UE23MA141A: Engineering Mathematics – I (4-0-0-4-4)

Unit 1: Partial Differentiation and Differential Equations of First Order

Introduction to Partial Differentiation, Geometrical Interpretation, Total Derivative, Chain rule, Partial Differentiation of Composite and Implicit functions, Homogeneous Functions and Euler's Theorem, Taylor & Maclaurin Series expansion for a function of two variables, Maxima and Minima for a function of two variables, Lagrange's method of undetermined multipliers for two variables, Bernoulli's Differential Equations, Exact Differential Equations.

Self-learning component: Errors and Approximations. **14 Hours (19 Sessions)**

Class work problems

Problems on Partial Differentiation

1) Find the first order partial derivatives of
   i. $f(x,y) = x^4 - x^2y^2 + y^4$ at $(-1,1)$. **Ans: -2, 2**
   ii. $f(x,y) = x^2e^{-y/x}$ at (4,2) **Ans:** $\boldsymbol{6e^{1/2}}$, $\boldsymbol{4e^{1/2}}$

2) Show that $\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x}$ for all $(x,y) \neq (0,0)$ when $f(x,y) = x^y$.

3) Find all the second order partial derivatives of $f(x,y) = log\left(\frac{1}{x} - \frac{1}{y}\right)$ at $(1,2)$.

   Ans: $\mathbf{f_{xx} = 0, f_{xy} = 1, f_{yy} = -3/4}$

4) If $f(x,y,z) = e^{x^2+y^2+z^2}$, then find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial z^2}$ at (-1, 1, -1) . **Ans:** $\boldsymbol{\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial z^2} = 6e^3}$

5) If $f(x,y) = e^x\, log\, y + cos\, y\, log\, x$, then find $f_{xxx}, f_{xxy}, f_{xyy}$ and f_{yyy} at $\left(1, \frac{\pi}{2}\right)$

   Ans: $\boldsymbol{f_{xxx} = e\, log\left(\frac{\pi}{2}\right), f_{xxy} = \left(\frac{2e}{\pi}\right) + 1, f_{xyy} = -\frac{4e}{\pi^2}}$ **and** $\boldsymbol{f_{yyy} = \frac{16e}{\pi^3}}$

6) For the point on the surface $x^x\, y^y\, z^z = c$, where $x = y = z$, show that $\frac{\partial^2 z}{\partial x\partial y} = -[x\log(ex)]^{-1}$.

7) Find the value of n so that $v = r^n(3cos^2\theta - 1)$ satisfies the equation $\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right) + \frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial v}{\partial\theta}\right)$ =0. **Ans: -3, 2**

8) If $u = x^2tan^{-1}(y/x) - y^2tan^{-1}(x/y)$ show that $\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 u}{\partial y\partial x} = \frac{x^2-y^2}{x^2+y^2}$ **.(Homework)**

Problems on Total Derivative, Chain rule and Implicit functions:

9) Find the total differential of i) $u = xy$ ii) $u = x/y$
   Ans: i) $\boldsymbol{du = ydx + xdy}$ **ii)** $\boldsymbol{du = \frac{ydx - xdy}{y^2}}$

10) Find $\frac{du}{dt}$, if $u = x^2 - y^2$ and $x = e^t\, cos\, t, y = e^t\, sin\, t$ at $t = 0$. **Ans: 2**

11) Find $\frac{df}{dt}$ at t=0 where $f(x,y,z) = x^3 + xz^2 + y^3 + xyz,\ x = e^t,\ y = cost,\ z = t^3$. **Ans: 3 (Homework)**

12) Find the rate at which the area of a rectangle is increasing at a given instant when the sides of the rectangle are 5 ft and 4 ft and are increasing at the rate of 1.5 ft/sec and 0.5 ft/sec respectively. **Ans:** $\boldsymbol{8.5 sq.ft/s}$

13) The altitude of a right circular cone is 15cm and is increasing at 0.2cm/s. The radius of the base is 10cm and is decreasing at 0.3cm/s. How fast is the volume changing?
Ans: $\boldsymbol{-70\pi/3(cm^3/s)}$ **(Homework)**

14) If $z = log(u^2 + v)$, where $u = e^{x+y^2}, v = x + y^2$, then find the value of $2y\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$. **Ans:** 0

15) Show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$, where u is a function of x, y where $x = r\cos\theta, y = r\sin\theta$.

16) Find $\frac{du}{dx}$, if $u = tan(x^2 + y^2)$ and x, y are connected by the relation $x^2 - y^2 = 2$.

