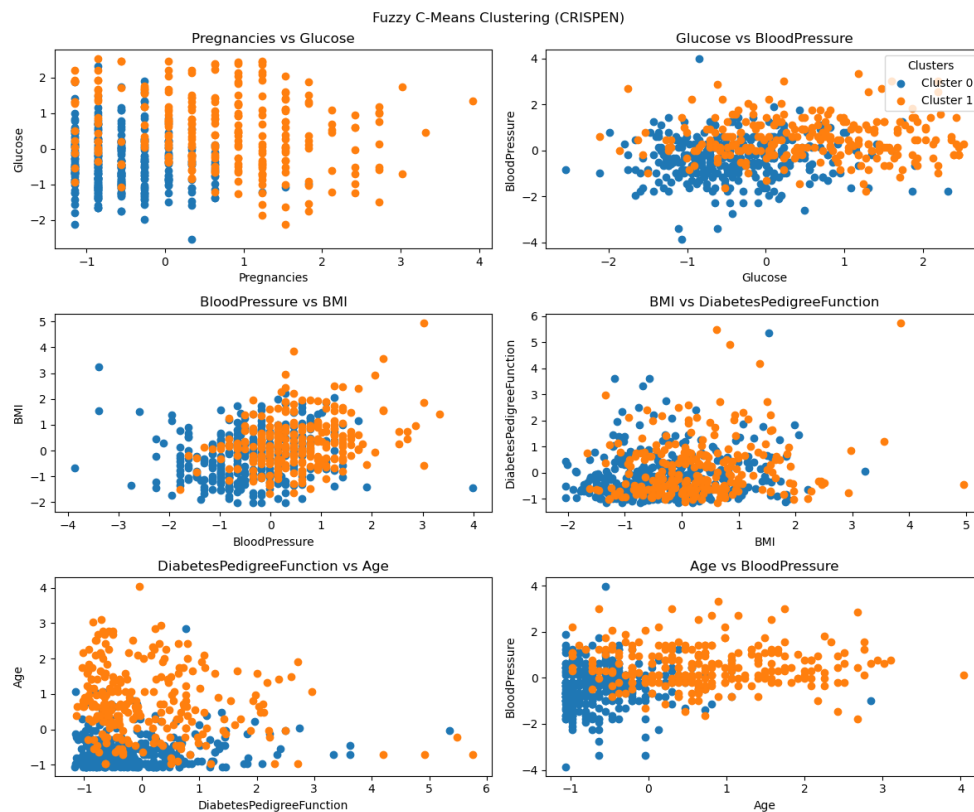
for ax, (i, j) in zip(axes.ravel(), pairs):
    # Plot 2 features with fuzzy membership
    for k in range(n_clusters):
        ax.scatter(
            Xexp[cluster_labels == k, i],
            Xexp[cluster_labels == k, j],
            label=f'Cluster {k}'
        )

    ax.set_xlabel(feature_names[i])
    ax.set_ylabel(feature_names[j])
    ax.set_title(f"{feature_names[i]} vs {feature_names[j]}")

fig.suptitle("Fuzzy C-Means Clustering (CRISPEN)")
axes[0,1].legend(title="Clusters", loc="upper right")
fig.tight_layout()
```

