Total Differential, Composite Functions

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TOTAL DIFFERENTIAL

The total or exact differential of a function $F(x, y, z)$ is given by

$$dF = \frac{\partial F}{\partial x} \cdot dx + \frac{\partial F}{\partial y} \cdot dy + \frac{\partial F}{\partial z} \cdot dz$$

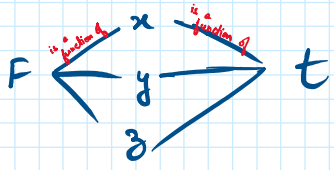
COMPOSITE FUNCTIONS

We need to use chain rule to differentiate.

F is a function of x, y, z and if $x = x(t)$, $y = y(t)$, $z = z(t)$, then derivative of F w.r.t. t is called the total derivative of F, and is denoted as $\frac{dF}{dt}$.

Type (1)

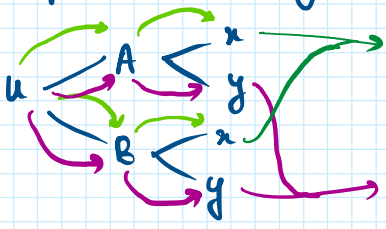
$$\frac{dF}{dt} = \frac{\partial F}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial F}{\partial z} \cdot \frac{dz}{dt}$$



This can be extended to any no. of variables.

Type (2)

If u is a function of A, B and A, B are functions of x, y, then u is a composite fn. of x, y through A and B.

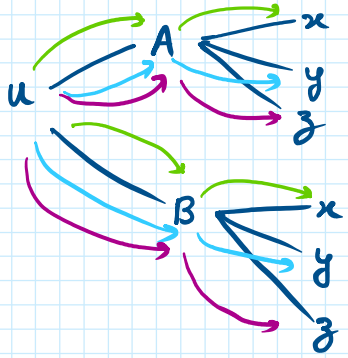


$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial A} \cdot \frac{\partial A}{\partial x} + \frac{\partial u}{\partial B} \cdot \frac{\partial B}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial A} \cdot \frac{\partial A}{\partial y} + \frac{\partial u}{\partial B} \cdot \frac{\partial B}{\partial y}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

Type ③ : 3 variables



$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial A}\cdot\frac{\partial A}{\partial x} + \frac{\partial u}{\partial B}\cdot\frac{\partial B}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial A}\cdot\frac{\partial A}{\partial y} + \frac{\partial u}{\partial B}\cdot\frac{\partial B}{\partial y}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial A}\cdot\frac{\partial A}{\partial z} + \frac{\partial u}{\partial B}\cdot\frac{\partial B}{\partial z}$$

$$\boxed{\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt}}$$

where u is a function of t.