

## Total Differential, Composite Functions

11 September 2023 09:11

### TOTAL DIFFERENTIAL

The total or exact differential of a function  $F(x, y, z)$  is given by

$$dF = \frac{\partial F}{\partial x} \cdot dx + \frac{\partial F}{\partial y} \cdot dy + \frac{\partial F}{\partial z} \cdot dz$$

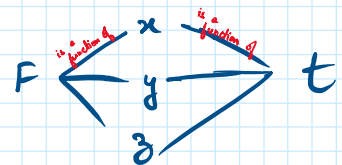
### COMPOSITE FUNCTIONS

We need to use chain rule to differentiate.

$F$  is a function of  $x, y, z$  and if  $x = x(t)$ ,  $y = y(t)$ ,  $z = z(t)$ , then derivative of  $F$  w.r.t.  $t$  is called the total derivative of  $F$ , and is denoted as  $\frac{dF}{dt}$ .

Type ①

$$\frac{dF}{dt} = \frac{\partial F}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial F}{\partial z} \cdot \frac{dz}{dt}$$



This can be extended to any no. of variables.

Type ②

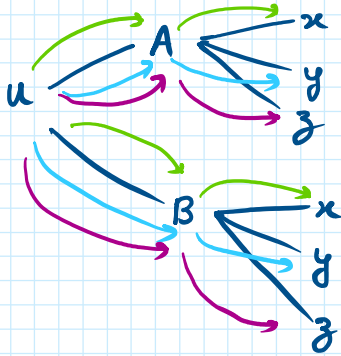
If  $u$  is a function of  $A, B$  and  $A, B$  are functions of  $x, y$ , then  $u$  is a composite fn. of  $x, y$  through  $A$  and  $B$ .

A diagram showing a central node 'u' with two arrows pointing to it from nodes 'A' and 'B'. To the right of 'A' and 'B' are nodes 'x' and 'y'. Arrows point from 'x' to 'A' and 'x' to 'B'. Arrows point from 'y' to 'A' and 'y' to 'B'.

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial A} \cdot \frac{\partial A}{\partial x} + \frac{\partial u}{\partial B} \cdot \frac{\partial B}{\partial x}$$
$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial A} \cdot \frac{\partial A}{\partial y} + \frac{\partial u}{\partial B} \cdot \frac{\partial B}{\partial y}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

Type ③: 3 variables



$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial A} \cdot \frac{\partial A}{\partial x} + \frac{\partial u}{\partial B} \cdot \frac{\partial B}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial A} \cdot \frac{\partial A}{\partial y} + \frac{\partial u}{\partial B} \cdot \frac{\partial B}{\partial y}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial A} \cdot \frac{\partial A}{\partial z} + \frac{\partial u}{\partial B} \cdot \frac{\partial B}{\partial z}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$$

where  $u$  is a function of  $t$ .