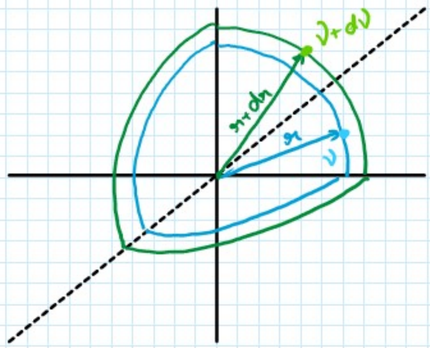
2. Modes, Rayleigh-Jean, Planck's Model, Compton Effect, De-Broglie's Theory, ????? i.e. Quantum Breaks All Common Sense

14 February 2024 10:19

No. of modes lying between ν and $\nu + d\nu$

$$= \frac{\pi}{6}\left[3r^2\, dr\right]$$



$$r^2 = n_x^2 + n_y^2 + n_z^2$$

$$\nu = \frac{c}{2a} r \qquad \frac{d\nu}{dr} = \frac{c}{2a}$$

$$\boxed{r^2 = \frac{4a^2\nu^2}{c^2}} \qquad \boxed{dr = \frac{2a}{c}\, d\nu}$$

Substituting into no. of modes eqn:

$$\frac{\pi}{\cancel{6}_{\cancel{2}}}\left[\cancel{3}\ \frac{\cancel{4}^{2} a^2\nu^2}{c^2}\ \frac{2a}{c}\, d\nu\right]$$

No. of modes $= \dfrac{4\pi a^3 \nu^2\, d\nu}{c^3}$

We're trying to find no. of modes between ν and $d\nu$ for unit volume of the cube with polarisation accounted for.

$$dN = \frac{\frac{4\pi a^3 \nu^2 d\nu}{c^3}}{a^3} \times 2$$

$$\boxed{dN = \frac{8\pi\nu^2}{c^3}\, d\nu}$$ till here everything works

Rayleigh - Jeans Expression → wrong

The avg. energy / mode of a harmonic oscillator under thermal equilibrium

$$\bar{E} = kT$$

The energy density of modes between ν & $\nu + d\nu$

$$U(\nu) = dN\, \bar{E}$$

$$\boxed{U(\nu) = \frac{8\pi\nu^2}{c^3} d\nu\, kT = \frac{8\pi kT\nu^2}{c^3} d\nu}$$

$$c = \nu\lambda \implies \nu = \frac{c}{\lambda}$$

$d\nu = -\dfrac{c}{} \, d\lambda$ ← unnecessary; we are only interested in magnitude

$c = \nu\lambda \implies \nu = \dfrac{c}{\lambda}$

$$d\nu = \underbrace{-}_{\text{unnecessary; we are only interested in magnitude}}\frac{c}{\lambda^2}\,d\lambda$$

Substituting,

$$U(\lambda) = \frac{8\pi kT}{\cancel{c^3}}\left(\frac{\cancel{c^2}}{\lambda^2}\right)\left(\frac{\cancel{c}}{\lambda^2}\right)d\lambda$$

$$\boxed{U(\lambda) = \frac{8\pi kT}{\lambda^4}\,d\lambda}$$

Failure of this theorem

$$\int_{\nu=0}^{\infty} \frac{8\pi kT\nu^2}{c^3}\,d\nu \longrightarrow \infty$$

UV catastrophe

PLANCK'S EXPRESSION → completely braindead fluke

Given some oscillator, it can only generate frequencies which are integral multiples of some frequency.

$E \propto \nu$

$E = h\nu,\ 2h\nu,\ 3h\nu \ldots$ → He did not know the value of h

Average energy → Boltzmann equation

$$\bar{E} = \frac{\sum E_n e^{-\frac{E_n}{kt}}}{\sum e^{-\frac{E_n}{kt}}}$$

Substituting E_n with $nh\nu$ → absolutely braindead assumption

$$\bar{E} = \frac{\sum nh\nu\, e^{-\frac{nh\nu}{kt}}}{\sum e^{-\frac{nh\nu}{kt}}}$$

$$= \frac{h\nu \sum_{n=0}^{\infty} nx^n}{\sum_{n=0}^{\infty} x^n} \quad \rightarrow \quad e^{-\frac{h\nu}{kt}} = x$$

$$= h\nu\frac{[1x + 2x^2 + 3x^3 + \ldots]}{[1 + x + x^2 + x^3 + \ldots]} = \frac{h\nu x[1 + 2x + 3x^2 + 4x^3 + \ldots]}{[1 + x + x^2 + x^3 + \ldots]}$$

$$\bar{E} = \frac{h\nu x\left(\frac{1}{1-x}\right)^2}{\left(\frac{1}{1-x}\right)}$$