```
fig.show()
```

```
fig.savefig("Fuzzy C-Means Clustering (CRISPEN) CLF.pdf", format="pdf",
            bbox_inches="tight")
```



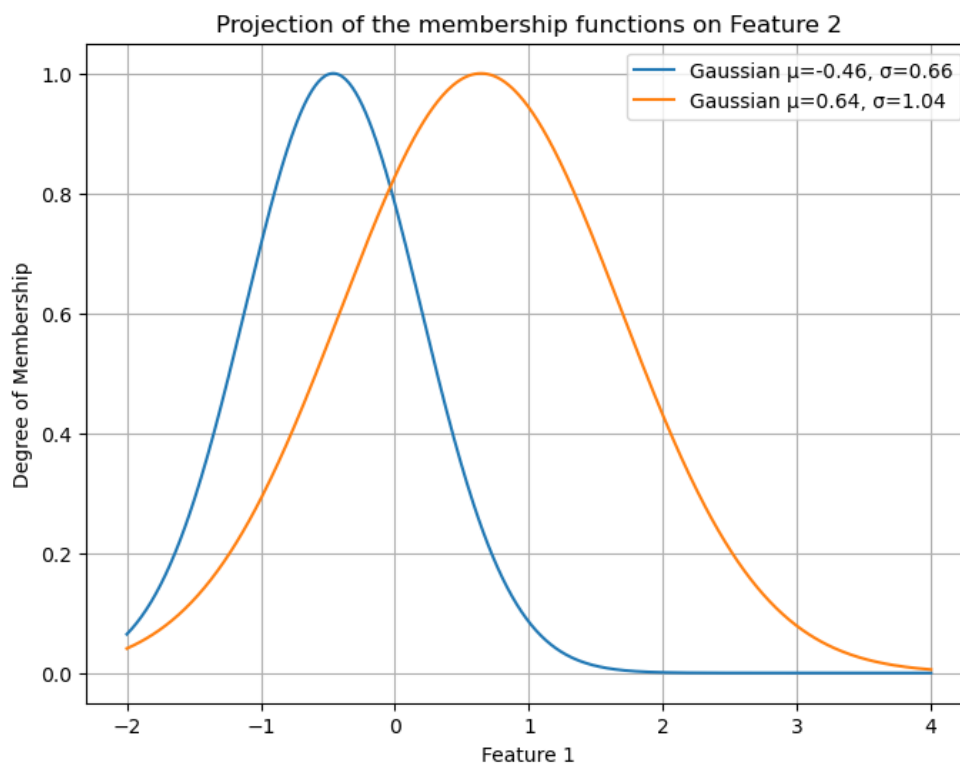
```
[10]: # Gaussian formula
def gaussian(x, mu, sigma):
    return np.exp(-0.5 * ((x - mu)/sigma)**2)

lin=np.linspace(-2, 4, 500)
plt.figure(figsize=(8,6))

y_aux=[]
feature=0
for j in range(n_clusters):
    # Compute curves
    y_aux.append(gaussian(lin, centers[j,feature], sigmas[j,feature]))
```

```
# Plot
plt.plot(lin, y_aux[j], label=f"Gaussian  $\mu$ ={np.
round(centers[j,feature],2)},  $\sigma$ ={np.round(sigmas[j,feature],2)}")

plt.title("Projection of the membership functions on Feature 2")
plt.xlabel("Feature 1")
plt.ylabel("Degree of Membership")
plt.legend()
plt.grid(True)
plt.show()
```



```
[11]: # -----
# Gaussian Membership Function
# -----
class GaussianMF(nn.Module):
    def __init__(self, centers, sigmas, agg_prob):
        super().__init__()
        self.centers = nn.Parameter(torch.tensor(centers, dtype=torch.float32))
        self.sigmas = nn.Parameter(torch.tensor(sigmas, dtype=torch.float32))
```

```

        self.agg_prob=agg_prob

    def forward(self, x):
        # Expand for broadcasting
        # x: (batch, 1, n_dims), centers: (1, n_rules, n_dims), sigmas: (1,
        n_rules, n_dims)
        diff = abs((x.unsqueeze(1) - self.centers.unsqueeze(0))/self.sigmas.
        unsqueeze(0)) #(batch, n_rules, n_dims)

        # Aggregation
        if self.agg_prob:
            dist = torch.norm(diff, dim=-1) # (batch, n_rules) # probabilistic
            intersection
        else:
            dist = torch.max(diff, dim=-1).values # (batch, n_rules) # min
            intersection (min intersection of normal function is the same as the max on
            dist)

        return torch.exp(-0.5 * dist ** 2)

# -----
# TSK Model
# -----
class TSK(nn.Module):
    def __init__(self, n_inputs, n_rules, centers, sigmas,agg_prob=False):
        super().__init__()
        self.n_inputs = n_inputs
        self.n_rules = n_rules

