CW Problems

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① $X \in \{0, 1, 2, \dots, 50\}$

② $p = 2/6 = 1/3$

Mean $= np = 1$

Variance $= np(1-p) = 2/3$

③ $p = 1/2$

$\mu = np = 3/2$

$\sigma^2 = np(1-p) = 3/4$

④

X:	0	1	2	3	4	5	6
p(x):	k	3k	5k	7k	9k	11k	13k

$k = 0.02$

(i) $P(X<4) = 1 - P(X \geq 4)$
$= 1 - [9k + 11k + 13k]$
$= 0.34$

$P(X \geq 5) = 11k + 13k$
$= 0.48$

$P(3 < X \leq 6) = 9k + 11k + 13k$
$= 0.66$

(ii) $P(X \leq 2) < 0.3$

$9k < 0.3$

$k < \frac{0.3}{9}$

$k < \frac{1}{30}$

Doubt

5. **Home work problem**: The probability density function of a random variable X is given as follows:

x:	0	1	2	3	4	5	6	7
p(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find the value of k. Evaluate $P(X < 6)$; $P(X \geq 6)$; and $P(0 < X < 5)$.

$k = 0.1$

$P(X<6) = 1 - P(X \geq 6)$
$= 1 - [2k^2 + 7k^2 + k]$
$= 0.81$

$P(X \geq 6) = 0.19$

$P(0 < X < 5) = P(X<6) - P(X=5)$
$= 0.8$

6. The Sample space of a random experiment is {a, b, c, d, e, f} and each outcome is equally likely. A random variable is defined as folows:

Outcome	a	b	c	d	e	f
X	0	0	1.5	1.5	2	3

Determine the probability mass function of X. Use the probability mass function to determine the following probabilities. $a)$ $P(X = 1.5)$ $b)$ $P(0.5 < X < 2.7)$ $c)$ $P(X > 3)$
$d)$ $P(0 \leq X < 2)$ $e)$ $P(X = 0 \text{ or } X = 2)$.

$P(\text{outcome}) = 1/6$

$$f_X(x) = \begin{cases} 1/3, & x = 0 \\ 1/3, & x = 1.5 \\ 1/6, & x = 2 \\ 1/6, & x = 3 \end{cases}$$

(a) $P(X = 1.5) = 1/3$

(b) $P(0.5 < X < 2.7) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

(c) $P(X > 3) = 0$

(d) $P(0 \leq X < 2) = \frac{1}{3} + \frac{1}{3} = 2/3$

(e) $P(X = 0 \text{ or } X = 2) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

7. The space shuttle flight control system called PASS[primary Avionics software set] uses four independent computers working in parallel. At each critical step, the computers 'vote' to determine the appropriate step. The probability that a computer will ask for roll to the left when a roll to the right is appropriate is 0.0001. let X denote the number of computers that vote for a left roll when a right roll is appropriate. what is the probability mass function of X? what is mean variance of X?

$p = 0.0001$

$P(X = x) = {}^4C_x (0.0001)^x (0.9999)^{4-x}$

$\mu = np = 0.0004$

$\sigma^2 = np(1-p) = 3.996 \times 10^{-4}$

8. Is the function defined as follows a density function?

$$\begin{aligned} f(x) &= e^{-x} \quad (x \geq 0) \\ &= 0 \quad (x < 0) \end{aligned} \tag{0.1}$$

If so, determine the probability that random variable X having this density will fall in the interval $(1, 2)$.

$$\int_0^\infty e^{-x}\,dx = \left[\frac{e^{-x}}{-1}\right]_0^\infty = 0 - (-1) = 1$$

$\Rightarrow$ It is a density function

$$\int_1^2 e^{-x}\,dx = (-1)\left[e^{-x}\right]_1^2$$

$$= -1\left[e^{-2} - e^{-1}\right]$$

$$= 0.2325$$

9. Let X be a continuous random variable with probability density function given by

$$\begin{aligned} f(x) &= kx \quad (0 \le x \le 2) \\ &= 2k \quad (2 \le x < 4) \\ &= -kx + 6k \quad (4 \le x < 6) \end{aligned}$$

Find the value of k and the mean value of X.

$$\int_0^2 kx + \int_2^4 2k + \int_4^6 -kx + 6k = 1$$

$k = 0.125$

$$E(X) = \int_{supp(X)} x_i \cdot f(x_i) = 3$$

10. A coin has a probability of 0.5 of landing heads when tossed. Let X = 1 if the coin comes up heads, and X = 0 if the coin comes up tails. What is the distribution of X?

