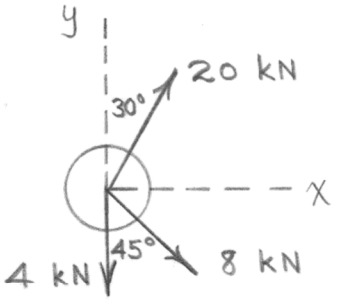
2/79



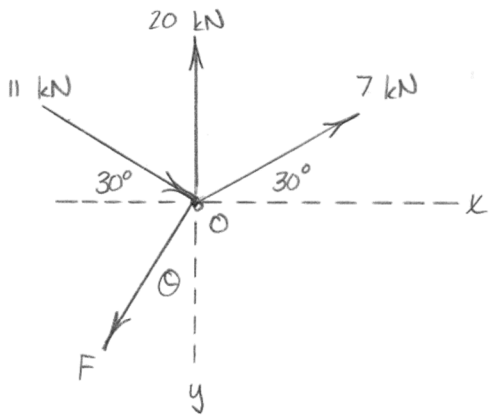
$R_x = \Sigma F_x = 20 \sin 30° + 8 \sin 45° = 15.66 \text{ kN}$

$R_y = \Sigma F_y = 20 \cos 30° - 8 \cos 45° - 4 = 7.66 \text{ kN}$

$R = \sqrt{R_x^2 + R_y^2} = \underline{17.43 \text{ kN}}$

$\theta_x = \tan^{-1}\left(R_y / R_x\right) = \underline{26.1°}$

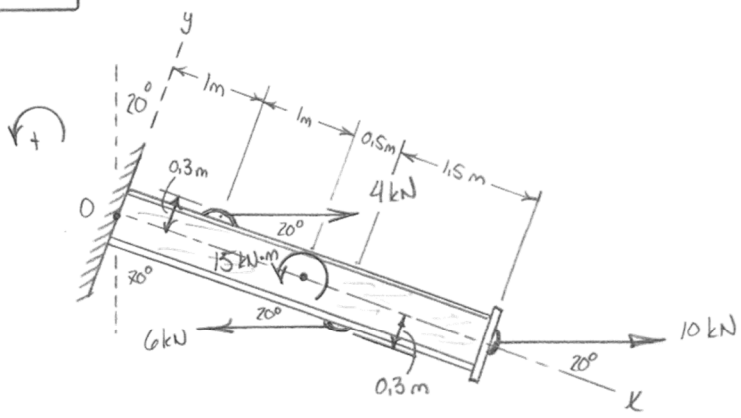
2/80 $R = 9$ kN, RIGHTWARD



$$\begin{cases} R_x = 9 = 11\cos 30^\circ + 7\cos 30^\circ - F\sin\theta \\ R_y = 0 = 11\sin 30^\circ - 7\sin 30^\circ + F\cos\theta - 20 \end{cases}$$

SOLVING ... $F = 19.17$ kN AND $\theta = 20.1^\circ$

2/81



$$\begin{cases} R = 10+4-6 \longrightarrow R = 8\ \text{kN} \\ \underline{R} = 8\cos 20^\circ \underline{i} + 8\sin 20^\circ \underline{j} \longrightarrow \underline{R} = 7.52\,\underline{i} + 2.74\,\underline{j}\ \text{kN} \end{cases}$$

$$M_O = 15 + 4\sin 20^\circ(1) - 6\sin 20^\circ(2) + 10\sin 20^\circ(4) - 4\cos 20^\circ(0.3) - 6\cos 20^\circ(0.3)$$

$$\therefore M_O = 22.1\ \text{kN·m CCW}$$

· LINE-OF-ACTION:

$$\underline{r} \times \underline{R} = \underline{M}_O \longrightarrow (x\,\underline{i} + y\,\underline{j}) \times (7.52\,\underline{i} + 2.74\,\underline{j}) = 22.1\,\underline{k}$$

$$\underline{k}: \quad 2.74x - 7.52y = 22.1$$

$$\therefore \quad y = 0.364x - 2.94\ (\text{m})$$

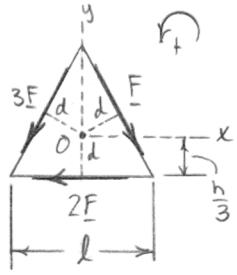
2/82 (a) $\underline{R} = -2F\underline{j}$, $\underline{M}_O = \underline{0}$

