

PES UNIVERSITY, BANGALORE

(Established Under Karnataka Act 16 of 2013)

UE21MA141A

March 2022: **END SEMESTER ASSESSMENT (ESA) B. TECH I SEMESTER**
UE21MA141A – ENGINEERING MATHEMATICS - I

Time: 3 Hours

Answer All Questions

Max Marks: 100

1.	a)✓	Test the convergence of the series $\frac{1^2}{4^2} + \frac{1^2 \cdot 5^2}{4^2 \cdot 8^2} + \frac{1^2 \cdot 5^2 \cdot 9^2}{4^2 \cdot 8^2 \cdot 12^2} + \dots$	7
	b)✓	Test the convergence of the series $\sum \frac{n^2-1}{n^2+1} \cdot x^n$	7
	c)	Discuss the convergence of the series $\sum \left[\frac{1}{n} \cdot \tan \left(\frac{1}{n} \right) \right]$	6
2.	a)✗	If $x^x y^y z^z = c$, find the value of $\frac{\partial^2 z}{\partial x \partial y}$ at $x = y = z$.	6
	b)	If $f(x, y) = \tan^{-1} \left(\frac{y}{x} \right)$, find the approximate value of $\tan^{-1} \left(\frac{0.9}{1.1} \right)$ to four decimal places using Taylor series up to second degree.	7
	c)✗	If x, y, z are the length of the perpendiculars dropped from any point P to the three sides of a triangle of constant area A , find the minimum value of $x^2 + y^2 + z^2$.	7
3.	a)	Solve the differential equation $(y \log x - 2)y dx = x dy$.	7
	b)✓	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter.	6
	c)✗	Solve the non-linear differential equation $y = x + 2 \tan^{-1} p$.	7
4.	a)	Solve the initial value problem $\frac{d^3 y}{dx^3} + 3 \frac{d^2 y}{dx^2} - 4y = 0, y(0) = 1; y'(0) = 0; y''(0) = \frac{1}{2}$.	7
	b)✓	Solve the differential equation $\frac{d^2 y}{dx^2} - y = 2(1 - e^{-2x})^{-\frac{1}{2}}$ by using the method of variation of parameters.	6
	c)✓	Solve the differential equation $x^3 \frac{d^3 y}{dx^3} - 3x \frac{dy}{dx} + 3y = 16x + 9x^2 \log x, x > 0$.	7

5.	a)	Prove that $\int_0^1 x^m (1 - x^p)^n dx = \frac{1}{p} \beta\left(\frac{m+1}{p}, n+1\right)$ and hence evaluate the integral $\int_0^1 x^{\frac{3}{2}} (1 - \sqrt{x})^{\frac{1}{2}} dx$. Here, \sqrt{x} is the square root of x .	7
	b)	Prove that $x[J_{v-1}(x) + J_{v+1}(x)] = 2vJ_v(x)$.	6
	c)	If α and β are distinct roots of the equation $J_n(ax) = 0$, prove that $\int_0^a x J_n(\alpha x) J_n(\beta x) dx = 0$.	7