$p = 0.5$

$X \sim$ Bernoulli(0.5)

11. A die has a probability $\frac{1}{6}$ of coming up 6 when rolled. Let X = 1 if the die comes up 6, and X = 0 otherwise. What is the distribution of X?

$p = 1/6$

$X \sim$ Bernoulli$(1/6)$

12. Ten percent of the components manufactured by a certain process are defective. A component is chosen at random. Let X = 1 if the component is defective, and X = 0 otherwise. What is the distribution of X?

$p = 0.1$

$X \sim$ Bernoulli(0.1)

$\mu = p = 0.1$

$\sigma^2 = p(1-p) = 0.09$

$\mu = p = 0.1$

$\sigma^2 = p(1-p) = 0.09$

13. The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on succesive calls are independent. Assume that 10 calls are placed to the airline.
(i) what is the probability that for exactly three calls the lines are occupied?
(ii) what is the probability that for atleast one call the lines are not occupied?
(iii) what is the expected number of calls in which the lines are all occupied.

$p = 0.4$, $n = 10$

$X \to$ no. of lines occupied

(i) $P(X=3) = {}^{10}C_3 (0.4)^3 (0.6)^7$

≈ 0.2149

(ii) $P(Y \geq 1) = 1 - P(Y<1)$

$p = 0.6$

$= 1 - [P(0)]$

$\approx 1 - [1.048 \times 10^{-4}]$

$= 0.9999$

(iii) $p = 0.4$

$E(X) = np = 4$ calls

14. Heart failure is due to either natural occuerence(87%) or outside factors(13%). outside factors are releated to induce substances or foreign objects. Natural occurence are caused by arterial blockage, disease and infection. Suppose that 20 patients will visit an emergency room with heart failure. Assume that cause of heart failure between individuals are independent.
(a) what is the probability that three individuals have conmditions caused by outtside factors?
(b) what is the probability that three or more individuals have conditions caused by outside factors?
(c) what is the mean and standard deviation of the number of individuals with conditions caused by outside factors?

Whoever wrote this question, please work on your English smh

$p = 0.13$, $n = 20$

$X \to$ no. of patients due to outside factors

(a) $P(X=3) = {}^{20}C_3 (0.13)^3 (0.87)^{17}$

$= 0.2347$

(b) $P(X \geq 3) = 1 - P(X<3)$

$\approx 1 - [P(0) + P(1) + P(2)]$

$\approx 1 - [0.5079]$

$= \underline{\underline{0.4921}}$

(c) $\mu = np = 20(0.13) = \underline{\underline{2.6}}$

$\sigma^2 = np(1-p) = 20(0.13)(0.87)$

$\sigma = \underline{\underline{1.5039}}$

15. In eight throws of a fair dies, 5 or 6 is considered a success. Find the mean of the number of success and the standard deviation.

$p = 2/6 = 1/3 \;;\quad n = 8$

$\mu = np = \underline{\underline{\frac{8}{3}}}$

$\sigma = \sqrt{\frac{8}{3}\left(\frac{2}{3}\right)} = \sqrt{\frac{16}{9}} = \underline{\underline{\frac{4}{3}}}$

16. The probability that a man hits a target is $\frac{1}{3}$. How many times must he fire so that the probability of hitting the target atleast once is more than 90% ?