        # Antecedents (Gaussian MFs)

        self.mfs=GaussianMF(centers, sigmas,agg_prob)

        # Consequents (linear functions of inputs)
        # Each rule has coeffs for each input + bias
        self.consequents = nn.Parameter(
            torch.randn(n_inputs + 1,n_rules)
        )

    def forward(self, x):
        # x: (batch, n_inputs)
        batch_size = x.shape[0]

        # Compute membership values for each input feature
        # firing_strengths: (batch, n_rules)
        firing_strengths = self.mfs(x)

```



```

        # Normalize memberships
        # norm_fs: (batch, n_rules)
        norm_fs = firing_strengths / (firing_strengths.sum(dim=1, keepdim=True)
        ↵ + 1e-9)

        # Consequent output (linear model per rule)
        x_aug = torch.cat([x, torch.ones(batch_size, 1)], dim=1) # add bias

        rule_outputs = torch.einsum("br,rk->bk", x_aug, self.consequents) # ↵
        ↵ (batch, rules)
        # Weighted sum
        output = torch.sum(norm_fs * rule_outputs, dim=1, keepdim=True)

        return output, norm_fs, rule_outputs

```

```

[12]: # -----
# Least Squares Solver for Consequents (TSK)
# -----
def train_ls(model, X, y):
    with torch.no_grad():
        _, norm_fs, _ = model(X)

        # Design matrix for LS: combine normalized firing strengths with input
        X_aug = torch.cat([X, torch.ones(X.shape[0], 1)], dim=1)

        Phi = torch.einsum("br,bi->bri", X_aug, norm_fs).reshape(X.shape[0], -1)

        # Solve LS: consequents = (Phi^T Phi)^-1 Phi^T y

        theta = torch.linalg.lstsq(Phi, y).solution

        model.consequents.data = theta.reshape(model.consequents.shape)

```

```

[13]: # -----
# Gradient Descent Training
# -----
def train_gd(model, X, y, epochs=100, lr=1e-3):
    optimizer = optim.Adam(model.parameters(), lr=lr)
    criterion = nn.MSELoss()
    for _ in range(epochs):
        optimizer.zero_grad()
        y_pred, _, _ = model(X)
        loss = criterion(y_pred, y)

```

```

print(loss)
loss.backward()
optimizer.step()

```

```

[14]: # -----
# Hybrid Training (Classic ANFIS)
# -----
def train_hybrid_anfis(model, X, y, max_iters=10, gd_epochs=20, lr=1e-3):
    train_ls(model, X, y)
    for _ in range(max_iters):
        # Step A: GD on antecedents (freeze consequents)
        model.consequents.requires_grad = False
        train_gd(model, X, y, epochs=gd_epochs, lr=lr)

        # Step B: LS on consequents (freeze antecedents)
        model.consequents.requires_grad = True
        model.mfs.requires_grad = False
        train_ls(model, X, y)

        # Re-enable antecedents
        model.mfs.requires_grad = True

[15]: # -----
# Alternative Hybrid Training (LS+ gradient descent on all)
# -----
def train_hybrid(model, X, y, epochs=100, lr=1e-4):
    # Step 1: LS for consequents
    train_ls(model, X, y)
    # Step 2: GD fine-tuning
    train_gd(model, X, y, epochs=epochs, lr=lr)

[16]: # Build model
model = TSK(n_inputs=Xtr.shape[1], n_rules=n_clusters, centers=centers[:, :-1],
           sigmas=sigmas[:, :-1])

Xtr = torch.tensor(Xtr, dtype=torch.float32)
ytr = torch.tensor(ytr, dtype=torch.float32)
Xte = torch.tensor(Xte, dtype=torch.float32)
yte = torch.tensor(yte, dtype=torch.float32)

# Training with LS:
train_ls(model, Xtr, ytr.reshape(-1,1))

[17]: y_pred, _, _=model(Xte)
#performance metric for classification
print(f'ACC:{accuracy_score(yte.detach().numpy(),y_pred.detach().numpy())>0.5}')

```

ACC:0.8