Verification and Validation of a 2D energy based peridynamic state-based failure criterion

Christian Willberg, Martin Rädel and Falk Heinecke

christian.willberg@dlr.de

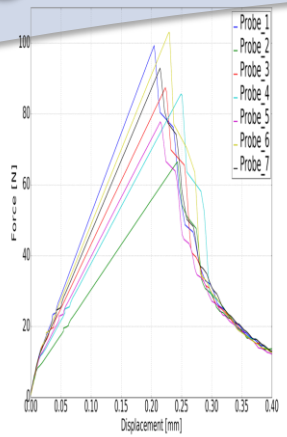
DLR

Motivation – State-of-the-art design process in aeronautics

Design criteria

- **Fatigue**
- Stability
- ***Damage tolerance***
- Plain and bearing strength
- ...

$[45 / -45 / 0 / 90]_s$
$[0 / 60 / -60 / 0]_s$
...

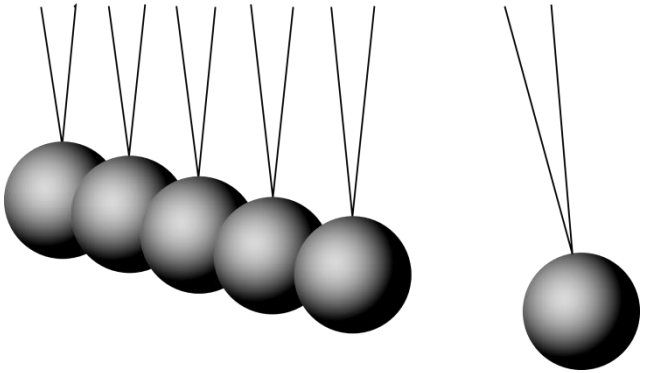


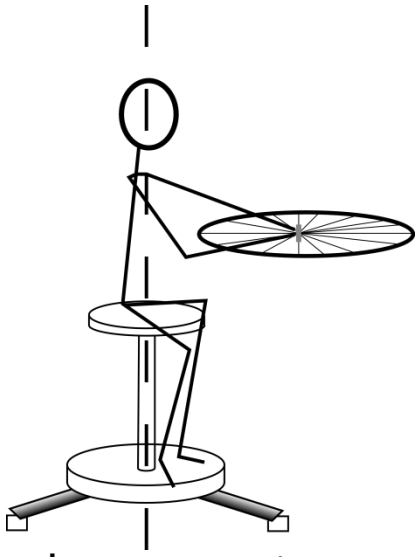
Motivation - Summary

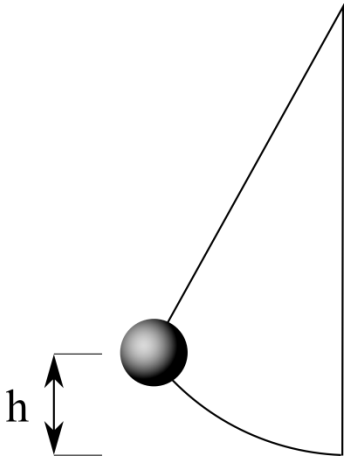
- Micromechanical or damage models are not directly used in the design process
- These models can be used to verify simplified criteria
- Robustness of damaged structures can be evaluated
- Reduction of cost-intensive experiments

- **A better understanding of damage initiation can be used to improve criteria and avoid expensive experiments.**

Physically motivated material modeling







Conservation of momentum, angular momentum and energy

- If the conservation equations are fulfilled + if the material behaviour is described, it is a physically motivated modeling

