PES University, Bengaluru

(Established under Karnataka Act No. 16 of 2013)

UE20MA101

MAY 2022: END SEMESTER ASSESSMENT (ESA) B TECH I SEMESTER

UE20MA101 – ENGINEERING MATHEMATICS - I

Time: 3 Hrs	Answer All Questions	Max Marks: 100

1	a)	Test the convergence of the series $\sum_{n=1}^{\infty} \sqrt{n^4+1} - \sqrt{n^4-1}$	6
	b)	Discuss the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots \ldots \ldots \ldots \ldots$	7
	c)	Obtain the nature of the series $\frac{2^2}{3.4}x^2 + \frac{2^2.4^2}{3.4.5.6}x^4 + \frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^6 + \cdots \ldots \ldots \ldots \ldots$	7
2	a)	If $u = \sqrt{x^4+y^4}\, tan^{-1}(y/x)$ prove the following : i. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$ ii. $x^2\frac{\partial^2 u}{\partial^2 x} + 2xy\frac{\partial^2 u}{\partial x\, \partial y} + y^2\frac{\partial^2 u}{\partial^2 y} = 2u$	7
	b)	Expand $e^{ax}\sin by$ about the origin upto third degree terms	6
	c)	Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.	7
3	a)	Solve : $\frac{dy}{dx} - \frac{tany}{1+x} = (1+x)e^x secy$	6
	b)	Show that the family of curves $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, where '$\lambda$' is a parameter is self orthogonal.	7
	c)	Solve : $p^3 - 4xyp + 8y^2 = 0$	7
4	a)	Solve : $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$ by the method of variation of parameters.	7
	b)	Solve : $(5+2x)^2 y'' - 6(5+2x)y' + 8y = log(5+2x)$	7
	c)	At $t = 0$, a current flows in an LCR circuit with resistance $R = 40$ohms, inductance $L = 0.2$ henrys and capacitance $C = 10^{-5}$ farads. Find the current flowing in the circuit at $t > 0$ if the initial change on the capacitor is 1 coulomb. Assume that $E(t) = 0$ for $t > 0$.	6
5	a)	Prove that $\int_0^{\infty} \frac{x}{1+x^6} dx = \frac{\pi}{3}$	7
	b)	Prove that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(\frac{24}{x^2} - 1\right) J_0(x)$	6
	c)	Show that $J_0^2 + 2J_1^2 + 2J_2^2 + 2J_3^2 + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots = 1$	7