TAYLOR SERIES

Shankar

$$\bar{E} = \frac{h\nu x}{1-x} = \frac{h\nu}{x^{-1}(1-x)} = \frac{h\nu}{x^{-1}-1}$$

$$\bar{E} = \frac{h\nu x}{1-x} = \frac{h\nu}{x^{-1}(1-x)} = \frac{h\nu}{x^{-1}-1}$$

TAYLOR SERIES

Ramamurthy Shankar
Yale University

$$\boxed{\bar{E} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}}$$

Energy density

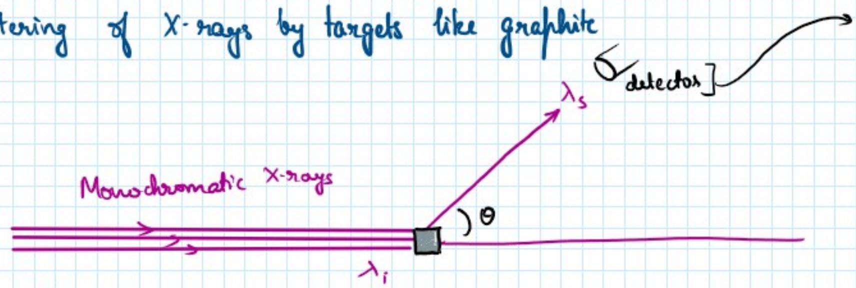
$$U(\nu) = dN\,\bar{E}$$

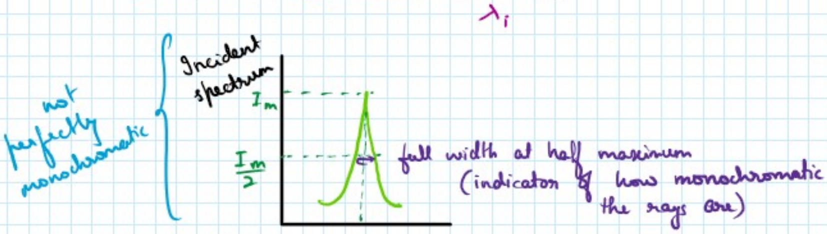
$$= \frac{8\pi\nu^2\,d\nu}{c^3}\left(\frac{h\nu}{e^{h\nu/kT}-1}\right)$$

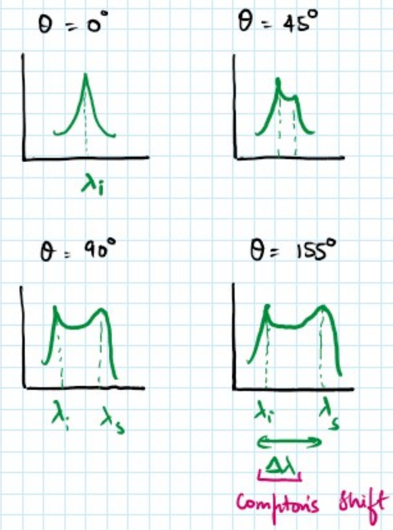
$$\boxed{U(\nu) = \frac{8\pi h\nu^3}{c^3}\,\frac{1}{e^{h\nu/kT}-1}\,d\nu}$$

COMPTON EFFECT

Scattering of X-rays by targets like graphite







Compton's Effect was not observed in all materials, but in those that it was observed in:

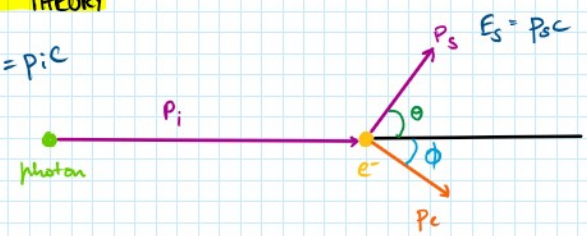
- $\Delta\lambda$ independent of λ_i
- $\Delta\lambda$ independent of target material

$$\Delta\lambda(\theta) = ?$$

Some photons change wavelength after scattering while others do not. Why?

