2. Boolean Laws, Realisation of Boolean Gates using Universal Gates

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BOOLEAN LAWS

Inversion law

If $A = 1 \longrightarrow A' = 0$

$A = 0 \longrightarrow A' = 1$

Double Inversion law

$(A')' = A$

AND law

$A \cdot 0 = 0$

$A \cdot 1 = A$

$A \cdot A = A$

$A \cdot A' = 0$

OR law

$A + 0 = A$

$A + 1 = 1$

$A + A = A$

$A + A' = 1$

Principle of duality / Duality theorem

- Each AND sign is changed to an OR sign, 0s are changed to 1s and vice versa for both.

Dual of AND gate $\Rightarrow$ OR gate

$A \cdot B \Longrightarrow A + B$

Dual of NAND gate $\Rightarrow$ NOR gate

$\overline{A \cdot B} \Longrightarrow \overline{A + B}$

Dual of XOR gate $\Rightarrow$ XNOR gate

Commutative law

$$\boxed{\begin{aligned} A + B &= B + A \\ A \cdot B &= B \cdot A \end{aligned}}$$

Associative law

$(A + B) + C = A + (B + C)$

From principle of duality:

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Distributive law

$$\boxed{A + B\cdot C = (A+B)\cdot(A+C)}$$

$$A + \underbrace{\bar{A}}_{\text{redundant}}\cdot B = (A+\bar{A})\cdot(A+B) = A+B$$

$$\bar{A} + \underbrace{A}_{\text{redundant}}\cdot\bar{B} = \bar{A}+\bar{B}$$

From principle of duality,
$A\cdot(B+C) = A\cdot B + A\cdot C$

De Morgan's Theorem

"Break the line, change the sign"

$$\overline{A+B} = \bar{A}\cdot\bar{B}$$

$$\overline{A\cdot B} = \bar{A}+\bar{B}$$

Absorption Theorem

(i) $\boxed{A + AB = A}$

$\because A + AB = A\underbrace{(1+B)}_{=1}$

$\Rightarrow A+AB = A$

(ii) $\boxed{A(A+B) = A}$

$\underbrace{A\cdot A}_{=A} + A\cdot B$

$\Rightarrow A + AB = A\underbrace{(1+B)}_{=1} = A$

Redundancy Laws

(i) $\boxed{A + \bar{A}B = A+B}$

(ii) $\boxed{A\cdot(\bar{A}+B) = AB}$

$$A + \bar{A}\cdot B = (A+\bar{A})\cdot(A+B) = A+B$$

redundant since $(A+\bar{A})$ is 1

Consensus Theorem

$$Y = AB + \underbrace{BC}_{\text{redundant}} + \bar{A}C = AB + \bar{A}C$$

$AB + A\bar{C} + BC(A+\bar{A})$

If you have an expression with no single variable :

- Check terms for any variable X and

$AB + A\bar{C} + BC(A+\bar{A})$

$= AB + A\bar{C} + ABC + \bar{A}BC$

$= AB + ABC + \bar{A}C + \bar{A}BC$

$= AB\underbrace{(1+C)} + \bar{A}C\underbrace{(1+B)}$

$= AB + \bar{A}C$

redundant since both equal one

variable.

- Check terms for any variable X and its complement $\bar{X}$.
- If such terms exist, then check their coefficients.
  Eg: XA, $\bar{X}B$
  Coefficients: A, B
- Check for terms consisting of only the coefficient variables (eg: AB).
  Such terms are taken as redundant

Example

1. $Y = \bar{A}B + \bar{B}\bar{C} + BC + A\bar{B} + AC$

$= \bar{A}B + \bar{B}\bar{C} + A\bar{B} + AC$

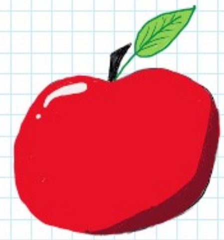
$= \bar{A}B + \bar{B}\bar{C} + AC$

2. $Y = AB + A\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$

$= AB + A\bar{C} + \bar{A}\bar{B} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$

