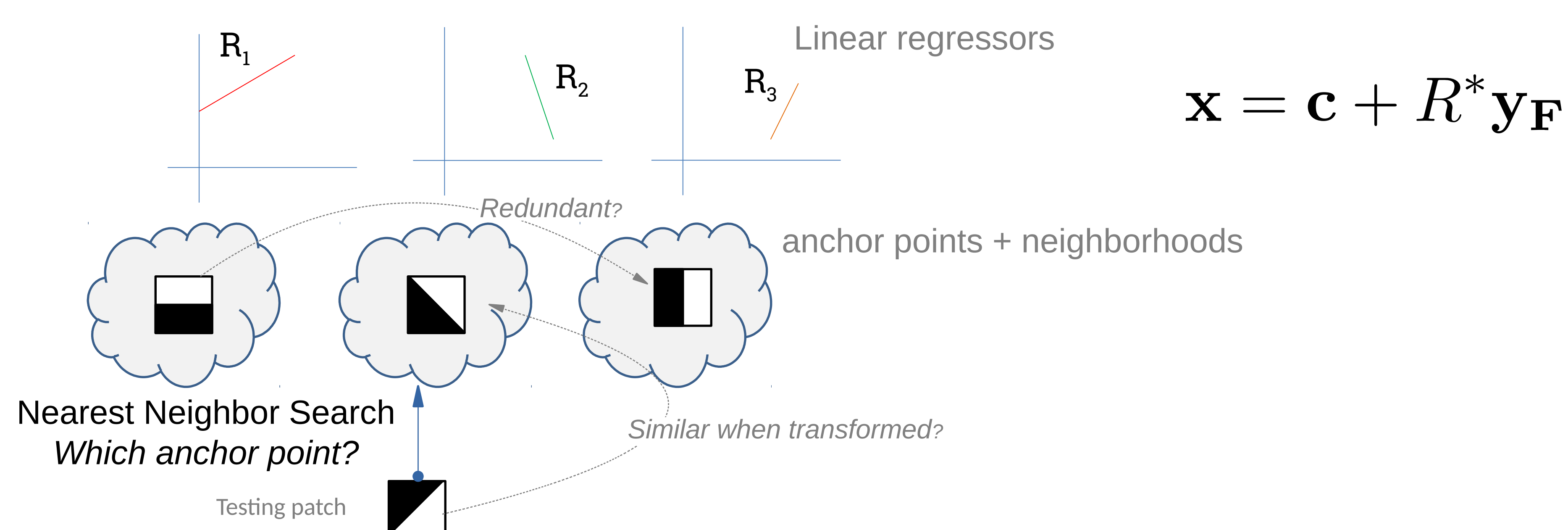


1. Regression-based SR

- Mapping between low-res and high-res patch manifolds is **non-linear**
 - Locally linear assumption
 - Ensemble of linear regressors to model non-linearity [1] (anchor points)



Some questions we asked ourselves :

- Is there redundancy among the regressors we learn?
- More generally, are the **manifolds** in which patches lie **redundant**?
- Can we obtain **transformation models** that describe that redundancy?
- Can we **get rid** of redundancy?

