3. Laplace Plots (unfinished), Inverse Laplace

3D PLOTS OF A GIVEN LAPLACE TRANSFORM

$$f(t) = e^{-3t} \cos(5t)$$

$$F(s) = s+3/(s+3)^2 + 25$$

INVERSE LAPLACE TRANSFORMS

Evaluate: $\mathcal{L}^{-1}\left[\dfrac{s-5}{s(s+2)^2}\right]$

```
syms t s
f=(s-5)/(s*(s+2)^2);
ilaplace(f)

ans =

(5*exp(-2*t))/4 + (7*t*exp(-2*t))/2 - 5/4
```

Evaluate $\mathcal{L}^{-1}\left[\dfrac{s+2}{s^2-4s+13}\right]$

```
syms t s
f=(s+2)/(s^2-4*s+13);
r=ilaplace(f,s,t)
```

```
r =

exp(2*t)*(cos(3*t) + (4*sin(3*t))/3)
```

Solving ODEs using Laplace

```
syms s t Y
f=exp(-t)
F=laplace(f,t,s)
Y1=s*Y-4
Y2=s*Y1-5
solve(Y2+3*Y1+2*Y-F,Y)
ilaplace(ans,s,t)
```

f =

exp(-t)

F =

1/(s + 1)

Y1 =

Y*s - 4

Y2 =

s*(Y*s - 4) - 5

ans =

(4*s + 1/(s + 1) + 17)/(s^2 + 3*s + 2)

ans =

12*exp(-t) - 8*exp(-2*t) + t*exp(-t)

Evaluate

$$y'' + 2y' + 2y = \sin 2t$$

```
1 syms s t Y
2 f=sin(2*t)
3 F=laplace(f,t,s)
4 Y1=s*Y-4
5 Y2=s*Y1+4
6 solve(Y2+2*Y1+2*Y-F,Y)
7 ilaplace(ans,s,t)
```

>> test

f =

sin(2*t)

F =

2/(s^2 + 4)

Y1 =

Y*s - 4

Y2 =

s*(Y*s - 4) + 4

ans =

(4*s + 2/(s^2 + 4) + 4)/(s^2 + 2*s + 2)

ans =

(21*exp(-t)*(cos(t) + (2*sin(t))/21))/5 - sin(2*t)/10 - cos(2*t)/5

Evaluate

$$y'' + 2y' + y = e^{-2t}$$
$$y(0) = 1 \ , \ y'(0) = 0$$

$$s^2 y'' - s \cdot y(o) - y'(0) + 2[s \cdot y' - y(0)]$$

```
syms s t Y
f=exp(-2*t)
F=laplace(f,t,s)
Y1=s*Y-1
Y2=s*s*Y-s
solve(Y2+2*Y1+Y-F,Y)
ilaplace(ans,s,t)
```

f =

exp(-2*t)

F =

1/(s + 2)

Y1 =

Y*s - 1

Y2 =

Y*s^2 - s

ans =

(s + 1/(s + 2) + 2)/(s^2 + 2*s + 1)

ans =

exp(-2*t) + 2*t*exp(-t)

Evaluate

$$y'' + 6y' + 8y = e^{-t}$$

$$y(0) = 0 \qquad y'(0) = 1$$

```
syms s t Y
f=exp(-t)
F=laplace(f,t,s)
Y1=s*Y
Y2=s*s*Y-1
solve(Y2+6*Y1+8*Y-F,Y)
ilaplace(ans,s,t)
```

```
f =

exp(-t)


F =

1/(s + 1)


Y1 =

Y*s


Y2 =

Y*s^2 - 1


ans =

1/(s^2 + 5*s + 4)


ans =

exp(-t)/3 - exp(-4*t)/3
```