Ans: $\boldsymbol{4x\, sec^2(2x^2 - 2)}$.

17) Find $\frac{dy}{dx}$ at the point$(1,1)$, for $e^y - e^x + xy = 1$. **Ans:** $\boldsymbol{\frac{dy}{dx} = \frac{e-1}{e+1}}$

18) Find $\frac{du}{dx}$, if $u = xlog(xy)$ and $x^3 + y^3 - 3xy - 1 = 0$ **Ans:** $\boldsymbol{\frac{du}{dx} = 1 + logxy + \frac{x}{y}\left(\frac{y - x^2}{y^2 - x}\right)}$ **(Homework)**

Problems on Homogeneous Functions and Euler’s Theorem

19) If $u = \frac{y^{\frac{1}{3}} - x^{\frac{1}{3}}}{x+y}$, then find the values of

i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ **Ans: i)** $\boldsymbol{-\frac{2}{3}u}$ **ii)** $\boldsymbol{\frac{10}{9}u}$

20) If $u = tan^{-1}\left(\frac{x^4+y^4}{xy}\right)$, then prove that

i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u(1 - 4\sin^2 u)$

21) If $u = \sqrt{y^2 - x^2}\, sin^{-1}\left(\frac{x}{y}\right)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. **Ans: u (Homework)**

22) If $u = x^3 sin^{-1}\left(\frac{y}{x}\right) + y^3 tan^{-1}\left(\frac{x}{y}\right)$, then evaluate $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ by using the extension of Euler’s theorem on homogeneous functions. **Ans: 6u**

23) If $u = x^3 sin^{-1}\left(\frac{y}{x}\right) + y^{-3} tan^{-1}\left(\frac{x}{y}\right)$, then evaluate $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ by using the Euler’s and its extension on homogeneous functions. **Ans : 9u (Homework).**

Problems on Taylor's & Maclaurin's Series expansion for a function of two variables

24) Expand $f(x,y) = \sin x \sin y$ in Taylor's series up to second order terms about the point $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$. **Ans:** $\boldsymbol{\sin x \sin y = \frac{1}{2} + \frac{1}{2}\left[\left(x-\frac{\pi}{4}\right)+\left(y-\frac{\pi}{4}\right)\right] - \frac{1}{4}\left[\left(x-\frac{\pi}{4}\right)^2 - 2\left(x-\frac{\pi}{4}\right)\left(y-\frac{\pi}{4}\right)+\left(y-\frac{\pi}{4}\right)^2\right] + \ldots\ldots}$

25) Expand $f(x,y) = \sin(x+2y)$ in Taylor's series up to third degree terms about the point(0,0). **Ans:** $\boldsymbol{\sin(x+2y) = (x+2y) - \frac{1}{6}(x+2y)^3 + \ldots\ldots}$

26) Compute $\tan^{-1}\left(\frac{0.9}{1.1}\right)$approximately.

Ans: $\boldsymbol{\tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{4} - \frac{1}{2}(x-1) + \frac{1}{2}(y-1) + \frac{1}{4}(x-1)^2 - \frac{1}{4}(y-1)^2 + \ldots\ldots}$

$$\boldsymbol{\tan^{-1}\left(\frac{0.9}{1.1}\right) = 0.6904}$$

27) Find the Taylor's expansion of $\sqrt{1+x+y^2}$ in powers of $(x-1)$ and $(y-0)$up to the second degree terms.

Ans: $\boldsymbol{\sqrt{1+x+y^2} = \sqrt{2}\left[1 + \frac{(x-1)}{4} - \frac{(x-1)^2}{32} + \frac{y^2}{4} + \ldots\ldots\right]}$

28) Expand $f(x,y) = xy^2 + y\cos(x-y)$ in Taylor's series up to second order terms about the point $(1,1)$. **(Homework)**

$$\boldsymbol{Ans: 2 + [(x-1) + 3(y-1)] + \frac{1}{2}[-(x-1)^2 + 6(x-1)(y-1) + (y-1)^2]}$$

Problems on Maxima and Minima for a Function of Two Variables

29) Find the maximum and minimum values of $f(x,y) = x^3 - 12x + y^3 + 3y^2 - 9y$. **Ans: Critical points: $(2,-3),(2,1),(-2,-3)$and$(-2,1)$. Max Value: 43 at $(-2,-3)$; MinValue: - 21 at $(2,1)$;$(-2,1)$ and $(2,-3)$ are the saddle points**.

