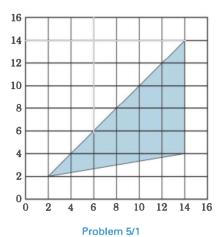
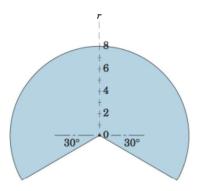
### **Introductory Problems**

5/1 Place your pencil on the position of your best visual estimate of the centroid of the triangular area. Check the horizontal position of your estimate by referring to the result of Sample Problem 5/2.

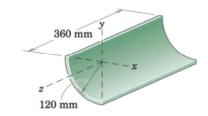


5/2 With your pencil make a dot on the position of your best visual estimate of the centroid of the area of the circular sector. Check your estimate by using the results of Sample Problem 5/3.



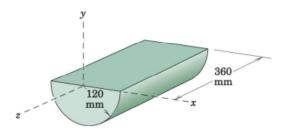
Problem 5/2

5/3 Specify the x-, y-, and z-coordinates of the mass center of the quarter-cylindrical shell.



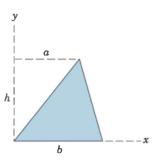
Problem 5/3

5/4 Specify the x-, y-, and z-coordinates of the mass center of the homogeneous semicylinder.



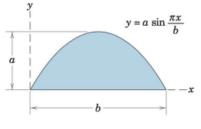
Problem 5/4

5/5 Determine the x-coordinate of the centroid of the shaded area.



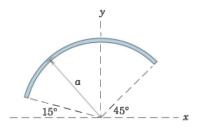
Problem 5/5

5/6 Determine the y-coordinate of the centroid of the area under the sine curve shown.



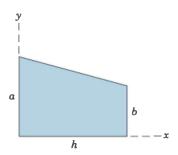
Problem 5/6

5/7 The homogeneous slender rod has a uniform cross section and is bent into a circular arc of radius a. Determine the x- and y-coordinates of the mass center of the rod by direct integration.



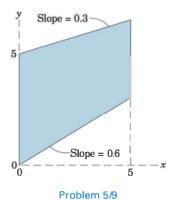
Problem 5/7

5/8 Determine the x- and y-coordinates of the centroid of the trapezoidal area.

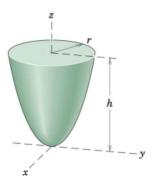


Problem 5/8

5/9 By direct integration, determine the coordinates of the centroid of the trapezoidal area.

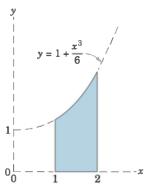


5/10 Determine the z-coordinate of the mass center of the homogeneous paraboloid of revolution shown.



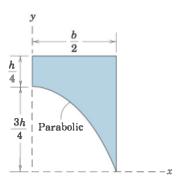
Problem 5/10

5/11 Determine the x- and y-coordinates of the centroid of the shaded area.



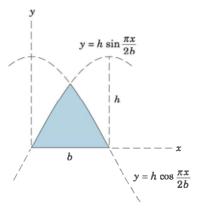
Problem 5/11

5/12 If the shaded area is revolved 360° about the y-axis, determine the y-coordinate of the centroid of the resulting volume.



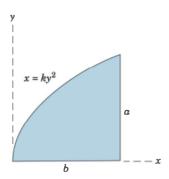
Problem 5/12

5/13 Determine the x- and y-coordinates of the centroid of the shaded area.



Problem 5/13

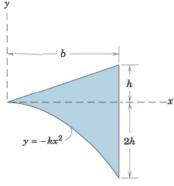
5/14 Determine the coordinates of the centroid of the shaded area.



Problem 5/14

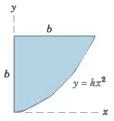
# Representative Problems

5/15 Determine the coordinates of the centroid of the shaded area.



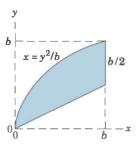
Problem 5/15

5/16 Determine the coordinates of the centroid of the shaded area.



