Calculation and Design of Resistance-Coupled Amplifiers Using Pentode Tubes

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RESISTANCE-COUPLED amplifiers employing pentode tubes are used more widely than any other type of voltage amplifier for audio frequencies. In spite of this, no method of analysis is available for correctly calculating the complete characteristics of such amplifiers and design procedures ordinarily followed involve considerable cut and try. The present paper presents (1) an analysis taking into account all of the factors of significance in determining the amplification and phase-shift characteristics of ordinary audio-frequency amplifiers employing pentode tubes, together with charts which make it a simple matter to use the analysis; and (2) a systematic procedure for proportioning the coupling network in the plate circuit when the main objective is: (*a*) maximum possible voltage gain, and (*b*) large output voltage.

Calculation of Voltage Amplification

The circuit of a typical resistance-coupled amplifier is shown in figure 1*a*. The magnitude and phase of the amplification obtained with such an arrangement is affected by the coupling network in the plate circuit, by the impedance in the screen circuit external to the tube, and by the impedance between cathode and ground.

If A_0 is the amplification on the assumption that the cathode-ground and screen circuit impedances are zero and the capacitors C_c and C_s in figure 1*a* have no effect, then from conventional amplifier theory[1]

$$A_0 = g_m R \tag{1}$$

where

g_m = mutual conductance of tube,

$$R = \frac{R_c}{1 + \dfrac{R_c}{R_{gl}} + \dfrac{R_c}{R_p}}$$

= equivalent resistance formed by coupling resistance R_c, grid-leak resistance R_{gl}, and plate resistance R_p, all in parallel.

The actual amplification will be less than A_0 according to the relation

$$\text{Actual amplification} = A = \alpha\,\beta\,\gamma\,A_0 \tag{2}$$

where

α = factor taking into account the effect of capacitors C_c and C_s in the coupling network in the plate circuit

β = loss of amplification due to impedance in the screen circuit

γ = loss in amplification due to impedance between cathode and ground

Effect of Plate Circuit Coupling Network on Amplification

The factor α that takes into account the effect of the coupling network in the plate circuit upon the amplification can be evaluated with the aid of the equivalent circuit of figure 1*b*. At low frequencies

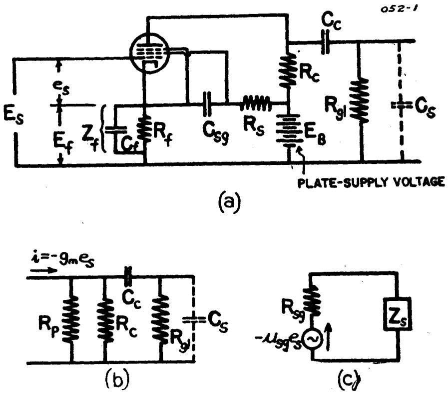

Figure 1. Actual and equivalent circuits of resistance-coupled amplifier using pentode tubes

(*a*)—Circuit of resistance-coupled amplifier using pentode tube
(*b*)—Equivalent plate circuit
(*c*)—Equivalent screen-grid circuit

the output falls off and leads in phase because of the loss of voltage in the coupling capacitor C_c, while at high frequencies, the output falls off and lags in phase because of the shunting capacitance C_s. In practical amplifiers the shunting effect of C_s is negligible at low frequencies, while C_c can be assumed to be a short circuit at

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1. For all numbered references, see list at end of paper.

Conclusions

The employment of carrier in the plant of the Western Union has been a large factor in reducing the new wire construction almost to the vanishing point. A further and perhaps almost as important economy will ultimately be realized in attendance costs of repeaters. Experience indicates that this charge is no more for a carrier repeater than for a single physical repeater handling the same class of service as is assigned to each carrier channel. Economy of repeater office space is another large factor, the carrier repeater requiring less space than a single physical repeater. The loading of cable sections adds to the fixed charges on lines and maintenance costs are increased, but these increases are not large. For long trunk circuits the balance is of course very much in favor of carrier operation and even for circuits of 200-mile length carrier is preferable to new line construction.

Four trunk systems varying in length from 250 to 1,000 miles with a total of 39 channels in operation radiate to the north, south, and west from New York City. In normal assignments 31 of these channels are multiplex operated, with the remainder held in reserve or operated in less important services which may be quickly released for multiplex working.

Numerous single- and double-channel systems operate over short distances where it has been found necessary to secure added facilities. The equipment employed for this class of service differs only in the carrier supply and frequently in the absence of the terminal repeaters.

Discussion

For discussion, see page 1132.