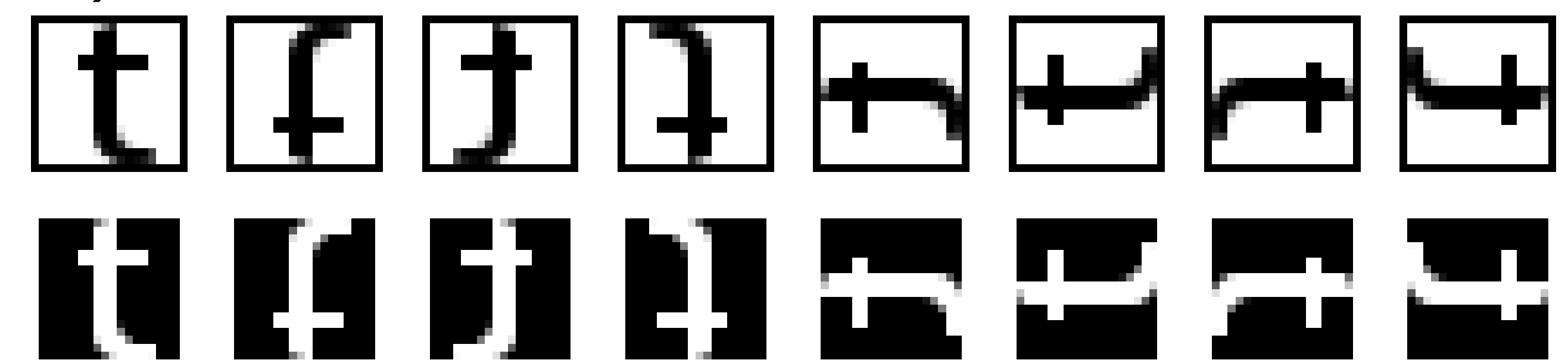
2. Extending the Search Space

- Extending the NN search space with transformation models solves partially the problem:
 - Rotation
 - Reflection
 - Antipodal Points [2]
 - Cross-scale [3]
 - Homography [3]
- More search candidates **increase** the **search cost**
- Problem: **Redundancy** persists within **dictionary atoms**
- Proposed Alternative**: **variability collapse**

3. Dihedral Group as Symmetries

- Dihedral group (i.e. rotations and reflections)
 - Exist a **lossless inverse transform**
 - Finite group**, can be composed by **three operators** g_x, g_y and g_τ
 - Defines a set of **8 high-dimensional** points for a given point in the manifold (with 8 additional **antipodal** points)

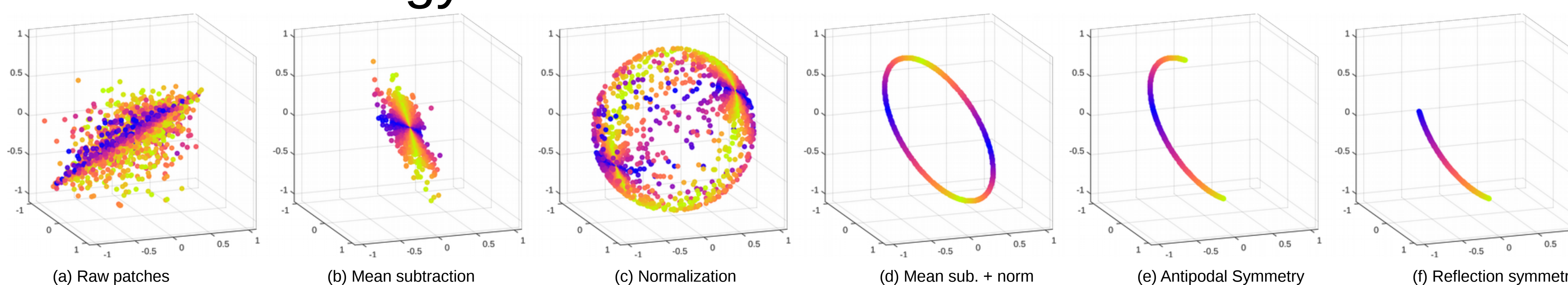
$$g_x = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, g_y = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, g_\tau = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