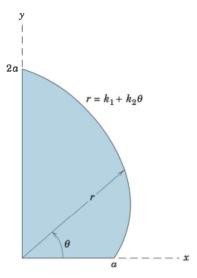
Problem 5/16

5/17 Determine the y-coordinate of the centroid of the shaded area.



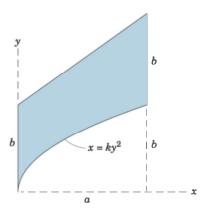
Problem 5/17

**5/18** Determine the *x*-coordinate of the centroid of the shaded area.



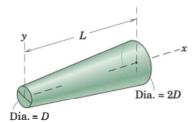
Problem 5/18

5/19 Determine the y-coordinate of the centroid of the shaded area.



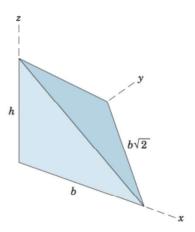
Problem 5/19

5/20 Determine the x-coordinate of the mass center of the tapered steel rod of length L where the diameter at the large end is twice the diameter at the small end.



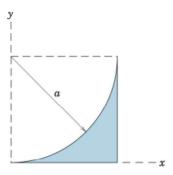
Problem 5/20

5/21 Determine by direct integration the coordinates of the centroid of the rectangular tetrahedron.



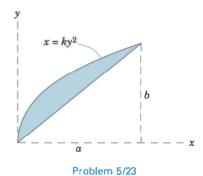
Problem 5/21

5/22 Locate the centroid of the area shown in the figure by direct integration. (Caution: Carefully observe the proper sign of the radical involved.)

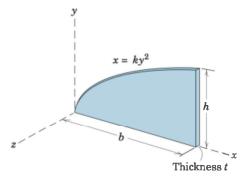


Problem 5/22

5/23 Determine the coordinates of the centroid of the shaded area.

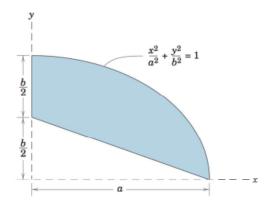


5/24 Determine the x- and y-coordinates of the mass center of the homogeneous plate of uniform thickness t.



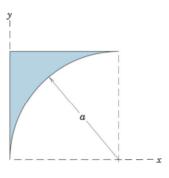
Problem 5/24

- 5/25 If the plate of Prob. 5/24 has a density that varies according to  $\rho = \rho_0(1 + \frac{x}{2b})$ , determine the x- and y-coordinates of the mass center.
- 5/26 Determine the x- and y-coordinates of the centroid of the shaded area.



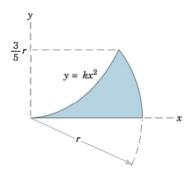
Problem 5/26

5/27 Locate the centroid of the area shown in the figure by direct integration. (*Caution:* Carefully observe the proper sign of the radical involved.)



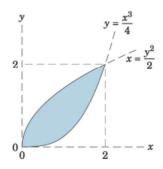
Problem 5/27

5/28 Determine the coordinates of the centroid of the shaded area.



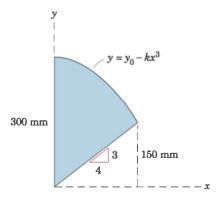
Problem 5/28

5/29 Locate the centroid of the shaded area between the two curves.



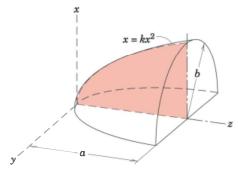
Problem 5/29

5/30 Determine the x- and y-coordinates of the centroid of the shaded area.



Problem 5/30

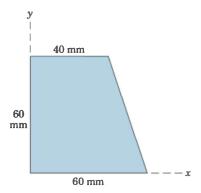
5/31 Determine the z-coordinate of the centroid of the volume obtained by revolving the shaded area under the parabola about the z-axis through 180°.