(b) $\underline{R} = \underline{0}$, $\underline{M}_O = Fd\underline{k}$ $(+\underline{k}$ is out$)$

(c) $\underline{R} = -F\underline{i} + F\underline{j}$, $\underline{M}_O = \underline{0}$

2/83 In each case, $h = \ell \sin 60 = \frac{\ell\sqrt{3}}{2}$ so.. $\frac{h}{3} = \frac{\ell\sqrt{3}}{6} = d$

a)



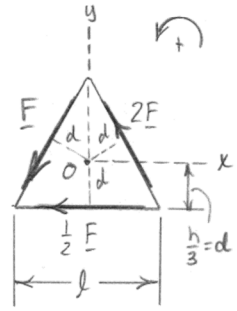
$$\underline{R} = (F\cos 60^\circ - 2F - 3F\cos 60^\circ)\underline{i} + (-F\sin 60^\circ - 3F\sin 60^\circ)\underline{j}$$

$$\underline{R} = -3F\underline{i} - 2\sqrt{3}F\underline{j}$$

$$M_O = 3Fd - Fd - 2Fd \longrightarrow \underline{M_O = 0}$$

$\underline{R}$ acts at $y = 0$.

b)



$$R = (-2F\cos 60^\circ - F\cos 60^\circ - \tfrac{1}{2}F)\underline{i} + (2F\sin 60^\circ - F\sin 60^\circ)\underline{j}$$

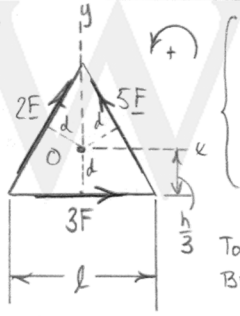
$$\underline{R} = -2F\underline{i} + \frac{\sqrt{3}}{2}F\underline{j}$$

$$M_O = 2Fd + Fd - \tfrac{1}{2}Fd \longrightarrow \underline{M_O = \frac{5\sqrt{3}}{12}F\ell \text{ CCW}}$$

To produce a CCW moment at O with negative R_x, R is placed above O.

$$R_x y = M_O \longrightarrow 2Fy = \frac{5\sqrt{3}}{12}F\ell \longrightarrow \underline{y = \frac{5\sqrt{3}}{24}\ell \text{ above O}}$$

c)



$$R = (3F - 5F\cos 60^\circ + 2F\cos 60^\circ)\underline{i} + (5F\sin 60^\circ + 2F\sin 60^\circ)\underline{j}$$

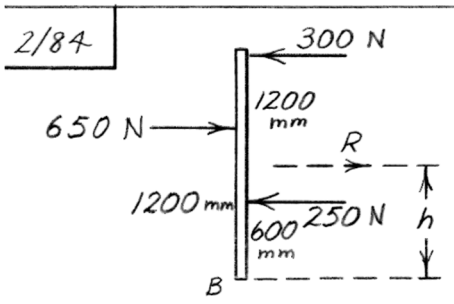
$$\underline{R} = \tfrac{3}{2}F\underline{i} + \frac{7\sqrt{3}}{2}F\underline{j}$$

$$M_O = 3Fd + 5Fd - 2Fd \longrightarrow \underline{M_O = \sqrt{3}F\ell \text{ CCW}}$$

To produce a CCW moment at O with positive R_x, R is placed below O.

$$R_x|y| = M_O \longrightarrow \tfrac{3}{2}F|y| = \sqrt{3}F\ell \longrightarrow \underline{|y| = \frac{2}{\sqrt{3}}\ell \text{ below O}}$$