Peridynamics

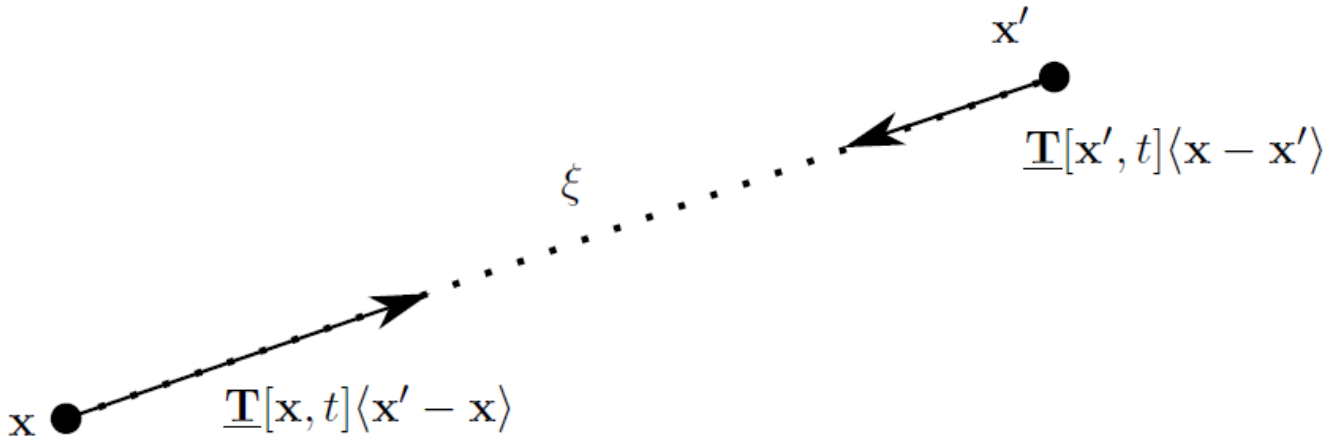
1. ~~The medium is continuous~~
2. ~~Internal forces are contact forces (interaction only with the neighbourhood)~~
3. ~~Deformations are twofold continuously derivable (in the weak formulation only simple)~~
4. **The conservation equations are fulfilled**

$$\mathrm{div}(\boldsymbol{\sigma}) + \mathbf{b} = \rho\ddot{\mathbf{u}}$$

$$\int_H (\underline{\mathbf{T}}(\mathbf{x}, t)\langle \mathbf{q} - \mathbf{x}\rangle - \underline{\mathbf{T}}(\mathbf{q}, \mathrm{t})\langle \mathbf{x} - \mathbf{q}\rangle) dV + \mathbf{b} = \rho\ddot{\mathbf{u}}$$

$$\lim_{H\to 0} \int_H (\underline{\mathbf{T}}\,(\mathbf{x}, t)\langle \mathbf{q} - \mathbf{x}\rangle - \underline{\mathbf{T}}(\mathbf{q}, \mathrm{t})\langle \mathbf{x} - \mathbf{q}\rangle) dV = \mathrm{div}(\boldsymbol{\sigma})$$

Peridynamics – ordinary state based formulation



$$\rho\left(\mathbf{x}\right)\ddot{\mathbf{u}}\left(\mathbf{x},t\right)$$
$$=\int_{\mathcal{H}}\left(\underline{\mathbf{T}}\left[\mathbf{x},t\right]\left\langle\mathbf{x}'-\mathbf{x}\right\rangle-\underline{\mathbf{T}}\left[\mathbf{x}',t\right]\left\langle\mathbf{x}-\mathbf{x}'\right\rangle\right)\mathrm{d}V+\mathbf{b}\left(\mathbf{x},t\right)$$

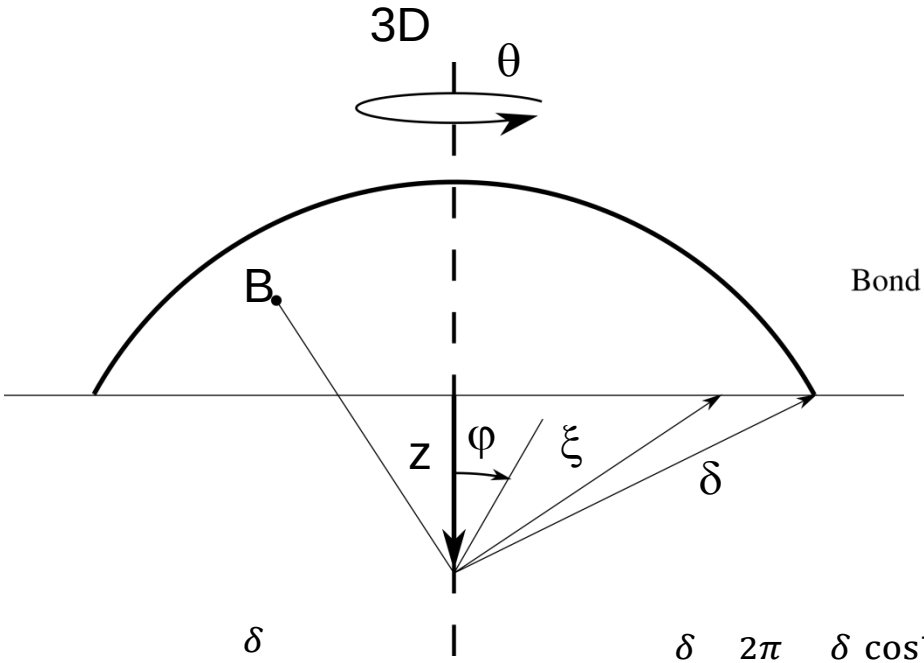
Peridynamics – 2D - ordinary state based formulation