30) Find the maximum and minimum values of $f(x,y) = 4x^2 + 2y^2 + 4xy - 10x - 2y - 3$ **Ans: Critical point: $(2,-3/2)$, Minimum value:$-23/2$. (Homework)**

31) Locate the stationary points of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. Discuss their nature. **Ans: Critical points: $(4,0),(6,0),(5,1),(5,-1)$; $(4,0)$is a point of maximum, $(6,0)$is a point of minimum; $(5,1)$and $(5,-1)$are the saddle points.**

32) Find the extreme value of $a^3x^2 + b^3y^2 + c^3z^2$ subject to the condition $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1$, where $a>0, b>0, c>0$. **Ans: Extreme Value $(a+b+c)^3$, at $(t/a, t/b, t/c)$, where $t = a+b+c$.**

33) A rectangular box without top is to have a given volume. How should the box be made so as to use the least material. **Ans: $x = y = (2a)^{\frac{1}{3}}$ and $z = \frac{x}{2}$.**

34) Find the extreme value of $x^3 + 8y^3 + 64z^3$, when $xyz = 1$.**(Homework) Ans: Extreme value 24 at $(2,1,1/2)$**

35) Find the maximum and minimum distances of the point $(3,4,12)$from the unit sphere with centre at the origin. **Ans: Maximum 14, Minimum 12**.

Problems on Bernoulli's Differential Equations

36) Solve the differential equation $xy' + 2y = x^3y^2$.
    Ans: $\boldsymbol{\frac{1}{y} = -x^3 + cx^2}$
37) Solve the differential equation $y' + 4xy + xy^3 = 0$.
    Ans: $\boldsymbol{y = \left(ce^{4x^2} - \frac{1}{4}\right)^{-\frac{1}{2}}}$ **. (Homework)**
38) Solve the differential equation $y - cosx\frac{dy}{dx} = y^2(1 - sinx)cosx$ given $y = 2$, when $x = 0$.
    Ans: $\boldsymbol{\frac{secx+tanx}{y} = sinx + \frac{1}{2}}$
39) Solve the differential equation $\frac{dy}{dx} - y = y^2(sinx + cosx)$.
    Ans: $\boldsymbol{y = \frac{1}{ce^{-x}-sinx}}$. **(Homework)**

Problems on Exact Differential Equations

40) Check the equation $\{3x^2 + 2e^y\}dx + \{3y^2 + 2xe^y\}dy = 0$ for exactness. If it is exact, find the solution.
    Ans: The given equation is exact and the solution is $\boldsymbol{x^3 + 2xe^y + y^3 = c}$
41) Solve: $\{2xycosx^2 - 2xy + 1\}dx + \{sinx^2 - x^2 + 3\}dy = 0$.
    Ans:$\boldsymbol{ysinx^2 - yx^2 + x + 3y = c}$.
42) Determine for what values of a and b, the following differential equation is exact and obtain the general solution of the exact equation $(y + x^3)dx + (ax + by^3)dy = 0$.
    Ans: $\boldsymbol{a = 1}$ **and the solution is** $\boldsymbol{xy + \frac{x^4}{4} + \frac{by^4}{4} = c}$ **for all b; and c is an arbitrary constant.**

Unit 2: Differential Equations of First Order and Higher Order

Equations reducible to Exact form, Orthogonal Trajectories (Cartesian and polar forms), Solution of first order Non-Linear Differential Equations - Equations solvable for p, Equations solvable for y, Application problems on Differential Equations, Solution to higher order Linear Differential Equations with constant coefficients - Complementary Function and Particular Integrals of standard functions.

Self-learning component: Equations solvable for x. **14 Hours (19 sessions)**

Class Work Problems

Problems on equations reducible to exact form

1) Solve the differential equation $(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$
   Ans: $\boldsymbol{x^5 + 3x^4 + 3x^2y^2 = c}$
2) Solve the differential equation $(xy + y^2)dx + (x + 2y - 1)dy = 0$

Ans: $\boldsymbol{e^x(xy - y + y^2) = c}$

3) Solve the differential equation $(3x^2y^3e^y + y^3 + y^2)dx + (x^3y^3e^y - xy)dy = 0$. **(Homework) Ans:** $\boldsymbol{x^3e^y + x + \frac{x}{y} = c}$

4) Solve $\left(\frac{y}{x}secy - tany\right)dx + (secylogx - x)dy = 0$.
Ans: $\boldsymbol{ylogx - xsiny = c}$

5) $(y^2e^x + 2xy)dx - x^2dy = 0$. **(Homework)**
Ans: $\boldsymbol{x^2 = y(-e^x + c)}$

