UE24MA141A → Engineering Mathematics

DATE

Partial Derivatives and Differential Equations

Ordinary Differential Eq^n: $\boxed{f(D)y = X}$

→ Homogeneous ⇒ $X = 0$ ⟶ $\boxed{y = y_c}$

→ Non-homogeneous ⇒ $X \neq 0$ ⟶ $\boxed{y = y_c + y_p}$

⇒ if Roots are: [for finding complementary func.]

1) Real & Distinct: (m_1, m_2)

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

2) Real & Repeated: $(m_1 = m_2)$

$$y_c = (c_1 + x c_2) e^{m_1 x}$$

3) Imaginary Roots: $(\alpha \pm i\beta)$

$$y_c = (c_1 \cos\beta x + c_2 \sin\beta x) e^{\alpha x}$$

4) Imaginary & Repeated:

$$y_c = \left[(c_1 + x c_2)\cos\beta x + (c_3 + x c_4)\sin\beta x\right] e^{\alpha x}$$

Types of Eq^n for 'y_p': [Particular Solution]

1) if $X = e^{ax+b}$

⇒ $y_p = \dfrac{1}{f(D)} e^{ax+b}$ $D \to a$; if $f(a) \neq 0$

→ $y_p = \dfrac{x}{f'(D)} e^{ax+b}$ [if $f(a) = 0$] $D \to a$

2) if $X = \sin(ax+b) / \cos(ax+b)$

$y_p = \dfrac{1}{f(D)} \sin(ax+b)$

$D^2 \longrightarrow -a^2$

if $f(a) \neq 0$

3) if X = a polynomial function $[2x+x^2,\ x^2+8x,\ \text{etc}]$

$$\Rightarrow y_p = \frac{1}{f(D)} X(x)$$

$\Rightarrow$ let $X = 2x + x^2$; $f(D) = 1 + 2D + D^2$

$\Rightarrow y_p = \ 1+2D+D^2 \,\big)\ 2x + x^2 \ \big(\ x^2 - 2x + 2$

$-(x^2 + 4x + 2)$

$0 - 2x - 2$

$-(-2x - 4)$

$0 + 2$

$-(+2)$

0

$$\boxed{\therefore y_p = x^2 - 2x + 2}$$

4) if $X = e^{ax} V(x)$

$$\Rightarrow y_p = \frac{1}{f(D)} e^{ax} V(x) \qquad D \longrightarrow D+a$$

$$= e^{ax} \frac{1}{f(D+a)} V(x)$$

5) if $X = x\, V(x)$

$[V(x) \rightarrow \cosh(x) / \sinh(x)]$

$$y_p = \left[\frac{x - f'(D)}{f(D)}\right] \frac{V(x)}{f(D)}$$

6) if $X = x^n \sin ax \ / \ x^n \cos ax$

$\hookrightarrow$ Im.P $(x^n e^{iax})$ $\quad \hookrightarrow$ Re.P $(x^n e^{iax})$

$$[e^{iax} = \cos ax + i \sin ax]$$

7) for 2nd order $\rightarrow f(D) \longrightarrow$ variation of parameter method.

$\Rightarrow$ if $\longrightarrow y'' + Py' + Q = X$

$\Rightarrow y_c = C_1 y_1 + C_2 y_2$

$$\boxed{\therefore y_p = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx}$$

if $W = 0 \rightarrow y_1$ & $y_2 \rightarrow$ linearly dependent $\quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$

$W \neq 0 \rightarrow y_1$ & $y_2 \rightarrow$ linearly independent.

DATE

Euler's Theorem:

if '$u = f(x, y)$' is a homogeneous function of degree of 'n'.

$\Rightarrow \quad x u_x + y u_y = n u$ ——— ①

$\Rightarrow \quad x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = n(n-1) u$ ——— ②

if '$u = f(u)$'

$\Rightarrow \quad x u_x + y u_y = n \dfrac{f(u)}{f'(u)} = g(u)$

$\Rightarrow \quad x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = g(u)\,[g'(u) - 1]$

Taylor's & Maclarin's Series Expansion:

Taylor's → about (a, b)
Maclarin's → about $(0, 0)$

Taylor's Expansion Series:

$\Rightarrow f(x, y) = f(a, b) + [(x-a) f_x(a, b) + (y-b) f_y(a, b)]$

$+ \dfrac{1}{2!}\left[(x-a)^2 f_{xx}(a, b) + 2(x-a)(y-b) f_{xy}(a, b) + (y-b)^2 f_{yy}(a, b)\right]$

$+ \ldots$

for Maclarin's Series → $(a = 0 \, ; \, b = 0)$

$\Rightarrow f(x, y) = f(0, 0) + [x f_x(0, 0) + y f_y(0, 0)]$

$+ \dfrac{1}{2!}\left[x^2 f_{xx}(0, 0) + 2xy\, f_{xy}(0, 0) + y^2 f_{yy}(0, 0)\right]$

DATE

Maxima & Minima of a (2-variable) function:

if we have a function as '$f(x, y)$'

$\Rightarrow$ $f_x = 0$ $\longrightarrow$
$f_y = 0$ $\longrightarrow$ $\rightarrow$ Gives two pais of critical points (x_1, y_1), (x_2, y_2).

$\therefore$ $f_{xx} = r$
$f_{xy} = f_{yx} = s$
$f_{yy} = t$
$\rightarrow$ Assumption [for easiness]

$\therefore$ $rt - s^2 > 0$, $r > 0$ $\longrightarrow$ f is min at (x, y)
$rt - s^2 > 0$, $r < 0$ $\longrightarrow$ f is max at (x, y)
$rt - s^2 < 0$ $\longrightarrow$ Saddle Point / Point of inflection.
$rt - s^2 = 0$ $\longrightarrow$ Don't know, fails to test & categorise the pt.

Hessian Matrix 'H':

$$\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = rt - s^2$$

PES UNIVERSITY

DATE

Linear Differential Equations with variable co-efficient:

1] Legrandre's Equation:

$$\Rightarrow a_0 (ax+b)^2 y'' + a_1 (ax+b) y' + a_2 y = \phi(x)$$

$\Rightarrow$ we take $\rightarrow$ $z = \log(ax+b)$

$\Rightarrow e^z = (ax+b)$

WKT,

$$\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx}$$

$$\rightarrow \frac{dz}{dx} = \frac{a}{ax+b}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz}\left(\frac{a}{ax+b}\right)$$

★ $\therefore (ax+b)\frac{dy}{dx} = a\frac{dy}{dz} = aDy$

★ $\Rightarrow (ax+b)^2 \frac{d^2y}{dx^2} = a^2 D(D-1)y$

★ $\Rightarrow (ax+b)^3 \frac{d^3y}{dx^3} = a^3 D(D-1)(D-2)y$

we substitute the value of '$\frac{dz}{dx}$', in higher degrees, & we get the above.

2] Cauchy's Equation:

The special case of Legrandre's Equation, $(a=1, b=0)$