

END SEMESTER ASSESSMENT (ESA) B.Tech. II SEMESTER- MAY 2015

UE14MA151- ENGINEERING MATHEMATICS -II

Time: 3 Hours		Answer All Questions	Max Marks: 100															
1.	a)	Find $L\left\{\sin 5t + \cosh 3t + \sqrt{t} + \frac{e^{-2t}-e^{-4t}}{t} + t e^{4t} \cos 2t\right\}$	7															
	b)	Derive the Laplace transform for the unit step function $u(t-a)$	5															
	c)	A rectifier sine wave is given by a periodic function $f(t) = f(t+2\pi) = \begin{cases} \sin t & 0 < t < \pi \\ -\sin t & \pi < t < 2\pi \end{cases}$. Obtain the Laplace transform of the periodic function.	5															
2.	a)	Find $L^{-1}\left\{\frac{2}{(s^2-4)(s+1)^2}\right\}$ using convolution theorem.	6															
	b)	Solve the following differential equation using Laplace transform $y'' + 4y' + 9y = 2\delta(t-1) + e^t$; $y(0) = 0$, $y'(0) = -1$	10															
3.	a)	A point moves along the intersection of the surfaces $z = x^2 + y^2$ with the plane $x + y = 2$ from the point $(2,0,4)$ to the point $(0,2,4)$ in such a way that $x = 2-t$. Find the unit tangent vector and the unit normal vector at the final point.	7															
	b)	Find the Directional derivative of $f(x,y,z) = x^2y^2z^2$ at the point $(1,1,-1)$ in the direction of the tangent to the curve $x = e^t, y = 1 + 2\sin t, z = t - \cos t$, where $-1 \leq t \leq 1$.	5															
	c)	Show that $r^n \cdot \vec{r}$ is an irrotational vector for any value n but is solenoidal if $n+3=0$, where $\vec{r} = xi + yj + zk$ and r is the magnitude of \vec{r}	5															
4.	a)	Verify Green's theorem for $\oint (2x - y^3)dx - xy dy$ over a curve c , where c is the boundary of the region bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.	7															
	b)	Let $\vec{F} = -yi + xj + zk$. Let S be the part of the paraboloid $z = 7 - x^2 - 4y^2$ that lies above the plane $z = 3$, oriented with upward pointing normal. Evaluate $\iint \text{curl } \vec{F} \cdot \hat{n} ds$ by finding the work done in moving once around the boundary of the surface in the counter clockwise direction (use Stoke's theorem)	5															
	c)	Find the flux of the vector field \vec{F} given by $\vec{F} = ye^zi + y^2j + xe^yk$ over the boundary of the region enclosed by the cylinder $x^2 + y^2 = 9$, $z = 0$ and $z = y - 3$ using Gauss divergence theorem.	5															
5.	a)	Find the Fourier Series for the function $f(x) = \begin{cases} 1, & 0 < x < \pi \\ 2, & \pi < x < 2\pi \end{cases}$ and hence find the sum of the infinite series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$,	7															
	b)	Compute half range cosine series expansion for the function $f(x) = \begin{cases} 0, & 0 \leq x \leq 1 \\ x-1, & 1 \leq x \leq 2 \end{cases}$	5															
	c)	Find the direct current part and Amplitude of the first harmonic from the following table consisting of the variations of periodic current <table><tr><td>t (secs)</td><td>0</td><td>T/6</td><td>T/3</td><td>T/2</td><td>2T/3</td><td>5T/6</td><td>T</td></tr><tr><td>A amps)</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr></table>	t (secs)	0	T/6	T/3	T/2	2T/3	5T/6	T	A amps)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98
t (secs)	0	T/6	T/3	T/2	2T/3	5T/6	T											
A amps)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98											
6.	a)	Prove that $x^2 J_n''(x) = (n^2 - n - x^2)J_n(x) + xJ_{n+1}(x)$ where $n = 0,1,2,3, \dots$	6															
	b)	Using the Jacobi series Prove that $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x\sin\theta) d\theta$, where n is an integer	5															
	c)	Show that $y = \sqrt{x} J_1(x)$ is a solution of the differential equation $y'' + \left(1 - \frac{3}{4x^2}\right)y = 0$	5															