Problem 5/31

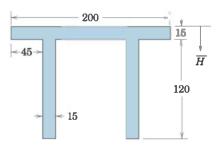
### **Introductory Problems**

5/47 Determine the coordinates of the centroid of the trapezoidal area shown.



Problem 5/47

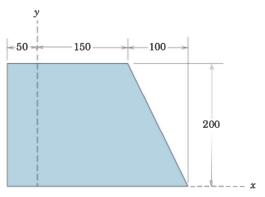
5/48 Determine the distance  $\overline{H}$  from the upper surface of the symmetric double-T beam cross section to the location of the centroid.



Dimensions in millimeters

Problem 5/48

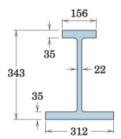
5/49 Determine the x- and y-coordinates of the centroid of the shaded area.



Dimensions in millimeters

Problem 5/49

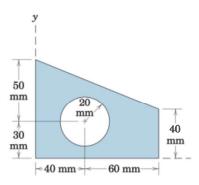
5/50 Determine the height above the base of the centroid of the cross-sectional area of the beam. Neglect the fillets.



Dimensions in millimeters

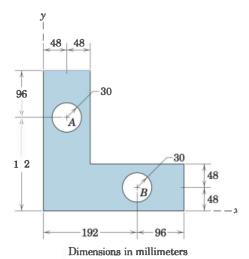
Problem 5/50

5/51 Determine the x- and y-coordinates of the centroid of the shaded area.



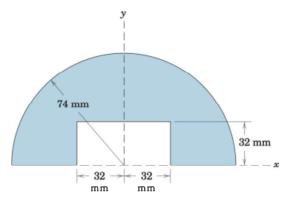
Problem 5/51

5/52 Determine the x- and y-coordinates of the centroid of the shaded area.



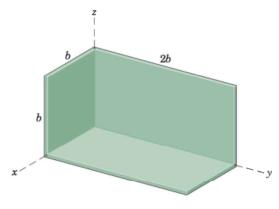
Problem 5/52

5/53 Calculate the y-coordinate of the centroid of the shaded area.



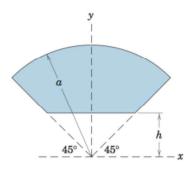
Problem 5/53

5/54 Determine the coordinates of the mass center of the body which is constructed of three pieces of uniform thin plate welded together.



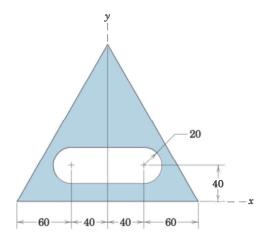
Problem 5/54

5/55 Determine the y-coordinate of the centroid of the shaded area.



Problem 5/55

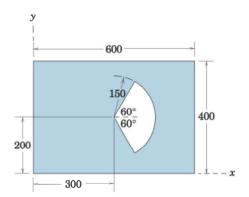
5/56 Determine the y-coordinate of the centroid of the shaded area. The triangle is equilateral.



Dimensions in millimeters

Problem 5/56

5/57 Determine the x- and y-coordinates of the centroid of the shaded area.

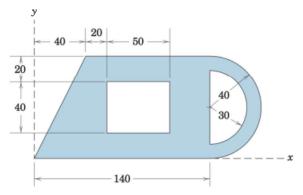


Dimensions in millimeters

Problem 5/57

#### Representative Problems

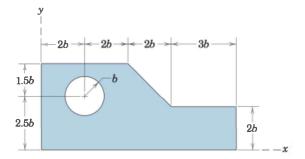
5/58 Determine the coordinates of the centroid of the shaded area.



Dimensions in millimeters

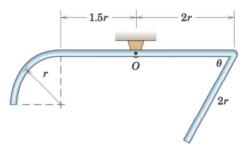
Problem 5/58

5/59 Determine the x- and y-coordinates of the centroid of the shaded area.



