Self Learning Component Assignment

Unit 1 :

1. $u = xyz = m^3$

$\log u = \log x + \log y + \log z$

$\frac{\partial v}{v} = \frac{\partial x}{x} + \frac{\partial y}{y} + \frac{\partial z}{z}$

$= \frac{1}{100} + \frac{1}{100} + \frac{1}{100} = \frac{3}{100}$

$\frac{\partial U}{36} = \frac{3}{100} \Rightarrow \partial U = \frac{108}{100} = 1.08\, m^3$

Total bricks $= 1.08 \times 450$

$= 486\, m^3$

Error $= 486 \times \frac{53\cancel{0}}{100\cancel{0}} = 257.58$

2. $v = \pi d^2 h$

$\log v = \log \pi + 2\log r + \log h$

$\frac{\partial v}{v} = 2\frac{\partial d}{d} + \frac{\partial h}{h}$

$d = 5 \quad h = 8$

$\frac{\partial v}{v} = \pm \frac{2(0.1)}{5} \pm \frac{0.1}{8}$

$= \frac{0.2}{5} \pm \frac{0.1}{8}$

$\frac{\partial v}{v} = \pm 0.0525$

Unit 2 :

3. $yp^2 - 2xp + y = 0 \longrightarrow$ (1)

$$\frac{y(p^2+1)}{2p} = x$$

$$\frac{dx}{dy} = \frac{1}{2}\left(p + y.\frac{dp}{dy} + \frac{1}{p} - \frac{y}{p^2}.\frac{dp}{dy} \right)$$

$$\frac{2}{p} = p + \frac{y}{p}.\frac{dp}{dy} + \frac{1}{p} - \frac{y}{p^2}.\frac{dp}{dy}$$

$$\left(1 + y.\frac{dp}{dy}\right)\left(\frac{1}{p} - p\right) = 0$$

$$1 = -\frac{y}{p}.\frac{dp}{dy}$$

$$\int \frac{dy}{y} + \int \frac{dp}{p} = 0$$

$$\log y + \log p = \log c$$

$$y.\frac{dy}{dx} = c$$

$$\boxed{\frac{dy}{dx} = \frac{c}{y}}$$

Now, In (1),

$$yp^2 - 2xp + y = 0$$

$$y\left(\frac{c}{y}\right)^2 - 2x\left(\frac{c}{y}\right) + y = 0$$

$$\frac{c^2}{y} - \frac{2cx}{y} + y = 0$$

$$c^2 - 2cx + y^2 = 0$$

$$\boxed{y^2 = 2cx - c^2}$$

4. $xp^2 - yp - y = 0$

$$x = \frac{yp + y}{p^2} = \frac{y}{p} + \frac{y}{p^2}$$

$$\frac{dx}{dy} = \left(\frac{1}{p} - \frac{y}{p^2}\cdot\frac{dp}{dy}\right) + \left(\frac{1}{p^2} - \frac{2y}{p^3}\cdot\frac{dp}{dy}\right)$$

$$\cancel{\frac{1}{p}} = \cancel{\frac{1}{p}} - \frac{y}{p^2}\cdot\frac{dp}{dy} + \frac{1}{p^2} - \frac{2y}{p^3}\cdot\frac{dp}{dy}$$

$$\left(\frac{y}{p^2} + \frac{2y}{p^3}\right)\cdot\frac{dp}{dy} = \frac{1}{p^2}$$

$$\frac{y}{\cancel{p^2}}\left(1 + \frac{2y}{p}\right)\frac{dp}{dy} = \frac{1}{\cancel{p^2}}$$

$$y\left(1 + \frac{2y}{p}\right)\cdot\frac{dp}{dy} = 1$$

$$\int \frac{dy}{y} = \int\left(1 + \frac{2y}{p}\right)dp$$

$$\log y = p + 2\log p - \log c$$

$$\log y + \log c = p + 2\log p$$

$$\frac{cy}{p^2} = e^p$$

$$y = cp^2e^p$$

$$x = \frac{c\cancel{p^2}e^p(p+1)}{\cancel{p^2}} = c(1+p)e^p$$

$$\boxed{\begin{aligned} x &= c(1+p)e^p \\ y &= cp^2e^p \end{aligned}}$$

Unit 3 :

5. $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^4 \sin x$

$x = e^t \Rightarrow x^4 = e^{4t}$

$t = \log x$

$D = \frac{d}{dt}$

$x^2\frac{d^2y}{dx^2} = D(D-1)y$

$x\frac{dy}{dx} = Dy$

$(D(D-1) - 4D + 6)y = e^{4t}\sin e^t$

$(D^2 - D - 4D + 6)y = e^{4t}\sin e^t$

$(D^2 - 5D + 6)y = e^{4t}\sin e^t$

$m^2 - 5m + 6 = 0$

$m = 2, 3$

$CF = c_1 e^{2t} + c_2 e^{3t} = c_1 x^2 + c_2 x^3$

$W = y_1 y_2' - y_2 y_1'$

$y_1 = e^{2z} \qquad y_1' = 2e^{2z}$

$y_2 = e^{3z} \qquad y_2' = 3e^{3z}$

$PI = -y_1\int \frac{y_2 X}{W} + y_2\int \frac{y_1 X}{W}$

$= -e^{2t}\int \frac{e^{4t}\sin(e^t)}{e^{2t}}\,dt + e^{3t}\int \frac{e^{4t}\sin e^t\,dt}{e^{3t}}$