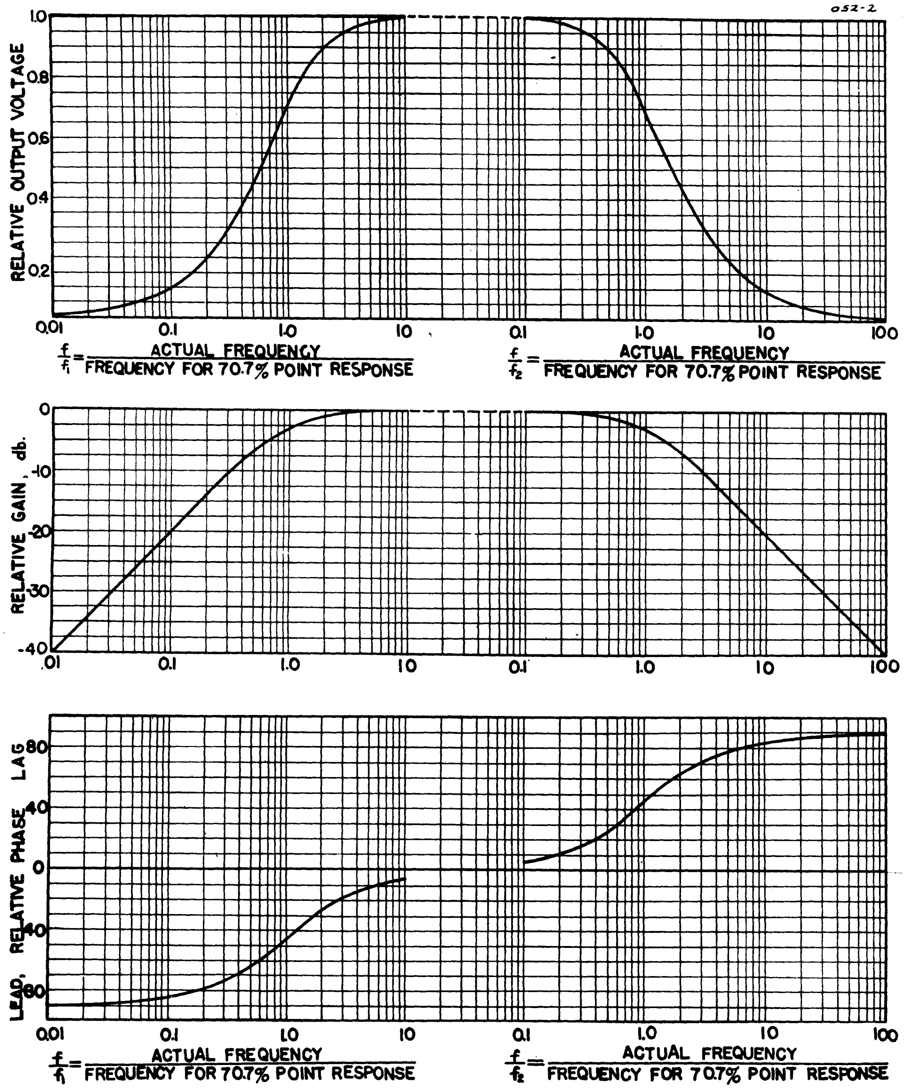


Figure 2. Curves giving effect on amplification of the coupling network in the plate circuit

high frequencies, giving[1]

For low frequencies

$$\alpha = \frac{1}{1 - j\dfrac{f_1}{f}} \tag{3}$$

For high frequencies

$$\alpha = \frac{1}{1 + j\dfrac{f}{f_2}} \tag{4}$$

where

f = actual frequency

$$f_1 = \frac{1}{2\pi C_c\left(R_{gl} + \dfrac{R_pR_c}{R_p+R_c}\right)}$$

= frequency at which the reactance of C_c equals the resistance formed by the grid leak in series with the combination of plate and coupling resistances in parallel

$$f_2 = \frac{1}{2\pi C_sR}$$

f_2 = frequency at which reactance of C_s equals the resistance R as defined in connection with equation 1

Values of α are plotted in figure 2 in a convenient form.

The phase shift and falling off in amplification represented by α is usually taken as the frequency-response characteristic of the amplifier. The actual characteristics may differ appreciably from this at low frequencies because of the impedances in the screen and cathode circuits.

Effect of Impedance in the Screen Circuit

An impedance in the screen-grid circuit such as supplied by the voltage-dropping resistance R_s and the by-pass capacitor C_{sg} in figure 1*a* affects the amplification by the following process: A signal voltage e_s acting between grid and cathode produces variations in screen current as well as variations in plate current. This causes a voltage to be developed between screen and cathode, which affects the plate current and so modifies the gain.