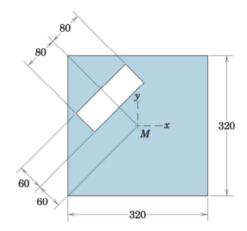
Problem 5/59

5/60 The uniform wire is bent into the shape shown and held by the frictionless pin at O. Determine the angle  $\theta$  which will allow the wire to hang in the orientation shown.



Problem 5/60

5/61 By inspection, state the quadrant in which the centroid of the shaded area is located. Then determine the coordinates of the centroid. The plate center is M.

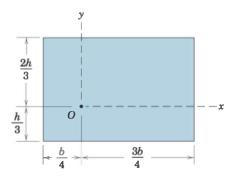


Dimensions in millimeters

Problem 5/61

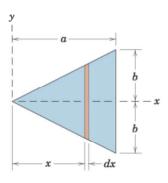
### **Introductory Problems**

A/1 Determine the moments of inertia of the rectangular area about the x- and y-axes and find the polar moment of inertia about point O.



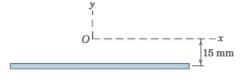
Problem A/1

A/2 Use the differential element shown to determine the moment of inertia of the triangular area about the x-axis and about the y-axis.



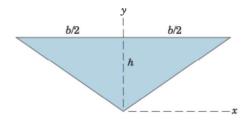
Problem A/2

A/3 The narrow rectangular strip has an area of 300 mm<sup>2</sup>, and its moment of inertia about the y-axis is 35(10<sup>3</sup>) mm<sup>4</sup>. Obtain a close approximation to the polar radius of gyration about point O.



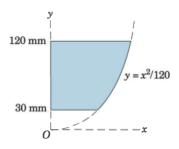
Problem A/3

A/4 Determine the ratio b/h such that  $I_x = I_y$  for the area of the isosceles triangle.



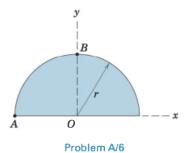
Problem A/4

A/5 Calculate the moment of inertia of the shaded area about the y-axis.

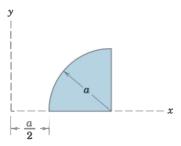


Problem A/5

A/6 Determine the polar moments of inertia of the semicircular area about points A and B.

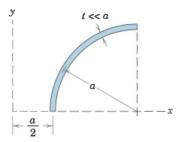


A/7 Determine the moment of inertia of the quartercircular area about the y-axis.



Problem A/7

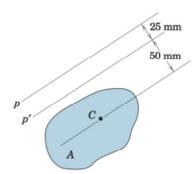
A/8 Determine the moment of inertia of the quartercircular strip about the y-axis.



Problem A/8

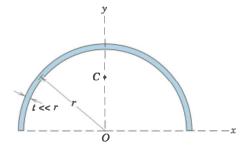
## Representative Problems

A/9 The moments of inertia of the area A about the parallel p- and p'-axes differ by  $15(10^6)$  mm<sup>4</sup>. Compute the area A, which has its centroid at C.



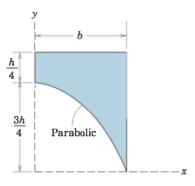
Problem A/9

A/10 Determine the moments of inertia  $I_x$  and  $I_y$  of the area of the thin semicircular ring about the x- and y-axes. Also find the polar moment of inertia  $I_c$  of the ring about its centroid C.



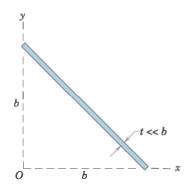
Problem A/10

A/11 Determine the moment of inertia of the shaded area about the y-axis.



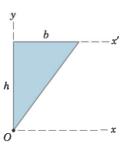
Problem A/11

- A/12 Determine the moment of inertia of the shaded area of the previous problem about the x-axis.
- A/13 Use the relationships developed and used in Sample Problem A/1 to determine expressions for the rectangular and polar moments of inertia  $I_x$ ,  $I_y$ , and  $I_O$  of the thin rectangular strip of area A where t is very small compared with b.