$= AB(1+C) + \bar{A}\bar{B}(1+C) + A\bar{C}(1+\bar{B})$

$= AB + \bar{A}\bar{B} + A\bar{C}$



REMEMBER: A P P L E

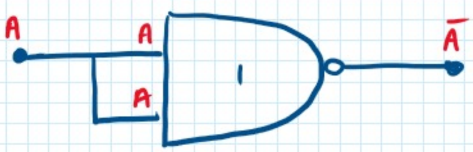
UNIVERSAL GATES

NAND | NOR

- Any digital logic circuit can be implemented using these

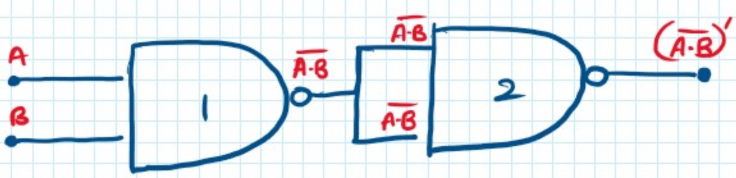
REALISATION OF LOGIC GATES USING NAND

① NOT GATE



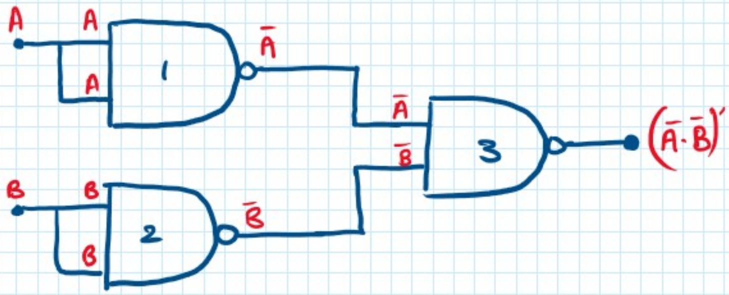
$Y = \overline{A \cdot A} = \bar{A} + \bar{A} = \bar{A}$

