SRN

PES University, Bangalore

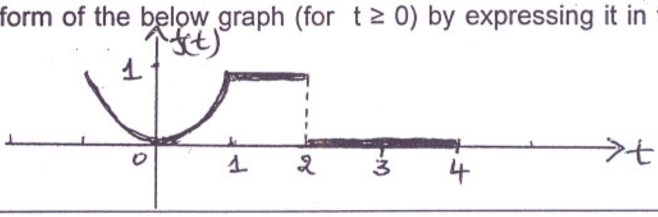
(Established under Karnataka Act No. 16 of 2013)

UE15MA151

MAY 2016: END SEMESTER ASSESSMENT (ESA) B.TECH. II SEMESTER

UE15MA151- ENGINEERING MATHEMATICS-II

Time: 3 Hrs | Answer All Questions | Max Marks: 100

<table>
<tr><td>1.</td><td>a)</td><td>Find the Laplace transform of $t^6e^{3t} + t^2cos2t + \frac{e^{-3t}sint}{t}$.</td><td>7</td></tr>
<tr><td></td><td>b)</td><td>If $f(t)$ is a periodic function with period T, then prove that $L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$.</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Find the Laplace transform of the below graph (for $t \geq 0$) by expressing it in terms of Unit step function.

</td><td>6</td></tr>
<tr><td>2.</td><td>a)</td><td>Find the inverse Laplace transform of $\frac{S}{S^4+S^2+1}$.</td><td>7</td></tr>
<tr><td></td><td>b)</td><td>Using convolution theorem , for the Laplace transform of product of two functions , establish Euler's formula for Beta function given by $\beta(a,b) = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}$
(choose $f(t) = t^{a-1}; g(t) = t^{b-1}$ in the definition of convolution theorem)</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Using the Laplace transform method solve $y''(t) + y(t) = u(t-1)$, given $y(0) = 0$ & $y'(0) = 1$.</td><td>6</td></tr>
<tr><td>3.</td><td>a)</td><td>Evaluate $\int_0^1 \log\Gamma(x)\,dx$.</td><td>7</td></tr>
<tr><td></td><td>b)</td><td>If n is a non negative integer & m is a real constant greater than -1, prove that $\int_0^1 x^m\,(logx)^n dx = \frac{(-1)^n n!}{(m+1)^{(n+1)}}$ and hence evaluate $\int_0^1 \frac{logx}{\sqrt{x}}dx$.</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>With usual notations prove that $J_n(x) = \frac{1}{\pi}\int_0^\pi cos(xsin\theta - n\theta)d\theta$, where n is a positive integer.</td><td>6</td></tr>
<tr><td>4.</td><td>a)</td><td>Show that $\vec{F} = (y^2 + 2xz^2 - 1)\hat{\imath} + 2xy\hat{\jmath} + 2x^2z\,\hat{k}$ is irrotational. Also find $\emptyset$ such that $\vec{F} = \nabla\emptyset$.</td><td>7</td></tr>
<tr><td></td><td>b)</td><td>Evaluate $\iint_s \vec{F}\,.\,\hat{n}\,ds$ where $\vec{F} = y\hat{\imath} + 2x\hat{\jmath} - z\hat{k}$ and s is the surface of the plane $2x + y = 6$ in the first octant cut by z=4.</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Using divergence theorem evaluate $\iint_s \vec{F}.\,\hat{n}\,ds$, where $\vec{F} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ where s is the surface of the sphere $x^2 + y^2 + z^2 = 4$.</td><td>6</td></tr>
<tr><td>5.</td><td>a)</td><td>Obtain the Fourier series expansion of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and hence deduce that
$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$</td><td>7</td></tr>
<tr><td></td><td>b)</td><td>Find the direct current part & amplitude of the first harmonic from the following table consisting of the variations of periodic current.
<table><tr><td>t(sec)</td><td>0</td><td>$T/6$</td><td>$T/3$</td><td>$T/2$</td><td>$2T/3$</td><td>$5T/6$</td><td>T</td></tr><tr><td>A(amp)</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr></table></td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Find half range Fourier cosine series for $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$ up to the first three non-zero terms.</td><td>6</td></tr>
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