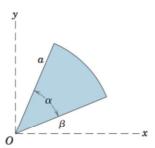


Problem A/13

A/14 By direct integration, determine the moments of inertia of the triangular area about the x- and x'-axes.

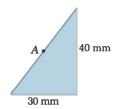


Problem A/14



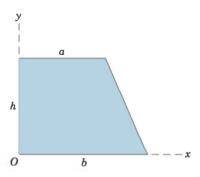
Problem A/15

A/16 Determine the radius of gyration about a polar axis through the midpoint A of the hypotenuse of the right-triangular area. (*Hint*: Simplify your calculation by observing the results for a 30  $\times$  40-mm rectangular area.)



Problem A/16

A/17 Determine by direct integration the moments of inertia of the trapezoidal area about the x- and y-axes. Find the polar moment of inertia about point O.



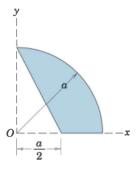
Problem A/17

A/18 Determine the polar radius of gyration of the area of the equilateral triangle of side b about its centroid C.



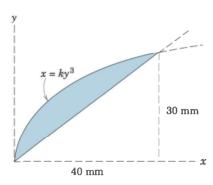
Problem A/18

A/19 Determine the moment of inertia of the shaded area about the x-axis.



Problem A/19

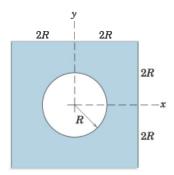
A/20 Calculate the moment of inertia of the shaded area about the x-axis.



Problem A/20

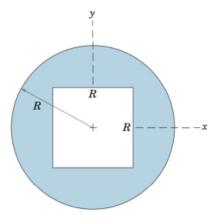
### **Introductory Problems**

A/35 Determine the moment of inertia about the x-axis of the square area without and with the central circular hole.



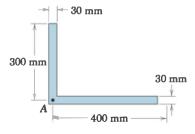
Problem A/35

A/36 Determine the polar moment of inertia of the circular area without and with the central square hole.



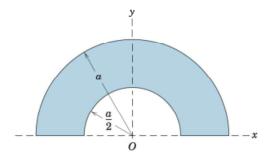
Problem A/36

A/37 Calculate the polar radius of gyration of the area of the angle section about point A. Note that the width of the legs is small compared with the length of each leg.



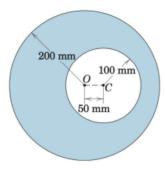
Problem A/37

A/38 By the method of this article, determine the rectangular and polar radii of gyration of the shaded area, repeated here from Prob. A/33, about the axes shown.



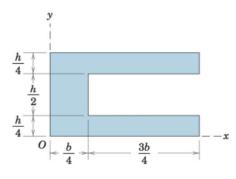
Problem A/38

A/39 Calculate the polar radius of gyration of the shaded area about the center O of the larger circle.



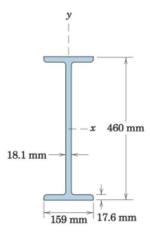
Problem A/39

A/40 Determine the percent reductions in both area and area moment of inertia about the y-axis caused by removal of the rectangular cutout from the rectangular plate of base b and height h.



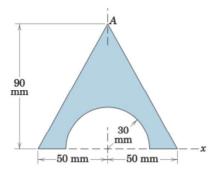
Problem A/40

A/41 The cross-sectional area of an I-beam has the dimensions shown. Obtain a close approximation to the handbook value of  $\bar{I}_x = 385(10^6) \text{ mm}^4$  by treating the section as being composed of three rectangles.



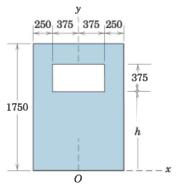
Problem A/41

A/42 Calculate the moment of inertia of the shaded area about the x-axis.



Problem A/42

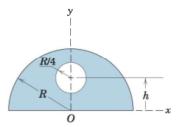
A/43 The variable h designates the arbitrary vertical location of the bottom of the rectangular cutout within the rectangular area. Determine the area moment of inertia about the x-axis for (a) h = 1000 mm and (b) h = 1500 mm.