A quantitative analysis can be carried out with the aid of the equivalent screen-grid circuit shown in figure 1*c*. Here the effect on the screen current of the signal voltage e_s applied between control grid and cathode is represented by an equivalent voltage $-\mu_{sg}e_s$ acting in a circuit consisting of the dynamic screen-grid resistance R_{sg} in series with the impedance Z_s in the screen circuit. This equivalent circuit is analogous to the equivalent plate circuit of a triode, with the constant μ_{sg} representing the μ-factor of the screen-grid relative to the control grid with respect to the screen current.[2] The only approximation in this equivalent circuit is that it assumes the screen-grid current is independent of the plate potential, a requirement realized under the practical operating condition of plate current independent of plate potential.

Referring to figure 1*c*, the voltage developed across the impedance Z_s is $-\mu_{sg} \times e_sZ_s/(R_{sg}+Z_s)$. Now this voltage which exists between screen and cathode has the same effect on the plate current as a voltage $[-\mu_{sg}e_sZ_s/(R_{sg}+Z_s)]/\mu_s$ acting on the control grid, where μ_s is the μ-factor of the screen grid relative to the control grid with respect to the plate current. The effect of the impedance in the screen circuit on the gain is accordingly

$$\beta = \frac{e_s - (\mu_{sg}/\mu_s)e_sZ_s/(R_{sg}+Z_s)}{e_s}$$

or

$$\beta = 1 - \left(\frac{\mu_{sg}}{\mu_s}\right)\frac{Z_s}{R_{sg}+Z_s} \tag{5}$$

The ratio μ_{sg}/μ_s appearing in equation 5 represents the relative effectiveness of the control and screen grids on the screen current compared to their relative effectiveness on the plate current. Under the usual pentode operating conditions of plate current independent of plate voltage, this ratio is unity.

In the common case where the impedance in the screen circuit is a resistance-capacitor combination as in figure 1. then for $\mu_{sg} = \mu_s$, equation 5 becomes*

$$\beta = \frac{\dfrac{K}{1+K} + j\dfrac{f}{f_3}}{1 + j\dfrac{f}{f_3}} \tag{6}$$

where

$$K = \frac{R_{sg}}{R_s} = \frac{\begin{pmatrix}\text{dynamic resistance of screen-}\\ \text{grid circuit of tube}\end{pmatrix}}{\begin{pmatrix}\text{voltage dropping resistance in}\\ \text{screen circuit}\end{pmatrix}}$$

* This assumes that the impedance between the cathode and ground is negligible in comparison with R_s, a condition always satisfied in ordinary designs.

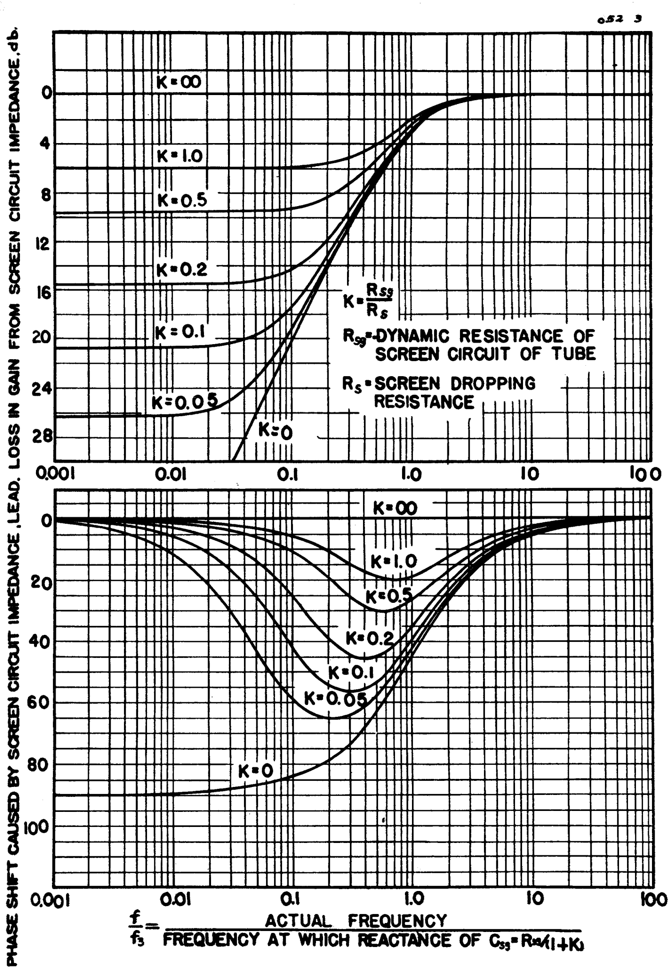


Figure 3. Curves giving effect on amplification of resistance-capacitor combination in screen circuit