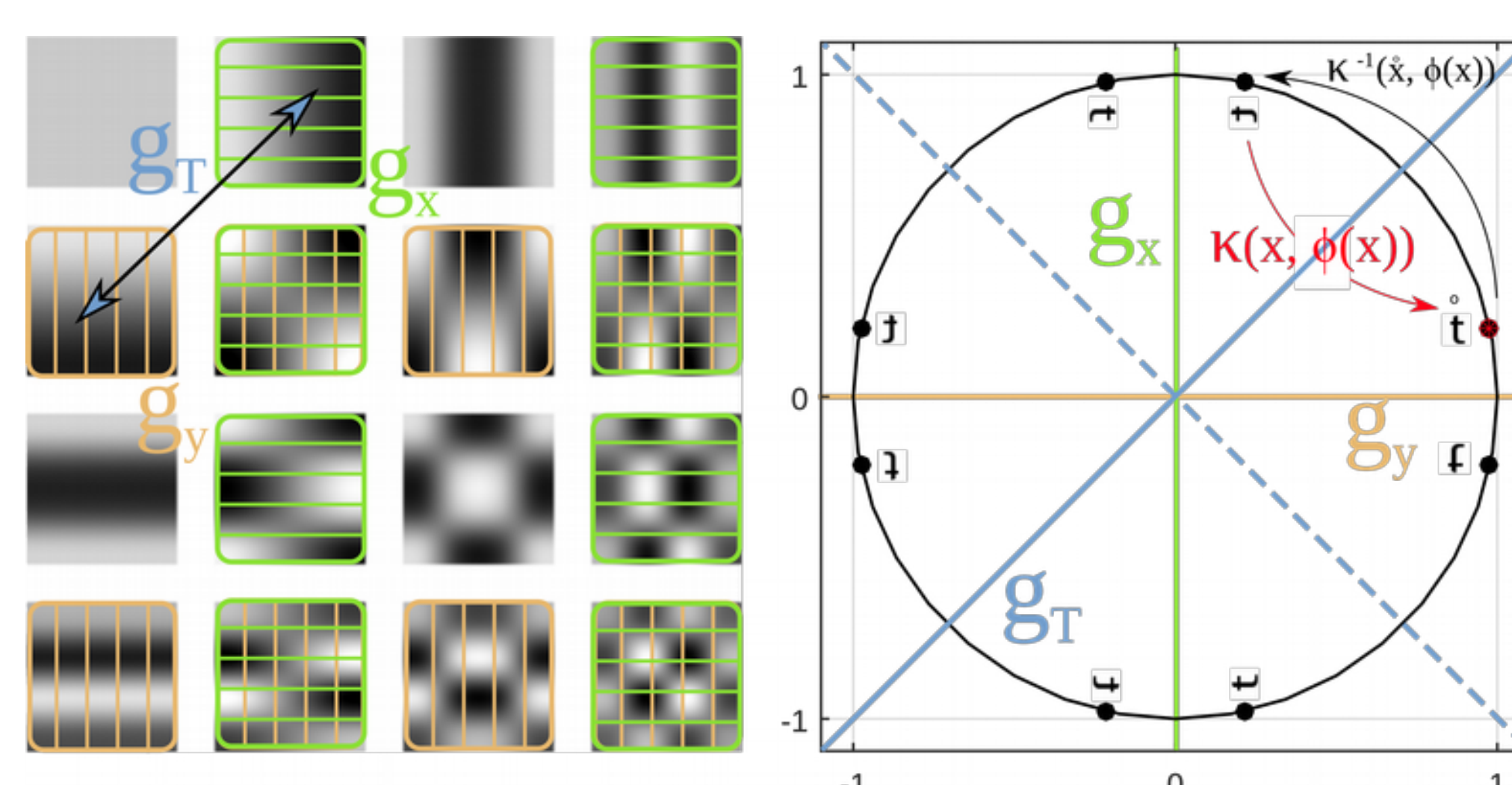
- Symmetry is defined as non-trivial group of action that defines isomorphism

4. Patch Symmetry Collapse

- Similar strategy as normalization or mean subtraction:



- Dihedral group of transforms in the DCT: sign change and transpose
 - Low-dimensional representation enables to find symmetry axes.
 - Collapse transform K based on the Symmetric Distance of [4]



$$\hat{c} = \kappa(c, \varphi(c))$$

$$g_j = \varphi(c)$$

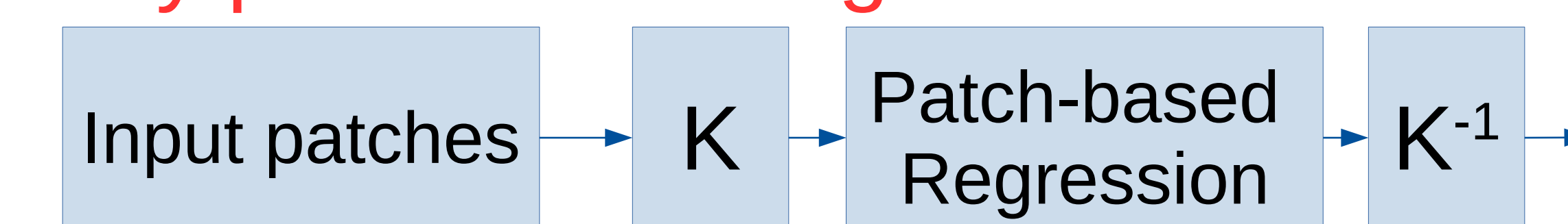
$$c = \kappa^{-1}(\hat{c}, g_j)$$

$$R_i = (1 + \lambda)(\hat{\mathbf{X}}_i - \hat{\mathbf{C}}_i) \hat{\mathbf{C}}_i^\top (\hat{\mathbf{C}}_i \hat{\mathbf{C}}_i^\top + \lambda \mathbf{I})^{-1}$$