Dimensions in millimeters

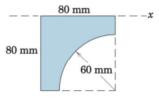
Problem A/43

A/44 The variable h designates the arbitrary vertical location of the center of the circular cutout within the semicircular area. Determine the area moment of inertia about the x-axis for (a) h = 0 and (b) h = R/2.



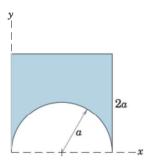
Problem A/44

A/45 Calculate the moment of inertia of the shaded area about the x-axis.



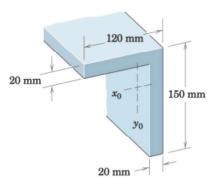
Problem A/45

A/46 Determine the moments of inertia of the shaded area about the x- and y-axes.



Problem A/46

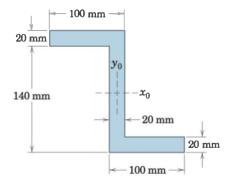
A/47 Calculate the moment of inertia of the cross section of the beam about its centroidal x<sub>0</sub>-axis.



Problem A/47

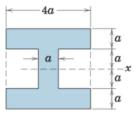
#### Representative Problems

A/48 Determine the moments of inertia of the Z-section about its centroidal  $x_0$ - and  $y_0$ -axes.



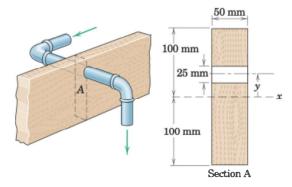
Problem A/48

A/49 Determine the moment of inertia of the shaded area about the x-axis in two different ways.



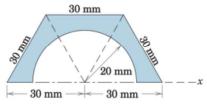
Problem A/49

A/50 A floor joist which measures a full 50 mm by 200 mm has a 25-mm hole drilled through it for a water-pipe installation. Determine the percent reduction n in the moment of inertia of the cross-sectional area about the x-axis (compared with that of the undrilled joist) for hole locations in the range  $0 \le y \le 87.5$  mm. Evaluate your expression for y = 50 mm.



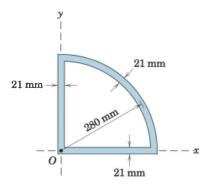
Problem A/50

A/51 Calculate the moment of inertia of the shaded area about the x-axis.



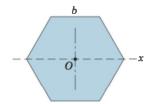
Problem A/51

A/52 Calculate the polar radius of gyration about point O of the area shown. Note that the widths of the elements are small compared with their lengths.



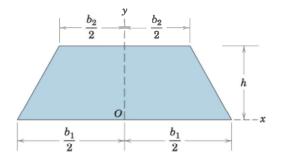
Problem A/52

A/53 Develop a formula for the moment of inertia of the regular hexagonal area of side b about its central x-axis.



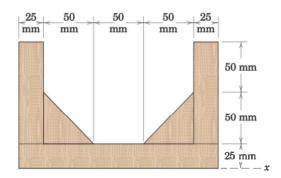
Problem A/53

A/54 By the method of this article, determine the moments of inertia about the x- and y-axes of the trapezoidal area.



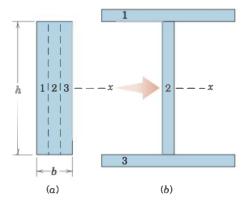
Problem A/54

A/55 Determine the moment of inertia of the cross-sectional area of the reinforced channel about the x-axis.



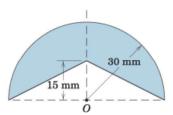
Problem A/55

A/56 The rectangular area shown in part a of the figure is split into three equal areas which are then arranged as shown in part b of the figure. Determine an expression for the moment of inertia of the area in part b about the centroidal x-axis. What percent increase n over the moment of inertia for area a does this represent if b = 200 mm and b = 60 mm?



Problem A/56

A/57 Calculate the polar moment of inertia of the shaded area about point O.



Problem A/57