$= -e^{2t}\int e^{2t}\sin e^t\,dt + e^{3t}\int e^t \sin e^t\,dt$

$= -e^{2t}\int z\sin z\,dz + e^{3t}\int \sin z\,dz$

$= -z^2(-z\cos z + \sin z) - z^3\cos t$

$= -z^2\sin z$

$= -x^2\sin x$

6. $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = \frac{x^3}{1+x^2}$$

$$x = e^t$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x\frac{dy}{dx} = Dy$$

$$(D^2 - \cancel{D} + \cancel{D} - 1)\, y = \frac{e^{3t}}{1+e^{2t}}$$

$$(D^2 - 1)\, y = \frac{e^{3t}}{1+e^{2t}}$$

$m = \pm 1$

$$y = c_1 e^t + c_2 e^{-t} = c_1 x + \frac{c_2}{x}$$

$$\underset{y_1}{\downarrow} \qquad \underset{y_2}{\downarrow}$$

$$W = y_1 y_2' - y_2 y_1'$$
$$= e^t(-e^{-t}) - e^{-t}(e^t)$$
$$= -2$$

$$\Rightarrow PI = -e^t \int \frac{e^{-t}\cdot e^{3t}}{-2(1+e^{2t})} dt + e^{-t}\int \frac{e^t \cdot e^{3t}}{-2(1+e^{2t})} dt$$

$$= e^t \int \frac{e^{2t}}{2(1+e^{2t})} dt - \frac{e^{-t}}{2}\int \frac{e^{4t}\,dt}{1+e^{2t}}$$

$$= \frac{e^t}{4}\log(1+e^{2t}) - \frac{e^{-t}}{4}\int \frac{z}{1+z} dz$$

$$= \frac{x\log(1+x^2)}{4} - \frac{e^{-t}}{4}\int \frac{z+1-1}{1+z} dz$$

$$= \frac{x\log(1+x^2)}{4} - \left(\frac{e^{-t}}{4}\int dz - \frac{e^{-t}}{4}\int \frac{1}{1+z} dz\right)$$

$$= \frac{x\log(1+x^2)}{4} - \left[\frac{e^{-t}z}{4}\right] - \frac{e^{-t}}{4}\log(1+z)$$

$$= \frac{x\log(1+x^2)}{4} - \left[\frac{e^{-t}e^{2t}}{4}\right] - \frac{e^{-t}}{4}\log(1+e^{2t})$$

$$= \frac{x\log(1+x^2)}{4} - \left[\frac{e^{t}}{4}\right] - \frac{1}{4x}\log(1+x^2)$$

$$PI = \frac{1}{4}\log(1+x^2)\left(x - \frac{1}{x}\right) - \frac{x}{4}$$

$$y = c_1 x + \frac{c_2}{x} + \frac{1}{4}\log(1+x^2)\left(x - \frac{1}{x}\right) - \frac{x}{4}$$

7. eq of sphere $\Rightarrow$ $x^2+y^2+z^2=r^2$

Since in xy plane, $(x-a)^2+(y-b)^2+z^2=0 \longrightarrow (1)$

Diff wrt x $\Rightarrow$ $\cancel{2}(x-a)+0+\cancel{2}z\cdot\frac{\partial z}{\partial x}=0$

Diff wrt y $\Rightarrow$ $\cancel{2}(y-b)+0+\cancel{2}z\cdot\frac{\partial z}{\partial y}=0$

$zp=a-x \qquad zq=b-y$

$x=a-zp \qquad y=b-zq$

$\hookrightarrow (2) \qquad \hookrightarrow (3)$

Put (2) & (3) in (1)

$(a-zp-a)^2+(b-zq-b)^2+z^2=r^2$

$(zp)^2+(zq)^2+z^2=r^2$

$z^2(p^2+q^2+1)=r^2$

8. $z = (x+y)\,\phi(x^2-y^2)$

$$\frac{\partial z}{\partial x} = \phi(x^2-y^2) + (x+y)\cdot\phi'(x^2-y^2)\,2x = p$$

$$\frac{\partial z}{\partial y} = \phi(x^2-y^2) + (x+y)\cdot\phi'(x^2-y^2)(-2y) = q$$

$$\frac{p-\phi(x^2-y^2)}{\cancel{(x+y)}} \times \frac{1}{\cancel{2}x} = \frac{q-\phi(x^2-y^2)}{\cancel{(x+y)}} \times \frac{1}{-\cancel{2}y}$$

$$-py + y\phi(x^2-y^2) = xq - x\,\phi(x^2-y^2)$$

$$qx + py = (x+y)\,\phi(x^2-y^2)$$

$$py + qx = z$$