② AND GATE



$Y = (\overline{A \cdot B})' = AB$

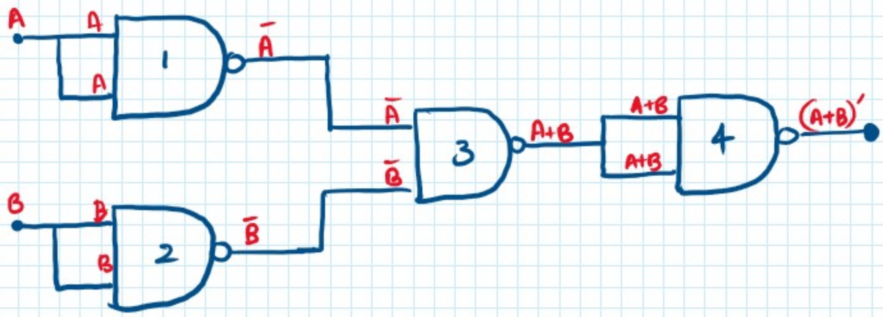
③ OR GATE



$Y = (\bar{A} \cdot \bar{B})' = (\overline{A+B})'$

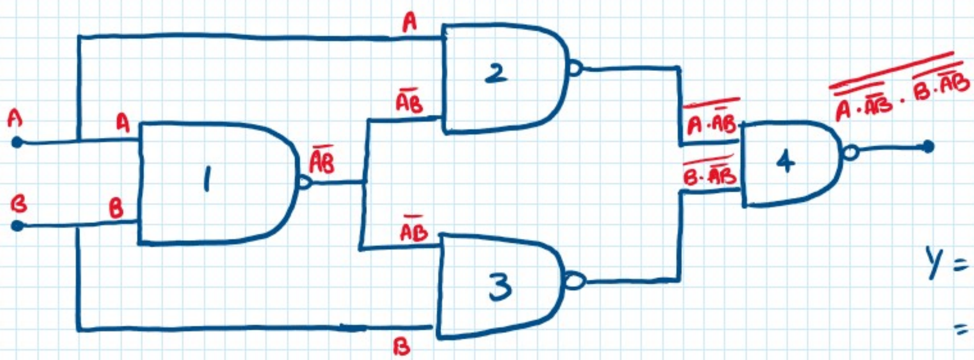
$Y = A+B$

④ NOR GATE



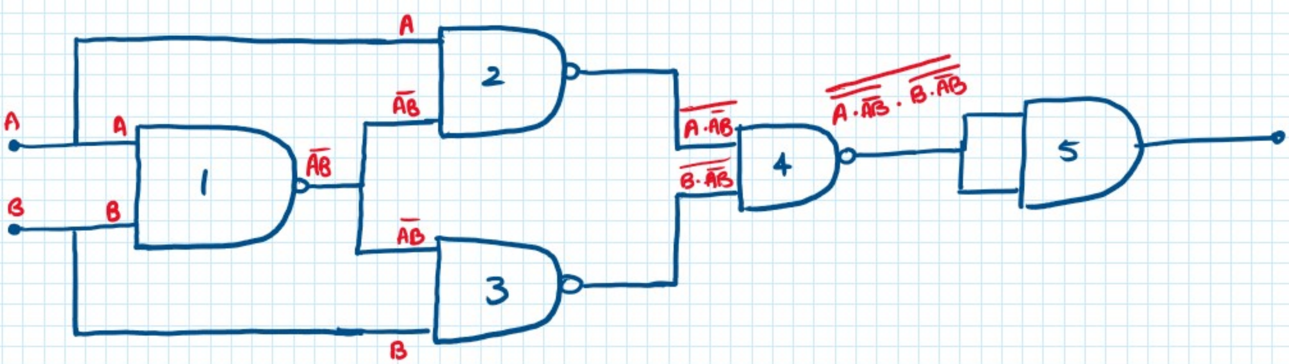
$Y = \overline{A+B}$

⑤ XOR GATE



$$
\begin{aligned}
Y &= \overline{\overline{A \cdot \overline{AB}} \cdot \overline{B \cdot \overline{AB}}} \\
&= \overline{\overline{A \cdot \overline{AB}}} + \overline{\overline{B \cdot \overline{AB}}} \\
&= A \cdot \overline{AB} + B \cdot \overline{AB} \\
&= A \cdot (\bar{A}+\bar{B}) + B(\bar{A}+\bar{B}) \\
&= A\bar{B} + \bar{A}B
\end{aligned}
$$

