NON-LINEAR DIFFERENTIAL EQNS.

First Order Higher Degree D.E.

Say

$$a_0\left(\frac{dy}{dx}\right)^n + a_1\left(\frac{dy}{dx}\right)^{n-1} + \dots + a_n = 0$$

$a_0, a_1, \dots, a_n \longrightarrow$ constants (or) functions of x and y

For simplicity, consider $\frac{dy}{dx} = p$

Case ①: Solvable for p

① Factorise the given n^{th} degree differential equation into 'n' linear factors.

② Equate the linear factors to zero to get 'n' diff. eqns. of first degree and first order.

③ Solve.

Let the general solutions be:

$$f_1(x,y,c)=0,\quad f_2(x,y,c)=0,\ \dots,\ f_n(x,y,c)=0$$

Thus GS of entire eqn.:

$$f_1(x,y,c)\times f_2(x,y,c)\times \dots \times f_3(x,y,c) = 0$$

Example: Solve -

$$x^2p^2 + xyp - 6y^2 = 10$$

Soln: $x^2p^2 + 2\frac{xy}{2}p + \frac{y^2}{4} - \frac{y^2}{4} - 6y^2 = 0$

$$\left(xp+\frac{y}{2}\right)^2 - \left(\frac{5y}{2}\right)^2 = 0$$

$$\left(xp+\frac{y}{2}+\frac{5y}{2}\right)\left(xp+\frac{y}{2}-\frac{5y}{2}\right) = 0$$

$$x\frac{dy}{dx} + 3y = 0 \qquad\qquad x\frac{dy}{dx} - 2y = 0$$

$$yx^3 - c = 0 \qquad\qquad \frac{y}{x^2} - c = 0$$

GS:

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$$\left(yx^3-c\right)\left(\frac{y}{x^2}-c\right)=0$$

Case (2) and Case (3): Solvable for y, x

Solvable for y

When?

$$y=f(x,p) \quad \text{—} \ (1)$$

$$\frac{dy}{dx}=p=\underbrace{f\left(x,p,\frac{dp}{dx}\right)}$$

considering p as a variable: first order, first degree DE in p

Solving DE, GS:

$$f_1(x,p,c)=0 \quad \text{—} \ (2)$$

Solve (1), (2) to eliminate p and get your answer.

Solvable for x

When?

$$x=f(y,p) \quad \text{—} \ (1)$$

$$\frac{dx}{dy}=\frac{1}{p}=\underbrace{f\left(y,p,\frac{dp}{dx}\right)}$$

considering p as variable
first order, first degree DE in y,p

Solve DE, GS:

$$f_2(y,p,c)=0 \quad \text{—} \ (2)$$

Solve (1), (2) to eliminate p.