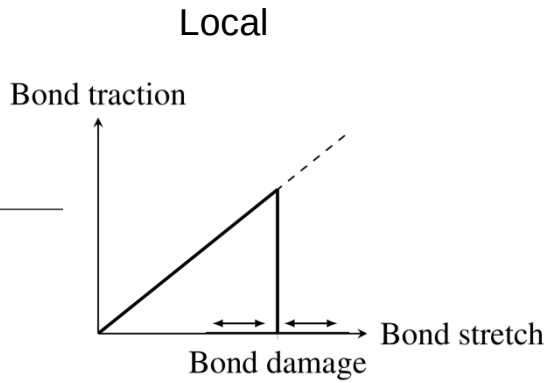
$$\underline{t}\langle \boldsymbol{\xi}, t \rangle = \frac{\underline{\omega}\langle \boldsymbol{\xi} \rangle}{m_V} \left[3K\theta \underline{x} + 15G\underline{e}^d \right]$$

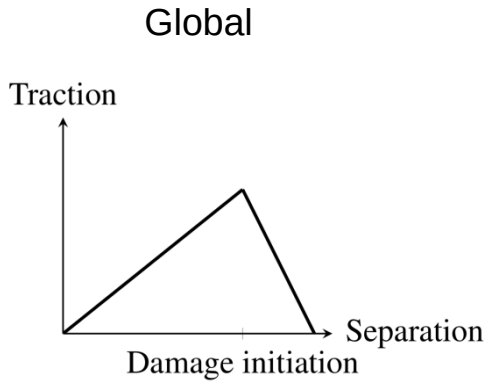
$$\underline{t}_{planestress}\langle \boldsymbol{\xi}, t \rangle = \frac{\underline{\omega}\langle \boldsymbol{\xi} \rangle}{m_V} \left[\frac{4KG}{3K+4G}\theta \underline{x} + 8G\underline{e}^d \right]$$

$$\underline{t}_{planestrain}\langle \boldsymbol{\xi}, t \rangle = \frac{\underline{\omega}\langle \boldsymbol{\xi} \rangle}{m_V} \left[\frac{4(3K-G)}{9}\theta \underline{x} + 8G\underline{e}^d \right]$$

$$\underline{\mathbf{T}} = \underline{t}\frac{\underline{\mathbf{Y}}}{|\underline{\mathbf{Y}}|}$$