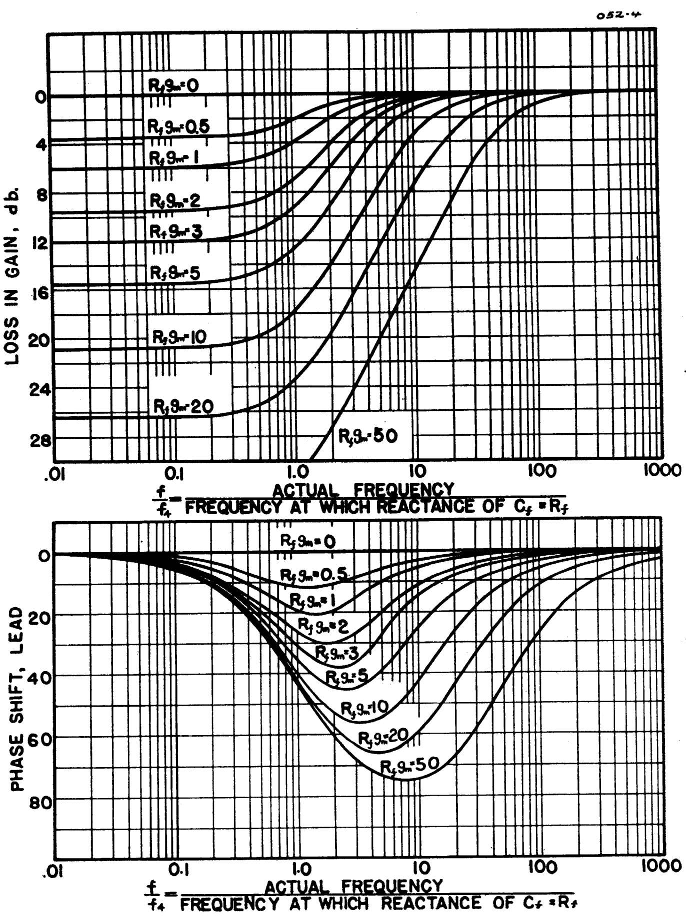


Figure 4. Curves giving effect on amplification of a resistance-capacitor combination in the cathode circuit when the screen impedance has negligible effect

$$f_3=\frac{1+K}{2\pi C_{sg}R_{sg}}$$

= frequency at which the reactance of C_{sg} equals the resistance formed by R_s and R_{sg} in parallel

Values of β are given in convenient form in figure 3 for this case, under the assumption that $\mu_{sg}/\mu_s=1$.

Examination of figure 3 shows that the resistance-capacitor combination in the screen circuit can cause a large loss in amplification at low frequencies and give rise to considerable phase shift. *In order for these effects to be negligible at a given frequency, it is necessary practically that the reactance of the screen by-pass capacitor* C_{sg} *at this frequency be small compared with the dynamic screen resistance* R_{sg} *of the tube.*

The lack of appreciation of the influence of impedance in the screen circuit on the amplification characteristic is indicated by the fact that information upon the essential constant R_{sg} of the pentode tube is not given in any of the tube data books. The designer must accordingly either measure this constant himself or infer its value from the plate resistance of the same tube operated as a triode. If m denotes the ratio of plate to screen d-c currents for pentode operation then

$$R_{sg}=(1+m)R_p \tag{7}$$

where R_p is the plate resistance of the tube connected for triode operation with the plate current the same as the total d-c space current in pentode operation.

Effect of Bias Impedance on Amplification

Impedance between the cathode and ground such as supplied by R_fC_f in figure 1*a* affects the amplification as a result of the fact that amplified signal currents flow through this impedance. This develops a voltage drop between cathode and ground that is superimposed upon the signal voltage and modifies the amplifier output.

The effect of an impedance Z_f between cathode and ground can be determined as follows:* The current through the impedance Z_f produced by the voltage e_s between cathode and control grid of a pentode tube is $g_me_s\beta$, which develops a voltage E_f across the impedance Z_f that is $g_me_s\beta Z_f$. The voltage e_s acting between grid and cathode is related to E_f and the applied signal E_s by the expression $e_s=E_s-E_f$. This gives

$$\frac{e_s}{E_s}=\gamma=\frac{1}{1+g_mZ_f\beta} \tag{8}$$

In the usual case where the impedance Z_f is supplied by a resistance-capacitor combination, as in figure 1*a*, equation 8 can be written

$$\gamma=\frac{1}{1+\dfrac{g_mR_f\beta}{1+j\dfrac{f}{f_4}}} \tag{9}$$

where

$$f_4=\frac{1}{2\pi C_fR_f}$$

= frequency at which the reactance of C_f equals the resistance R_f

* This derivation assumes that the screen grid is bypassed to the cathode so that the signal currents flowing in the screen circuit do not pass through the bias impedance, and so that voltage developed between cathode and ground does not produce voltage between screen and cathode. This is reasonably near the actual case under most conditions, but even when not the case the error will not be important under most conditions.

