4. Higher Order LDE - Intro

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Higher Order Linear DE

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_n y = \phi(x)$$

$a_0, a_1, a_2, \dots, a_n$
- constants → LDE with constant coefficients
- functions of x → LDE with variable coefficients

Conditions for a DE to be linear:

- Dependent variables and derivatives ⟶ highest power should be 1
- Dependent variable not in product with its derivative(s)

LDE with Constant Coefficients

- Homogeneous LDE: $\phi(x) = 0$
- Non-homogeneous LDE: $\phi(x) \neq 0$

General soln for homogeneous LDE

$$\boxed{y = \underbrace{CF}_{\text{Complementary Function}} \text{ (or) } \underbrace{y_c}_{\text{Complementary Function}}}$$

General soln for non-homogeneous LDE

$$\boxed{y = \underbrace{CF}_{y_c} + \underbrace{PI}_{y_p}}$$

y_c → Complementary Function

y_p → Particular Integral

Consider the following LDE:

$$a_0 \frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

Taking $\frac{d}{dx} = D$

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$$\frac{d^2}{dx^2} = \frac{d}{dx}\left(\frac{d}{dx}\right) = D^2$$

$$\Rightarrow a_0 D^2 y + a_1 D y + a_2 y = 0$$

$$(a_0 D^2 + a_1 D + a_2) y = 0$$

$F(D) \cdot y = 0$ where $\underbrace{F(D)}_{\text{Linear differential operator}} = a_0 D^2 + a_1 D + a_2$

Operator form

$\boxed{\text{An } n^{th} \text{ order DE should have "n" arbitrary constants in its GS}}$