1 Mark questions

1. Find $\frac{\partial^2 u}{\partial x \partial y}$, when $u = e^{xyz}$
2. Find $\frac{\partial^3 z}{\partial x^2 \partial y}$, when $z = \sin(xy)$
3. If $u = x^y$, then find $\frac{\partial u}{\partial x}$
4. If $u = \sin^{-1}\left(\frac{y}{x}\right)$, then what is the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{y}$?
5. If $u = x^y$, then find $\frac{\partial u}{\partial y}$
6. What is the degree of homogeneous function $u(x, y) = \frac{x^2 y^2}{x + y}$?
7. What is the degree of homogeneous function $u(x, y)$ $\frac{\sqrt{x^2 + y^2}}{x + y}$
8. What is the degree of homogeneous function $u(x, y) = \frac{x + y}{\sqrt{x} + \sqrt{y}}$?
9. If $u = x^2 y + xy^2$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{y}$.
10. If $u = xy$, where $x = t^2$, $y = t$, find $\frac{du}{dt}$

Answers: (1) $e^{xy}(1 + xy)$ (2) $-xy^2 \cos(xy) - 2y\sin(xy)$ (3) $u\frac{y}{x}$ (4) 0
(5) $u\log_e x$ (6) $n = 3$ (7) $n = 0$ (8) $n = \frac{1}{2}$ (9) $3u$
(10) $3t^2$

2 Mark Questions

1. If $u = f(x + ay) + g(x - xy)$, then what is $\frac{\partial^2 u}{\partial y^2}$?
2. If $u = x^m y^n$, then find the value of $\frac{\partial^2 u}{\partial y \partial x}$

3. If $u = \tan^{-1}\left(\frac{y}{x}\right)$, then what is $\frac{\partial u}{\partial x}$ at $(1,1)$

4. If $u = e^x \log(1+y)$, then find the value of $\frac{\partial^3 u}{\partial y \partial x^2}$ at $(0,0)$;

5. If $\sin u = \frac{x^2 y^3}{x+y}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{y}$

6. If $u = \frac{x^2 y^3}{x+y}$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

7. What is the degree of homogeneous function $u = \left[\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}\right]^{1/2}$?

8. If $x^3 + y^3 - 3axy = 1$, find $\frac{dy}{dx}$ using partial derivatives

9. If $u = \log\left(x^3 + y^3 - x^2 y - xy^2\right)$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{y}$

10. At what rate is the area of a rectangle changing if its length is 15 mts and increasing at 3 mts/sec while its width is 6 mts and increasing at 2 mts/sec. **Ans:48**

Answers: (1) $f''(x+ay)a^2 + g''(x-xy)x^2$ (2) $mn\,x^{m-1}y^{n-1}$ (3) $-\frac{1}{2}$ (4) 1

(5) $4\tan u$ (6) $12u$ (7) $n = -\frac{1}{12}$ (8) $\frac{ay - x^2}{y^2 - ax}$ (9) $n = 3$

(10) 48mts/sec

4 Mark questions

1. If $u = x^y$, then show that $\frac{x}{y}\frac{\partial u}{\partial x} + \frac{1}{\log x}\frac{\partial u}{\partial y} = 2u$

2. If $u = f(y+ax) + \phi(y-ax)$, then show that $\frac{\partial^2 u}{\partial x^2} - a^2\frac{\partial^2 u}{\partial y^2} = 0$

3. Verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for $u = x^3 + y^3 + 3axy$

4. If $x = r\cos\theta,\ y = r\sin\theta$ show that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}$

5. If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

6. If $x = r\cos\theta,\ y = r\sin\theta$ show that $\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial x}\right)^2\right] = \frac{1}{r}$

7. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ show that,

   i. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

   ii. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \left(1 - 4\sin^2 u\right)\sin 2u$

8. If $u = \sin^{-1}\left(\sqrt{x^2 + y^2}\right)$ show that,

   i. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

   ii. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \tan^3 u$

9. If $u = \sin^{-1}\left[\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}\right]^{1/2}$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$; **Ans:** $-\frac{\tan u}{12}$

10. If $u = \tan^{-1}\left[\frac{x^2 + y^2}{x + y}\right]^{1/2}$, then find the value of $x^2 u_{xx} + 2xy\,u_{xy} + y^2 u_{yy}$; **Ans:** $\frac{\sin 4u}{16}$

11. If $u = x^3\tan^{-1}\left[\frac{y}{x}\right] + y^{-3}\sin^{-1}\left(\frac{x}{y}\right)$, then find the value of $x^2 u_{xx} + 2xy\,u_{xy} + y^2 u_{yy} + xu_x + yu_y$. Ans: $9u$

12. If $u = x^3\tan^{-1}(y/x) + y^3\cos^{-1}(x/y)$, then find the value of $x^2 u_{xx} + 2xy\,u_{xy} + y^2 u_{yy}$; **Ans:** $12u$.

