

MAY 2022: END SEMESTER ASSESSMENT (ESA) B TECH I SEMESTER

UE18MA101/UE19MA101 – ENGINEERING MATHEMATICS - I

Time: 3 Hrs	Answer All Questions	Max Marks: 100
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1	a)	Find the radius of curvature for the curve: $x = e^t + e^{-t}, y = e^t - e^{-t}$ at $t = 0$	6
	b)	Find the radius of curvature for the curve: $r = e^{2\theta}$ at $\theta = \log 2$	7
	c)	Prove that: $\sin x = 1 - \left(x - \frac{\pi}{2}\right)^2 \frac{1}{2!} + \left(x - \frac{\pi}{2}\right)^4 \frac{1}{4!} - \left(x - \frac{\pi}{2}\right)^6 \frac{1}{6!} + \dots$	7
2	a)	If $u = e^{x^2+y^2+z^2}$, then find $\frac{\partial^3 u}{\partial x \partial y \partial z}$.	6
	b)	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$	7
	c)	Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.	7
3	a)	Evaluate $\int \int r^2 \sin \theta \, dr \, d\theta$ over the cardioid $r = a(1 + \cos \theta)$ above the initial line.	6
	b)	Find the moment of inertia of the area bounded by the curve $r^2 = a^2 \cos 2\theta$ about its axis.	7
	c)	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 \frac{1}{\sqrt{x^2+y^2+z^2}} \, dz \, dy \, dx$ using spherical coordinates.	7
4	a)	Solve: $\frac{dy}{dx} + y = y^2(\sin x - \cos x)$.	6
	b)	Show that family of curves $x^2 + 4y^2 = c_1$ and $y = c_2 x^4$ are orthogonal to each other.	7
	c)	Solve: $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$, where $p = \frac{dy}{dx}$.	7
5	a)	Solve: $(D^2 + 4)y = \sin 3x + \cos 2x$, where $D = \frac{d}{dx}$.	6
	b)	Solve: $(2x + 5)^2 \frac{d^2 y}{dx^2} - 6(2x + 5) \frac{dy}{dx} + 8y = 6x$.	7
	c)	Solve: $(D^2 + 2D + 1)y = e^{-x} \log x$ by using the method of variation of parameters where $D = \frac{d}{dx}$.	7