6) Solve $(y + x)dx + (y - x)dy = 0$.
Ans: $\boldsymbol{\log\sqrt{x^2 + y^2} + tan^{-1}\frac{x}{y} = c}$

7) Solve: $(2xy + x^2)y' = 3y^2 + 2xy$
Ans: $\boldsymbol{x^3 = cy(x + y)}$

8) Solve $(xysinxy + cosxy)ydx + (xysinxy - cosxy)xdy = 0$
Ans: $\boldsymbol{xsecxy = cy}$

9) Solve: $y(1 + xy)dx + (1 - xy)xdy = 0$ **(Homework)**
Ans: $\boldsymbol{log\frac{x}{y} - \frac{1}{xy} = c}$

Problems on Orthogonal Trajectories

10) Find the orthogonal trajectories of the hyperbolas $x^2 - y^2 = c$.
Ans: $\boldsymbol{xy = c}$

11) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is a parameter.
Ans: $\boldsymbol{x^2 + y^2 - 2a^2logx = c}$.

12) Show that the one parameter family of curves $y^2 = 4c(c + x)$ are self-orthogonal.

13) Find the orthogonal trajectories of the family of the curves:
$(i)\ r^2 = csin(2\theta)$ $(ii)\ r = c(sec\theta + tan\theta)$.
Ans: $\boldsymbol{(i)\ r^2 = acos(2\theta)}$ $\boldsymbol{(ii)\ r = be^{-sin\theta}}$

14) Find the orthogonal trajectories of the family of the curves $r^ncosn\theta = a^n$, where a is the parameter **(Homework)**
Ans: $\boldsymbol{r^nsinn\theta = c^n}$

Problems on First Order Nonlinear Differential Equations

15) Solve: $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$
Ans. $\boldsymbol{(y - c)(y + x^2 - c)\left(\frac{1}{y} + x + c\right) = 0}$

16) Solve : $p(p + y) = x(x + y)$.
Ans: $\boldsymbol{\left(y - \frac{x^2}{2} - c\right)(e^x(x + y - 1) - c) = 0}$

17) Solve : $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$.
Ans: $\boldsymbol{(y - cx^2)(y^2 + 3x^2 - c) = 0}$

18) Solve: $x^2p^4 + 2xp - y = 0$.
Ans: $\boldsymbol{y = c^4 + 2c\sqrt{x}}$.

19) Solve: $xp^2 + ax = 2yp$. **(Homework)**

Ans: $2cy = c^2x^2 + a$

20) Solve $y = 2px + p^n$

Ans: $y = \frac{2c}{p} + \frac{1-n}{1+n}p^n$; $x = -\frac{np^{n-1}}{n+1} + \frac{c}{p^2}$

Application Problems – Newton's Law of Cooling

21) A thermometer is removed from a room where the air temperature is $70^\circ F$ to the outside, where the temperature is $10^\circ F$. After $\frac{1}{2}minute$ the thermometer reads $50^\circ F$. What is the reading of the thermometer at $t = 1$minute? How long will it take for the thermometer to reach $15^\circ F$?

Ans: i) The thermometer reading at $t = 1min$ is 36.6666

ii) When $T = 15^\circ F, t = 3.0642min$.

22) Water at temperature $10^\circ c$ takes 5min to warm up to $20^\circ c$ in a room at temperature $40^\circ c$.

(a) Find the temperature after 20min; after $\frac{1}{2}hr$

(b) When will the temperature be $25^\circ c$?

Ans: (a) $34.1^\circ c, 37.4^\circ$c (b) 8.5min.

Problems on Homogeneous Linear Differential Equations with Constant Coefficients

23) Solve: $y'' - 4y' - 12y = 0$. **Ans:** $y = c_1e^{6x} + c_2e^{-2x}$

24) Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$. **Ans:** $y = (c_1 + c_2x)e^{-2x}$

25) Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$, y (0) =2, $y'(0) = -1$. **Ans:** $y = e^{2x}(2cosx - 5sinx)$

26) Solve: $4y'' - 8y' + 3y = 0$, y (0) =1, $y'(0) = 3$. **(Home work) Ans:** $y = \frac{1}{2}\left(5e^{\frac{3x}{2}} - 3e^{\frac{x}{2}}\right)$

27) Solve: $\frac{d^4x}{dt^4} + 4x = 0$. **Ans:** $x = e^{-t}(c_1cost + c_2sint) + e^t(c_3cost + c_4sint)$

28) Solve: $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. **Ans:** $y = c_1e^{-x} + c_2e^{2x} + c_3e^{\frac{3}{2}x} + c_4e^{-\frac{1}{2}x}$

29) Solve: $y^{iv} + 50y'' + 625y = 0$. **Ans:** $y = (c_1 + c_2x)cos5x + (c_3 + c_4x)sin5x$

30) Solve: $y''' - 6y'' + 11y' - 6y = 0$,y(0)=0, $y'(0) = -4$ and $y''(0) = -18$.