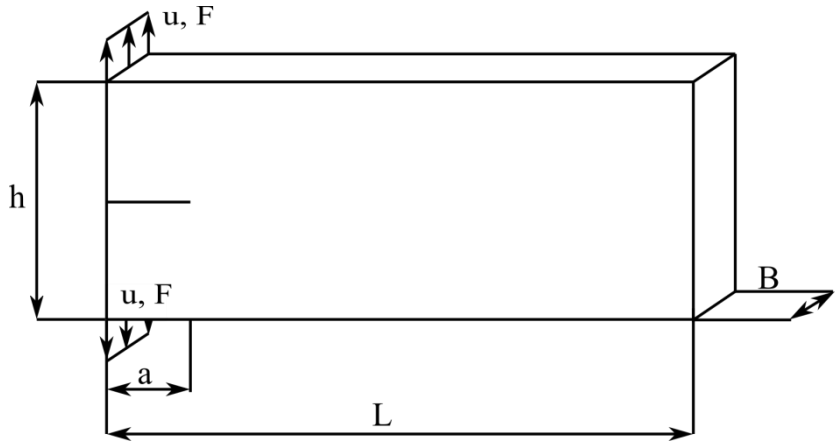




$$G = 2\int_0^{\delta}\int_{H_r} w_C \, dV_\xi \qquad G = \int_{z=0}^{\delta}\int_{\theta=0}^{2\pi}\int_{\xi=z}^{\delta}\int_{\varphi=0}^{\cos^{-1} z/\xi} w_C \xi^2 \sin\varphi \, d\varphi d\xi d\theta dz$$