2/84



$R = \Sigma F = 650 - 250 - 300$

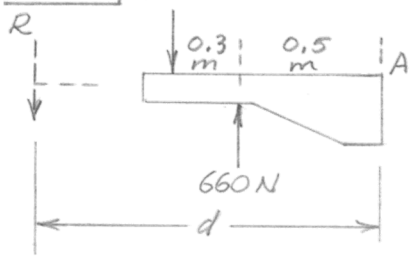
$= 100 \text{ N}$

$Rh = \Sigma M_B:$

$100h = 650(1.2) - 300(1.8) - 250(0.6)$

$h = 0.9 \text{ m}$

2/85



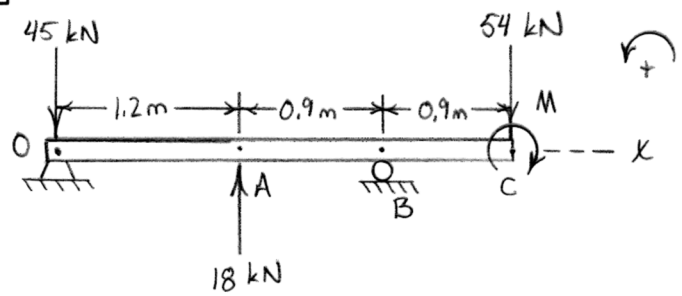
$R = \Sigma F = 680 - 660 = 20\ N$

$Rd = \Sigma M_A$

$20d = 680(0.8) - 660(0.5)$

$d = 10.70\ m$ to the left of A

2/86



$R = 81$ kN DOWN

$\Sigma M_B = 0$: $45(2.1) - 18(0.9) - 0.9(54) - M = 0$

so... $M = 29.7$ kN·m CW

$M_O = 18(1.2) - 54(3) - 29.7 = -170.1$ so... $M_O = 170.1$ kN·m CW

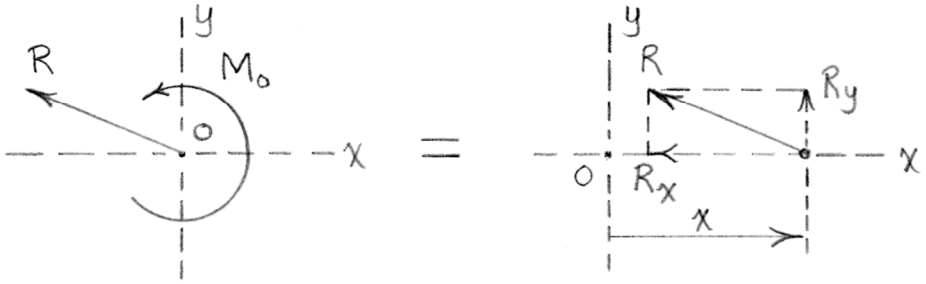
2/87 $M_O = 0$, so

$$\curvearrowleft_+ M - 400(0.150 \cos 30^\circ) - 320(0.300) = 0$$

$$M = 148.0 \text{ N·m}$$

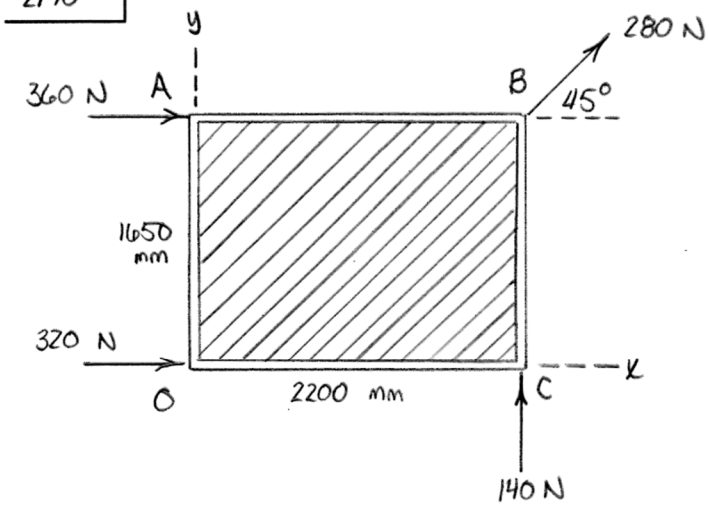
2/89 $\underline{R} = -200\underline{i} + 80\underline{j}$ N

$\curvearrowleft + M_O = -160(0.25) + 240(0.50) + 200(0.25) = 130$ N·m



$R_y x = M_O$, $x = \frac{130}{80} = \underline{1.625 \text{ m (off pipe)}}$

2/90



$\underline{R} = (360 + 320 + 280\cos 45°)\,\underline{i} + (140 + 280\sin 45°)\,\underline{j}$

$\underline{R} = 878\,\underline{i} + 338\,\underline{j}$ N

$M_O = 2.2(140 + 280\sin 45°) - 1.650(360 + 280\cos 45°) = -177.1$ N·m

$M_O = 177.1$ N·m CW

For CW moment about O, positive R_x is placed above O.