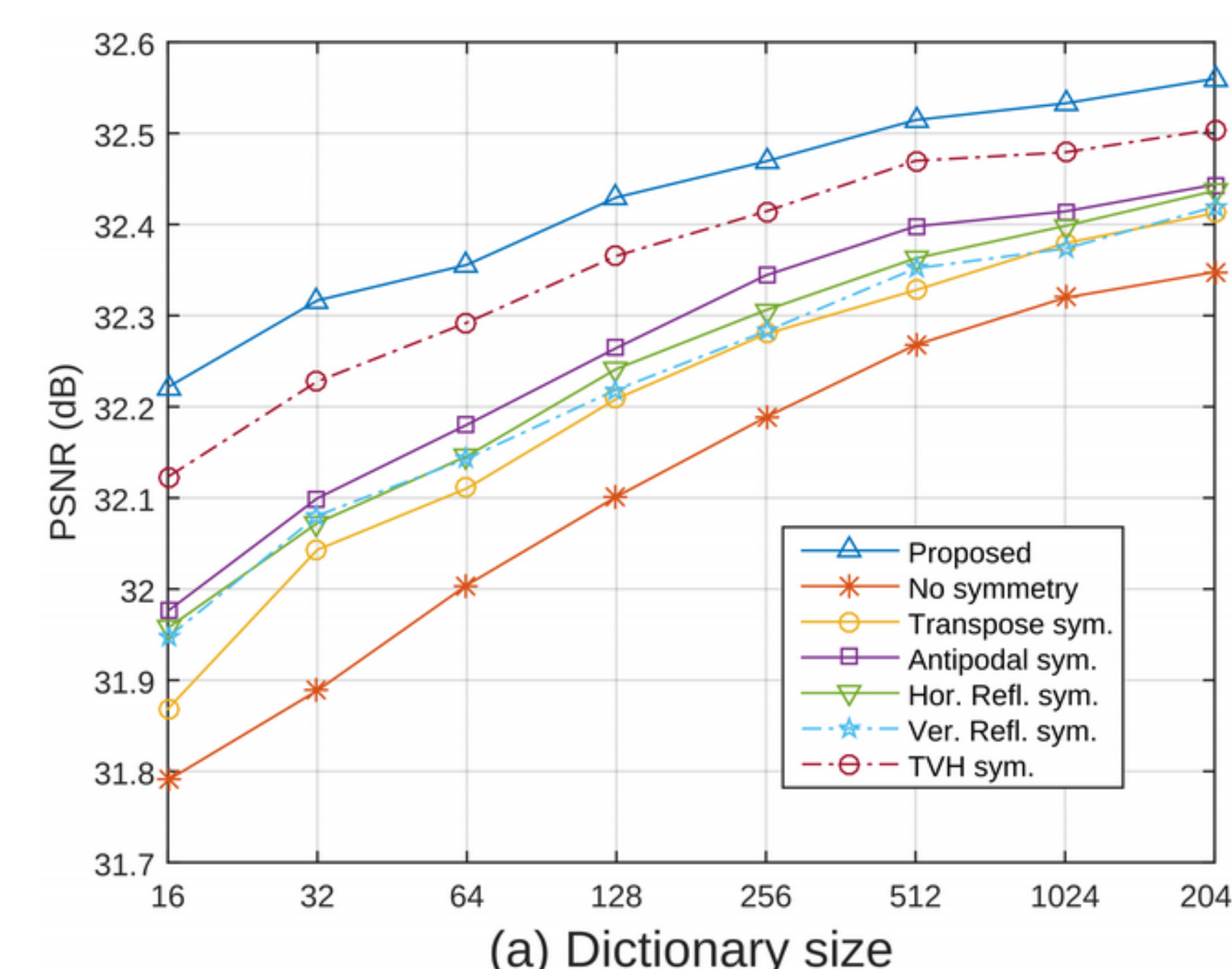
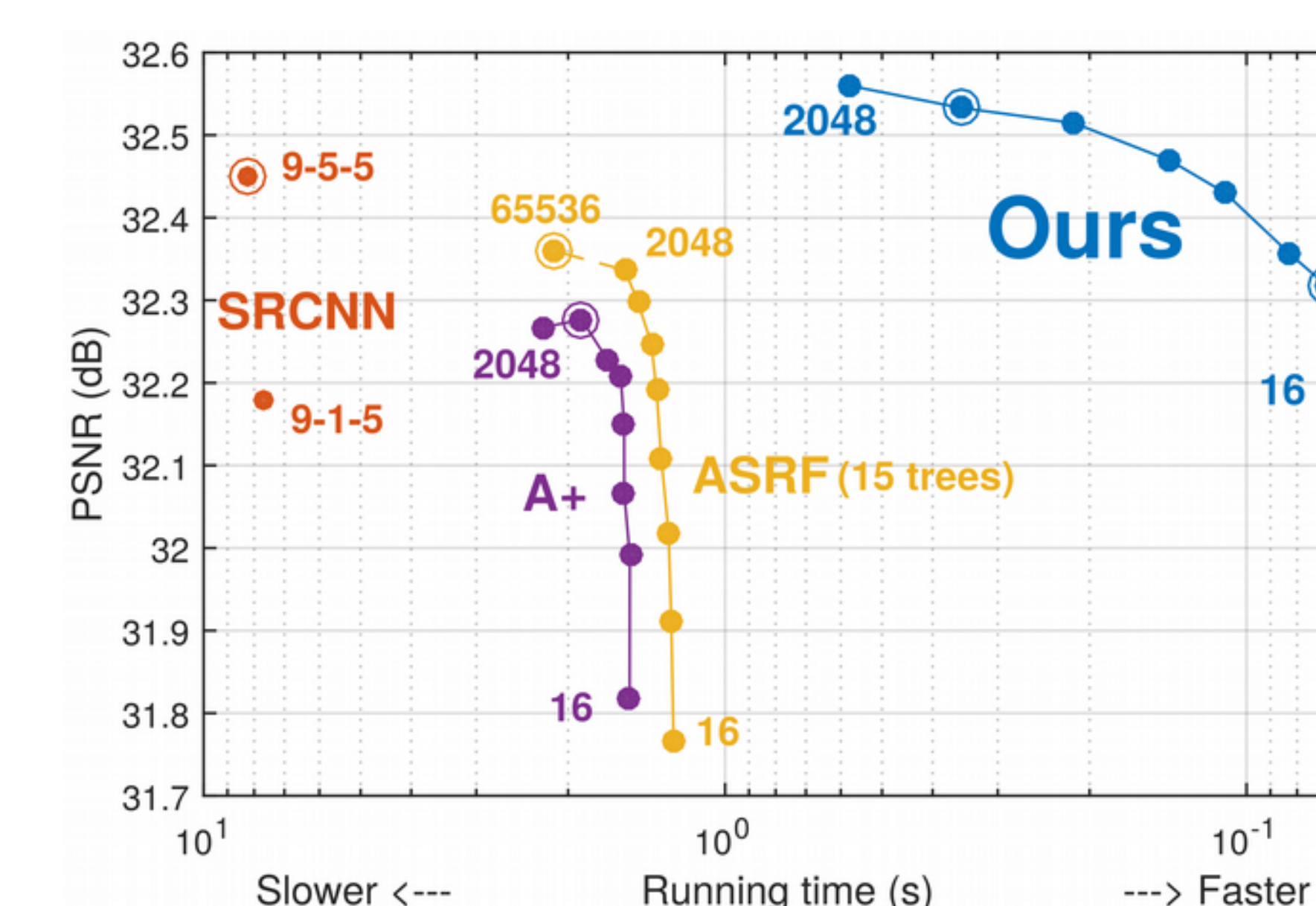
$$\tilde{\mathbf{x}} = \mathbf{c} + \kappa^{-1}(R^* \hat{\mathbf{c}}, \varphi(\hat{\mathbf{c}})),$$