$$w_{c3d} = \frac{4G}{\pi\delta^4} \qquad w_{c2d} = \frac{3G}{2\delta^3 h}$$

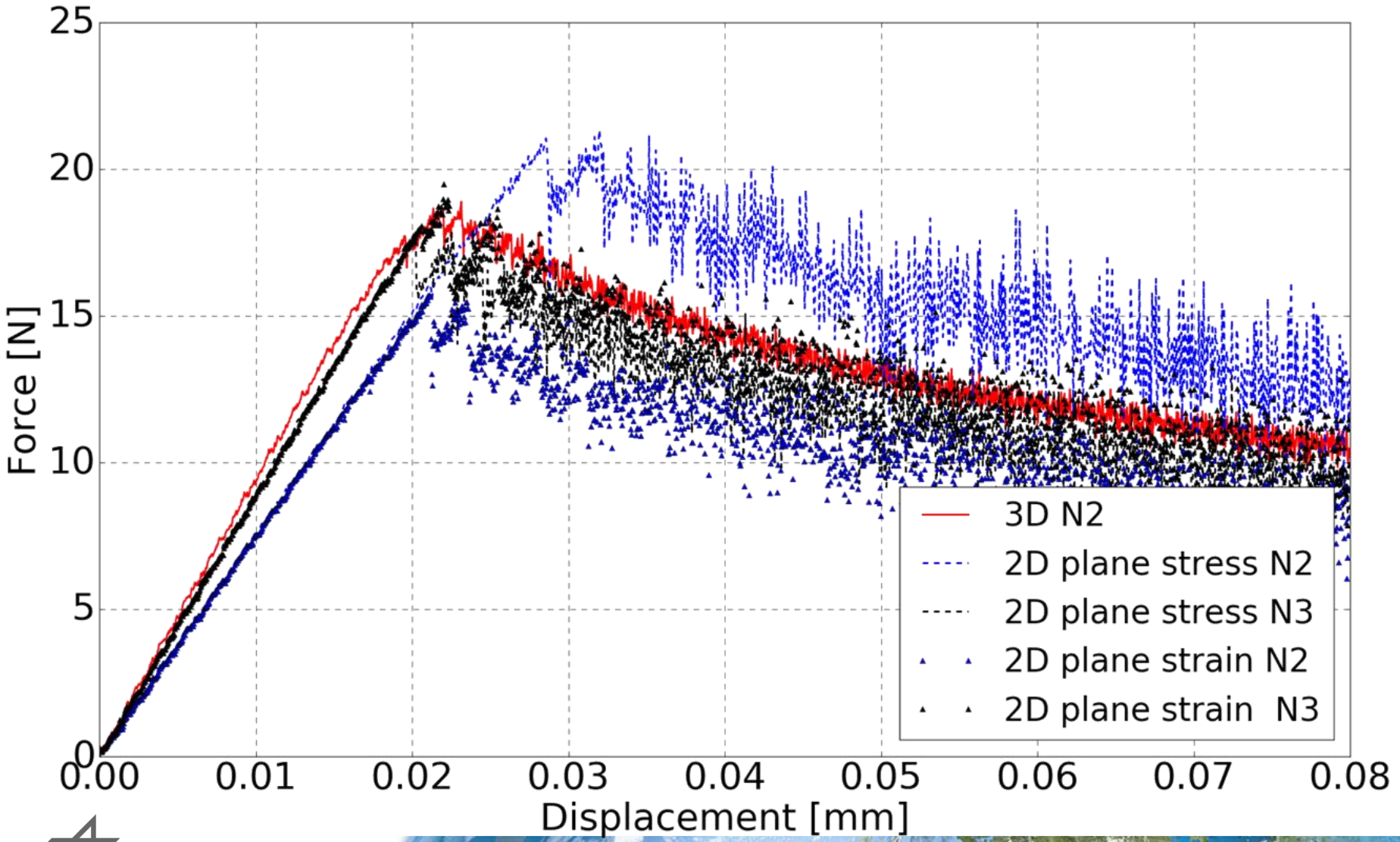
Verification



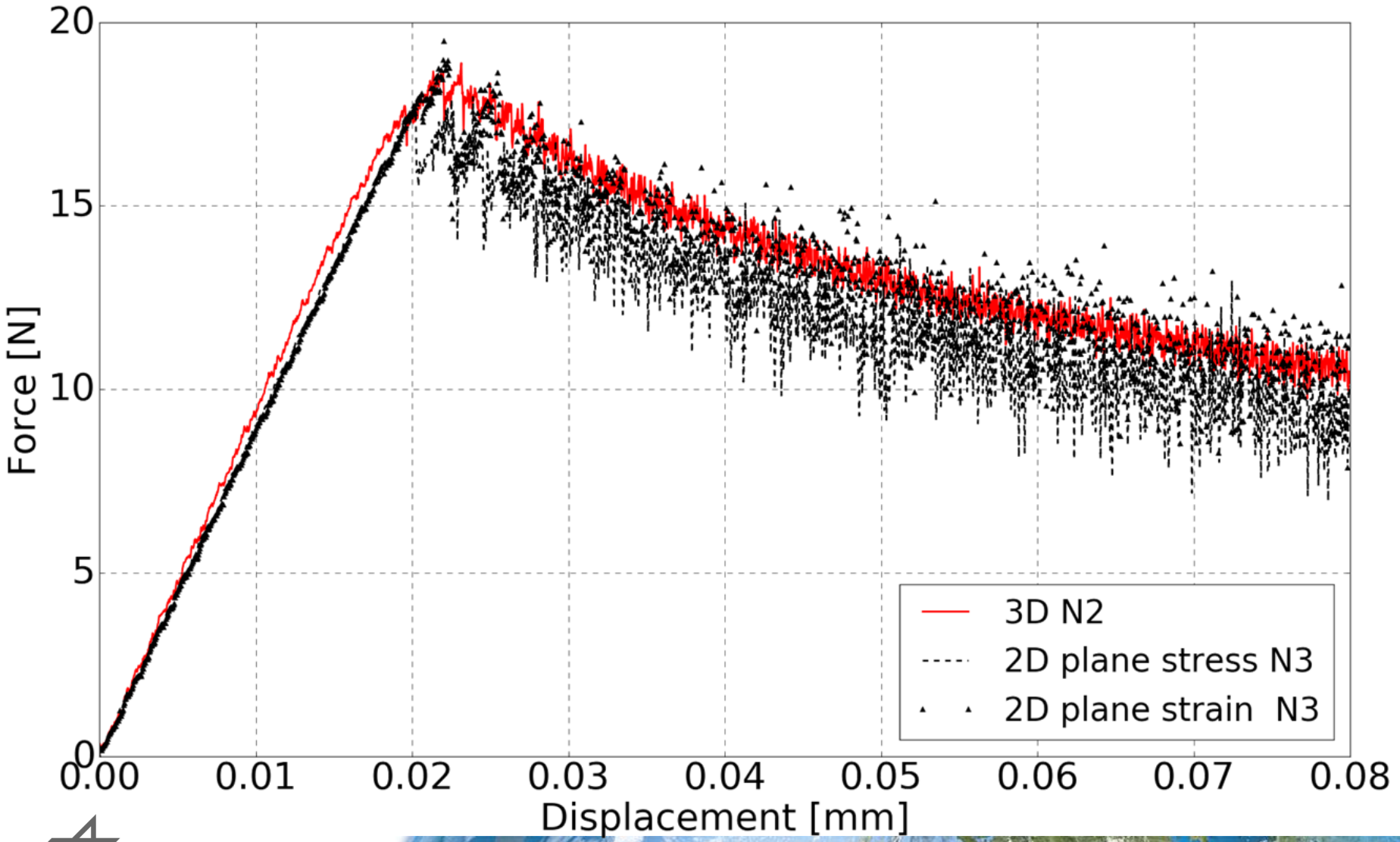
Geometry	a	h	L	B
	0.005m	0.02m	0.05m	0.003m