COMPTON'S THEORY

$$E_i = \frac{hc}{\lambda_i} = p_i c$$



$$E_i = \frac{hc}{\lambda_i} = p_i c \quad\rightarrow\quad p_i = \frac{h}{\lambda_i}$$

Conservation of momentum

X comp.: $p_i + 0 = p_s \cos\theta + p_e \cos\phi \implies p_i - p_s\cos\theta = p_e \cos\phi$ —— 1

Y comp.: $0 + 0 = p_s \sin\theta - p_e \sin\phi \implies p_s \sin\theta = p_e \sin\phi$ —— 2

$$p_e^2 = p_i^2 + p_s^2\cos^2\theta - 2p_ip_s\cos\theta + p_s^2\sin^2\theta$$

$$\boxed{p_e^2 = p_i^2 + p_s^2 - 2p_ip_s\cos\theta}$$

Conservation of energy

$$p_i c + \underbrace{m_e}_{\text{mass of } e^-} c^2 = \underbrace{p_s}_{\text{moment of photons}} c + \sqrt{p_e^2c^2 + m_e^2c^4}$$

$$(p_ic + m_ec^2 - p_sc)^2 = p_e^2c^2 + m_e^2c^4$$

$$p_i^2\cancel{c^2} + \cancel{m_e^2c^4} + 2p_icm_e\cancel{c^2} + p_s^2\cancel{c^2} - 2(p_ic + m_ec^2)p_sc = p_e^2\cancel{c^2} + \cancel{m_e^2c^4}$$

$$\cancel{p_i^2} + \cancel{2}p_icme + \cancel{p_s^2} - \cancel{2}(p_i + m_ec)p_s = \cancel{p_i^2} + \cancel{p_s^2} - \cancel{2}p_ip_s\cos\theta$$

$$p_icm_e - p_ip_s - m_ecp_s = -p_ip_s\cos\theta$$

$$\frac{h}{\lambda_i}cm_e - \frac{h^2}{\lambda_i\lambda_s} - m_ec\frac{h}{\lambda_s} = \frac{-h^2}{\lambda_i\lambda_s}\cos\theta$$

$$m_ec\left[\frac{\lambda_s - \lambda_i}{\cancel{\lambda_i\lambda_s}}\right] - \frac{h}{\cancel{\lambda_i\lambda_s}} = \frac{-h}{\cancel{\lambda_i\lambda_s}}\cos\theta$$

$$m_ec\,\Delta\lambda = h - h\cos\theta$$

$$\boxed{\Delta\lambda = \frac{h(1-\cos\theta)}{m_ec}}$$ } Compton's shift; only depends on θ

Some photons change their wavelength while others do not. WHY?

Photons scattered by external e^- ⟶ wavelength changes

Photons scattered by inner e^-/nucleus ⟶ taking on entire atom ⟶ m_{atom} instead of m_e

↓

no shift; extremely small negligent value of $\Delta\lambda$.

The final scattered particles have wavelengths λ_s as well as λ_i.

DE BROGLIE'S THEORY

In Compton's effect, for a photon:

$$E = \frac{hc}{\lambda}$$

$$p = \frac{h\nu}{c} = \frac{hc}{c\lambda} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} = \frac{h}{mc}$$

$$E = mc^2 = h\nu$$

$$mc^2 = \frac{hc}{\lambda}$$

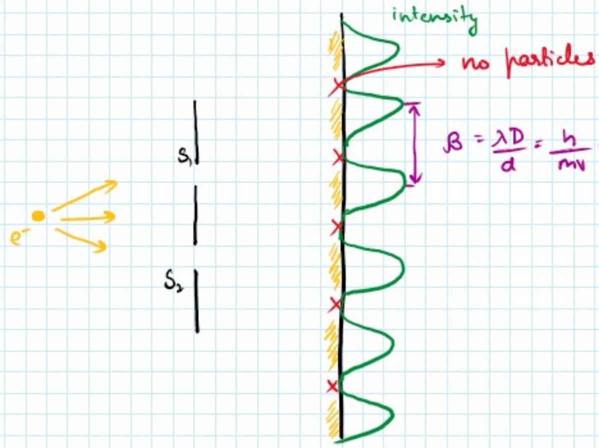
$$\lambda = \frac{h}{mc}$$ → for a photon

