1. Non - Homogeneous LDE

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NON- HOMOGENEOUS LINEAR PDE

A partial differential equation of the type:

$$a_0 \frac{\partial^2 z}{\partial x^2} + a_1 \partial$$

in which the order of derivatives is not the same across all terms is called a non-homogeneous PDE.

The method to find PI here is the same as in homogeneous linear PDE.

METHOD OF FINDING COMPLEMENTARY FUNCTION

Consider:

$$a_0 \frac{\partial z^2}{\partial x^2} + a_1 \frac{\partial z}{\partial x} + a_2 \frac{\partial z}{\partial y} + a_3 \frac{\partial^2 z}{\partial x \partial y} + a_4 \frac{\partial^2 z}{\partial y^2} + a_5 z = \phi(x,y)$$

AE is $F(D_x, D_y \mid z = \phi(x,y))$

$$AE: a_0 D_x^2 + a_1 D_x + a_2 D_y + a_3 D_x D_y + a_4 D_y^2 + a_5 = 0$$

Factorise AE into "n" linear factors where "n" is the highest power of derivative in AE.

Factorisation gives factors of the type:

$$(D_x - m_1 D_y - a_1)(D_x - m_2 D_y - a_2) = 0$$

From here,

$$\boxed{CF = e^{a_1 x} . F_1(y + m_1 x) + e^{a_2 x} . F_2(y + m_2 x)}$$

Say linear factors are repeated.

$\Rightarrow AE = (D_x - mD_y - a)(D_x - mD_y - a) = 0$

Then,

$$\boxed{CF = e^{ax} . F_1(y + mx) + x . e^{ax} . F_2(y + mx)}$$