Implicit Functions

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IMPLICIT FUNCTION

Function in which one variable cannot be explicitly written as a pure function of another.

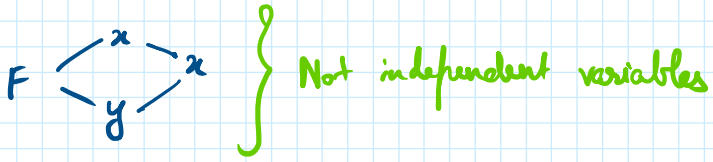
- Commonly of the form $F(x,y) = c$

Eg: $x^2 + xy + y^2 = 0$

$x^y = y^x$

Derivative of implicit function

Consider an implicit function $F(x,y) = c$



Not independent variables

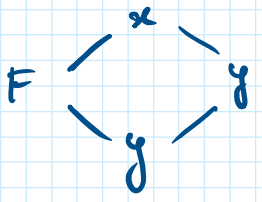
Thus we have

$$\frac{\partial F}{\partial x}(1) + \frac{\partial F}{\partial y}\cdot\frac{dy}{dx} = 0$$

$$\frac{\partial F}{\partial y}\cdot\frac{dy}{dx} = -\frac{\partial F}{\partial x}$$

$$\boxed{\frac{dy}{dx} = \frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}}$$

IIIly, for y being the independent variable,



$$\boxed{\frac{dx}{dy} = \frac{-\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}}$$

Hw: Find second order derivative for implicit function

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{-F_x}{F_y}\right)$$

$$\frac{dx}{dy} = \frac{-\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}$$