I	Solve the following PDEs by the method of direct integration :	
1	$\frac{\partial^2 u}{\partial x^2} = x + y$	$u = \frac{x^3}{6} + \frac{x^2 y}{2} + x f(y) + g(y)$
2	$\frac{\partial^2 u}{\partial x \partial y} = x^2 + y^2$	$u = \frac{x^3 y}{6} + \frac{x\, y^3}{3} + \int f(y)\, dy + g(x)$
3	$\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y} + c$	$z = \frac{x^2}{2} \log y + a x y + \int f(y) dy + g(x)$
4	$\frac{\partial^3 u}{\partial x^2 \partial y} = \cos(2x + 3y)$	$u = \frac{-1}{12} \sin(2x + 3y) + x f(y) + g(y) + h(x)$
5	$\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when x=0 and z=0 when y is an odd multiple of $\frac{\pi}{2}$.	$z = \cos x \cos y + \cos y = \cos y(1 + \cos x)$
6	$\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ given that u = 0 when t = 0 and $\frac{\partial u}{\partial t} = 0$ at x = 0. Also show that u → sin x as t →∞.	$u = (1 - e^{-t})\sin x$
7	. $\frac{\partial^2 u}{\partial x^2} = xy$ Subject to the condition that $\frac{\partial u}{\partial x} = \log(1 + y)$ when x = 1 and u = 0 when x = 0.	$u = \frac{x^3 y}{6} + x\left[\log(1 + y) - \frac{y}{2}\right]$
8	$\frac{\partial^2 u}{\partial x \partial y} = \frac{x}{y}$ given that $\frac{\partial u}{\partial x} = \log x$ when y = 1 and u = 0 when x = 1.	$u = \frac{1}{2} x^2 \log y + x \log x - x + 1 - \log\sqrt{y}$

II	Solve the following Linear PDEs:	
1	$\left(x^2-yz\right)p+\left(y^2-zx\right)q=z^2-xy$	$\varphi\left(\frac{x-y}{y-z},\frac{y-z}{z-x}\right)=0$
2	$(xy^3-2x^4)p+(2y^4-x^3y)q=9z(x^3-y^3)$	$f(x^3y^3z,\frac{y}{x^2}+\frac{x}{y^2})=0$
3	$(x^2-y^2-z^2)p+2xyq=2xz$	$f\left(\frac{y}{z},\frac{x^2+y^2+z^2}{z}\right)=0$
4	$x^2p+y^2q+z^2=0$	$f(\frac{1}{y}-\frac{1}{x},\frac{1}{y}+\frac{1}{z})=0$
5	$x^2p+y^2q=x+y$	$f(\frac{1}{y}-\frac{1}{x},e^{-z}(x-y))=0$
6	$y^2p-xyq=x(z-2y)$	$f(x^2+y^2,yz-y^2)=0$
7	$p-qy\log y=z\log y$	$yz=f(e^x\log y)$
8	$\frac{y^2z}{x}p+xzq=y^2$	$x^2-z^2=f(x^3-y^3)$
9	$(y^2+z^2)p-xyq+xz=0$	$x^2+y^2+z^2=f(y/z)$
10	$(x^2+2y^2)p-xyq=xz$	$f(yz,x^2y^2+y^4)=0$
11	$(z^2-y^2-2yz)p+(y+z)xq=xy-xz$	$x^2+y^2+z^2=f(y^2-2yz-z^2)$

III	Solve the following PDEs by the method of separation of variables :	
1	$\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u \quad where\ u(x,0)=6e^{-3x}$	$u=6e^{-(3x+2t)}$
2	$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=2(x+y)$	$u=ce^{x^2+y^2+k(x-y)}$
3	$x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}=0$	$u=ce^{k\left(\frac{1}{y}-\frac{1}{x}\right)}$
4	$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ given $u(0,y)=2e^{5y}$	$u=2e^{(\frac{-x}{2})+5y}$
5	$u_{xy}=u$	$u=ce^{kx+y/k}$
6	$\frac{\partial^2 z}{\partial x^2}=\frac{\partial z}{\partial y}+2z$	$z=(Ae^{kx}+Be^{-kx})e^{(k^2-2)y}$
7	$\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=0$	$u=ce^{kx}y^k$
8	$2x\frac{\partial u}{\partial x}-3y\frac{\partial u}{\partial y}=0$	$u=cx^{k/2}\ y^{k/3}$
9	$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3(x^2+y^2)u$	$u=ce^{x^3+y^3+k(x-y)}$

VI	Solve the following homogeneous linear PDE with constant coefficients:	
1	$(D_x^2 - 2D_xD_y + 5D_y^2)z = 0$	$z = \phi_1(y + (1+2i)x) + \phi_2(y + (1-2i)x)$
2	$(D_x^2 - D_xD_y - 6D_y^2)\, z = 0$	$z = f(y - 2x) + g(y + 3x)$
3	$(2D_x^2 + 5D_xD_y + 2D_y^2)\, z = 0$	$z = f(y - 2x) + g(2y - x)$
4	$(D_x^2 + 6D_xD_y + 9D_y^2)\, z = 0$	$z = f(y - 3x) + x\, g(y - 3x)$
5	$(9D_x^2 + 24D_xD_y + 16D_y^2)\, z = 0$	$z = f(3y - 4x) + x\, g(3y - 4x)$

VII	Solve the following non-homogeneous linear PDE with constant coefficients:	
1	$(4D_x^2 - 4D_xD_y + D_y^2)\, z = 16\log(x+2y)$	$z = f_1(2y+x) + xf_2(2y+x) + 2x^2\log(x+2y)$
2	$(D_x^2 + 5D_xD_y + 6D_y^2)\, z = e^{x-y}$	$z = f_1(y-2x) + f_2(y-3x) + \frac{1}{2}e^{x-y}$
3	$(D_x^2 + 3D_xD_y + 2D_y^2)\, z = x+y$	$z = f_1(y-x) + f_2(y-2x) + \frac{1}{36}(x+y)^3$

Questions on Self Learning Component:

I	**Form the PDE by eliminating the arbitrary constant(s):**	
1.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$	px + qy = 2z
2.	$z=a\log(x^2+y^2)+b$	py – qx = 0
3.	$ax^2+by^2+z^2=k$	$z^2-1=z(px+qy)$
4.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	$pz=x(rz+p^2)$
5	$z=xy+y\sqrt{x^2-a^2}\ +b$	pq = px + qy
II	**Form the PDE by eliminating the arbitrary function(s) :**	
1	$z=y^2+2f\left(\frac{1}{x}+\log y\right)$	$x^2p+yq=2y^2$
2	$z=yf(x)+xg(y)$	$sxy=px+qy-z$
3	$f(x^2+y^2+z^2,\ z^2-2xy)=0$	(p – q) z = y – x
4	$z=e^{mx}\ f(x+y)$	p – q = mz
5	$z=(x-y)\ f(x^2+y^2)$	(x – y)(py - qx)= z (x + y)
6	$z=f(x-z)+g(x+y)$	qr + (1 – p – q) s – (1 – p)t= 0.
7	$z=xy+f(x^2+y^2)$	py – qx = $y^2 - x^2$
8	$z=f\left(\sqrt{x^2+y^2}\right)$	py – qx = 0