Material	Bulk Modulus	Shear Modulus	Density	G_0
	1.75E+09 Nm^{-2}	8.08E+08 Nm^{-2}	2000 kgm^{-3}	12 Nm^{-1}

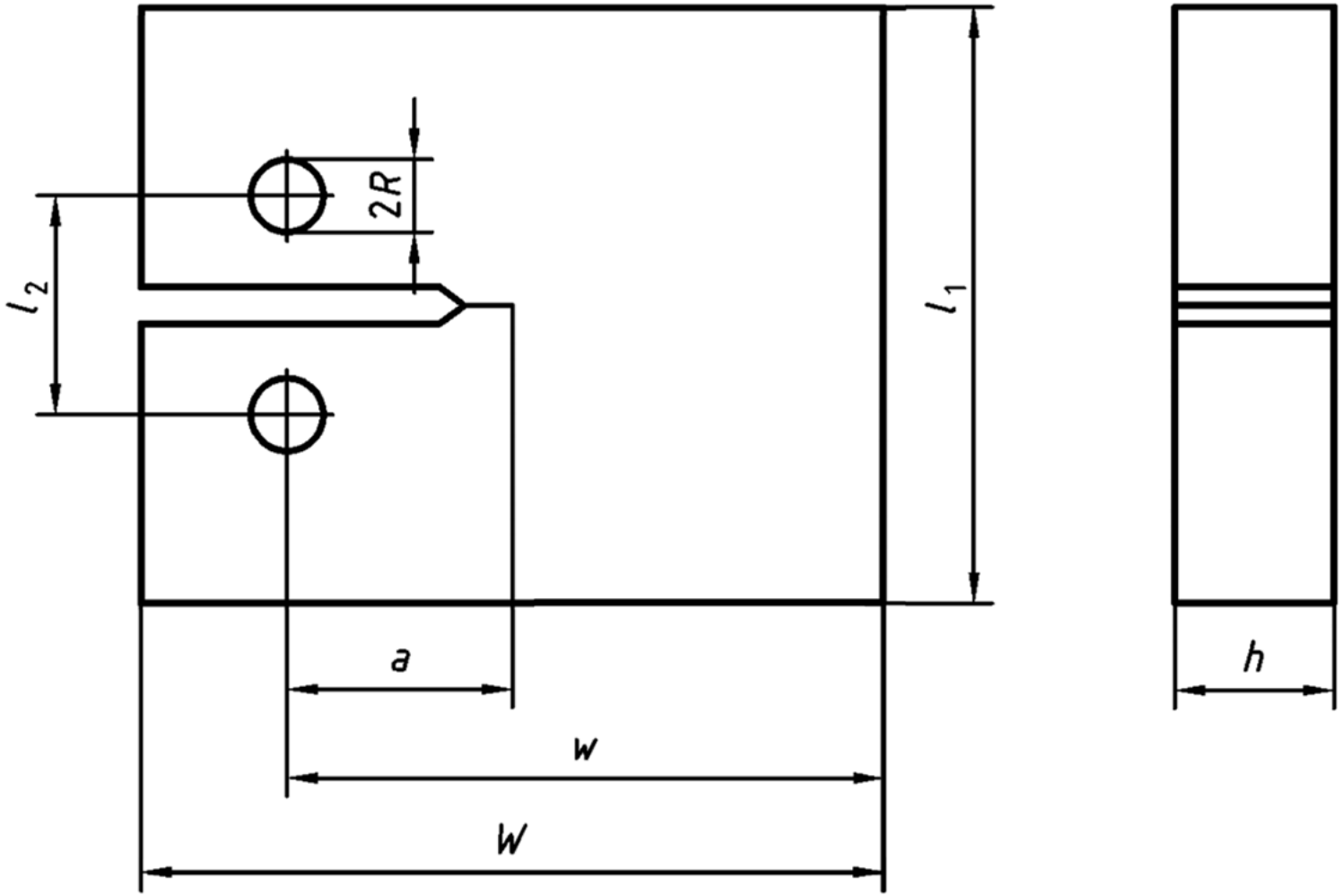
2D – 3D

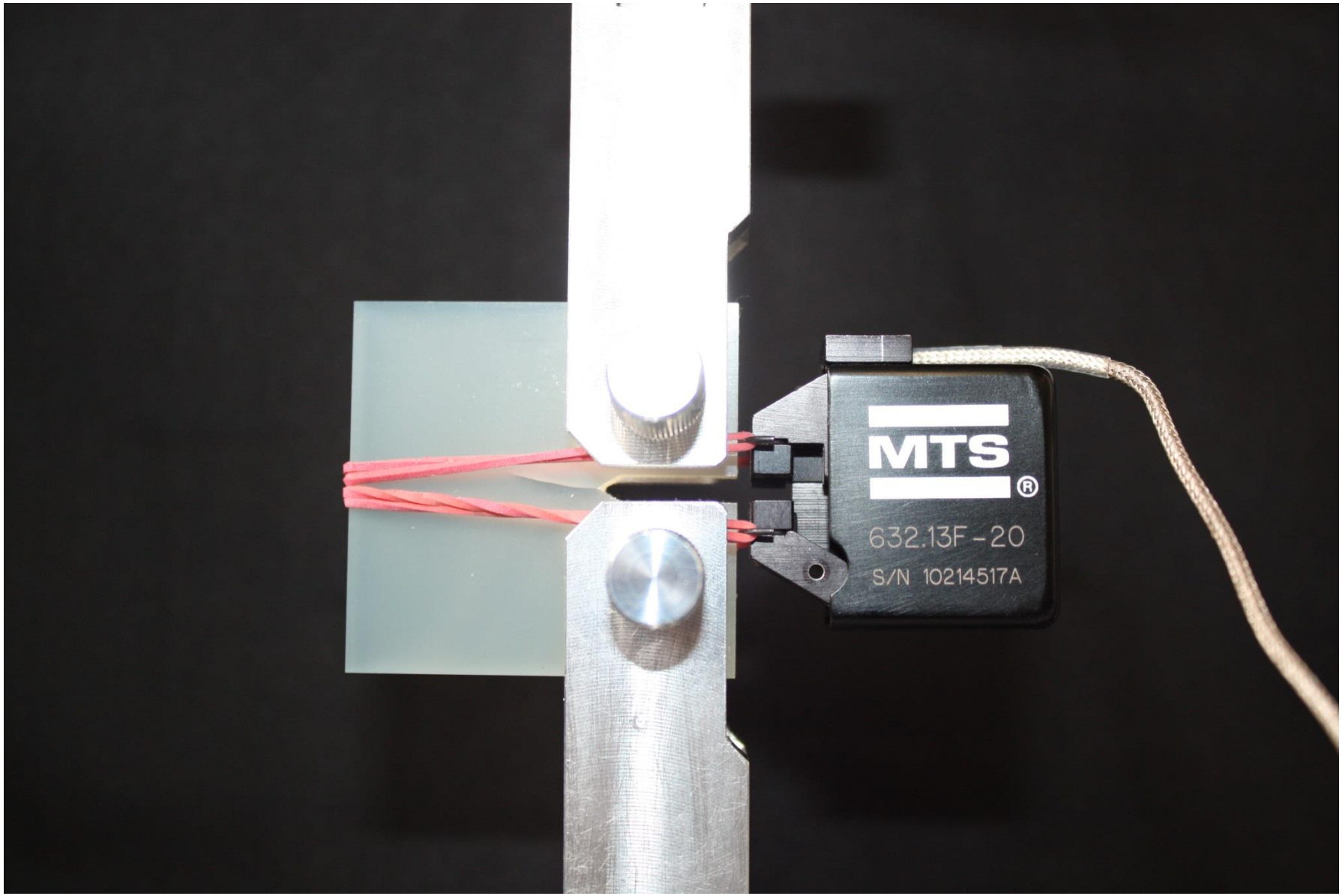

Force [N]
Displacement [mm]
0
5
10
15
20
25
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
3D N2
2D plane stress N2
2D plane stress N3
2D plane strain N2
2D plane strain N3