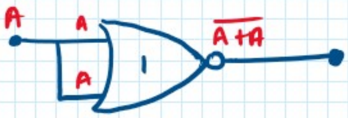
⑥ XNOR GATE



REALISATION OF LOGIC GATES USING NOR

NOT - 1
OR - 2
AND - 3
NAND - 4
XOR - 5
XNOR - 4

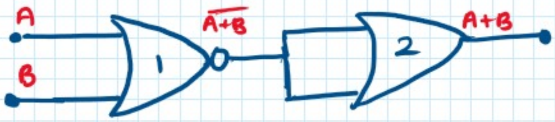
① NOT GATE



$Y = \overline{A+A} = \bar{A}\cdot\bar{A}$

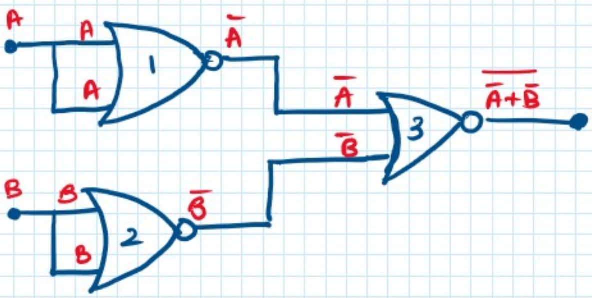
$Y = \bar{A}$

② OR GATE



$Y = A+B$

③ AND GATE

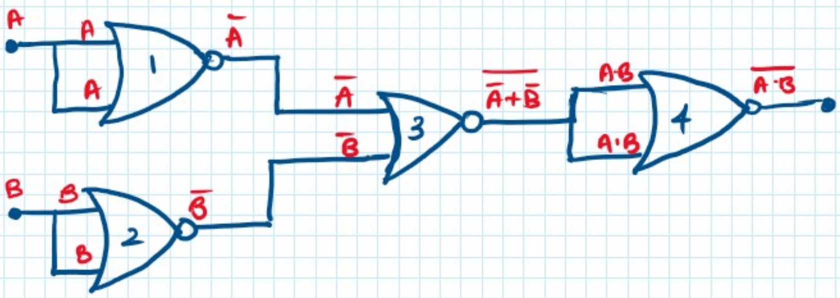


$Y = \overline{\bar{A}+\bar{B}}$

$= \overline{\overline{A\cdot B}}$

$Y = A\cdot B$

④ NAND GATE

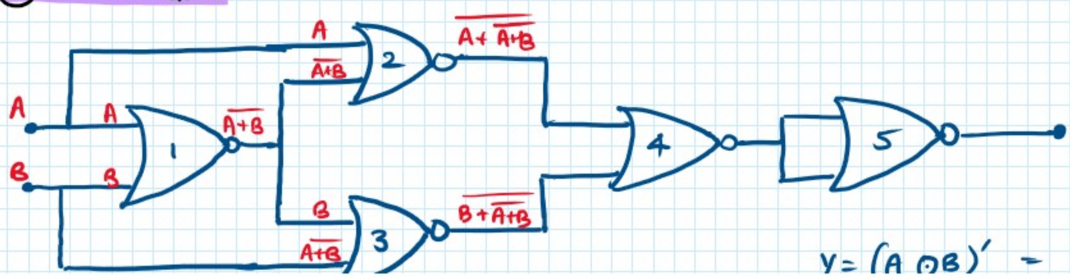


$Y = \overline{\overline{\overline{\bar{A}+\bar{B}}}}$

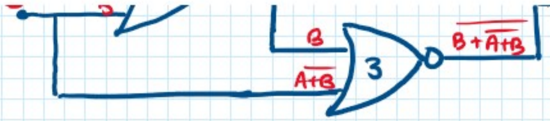
$= \overline{\overline{\overline{A\cdot B}}}$

$= \overline{A\cdot B}$

⑤ XOR GATE

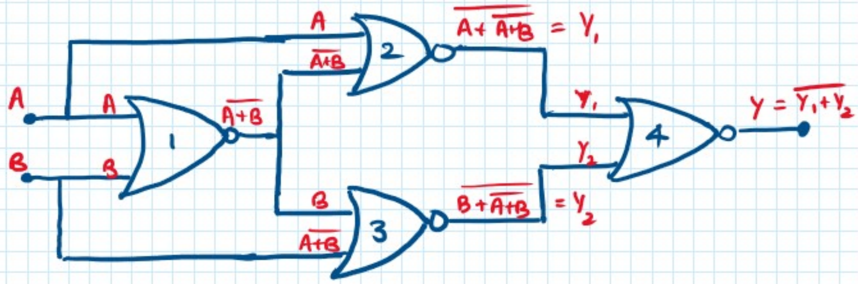


$Y = (A \odot B)' = A \oplus B$



$y = (A \odot B)' = A \oplus B$

⑥ XNOR GATE



$$Y_1 = \overline{A+\overline{A+B}} = \bar{A} \cdot \overline{\overline{A+B}} = \bar{A}(A+B) = \bar{A}B$$

$$Y_2 = \overline{B+\overline{A+B}} = \bar{B} \cdot \overline{\overline{A+B}} = \bar{B}(A+B) = A\bar{B}$$

