Class – 1

Solution of non-Homogeneous Linear Partial Differential Equations with constant co-efficients

1. Solve: $(D^2 - D'^2 + D - D')z = 0$

   Ans: $\boldsymbol{z = f_1(y + x) + e^{-x} f_2(y - x)}$

2. Solve: $(D - 2D' + 5)^2 z = 0$

   Ans: $\boldsymbol{e^{-5x} f_1(y + 2x) + x e^{-5x} f_2(y + 2x)}$

3. Solve: $(D^2 - DD' - 2D'^2 + 2D + 2D')z = 0$

   Ans: $\boldsymbol{z = f_1(y - x) + e^{-2x} f_2(y + 2x)}$

Class – 2

PI: $F(x,y) = e^{ax+by}$

1. Solve: $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$

   Ans: $z = e^{x}f_1(y+x) + e^{2x}f_2(y+x) + \frac{1}{2}e^{2x-y}$

2. Solve: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = e^{-x}$.

   Ans: $z = e^{-x}f_1(y) + e^{x}f_2(y-x) - \frac{xe^{-x}}{2}$

3. Solve: $(D^2 + 5DD' + 6D'^2 + D + 2D')z = 4e^{2x-y}$

   Ans: $z = f_1(y-2x) + e^{-x}f_2(y-3x) + 2x^2e^{2x-y}$ (or) $z = f_1(y-2x) + e^{-x}f_2(y-3x) + \frac{y^2}{3}e^{2x-y}$

Class – 3

PI: $\boldsymbol{F(x,y) = sin(ax+by)\ or\ \ cos(ax+by)}$

1. Solve: $(D^2 + DD' + D' - 1)z = \sin\,(x + 2y)$
   Ans: $\boldsymbol{z = e^x f_1(y) + e^{-x} f_2(y+x) + \frac{cos(x+2y)}{2}}$
2. Solve: $(D^2 - DD' - 2D)z = \cos(3x + 4y)$
   Ans: $\boldsymbol{z = f_1(y) + e^{2x} f_2(y+x) + \frac{[cos(3x+4y)-2sin(3x+4y)]}{15}}$

Class – 4

PI: $\boldsymbol{F(x,y) = x^m y^n,\ m\ and\ n\ being\ constants,}$ **PI:** $\boldsymbol{F(x,y) = e^{ax+by}V(x,y),\ where\ V(x,y)}$ **is any function of** $\boldsymbol{x}$ **and** $\boldsymbol{y}$

1. Solve: $(D^2 - D'^2 + 3D' - 3D)z = xy$

   Ans: $\boldsymbol{z = f_1(y+x) + e^{3x}f_2(y-x) - \frac{1}{3}(\frac{x^2y}{2} + \frac{x^2}{3} + \frac{xy}{3} + \frac{2x}{9} + \frac{x^3}{6})}$
2. Solve: $(D - 3D' - 2)^3 z = 6e^{2x}\sin(y+3x)$

   Ans: $\boldsymbol{z = e^{2x}f_1(y+3x) + xe^{2x}f_2(y+3x) + x^2e^{2x}f_3(y+3x) + x^3e^{2x}sin(y+3x)}$

Class 1: Gamma functions - definition, Graph and properties

1. Express $\int_0^\infty \frac{x^c}{c^x} dx$ in terms of gamma function. Ans: $\frac{\Gamma(c+1)}{(log\, c)^{c+1}}$
2. Evaluate: $\int_0^\infty x^7 e^{-x} dx$; Ans:5040
3. Evaluate: $\int_0^\infty x^3 e^{-4x} dx$; Ans:3/128

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Class 2: Definition of Beta function and its properties

1. Prove that $\beta(n,n)=\frac{1}{2^{2n-1}}\beta\left(n,\frac{1}{2}\right)$
2. Prove that $\int_0^1 \frac{xdx}{\sqrt{1+x^4}}=\frac{1}{5}\beta\left(\frac{2}{5},\frac{1}{2}\right)$
3. Show that $\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$

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Class 3: Relation between Beta and Gamma functions and Duplication formula.

1. Prove that $\beta\left(-\frac{1}{2},\frac{5}{2}\right)=-\frac{3}{2}\pi$
2. Prove that $\int_0^\infty \frac{xdx}{1+x^6}=\frac{\pi}{3}$
3. Evaluate $\int_{-1}^{1}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}dx$ Ans: π
4. Show that $\int_1^3 \frac{dx}{\sqrt{(x-1)(3-x)}}$ Ans: π

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Class 4: Problems on Beta and Gamma functions :

1. Show that $\int_0^\infty \frac{x^4}{4^x}dx = \frac{24}{(log\,4)^5}$
2. Prove that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx = \frac{1}{a^m b^n}\beta(m,n)$
3. Evaluate $\int_a^\infty e^{2ax-x^2}dx$ using beta function. Ans: $e^{a^2}\frac{\sqrt{\pi}}{2}$

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Class 5: Series solution of Bessel Differential Equation, Bessel function:

1. Prove that $J_n(-x) = (-1)^n J_n(x)$ when 'n' is a real number integer.
2. Compute $J_0(2)$ and $J_1(2)$ correct to 3 decimal places. Ans: 0.224, 0.44
3. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

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Class 6: Recurrence relations:

1. Prove that $J_n(-x) = (-1)^n J_n(x)$ when 'n' is a real number integer.
2. Compute $J_0(2)$ and $J_1(2)$ correct to 3 decimal places. Ans: 0.224, 0.44
3. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

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Class 7: Problems on Recurrence Relations

1. Prove that $J_n''(x) = \frac{1}{4}\left[J_{n-2} - 2J_n + J_{n+2}\right]$
2. Show that $\frac{d}{dx}\left\{xJ_n \cdot J_{n+1}\right\} = x\left[J^2{}_n - J^2{}_{n+1}\right]$
3. Prove that i) $\int xJ_0(x)dx = xJ_1(x) + c$

   ii) $\int J_1(x)dx = -J_0(x) + c$

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Class 8: Generating Functions, Jacobi series:

1. Prove the following recurrence relations using generating functions:

i) $$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$$

ii) $$J_n'(x) = \frac{1}{2}\left[J_{n-1}(x) + J_{n+1}(x)\right]$$

2. Using Jacobi Series prove the following results.

$$(i)\ \cos(x) = J_0 - 2J_2 + 2J_4 - + ---$$
$$(ii)\ \sin(x) = 2J_1 - 2J_3 + - + ---$$

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Class 9: Problems on generating functions and Jacobi Series:

1. Prove the following recurrence relation using generating functions:
   $$xJ_{n-1}(x) = nJ_n(x) + xJ_n'(x)$$

2. Using Jacobi Series prove the following results.

$$(i)\ \ x\cos(x) = 2\left(J_1 - 3^2 J_3 + 5^2 J_5 - - - - - \right)$$
$$(ii)\ \ x\sin(x) = 2\left(2^2 J_2 - 4^2 J_4 + 6^2 J_6 - - - - \right)$$

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Class 10: Bessel's Integral formula and Orthogonality of Bessel functions

1. If α and β are distinct roots of the equation $AJ_n(x) + BxJ_n{}'(x) = 0$ where α and β are constants, show that $\int_0^1 xJ_n(\alpha)J_n(\beta x)dx = 0$

2. Prove that $y = J_n(3x)$ is a solution of the equation $x^2 y'' + xy' + \left(9x^2 - n^2\right) y = 0$

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