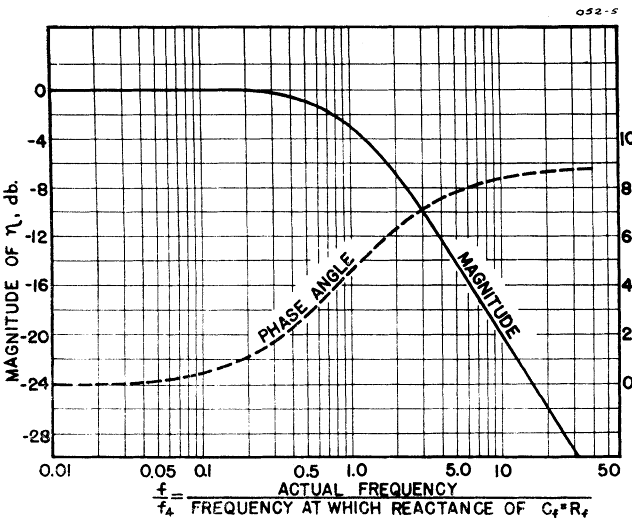


Figure 5. Value of factor η

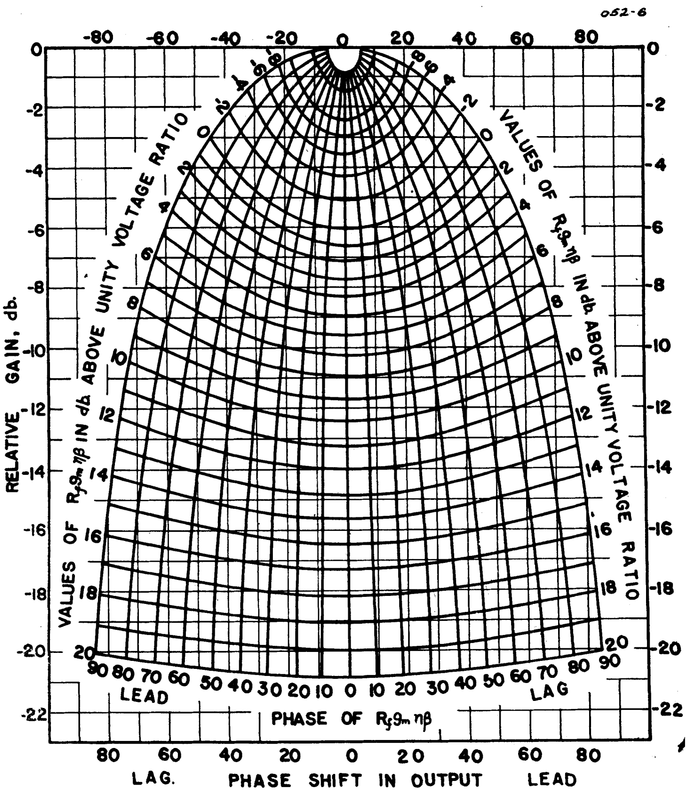


Figure 6 (right). Curves giving effect on amplification of a resistance - capacitor combination in the cathode circuit

Values of γ for the case $\beta = 1$ (zero impedance in the screen circuit) are presented in convenient form in figure 4. In the more general case, where $\beta \neq 1$, the value of γ can be obtained with the aid of figures 5 and 6 by first evaluating $\eta = 1/(1+jf/f_4)$ from figure 5, then forming the product $g_m R_f \eta\beta$, and finally evaluating γ from figure 6.

Examination of equation 9 shows that the effect of impedance between the cathode and ground in a pentode tube cannot be calculated independently of the impedance in the screen circuit, although this has been the customary procedure in the past.

Discussion

The application of the above analysis to a resistance-coupled amplifier with typical circuit proportions is illustrated in figure 7. Here curve 1 gives the magnitude and phase of the factor α. It represents the amplification characteristic as commonly calculated and is the actual characteristic when the screen and cathode impedances have negligible effect. Curve 2 gives the added phase shift and added loss in gain resulting from the screen impedance. Curve 3 gives the effect of the cathode impedance on the gain and phase shift, and is to be compared with number 4 which gives results that would be obtained by the usual procedure of neglecting the effect of the screen impedance on the action of the cathode impedance. Finally, number 5 gives the actual over-all characteristic, which is a summation of the first three curves. At low frequencies number 5 shows decidedly more falling off in amplification and greater phase shift than number 1.