$R_x y = M_O \rightarrow 878y = 177.1 \rightarrow$ $y = 0.202$ m or 202 mm above O

For CW moment about O, positive R_y is placed left of O.

$R_y x = M_O \rightarrow 338x = 177.1 \rightarrow$ $x = 0.524$ m or 524 mm left of O

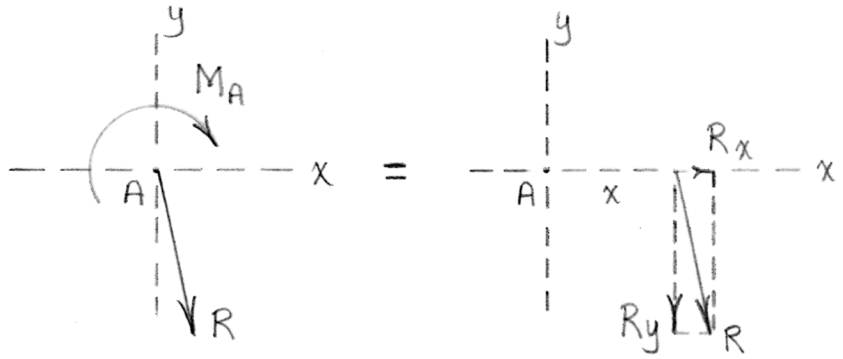
2/91 Equivalent force-couple system at A:

$$\underline{R} = -10\underline{j} - 4.8\underline{j} + 3.2(\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j})$$

$$= \underline{1.6\underline{i} - 12.03\underline{j}\ \text{kN}}$$

$$+\circlearrowright M_A = 10(1.2) + 4.8(1.2 + 1.2\cos 30^\circ + 0.9) - 3.2\sin 30^\circ(0.6\sin 30^\circ) - 3.2\cos 30^\circ(1.2 + 0.6\cos 30^\circ)$$

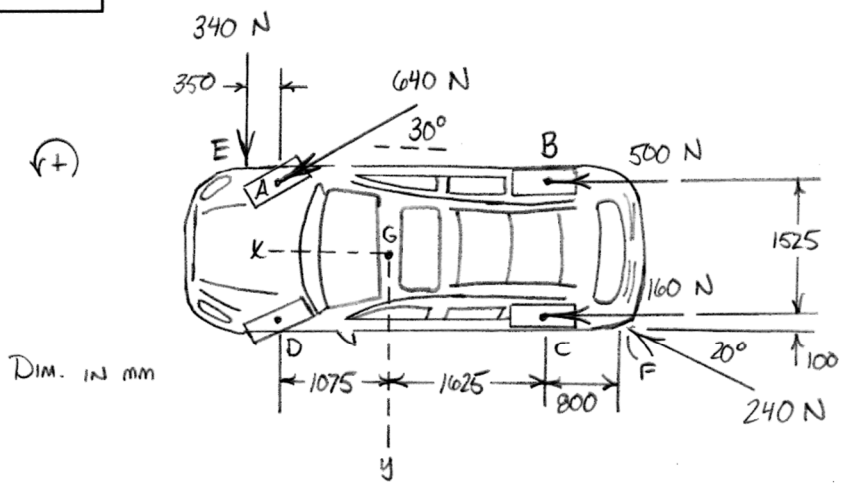
$$= \underline{21.8\ \text{kN}\cdot\text{m}\ \text{CW}}$$



Condition: $x|R_y| = M_A$

$$x = \frac{21.8}{12.03} = \underline{1.814\ \text{m}}$$

2/92



$$\underline{R} = (500 + 640\cos 30^\circ + 240\cos 20^\circ + 160)\underline{i} + (640\sin 30^\circ + 340 - 240\sin 20^\circ)\underline{j}$$

$$\underline{R} = 1440\underline{i} + 578\underline{j}\ \text{N}$$

$$\Sigma M_G = \frac{1.525}{2}(500 - 160 + 640\cos 30^\circ) + 340\left(\frac{1075+350}{1000}\right) + \frac{1075}{1000}(640\sin 30^\circ)$$
$$+ \frac{1625+800}{1000}(240\sin 20^\circ) - \left(\frac{1525/2}{1000} + \frac{100}{1000}\right)(240\cos 20^\circ) = 1515\ \text{N·m CCW}$$

FOR CCW M_G WITH POSITIVE R_x, R_x IS IN NEGATIVE y, ABOVE G.