Ans: $y = e^x + 2e^{2x} - 3e^{3x}$

31) Solve: $y'''' + 13y'' + 36y = 0$,y(0)=0, $y''(0) = 0$, y($\frac{\pi}{2}$)= -1, $y'\left(\frac{\pi}{2}\right) = -4$. **(Homework)**

Ans: $y = 2sin2x + sin3x$

Problems on Second and Higher Order Non-Homogeneous LDE with Constant Coefficients

32) Solve: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = e^x + 4\,e^{4x}$.**Ans:** $y = (c_1e^{-x} + c_2e^{4x}x) - \frac{e^x}{6} + \frac{4xe^x}{5}$

33) Solve: $y''' - 3y' + 2y = 2sinhx$. **Ans:** $y = (c_1 + c_2x)e^x + c_3e^{-2x} + \frac{x^2e^x}{5} - \frac{e^{-x}}{4}$

34) Solve: $(D^4 + D^3 - 3D^2 - 5D - 2)y = (e^{-x} + 2)^2 + e^{-x}coshx$ **(Homework)**

Ans: : $y = (c_1 + c_2x + c_3x^2)e^{-x} + c_4e^{2x} + \frac{3e^{-2x}}{8} - \frac{2x^3e^{-x}}{9} - \frac{9}{4}$

35) Solve: $y''' - y'' + 4y' - 4y = sin3x$. **Ans:** $y = c_1e^x + c_2cos2x + c_3sin2x + \frac{3cos2x+sin3x}{50}$

36) Solve: $(D^2+3)y = cos\sqrt{3}x$. **Ans:** $\boldsymbol{c_1 cos\sqrt{3}x + c_2 sin\sqrt{3}x + \frac{x sin\sqrt{3}x}{2\sqrt{3}}}$

37) Solve: $(D^2-2D-3)y = 2x^2+6x$. **Ans:** $\boldsymbol{y = c_1 e^{-x} + c_2 e^{3x} - \frac{1}{27}(18x^2+30x-8)}$

38) Solve: $y''' - y = x^5 + 3x^4 - 2x^3$.

Ans: $\boldsymbol{y = c_1 e^{x} + e^{-\frac{1}{2}x}\left(c_2 cos\frac{\sqrt{3}}{2}x + c_3 sin\frac{\sqrt{3}}{2}x\right) - x^5 - 3x^4 + 2x^3 - 60x^2 - 72x + 12.}$

39) Solve: $(D^2+4D+3)y = e^{2x}cosx$. Ans: $y = \boldsymbol{c_1 e^{-x} + c_2 e^{-3x} + e^{2x}(7cosx + 4sinx)/130}$

40) Solve: $(D^3-2D^2+D)y = x^2e^{-x} + sin^2x$

Ans: $y = c_1 + (c_2 + c_3 x)e^{x} - e^{-x}\left(\frac{x^4}{12} + \frac{x^3}{3} + x^2\right) + \frac{1}{2}\left(x + \frac{1}{100}(6\sin 2x + 8\cos 2x)\right)$

41) Solve: $(D^2-1)y = xsin3x$. **Ans:** $\boldsymbol{y = c_1 e^{x} + c_2 e^{-x} - \frac{xsin3x}{10} - \frac{3cos3x}{50}}$

42) Solve: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xcos2x$. **(Homework)**

Ans: $\boldsymbol{y = c_1 e^{2x} + c_2 e^{3x} + \frac{x}{104}(2cos2x - 10sin2x) + \frac{1}{2704}(-146sin2x - 80cos2x)}$

43) Solve: $(D-2)^2)y = 8(e^{2x} + sin2x + x^2)$.

Ans: $\boldsymbol{y = (c_1 x + c_2)e^{2x} + 4x^2e^{2x} + cos2x + 2x^2 + 4x + 3.}$

44) Solve: $(D-1)^2(D+1)^2y = sin^2\frac{x}{2} + 2^{-x} + x$. **(Homework)**

Ans: $\boldsymbol{y = (c_1 + c_2 x)e^{x} + (c_3 + c_4 x)e^{-x} + \frac{1}{2} - \frac{1}{8}cosx + \frac{1}{(log2)^4 - 2(log2)^2 + 1}2^{-x} + x}$