A full understanding of all of the factors controlling the magnitude and phase shift of the amplification opens the way for designs that will more exactly meet particular requirements. Examples of some of the characteristics obtainable when the screen and cathode impedances are resistance-capacitor combinations of various proportions are given in figure 8. It will be noted that a sharp low-frequency cutoff is obtained by proportioning the screen impedance so that f_3 approximates f_1, and minimizing cathode impedance effects. On the other hand, when minimum phase shift is important, as is the case in amplifiers employing negative feedback, the screen and cathode impedances should both be designed so that f_3 and f_4 are considerably less than f_1.

The formulas that are presented for α, β, and γ have been subjected to extensive experimental checks, both as to magnitude and phase angle. The observed results agreed with the theoretical calculations with an accuracy about the same as that obtained when making triode-amplifier calculations based on the equivalent plate circuit of the triode amplifier.

Design of Amplifiers for Maximum Gain and for Large Output Voltage

In the design of resistance-coupled amplifiers using pentode tubes, the principal problems center about the selection of the proper coupling resistance R_c and the proper d-c plate current. When these are fixed, it is possible to select a combination of bias voltage and screen potential that will give this current, and then to proportion the cathode and screen impedances to develop these voltages and give the required characteristics at low frequencies. The grid-leak resistance is taken as the highest resistance that it is permissible to place in the grid circuit of the succeeding tube, and the coupling capacitor C_c is chosen to give the desired low-frequency response.

In selecting the proper coupling resistance and plate current it is necessary to distinguish between designs in which the principal objectives are: (1) maximum possible voltage gain, and (2) largest possible output voltage without excessive distortion and with at least reasonable gain.

Design for Maximum Voltage Gain

With a given coupling resistance R_c and plate-supply voltage E_B the voltage gain of an amplifier will increase with d-c plate current up to the point where the voltage drop in the coupling resistance is so great that the remaining voltage available for the plate of the tube is insufficient to make the plate current substantially independent of plate voltage. Practically, a slightly larger plate potential is desirable to permit a reasonable tolerance in the coupling resistance R_c. A good practical rule to follow is to consume about 80 per cent of the total voltage in the coupling resistance.