5. Conclusions and Results

- Avoid **dihedral** and **antipodal redundancy** in the dictionary and nearest neighbor search
- Reduce manifold span to be learnt, thus **improving regressor specialization**
- Cover the whole space by the **exploitation of symmetries**
- Effectively obtain dictionaries **16 to 32 times smaller** (**faster search, less memory usage**)
- Avoid brute force solutions
- Applicable to any patch-based regression**:

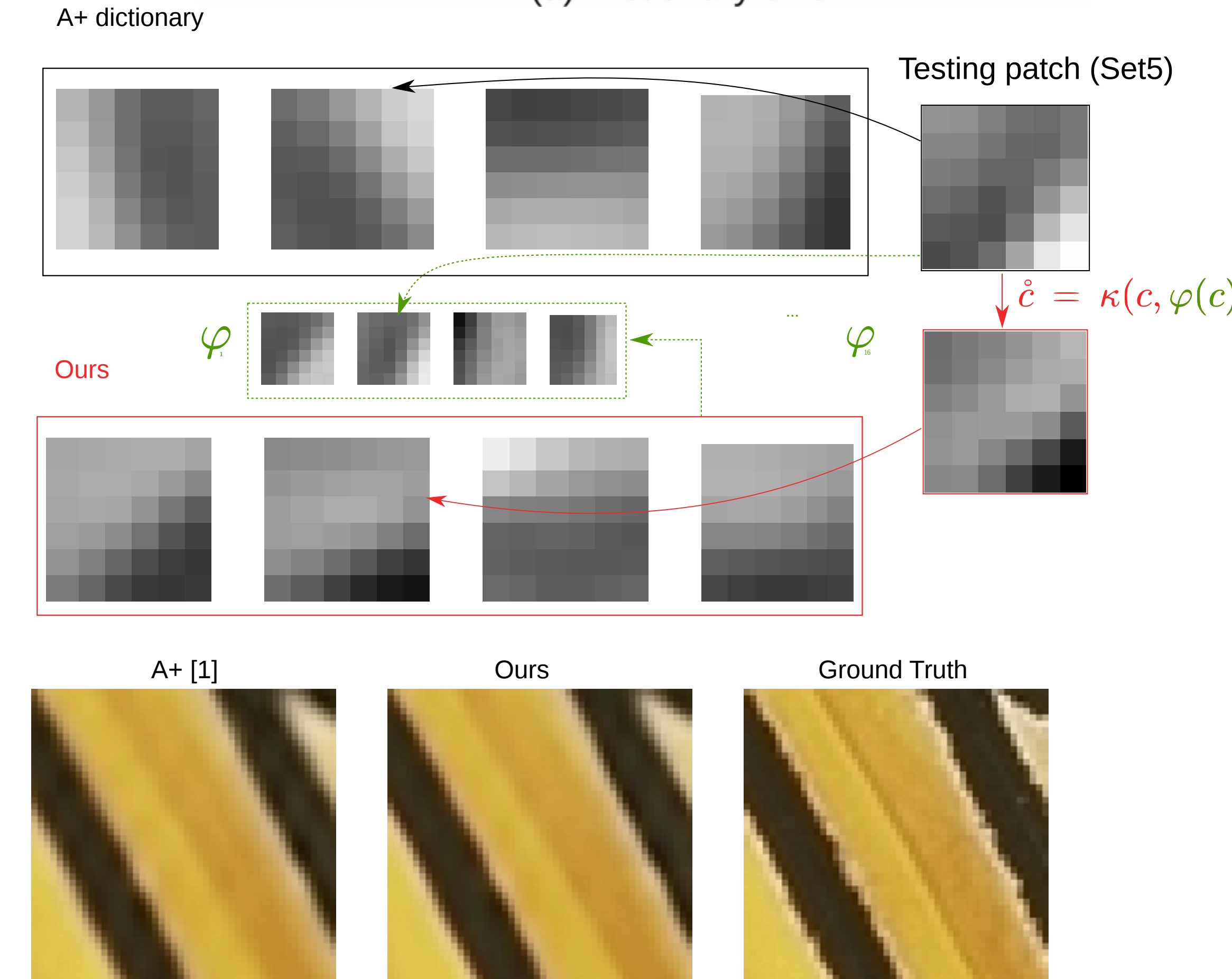


- Improve SR state of the art performance up to 0.2dB and 0.25 (IFC), fastest method by an order of magnitude



Experimental setup

- Datasets: Set5, Set14, Urban100, Kodak.
- MF: x2, x3, x4
- Measured PSNR, IFC, time.
- Compared with SRCNN, TselfEx, ASRF, A+



REFERENCES

- [1] R. Timofte, V. D. Smet, and L. V. Gool. A+: Adjusted anchored neighborhood regression for fast super-resolution. In ACCV, 2014.
- [2] E. Pérez-Pellitero, J. Salvador, J. Ruiz-Hidalgo, and B. Rosenhahn. Antipodally Invariant Metrics for Fast Regression-Based Super-Resolution. TIP, 2016.
- [3] J.-B. Huang, A. Singh, and N. Ahuja. Single image super-resolution from transformed self-exemplars. In CVPR, 2015.
- [4] H. Zabrodsky, S. Peleg, and D. Avnir. Symmetry as a continuous feature. TPAMI, 1995.
- [5] C. Dong, C. Loy, K. He, and X. Tang. Image super-resolution using deep convolutional networks. TPAMI, 2015.
- [6] S. Schuler, C. Leistner, and H. Bischof. Fast and accurate image upscaling with super-resolution forests. In CVPR, 2015.