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PES UNIVERSITY, BANGALORE

(Established Under Karnataka Act 16 of 2013)   **UE21MA141A**

March 2022: END SEMESTER ASSESSMENT (ESA) B. TECH I SEMESTER
UE21MA141A – ENGINEERING MATHEMATICS - I

Time: 3 Hours   **Answer All Questions**   Max Marks: 100

1.	a)	Test the convergence of the series $\frac{1^2}{4^2} + \frac{1^2 \cdot 5^2}{4^2 \cdot 8^2} + \frac{1^2 \cdot 5^2 \cdot 9^2}{4^2 \cdot 8^2 \cdot 12^2} + \cdots$	7
	b)	Test the convergence of the series $\sum \frac{n^2-1}{n^2+1} \cdot x^n$	7
	c)	Discuss the convergence of the series $\sum \left[\frac{1}{n} \cdot tan\left(\frac{1}{n}\right)\right]$	6
2.	a)	If $x^x y^y z^z = c$, find the value of $\frac{\partial^2 z}{\partial x \partial y}$ at $x = y = z$.	6
	b)	If $f(x,y) = tan^{-1}\left(\frac{y}{x}\right)$, find the approximate value of $tan^{-1}\left(\frac{0.9}{1.1}\right)$ to four decimal places using Taylor series up to second degree.	7
	c)	If x, y, z are the length of the perpendiculars dropped from any point P to the three sides of a triangle of constant area A, find the minimum value of $x^2 + y^2 + z^2$.	7
3.	a)	Solve the differential equation $(ylogx - 2)ydx = xdy$.	7
	b)	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is a parameter.	6
	c)	Solve the non-linear differential equation $y = x + 2tan^{-1}p$.	7
4.	a)	Solve the initial value problem $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y = 0, y(0) = 1$; $y'(0) = 0; y''(0) = \frac{1}{2}$.	7
	b)	Solve the differential equation $\frac{d^2y}{dx^2} - y = 2(1 - e^{-2x})^{-\frac{1}{2}}$ by using the method of variation of parameters.	6
	c)	Solve the differential equation $x^3\frac{d^3y}{dx^3} - 3x\frac{dy}{dx} + 3y = 16x + 9x^2logx, \ x > 0$.	7

5.	a)	Prove that $\int_0^1 x^m (1-x^p)^n \, dx = \frac{1}{p}\beta\left(\frac{m+1}{p}, n+1\right)$ and hence evaluate the integral $\int_0^1 x^{\frac{3}{2}}\left(1-\sqrt{x}\right)^{\frac{1}{2}} dx$. Here, $\sqrt{x}$ is the square root of x.	7
	b)	Prove that $x[J_{v-1}(x) + J_{v+1}(x)] = 2vJ_v(x)$.	6
	c)	If α and β are distinct roots of the equation $J_n(ax) = 0$, prove that $\int_0^a x J_n(\alpha x) J_n(\beta x) dx = 0$.	7