With a specified voltage drop in the

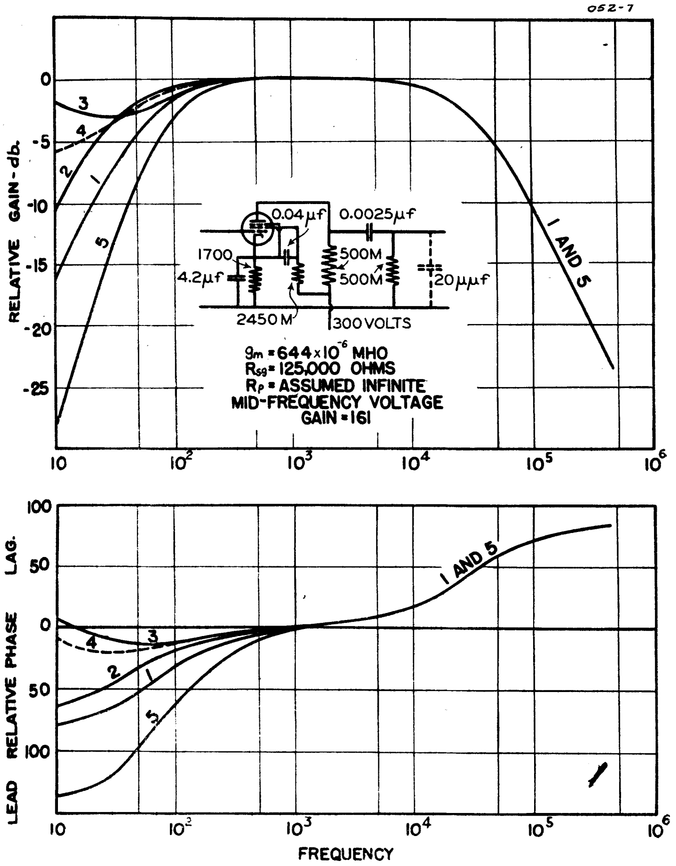


Figure 7. Characteristics of actual amplifier

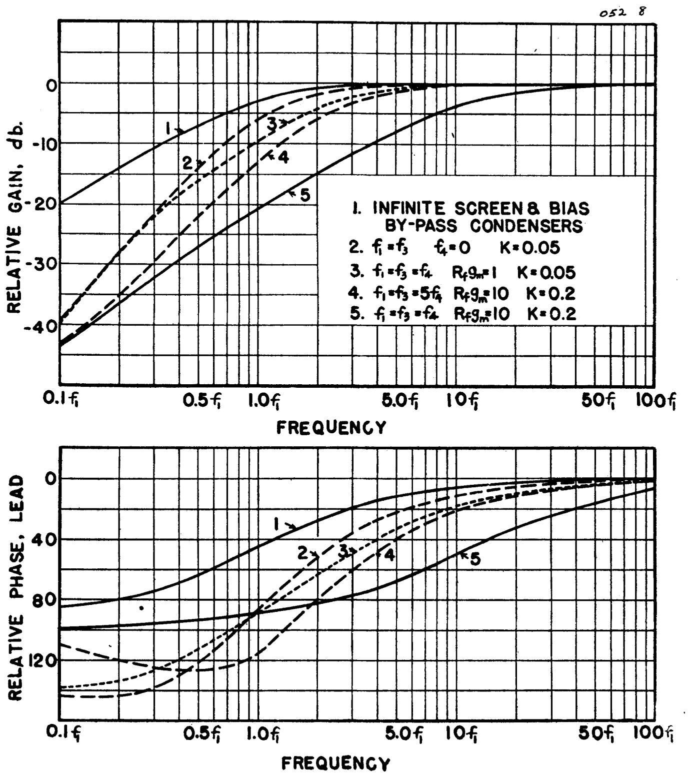


Figure 8. Typical low-frequency amplification characteristics obtainable with different resistance-capacitor combinations in screen and cathode

coupling resistance R_c, maximum gain is obtained when the coupling resistance has a definite relation to the grid-leak resistance. To obtain this relation, one first rewrites equation 1 considering the plate resistance R_p so great as to have negligible effect, an assumption very closely realized in practice.* This gives

$$A_0 = g_m \frac{R_c R_{gl}}{R_c + R_{gl}} \tag{10}$$

One also has

$$\text{Plate current} = i_p = k\left(e_g + \frac{e_{sg}}{\mu_s}\right)^n \tag{11}$$

$$g_m = \frac{di_p}{de_g} = nk\left(e_g + \frac{e_{sg}}{\mu_s}\right)^{n-1} \tag{12}$$

Voltage drop in coupling resistance =

$$E_d = i_p R_c \tag{13}$$

In these equations n and k are constants, e_g and e_{sg} are the direct voltages applied to the control and screen grids respectively, and μ_s has the same meaning as in equation 5. Substitution of (12) into (10), elimination of $(e_g + e_{sg}/\mu_s)$ by (11), and using (13) to eliminate i_p gives

$$\text{Amplification} = A_0 = nk^{\frac{1}{n}} E_d^{\frac{n-1}{n}} R_g^{\frac{1}{n}} \frac{(R_c/R_{gl})^{1/n}}{1 + \frac{R_c}{R_{gl}}} \tag{14}$$

Differentiation of A_0 with respect to R shows that

$$R_c \text{ for maximum gain} = \frac{R_{gl}}{n-1} \tag{15}$$

Equations 14 and 15 express a number of important properties of resistance-coupled amplifiers using pentode tubes. The exponent n in these equations can be expected to have values of the order of 1.5 to 2.25, with values of 2 or a little less typical for the usual operating conditions where the plate current is somewhat less than the rated value. Accordingly, the optimum coupling resistance R_c is roughly equal to the grid-leak resistance R_{gl} although the gain is not critical with respect to the ratio R_c/R_{gl}, as shown in figure 9.

The proper design procedure to obtain maximum gain is to make $R_c = R_{gl}$ and then check f_2 to see if the high-frequency response is satisfactory. If the high-frequency response is poorer than desired, R_c is lowered as necessary, and the resulting loss in gain accepted as the price paid to obtain the desired high-frequency characteristics.

Design for Large Voltage Output

The maximum output voltage is limited on the negative half cycle of the applied signal by the fact that the instantaneous plate current can never be less than zero, and on the positive half cycle by the fact that the instantaneous potential at the plate of the tube (direct voltage at plate minus peak a-c output voltage) must not be so low as to cause the plate current to cease to be independent of plate potential. With a given ratio R_c/R_{gl}, maximum output voltage occurs when these two limits are simultaneously reached with a sine-wave signal. The relations involved are approximated by the following analysis. Let

E_B = plate-supply voltage

$$r = \frac{\begin{pmatrix}\text{lowest plate potential at which plate} \\ \text{current is independent of plate voltage} \\ \text{when plate current is twice the d-c} \\ \text{value}\end{pmatrix}}{E_B}$$

$E_d = i_p R_c$ = direct-voltage drop in coupling resistance

i_p = d-c plate current

If the instantaneous plate current is to

* This approximation can be avoided by interpreting R_{gl} in equations 10, 14, and 15 to be the resistance formed by the plate resistance of the tube in parallel with the actual grid-leak resistance. The results then obtained are correct to the extent that the combined resistance varies much more slowly with plate current than does the transconductance. These remarks also apply to equations 16 and 17 and the related text.