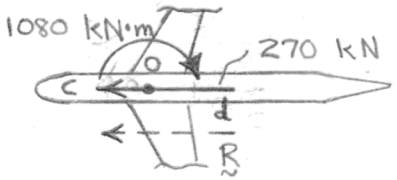
$R_x|y| = M_G \rightarrow 1440|y| = 1515 \rightarrow |y| = 1.052\ \text{m}$ SO... $\underline{(0, -1.052)\ \text{m}}$

FOR CCW M_G WITH POSITIVE R_y, R_y IS IN POSITIVE x, LEFT OF G.

$R_y x = M_G \rightarrow 578x = 1515 \rightarrow x = 2.62\ \text{m}$ SO... $\underline{(2.62, 0)\ \text{m}}$

2/93 Force – Couple system at point O:

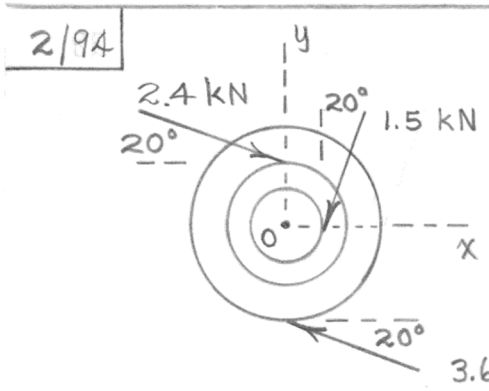
$$\begin{cases} R = 3(90) = 270 \text{ kN } (\leftarrow) \\ +\circlearrowleft M_O = 12(90) = 1080 \text{ kN·m} \end{cases}$$



$$d = \frac{M_O}{R} = \frac{1080}{270}$$

$$= 4 \text{ m}$$

2/94



$r_1 = 0.120$ m
$r_2 = 0.200$ m
$r_3 = 0.300$ m

$$\underline{R} = \Sigma \underline{F} = 2.4(c20°\underline{i} - s20°\underline{j}) + 1.5(-s20°\underline{i} - c20°\underline{j}) + 3.6(-c20°\underline{i} + s20°\underline{j}) = -1.641\underline{i} - 0.999\underline{j} \text{ kN}$$

$$\Sigma M_O = (2.4(0.2) + 1.5(0.12) + 3.6(0.3))\cos 20° = 1.635 \text{ kN·m}$$

$$\underline{r} \times \underline{R} = \underline{M}_O: \quad (x\underline{i} + y\underline{j}) \times (-1.641\underline{i} - 0.999\underline{j}) = -1.635$$

$$\Rightarrow -0.999x + 1.641y = -1.635$$

Axis intercepts: $x = 1.637$ m, $y = -0.997$ m

2/96 Equivalent force - couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (-25 + 20 \sin 30^\circ)\,\underline{i} + (-30 - 20 \cos 30^\circ)\,\underline{j} = \underline{-15\,\underline{i} - 47.3\,\underline{j}\ \text{kN}}$$

$$\circlearrowleft + M_O = 25(5) - 30(9) - (20 \cos 30^\circ)9 - (20 \sin 30^\circ)5 = -351\ \text{kN}\cdot\text{m}$$