$p = 1/3 \qquad n = ?$

$P(x \geq 1) > 0.9$

$1 - P(x < 1) > 0.9$

$1 - P(0) = 0.9$

$P(0) = 0.1$

${}^nC_0 (1/3)^0 (2/3)^n = 0.1$

$\Rightarrow n = 5.67$

$\Rightarrow$ He needs to fire at <u>least 6 times</u>

17. Suppose that X has poisson distribution with a mean of 4. Determine the following probabilities:
(a)$P(X = 0)$ (b) $P(X \leq 2)$, (c) $P(X = 4)$, (d) $P(X = 8)$

$\lambda = 4$

(a) $P(x=0) = \frac{e^{-\lambda}\lambda^0}{0!} = \underline{\underline{0.0183}}$

(b) $P(x \leq 2) = P(0) + P(1) + P(2) = \underline{\underline{0.2381}}$

(c) $P(x=4) = e^{-4}.\, 4^4 = 0.1953$

(c) $P(X=4) = e^{-4} \cdot \dfrac{4^4}{4!} = \underline{\underline{0.1953}}$

(d) $P(X=8) = \underline{\underline{0.0297}}$

18. The number of telephone calls that arrive at a phone exchange is often modelled as a Poisson random variable. Assume that on the average there are 10 calls per hour.
(a) What is the probability that there are exactly 5 calls in one hour?
(b) What is the probability that there are 3 or fewer calls in one hour?
(c) What is the probability that there are exactly 15 calls in two hour?
(d) What is the probability that there are exactly 5 calls in 30 minutes?

$\lambda = 10$

(a) $P(X=5) = e^{-10} \cdot \dfrac{10^5}{5!}$

$= \underline{\underline{0.0378}}$

(b) $P(X \le 3) = P(0) + P(1) + P(2) + P(3)$

$= 4.539 \times 10^{-5} + 4.539 \times 10^{-4} + 2.269 \times 10^{-3} + 7.566 \times 10^{-3}$

$= \underline{\underline{0.0103}}$

(c) For 2 hours, $\lambda = 20$

$P(Y=15) = \underline{\underline{0.0516}}$

(d) For 30 minutes, $\lambda = 5$

$P(Z=5) = \underline{\underline{0.1754}}$

19. Fit a Poisson distribution for the following data and calculate the theoretical frequencies

important question

X	0	1	2	3	4
f	122	60	15	2	1

$\mu = \lambda = \dfrac{\sum f_i x_i}{\sum f_i} = 0.5$

$n = \sum f_i = 200$

$P_X(0) = 0.6065$ $P_X(1) = 0.3032$ $P_X(2) = 0.0758$ $P_X(3) = 0.0126$

$TF_0 = \underline{\underline{121}}$ $TF_1 = \underline{\underline{61}}$ $TF_2 = \underline{\underline{15}}$ $TF_3 = \underline{\underline{3}}$

$P_X(4) = 1.579 \times 10^{-3}$

$p_X(4) = 1.579 \times 10^{-3}$

$TF_4 = 0$

20. In a certain factory turning out razor blades there is a small probability of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing
(i) no defective (ii) one defective (iii) two defective blades, in a consignment of 10000 packets.

$\lambda = np = \frac{1}{500} \times 10 = \frac{1}{50} =$

(i) $P(X=0) = 0.980198$

No of packets $= 10000 \times 0.980198 = 9801.98$

$\approx$ 9802 packets

(ii) $P(X=1) = 0.0196$

No of packets = 196 packets

(iii) $P(X=2) = 1.96 \times 10^{-4}$

No of packets $= 1.96$

$\approx$ 2 packets

Outcome	a	b	c	d	e	f
X	0	0	1.5	1.5	2	3

$$f_X(x) = \begin{cases} \frac{1}{3}, & x = 0 \\ \frac{1}{3}, & x = 1.5 \\ \frac{1}{6}, & x = 2 \\ \frac{1}{6}, & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

(i) $P(X = 1.5) = \frac{1}{3}$

(ii) $P(0.5 < X < 2.7)$

$= P(X=1.5) + P(X=2) = \frac{1}{2}$

(iii) $P(X > 3) = 0$

(iv) $P(0 \le X < 2) = P(X=0) + P(X=1.5) = \frac{2}{3}$

(v) $P(X=0 \text{ or } X=2) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$