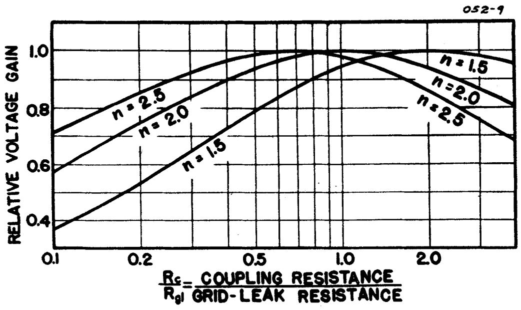
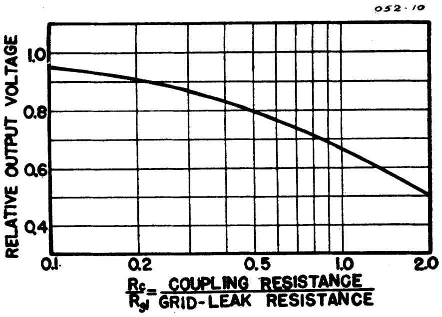


Figure 9 (left). Relative gain obtained with constant voltage drop in coupling resistance

Figure 10 (right). Effect of R_c/R_{gl} on output voltage obtainable



swing from normal to zero on the negative half-cycles, then the peak alternating current developed in the plate circuit of the tube is equal to the d-c plate current, and one can write the following relation for the minimum plate potential reached during the cycle:

Minimum plate potential =

$$E_B - E_d - i_p \frac{R_c R_{gl}}{R_c + R_{gl}} = rE_B$$

This gives the voltage drop E_d for maximum output voltage as

$$E_d = (1-r)\left(\frac{1+\frac{R_c}{R_{gl}}}{2+\frac{R_c}{R_{gl}}}\right) E_B \qquad (16)$$

The corresponding output voltage is $\frac{R_{gl}R_c}{R_c+R_{gl}} i_p$, or

$$\text{Maximum output voltage} = \frac{(1-r)}{2+\frac{R_c}{R_{gl}}} E_B \qquad (17)$$

This equation shows that the relative output voltage increases as R_c/R_{gl} is reduced, with the exact relation given by figure 10.

The practical procedure for obtaining maximum output voltage is to select coupling resistance R_c that makes R_c/R_{gl} a reasonable compromise between maximum gain and maximum power output. Usual values are in the range 0.2 to 0.5, corresponding to 91 per cent and 80 per cent, respectively, of the maximum possible output voltage. A value of r of the order of 0.1 to 0.2 is then assumed, and the plate current adjusted so that the voltage drop in this coupling resistance will satisfy equation 16. With these proportions the peak output voltage will be roughly $0.35E_B$, and the voltage drop in the coupling resistance will approximate 0.5 E_B.

A comparison of the amplification obtained when the amplifier is designed for maximum gain as contrasted with the design for large voltage output shows that in the former case the voltage amplification is about one-third greater.

Effect of Plate-Supply Voltage and Grid Leak Resistance on Amplification

Examination of equation 14 shows that for a fixed value of R_c/R_{gl} the gain is proportional to the voltage drop $E_d^{(n-1)/n}$ (and hence to $E_B^{(n-1)/n}$), and also to $R_{gl}^{1/n}$. In practice this means that the amplification obtainable from a resistance-coupled amplifier, irrespective of whether designed for maximum gain or large output voltage, can be expected to be proportional to something between the square root and cube root of the plate-supply voltage, and between the square root and two-thirds power of the grid-leak resistance.

Discussion

The relations represented by equations 14–17 should be regarded as approximations indicating the main factors and trends without giving highly accurate answers. Experimental results show that the formulas do this very satisfactorily, and that the quantitative results are fairly close to those considered typical in ordinary practice. Exact results could be obtained only by specification of the allowable distortion, and a study of the dynamic characteristic of the amplifier.

The designs for resistance-coupled amplifiers with pentode tubes commonly suggested in tube manuals[2] approximate the proportions recommended here for large output voltage, and lead to allowable output voltages that are generally within ten per cent of those calculated by equation 17.

References

1. Radio Engineering (a book), F. E. Terman. Second edition, pages 174–81.

2. See IRE Standards on Electronics, 1938, page 6.

3. RCA Receiving Tube Manual, *RC*-13, page 176.

Discussion

For discussion, see page 1133.