5. Complementary Function, Auxiliary Equation

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AUXILIARY EQN → Replace D^n by m^n, y by 1

COMPLEMENTARY FUNCTION FOR SECOND ORDER DE

CASE 1 - Roots are real, distinct → m_1, m_2

$$GS = y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$y = c_1 e^{m_1 x} + \ldots + c_n e^{m_n x}$$

CASE 2 - Roots are real and equal → $m_1 = m_2 = m$

$$GS = y = (c_1 + c_2 x) e^{mx}$$

$$y = (c_1 + c_2 \cdot x + \ldots + c_n x^{n-1}) e^{mx}$$

$$y = (c_1 + c_2 x + c_3 x^2 + c_4 x^3) e^{mx} + c_5 e^{m_5 x} + c_6 e^{m_6 x} + \ldots + c_n e^{m_n x}$$

CASE 3 - Roots are complex

$m_1 = \alpha + i\beta$ } conjugate pairs
$m_2 = \alpha - i\beta$ }

$$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$$

Say there are two pairs of imaginary roots $\alpha \pm i\beta$, $\gamma \pm i\delta$ and rest are real, distinct

$$y = (c_1 \cos \beta x + c_2 \sin \beta x) e^{\alpha x} + (c_3 \cos \delta x + c_4 \sin \delta x) e^{\gamma x} + c_5 e^{m_5 x} + \ldots + c_n e^{m_n x}$$

Say a pair of complex roots is repeated.

$$y = ((c_1 + c_2 x) \cos \beta x + (c_3 + c_4 x) \sin \beta x) e^{\alpha x} + c_5 e^{m_5 x} + \ldots + c_n e^{m_n x}$$