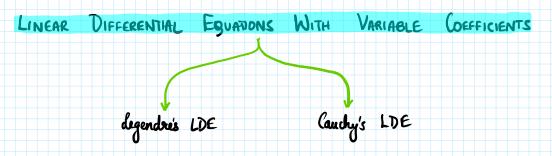
1. Legendre's LDE, Cauchy's LDE

06 November 2023 08:10



LEGENDRE'S LDE

$$a_0(ax+b)^2 \frac{d^2y}{dx^2} + a_1(ax+b) \frac{dy}{dx} + a_2y = \phi(x)$$

$$\begin{cases} y \text{ overal form } y \text{ degree } = 2 \\ y \text{ degree } = 2 \end{cases}$$
Can be extended to any degree

First reduce to LDE with constant coefficient and then solve.

$$t = log (ax+b)$$

$$ax+b = e^t \implies x = e^t - b$$

Joking derivative,

$$\frac{dy}{dx} = \frac{dy}{dt} \left(\frac{dt}{dx} \right) = \frac{dy}{dt} \left(\frac{a}{ax+b} \right)$$

Differentiating wort x,

$$(ax+b) \frac{d^2y}{dx^2} + \frac{dy}{dx}(a) = a\left(\frac{d}{dx}\left(\frac{dy}{dx}\right)\right)$$

(an+b)
$$\frac{d^2y}{dx^2} + a\left(\frac{dy}{dx}\right) = a\left(\frac{d}{dx}\left(\frac{dy}{dx}\right)\right)\left(\frac{dt}{dx}\right)$$
 \(\frac{d}{dx} = \frac{d}{dx}\left(\frac{dt}{dx}\right)\left\text{ by chain stule}

$$(ax+b)\frac{d^2y}{dx^2} + a\left(\frac{dy}{dn}\right) = \frac{d^2y}{dt^2}\left(\frac{a^2}{ax+b}\right)$$

$$(ax+b)^2 \left(\frac{d^2y}{dx^2}\right) + a(ax+b) \left(\frac{dy}{dx}\right) = a^2 \cdot \frac{d^2y}{dt^2}$$

$$(ax+b)^{2} \left(\frac{d^{2}y}{dx^{2}}\right) = a^{2} \left(\frac{d^{2}y}{dx^{2}}\right) - a\left(ax+b\right) \frac{dy}{dx}$$

$$(ax+b)^{2} \left(\frac{d^{2}y}{dx^{2}}\right) = a^{2} \left(\frac{d^{2}y}{dx^{2}}\right) - a\left(ax+b\right) \frac{dy}{dx}$$

$$(ax+b)^{2} \left(\frac{d^{2}y}{dx^{2}}\right) = a^{2} \left(\frac{d^{2}y}{dx^{2}}\right)$$

$$(ax+b)^{2} \left(\frac{d^{2}y}{dx^{2}}\right) = a^{2} \left(\frac{d^{2}y}{dx^{2}}\right)$$

$$(ax+b)^{2} \left(\frac{d^{2}y}{dx^{2}}\right) = a^{2} \left(\frac{d^{2}y}{dx^{2}}\right)$$

$$(ax+b)^2 \left(\frac{d^2y}{dx^2}\right) = a^2 \left(\frac{d^2y}{dx^2}\right) - a^2 \left(\frac{dy}{dx^2}\right)$$

$$(ax) \qquad (ax) \qquad (ax) \qquad = a^{2} (D_{1}^{2} - D_{1}) y$$

$$(ax + b)^{2} \frac{d^{2}y}{dx^{2}} = a^{2} D_{1} (D_{1} - 1) y \qquad 3$$

Use egns. (1), (2), (3) to solve the original equation

Illy, for third order derivative:

$$(ax+b)^3 \frac{d^2y}{dx^{13}} = a^3 D_1 (D_1-1)(D_1-2)y$$

CAUCHY'S LDE

- · Special case of degendres LDE
 · In ax+b, a=1 and b=0

 => ax+b=x

$$a_0 \cdot x^2 \cdot \frac{d^2y}{dx^2} + a_1 \cdot x \cdot \frac{dy}{dx} + a_2 y = \phi(n)$$

$$t = \log x \Rightarrow x = e^t$$

$$x \frac{dy}{dx} = D_{i}(y)$$

$$D_{i} = \frac{d}{dt}$$

$$D_i = \frac{d}{dt}$$

$$\frac{\lambda^2}{dx^2} = D_1(D_1 - 1) y$$