$$Y = \overline{\bar{A}B + A\bar{B}}$$

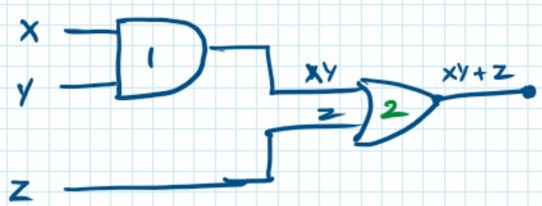
PROBLEMS

① Realise $F = XY + Z$ using only NAND gates

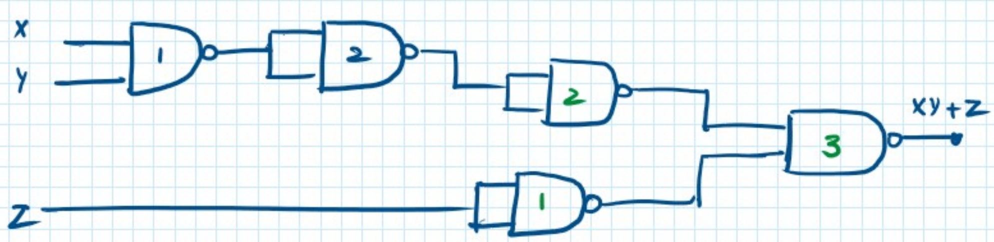
Solution:

Method ①

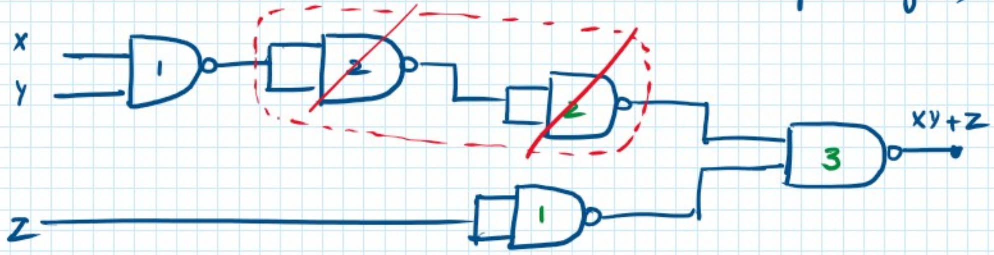
Step ①: Draw basic gates expression



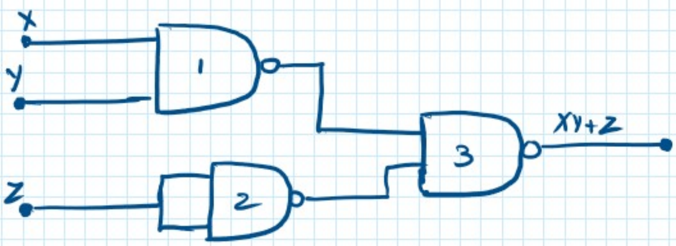
Step ②: Replace each gate by its NAND equivalent



Step ③: Remove double successive inversions (doubled up NOT gates)

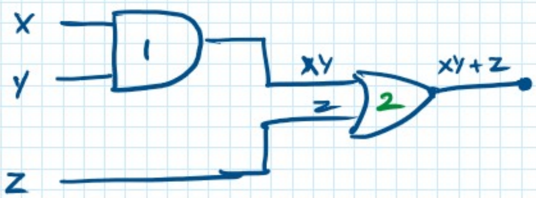


Solution:

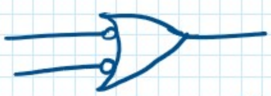


Method (2)

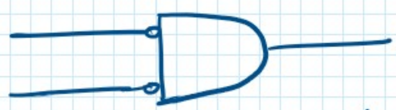
Step (1): Draw basic gate expression



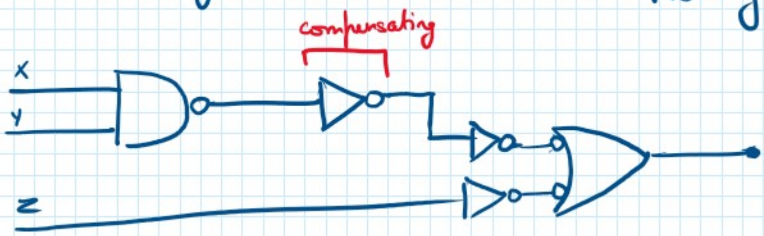
Step (2): Replace each gate symbol with NAND gate symbol



Bubbled OR gate
=
NAND gate



Bubbled AND gate
=
NOR gate



Step (3): Remove double successive inversions and ensure every diode is in terms of NAND