13. Find the exact differential of i) $u = \log_e\left(x^2 + y^2\right)$ ii) $u = \pi x^2 y$

Ans: i) $du = \dfrac{2(xdx + ydy)}{x^2 + y^2}$ ii) $du = \pi\left[2xydx + x^2dy\right]$

14. If $u = xy(x+y)$ where, $x = at^2, y = 2at$ then find $\dfrac{du}{dt}$; **Ans:** $4a^3t^3(3t+4)$

15. If $u = f(y-z, z-x, x-y)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$

16. If $u = f(x,y), x = s+t, y = s-t$, then find the value of $\dfrac{\partial u}{\partial s} + \dfrac{\partial u}{\partial t}$ **Ans:** $2\dfrac{\partial u}{\partial x}$

17. If $u = x^2y$ and $x^2 + xy + y^2 = 1$, find $\dfrac{du}{dx}$; **Ans:** $(2xy) + (x^2)\dfrac{(-2x-y)}{(2y+x)}$

18. If $u = e^{xy}$ and $x + xy + y = 1$, find $\dfrac{du}{dx}$. **Ans:** $\dfrac{du}{dx} = (e^{xy}y) + (e^{xy}x)\dfrac{(-y-1)}{(x+1)}$

19. $w = e^{x+y}\cos 2z, x = \log t, y = \log(t^2+1)$ and $z = t$, find $\dfrac{du}{dx}$

Ans: $t(t^2+1)\left[\dfrac{\cos 2t}{t} + \dfrac{2t}{t^2+1}\cos 2t - 2\sin 2t\right]$

20. Obtain the Taylor's series for the function $f(x,y) = xy^2 + y\cos(x-y)$ about the point $(1, 1)$.

Ans:

$$xy^2 + y\cos(x-y) = 2 + \left[(x-1) + 3(y-1)\right] + \frac{1}{2}\left[-(x-1)^2 + 6(x-1)(y-1) + (y-1)^2\right] + \ldots\ldots$$

21. Expand $f(x,y) = \sin(xy)$ in powers of (x – 1) and $\left(y - \dfrac{\pi}{2}\right)$ up to the second degree term.

Ans: $\sin(xy) = 1 - \dfrac{\pi^2}{8}(x-1)^2 - \dfrac{\pi}{2}(x-1)\left(y - \dfrac{\pi}{2}\right) - \dfrac{1}{2}\left(y - \dfrac{\pi}{2}\right)^2 + \ldots\ldots\ldots$

22. Expand $f(x,y) = e^x \log(1+y)$ in powers of x and y up to terms of third degree.

Ans: $e^x \log(1+y) = y + xy - \dfrac{1}{2}y^2 + \dfrac{1}{2}(x^2y - xy^2) + \dfrac{1}{3}y^3 + \ldots\ldots\ldots$

23. Find the Minimum and Maximum values of the

i) $f(x,y)=2(x^2-y^2)-x^4+y^4$.

Ans: Max. at $(\pm 1, 0)$, Min. at $(0, \pm 1)$

ii) $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$.

Ans: Max. at $(4, 0)$, Min. at $(6, 0)$.

24. Find the extreme value of xyz, when $x+y+z=a,\ a>0$.

Ans: Extreme Value is $a^3/27$ at $\left(\frac{a}{3},\frac{a}{3},\frac{a}{3}\right)$.

25. Find the extreme value of $x^p+y^p+z^p$ on the surface $x^q+y^q+z^q=1$, where $0<p<q,\ x>0, y>0, z>0$.

Ans: Extreme Value is $3^{(q-p)/q}$ at $\left(3^{-\frac{1}{q}},3^{-\frac{1}{q}},3^{-\frac{1}{q}}\right)$.