De Broglie said that this relation for λ is true for any mass with velocity v, i.e.,

$$\boxed{\lambda = \frac{h}{mv}}$$

Richard Feynman Vol. 3
Electron Waves

YOUNG'S DOUBLE SLIT EXPERIMENT

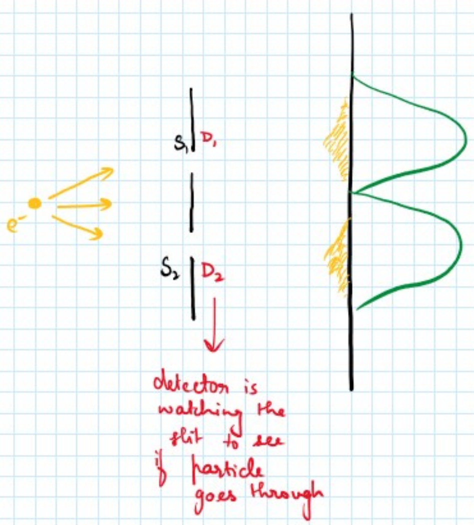


$$\beta = \frac{\lambda D}{d} = \frac{h}{mv}\frac{D}{d}$$

Particle exhibits wave-like properties and forms interference patterns even when shot one by one

???????

What wave ??



If slits are being watched by detectors, no interference pattern formed

???????

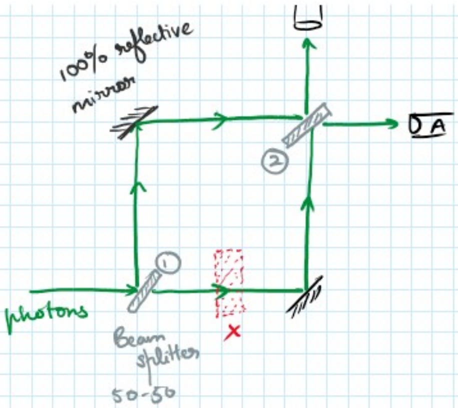
What is happening ??

God only knows

Wave function collapse

ANOTHER ???? EXPERIMENT

When ② is there, all particles go to A instead of B.



When ② is there, all particles go to A instead of B.
When ② isn't there, half go to A, half go to B

Path to B	Path to A
Reflect - Reflect - Reflect	Reflect - Reflect - Transmit
Transmit - Reflect - Transmit	Transmit - Reflect - Reflect

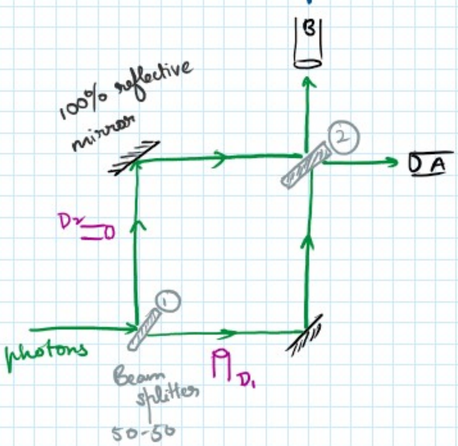
Using wave nature we can explain this, as there is destructive interference to B.

But what about sending 1 particle at a time?

All particles still go to A! There's nothing to cause interference!!

WHY???

When X is blocked, then 50% go to B and 50% go to A!
How does a particle know the path was closed???



When you put detectors, 50% go to B and 50% go to A

??????????

We assume that every particle has a ghost wave character. This is described by a mathematical function $\Psi(x,t)$ called the wave function.

We see evidence for the wave's existence, but we have not observed any wave.

Somehow, observing the particles destroys its wave function... ???

Consider some Ψ_1, Ψ_2 such that

$$\Psi = \Psi_1 + \Psi_2$$

$|\Psi|^2 \longrightarrow$ Probability density $\longrightarrow$ indicates likelihood of particle being there $\longrightarrow$ directly corresponds to intensity