PES University, Bengaluru
(Established under Karnataka Act No. 16 of 2013)

UE20MA101

APRIL 2021: END SEMESTER ASSESSMENT (ESA) B Tech 1 SEMESTER
(Chemistry Cycle)
UE20MA101 : Engineering Mathematics - I

Time: 3 Hrs	Answer All Questions	Max Marks: 100

1	a)	Prove that the series $1+\frac{1}{2}\frac{a}{b}+\frac{1.3}{2.4}\frac{a(a+1)}{b(b+1)}+\frac{1.3.5}{2.4.6}\frac{a(a+1)(a+2)}{b(b+1)(b+2)}+\cdots\infty$ is convergent if a>0, b>0 and b >a+$\frac{1}{2}$.	7 M
	b)	Examine the convergence or divergence of the series $1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\cdots+\frac{2^{n+1}-2}{2^{n+1}+1}x^n+\cdots\infty\ \ (x>0)$	7 M
	c)	Test the convergence of the series $\sum\frac{[(n+1)x]^n}{n^{n+1}}$.	6 M
2	a)	If $(\sqrt{x}+\sqrt{y})cotu-x-y=0,$ prove that $4x\frac{\partial u}{\partial x}+4y\frac{\partial u}{\partial y}+sin2u=0.$	4 M
	b)	If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right), show\ that\ x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0$.	4 M
	c)	Find the Taylor's expansion of $e^{ax}sinby$ about the origin upto third degree terms.	6 M
	d)	A tent of a given volume has a square base of side 2a, has its four-side vertical of length b and is surmounted by a regular pyramid of height h. Find the values of a and b interms of h such that the canvas required for its construction is minimum.	6 M
3	a)	Solve : $xsinx\frac{dy}{dx}+y(xcosx-sinx)=2$	7 M
	b)	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$,where λ is a parameter.	6 M
	c)	Solve: $y=2px+p^n$.	7 M
4	a)	Solve: $(D^2-4D+1)y=sin^2x+e^x+e^3x$	6 M
	b)	Solve the differential equation: $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=\frac{x^3}{1+x^2}$	7 M

	c)	A circuit consists of an inductance of 0.5 henrys, resistance of 6 ohms, capacitance of 0.02 farads and an e.m.f of voltage E=24sin10t. Find the charge and the current at time t > 0 given that the circuit carries no charge and no current at time t=0.	7 M
5	a)	Show that for $m,n>0$ $\int_0^1 x^{m-1}\left(log\frac{1}{x}\right)^{n-1} dx = \frac{\Gamma(n)}{m^n}$.	5 M
	b)	Show that for $m,n>0$, $\int_{-a}^{a}(a+x)^{m-1}(a-x)^{n-1}dx = (2a)^{m+n-1}\beta(m,n)$.	5 M
	c)	Use Jacobi series to derive the Bessel's integral formula $J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(n\theta - xsin\theta)\, d\theta$ where n is a positive integer.	6 M
	d)	Evaluate $\int J_4(x)dx$.	4 M