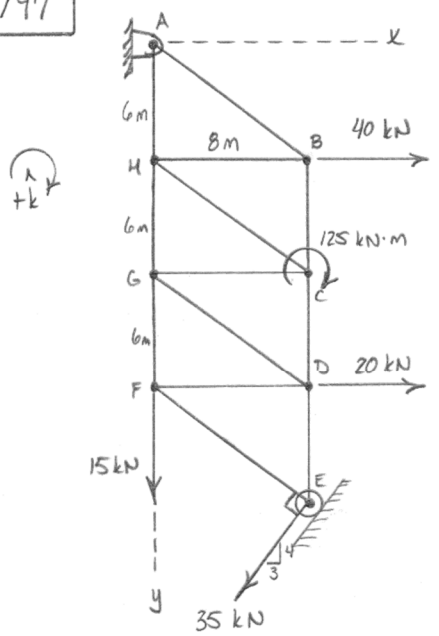
For final location of $\underline{R}$:

$$\underline{r} \times \underline{R} = \underline{M}_O, \quad (x\,\underline{i} + y\,\underline{j}) \times (-15\,\underline{i} - 47.3\,\underline{j}) = -351\,\underline{k}$$

$$-47.3x + 15y = -351$$

Axis intersections: $\underline{x = 7.42\ \text{m},\ y = -23.4\ \text{m}}$

2/97



$\underline{R} = \left(40 + 20 - \frac{3}{5}35\right)\underline{i} + \left(15 + \frac{4}{5}35\right)\underline{j}$

$\underline{R} = 39\underline{i} + 43\underline{j}$ kN

$\underline{M}_A = \left[-40(6) - 20(18) + 125 + 35(10) + \frac{3}{5}35(18)\right]\underline{k} \longrightarrow \underline{M}_A = 253\,\underline{k}$ kN·m

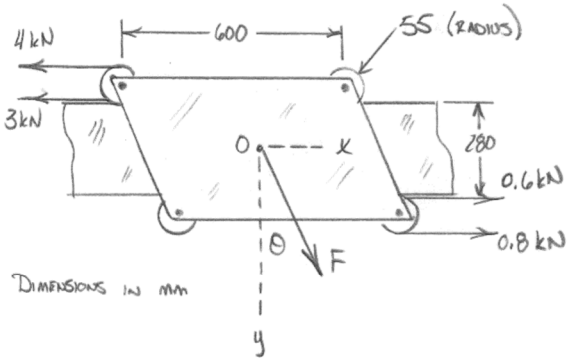
$\underline{r} \times \underline{R} = \underline{M}_A \longrightarrow (x\underline{i} + y\underline{j}) \times (39\underline{i} - 43\underline{j}) = 253\underline{k}$

$\underline{k}$: $43x - 39y = 253 \longrightarrow y = 1.103x - 6.49$ (m)

x-Axis: $y = 0 = 1.103x - 6.49 \longrightarrow x = 5.88$ m so $(5.88, 0)$ m

y-Axis: $x = 0 \longrightarrow y = -6.49$ m so $(0, -6.49)$ m

2/98 $F = 5\ \text{kN}, \quad \Theta = 30^\circ$



$$\underline{R} = (0.8 + 0.6 + 5\sin 30^\circ - 4 - 3)\,\underline{i} + 5\cos 30^\circ\,\underline{j} \longrightarrow \underline{R} = -3.10\,\underline{i} + 4.33\,\underline{j}\ \text{kN}$$

$$\Sigma M_O = 0.6\left(\frac{140}{1000}\right) + 0.8\left(\frac{140+110}{1000}\right) + 3\left(\frac{140}{1000}\right) + 4\left(\frac{140+110}{1000}\right) =$$

$$\therefore \Sigma M_O = 1.704\ \text{kN·m CCW}$$

FOR A CCW M_O WITH NEGATIVE R_x, R IS PLACED ABOVE O IN MINUS y.

$$R_x\,y = M_O \longrightarrow 3.10|y| = 1.704 \longrightarrow |y| = 0.550$$

$$\therefore \underline{|y| = 550\ \text{mm ABOVE O OR } (0, -550)\ (\text{mm})}$$

2/100 For a zero force-couple system at point O: (y, x)

$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30^\circ + F_D \sin 30^\circ)\,\underline{i} + (50 - 10 - 100 - 50 + F_B + F_C \cos 30^\circ + F_D \cos 30^\circ)\,\underline{j} = \underline{0}$$

$$\Rightarrow F_C = F_D = F$$

$$\circlearrowleft M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F \cos 30^\circ (2.9) = 0$$

$$F = F_C = F_D = 6.42 \text{ N}, \quad F_B = 98.9 \text{ N}$$