1. Boolean Algebra & Logic Gates

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BOOLEAN ALGEBRA, LOGIC GATES – INTRO

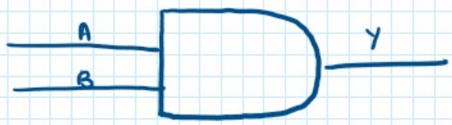
- Mathematics used to analyse and simplify logic/digital circuits ⟶ boolean algebra
- Digital circuits ⟶ constructed using logic gates
- Logic gates: one or more inputs, only one output
- Number of possible input states = 2^n ⟶ no. of inputs

Logic Gates
- ⟶ Basic gates
  AND, OR, NOT
- ⟶ Derived/special gates
  NAND, NOR, XOR, XNOR
  (NAND, NOR) ⟶ universal gates

AND GATE

All inputs are 1 ⟶ output is 1

$\boxed{Y = A \cdot B}$



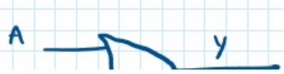
Truth Table

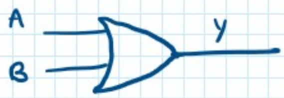
INPUTS		OUTPUT
A	B	Y
0	1	0
0	0	0
1	1	1
1	0	0

OR GATE

Any input is 1 ⟶ Output is 1

$\boxed{Y = A + B}$





Truth Table

INPUTS		OUTPUT
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT GATE

Any one input ⟶ opposite output

$$\boxed{Y = \bar{A}}$$

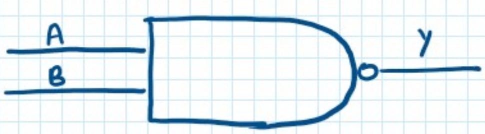
Truth Table

INPUT A	OUTPUT Y
0	1
1	0

NAND GATE

NOT (AND) = NAND.

Active low: If any input is 0 ⟶ Output is 1



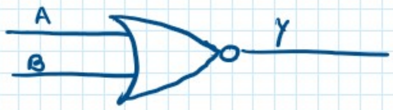
Truth Table

INPUTS		OUTPUT
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR GATE

NOT (OR) = NOR

Active high: If any input is 1 ⟶ output is 0



NOTE: Active high, active low

low, high ⟶ refers to the input the gate is "active" for

active ⟶ if the gate gets a certain input, not necessary to look at the other to get output, i.e., gate actively looks for certain input

Truth Table

INPUTS		OUTPUT
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XOR GATE

ODD function gate: If there is an odd no. of 1s in input ⟶ Output is 1

Truth Table

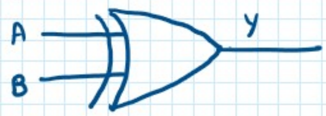
INPUTS		OUTPUT
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

MINTERMS: $\bar{A}B + A\bar{B}$ — combination of inputs in SOP that give output as 1

MAXTERMS: $(\bar{A}+\bar{B})(A+B)$ — combination of inputs in POS that give output as 0

$$\boxed{\begin{aligned} Y &= \bar{A}B + A\bar{B} \\ Y &= A \oplus B \end{aligned}}$$

Symbol



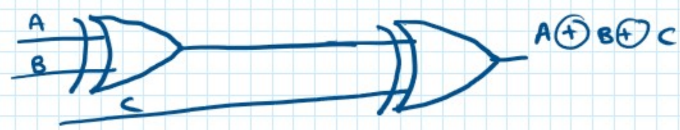
NOTE:

XOR gate performs modulo sum operation without including carry.

$0+0=0$, $1+0=1$, $0+1=1$, $1+1=0$

3- INPUT XOR GATE

INPUTS			OUTPUT
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	0
1	0	1	0
0	1	1	0
1	1	1	1



Associative law

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$ } SOP form (sum of products)

$$= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} - BC)$$

$$= \bar{A}(B \oplus C) + A(\overline{B \oplus C})$$

$$= \bar{A}X + A\bar{X}$$

$$= A \oplus X$$

$$= A \oplus B \oplus C$$

PROPERTIES OF XOR GATE

- Identity element : $A \oplus 0 = A$
- $A \oplus 1 = A'$
- $A \oplus A = 0$
- $A \oplus \bar{A} = 1$
- Commutative law : $A \oplus B = B \oplus A$

- Associative law : $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

XNOR GATE

not of XOR

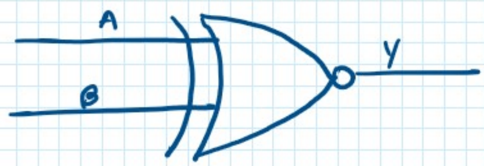
EVEN function gate: Even no. of 1s $\longrightarrow$ Output is 1

Truth Table

INPUTS		OUTPUT	MINTERMS	MAXTERMS
A	B	Y	M(A,B)	m(A,B)
0	0	1	$\bar{A}\bar{B}$	–
0	1	0	–	$A+\bar{B}$
1	0	0	–	$\bar{A}+B$
1	1	1	AB	–

$Y = A \cdot B + \bar{A} \cdot \bar{B}$

$Y = A \odot B$



THREE INPUT XNOR GATE

INPUTS			OUTPUT	MINTERMS	MAXTERMS
A	B	C	Y	M(A,B)	m(A,B)
0	0	0	1	ABC	–
1	0	0	0	–	$\bar{A}+B+C$
0	1	0	0	–	$A+\bar{B}+C$
0	0	1	0	–	$A+B+\bar{C}$
1	1	0	1	$AB\bar{C}$	–
0	1	1	1	$\bar{A}BC$	–
1	0	1	1	$A\bar{B}C$	–
1	1	1	0	–	$\bar{A}+\bar{B}+\bar{C}$

APPLICATIONS OF XOR AND XNOR

- To generate parity bits and error detection
- Equality detector
- XOR gate is used in processors' ALU (Arithmetic Logic Unit) for binary addition
- XOR gate is used to generate pseudorandom numbers