2D – 3D


Force [N]
Displacement [mm]
0
5
10
15
20
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
3D N2
2D plane stress N3
2D plane strain N3

Validation - Experiment – ISO 13586:2000(E)





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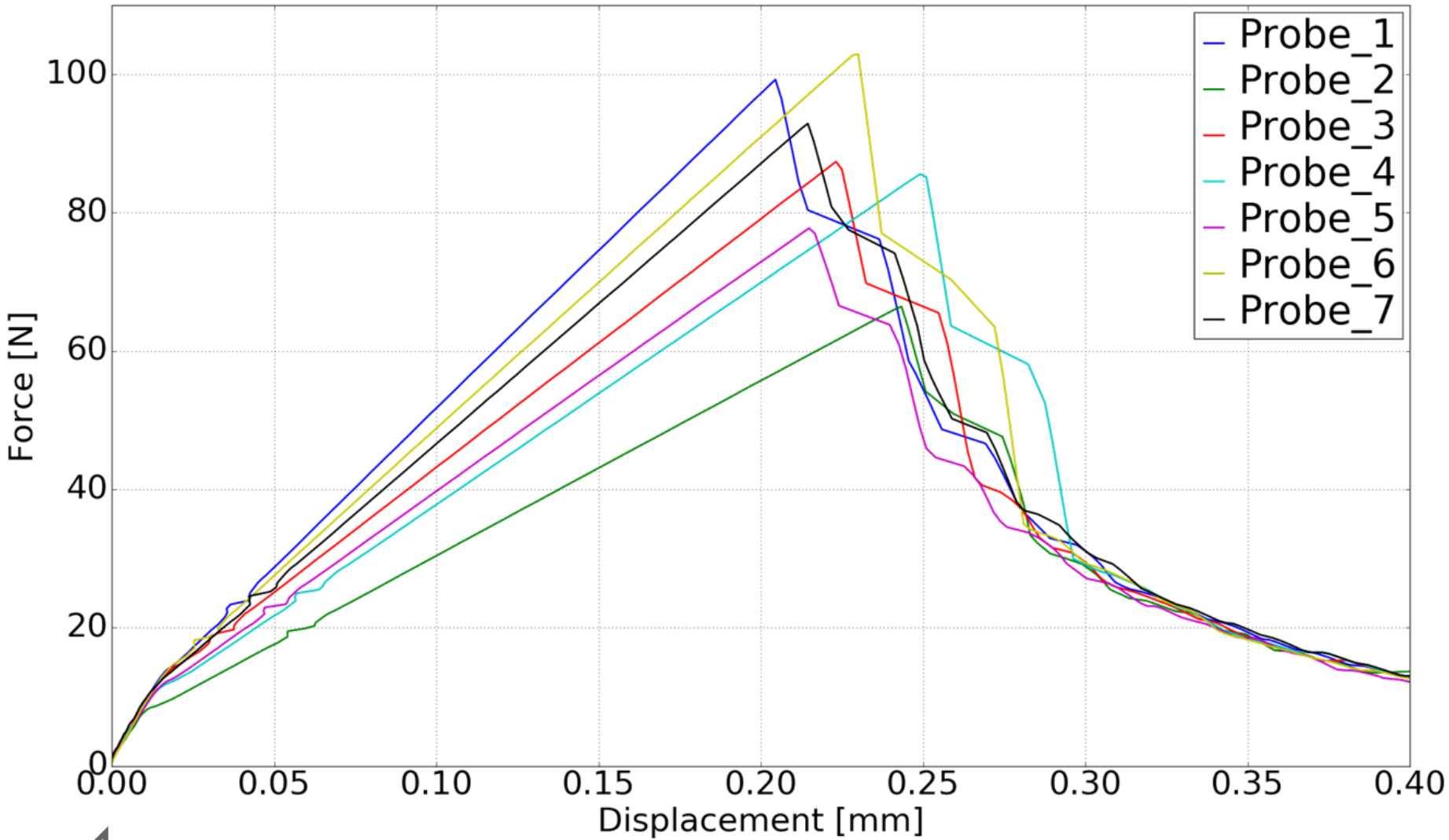
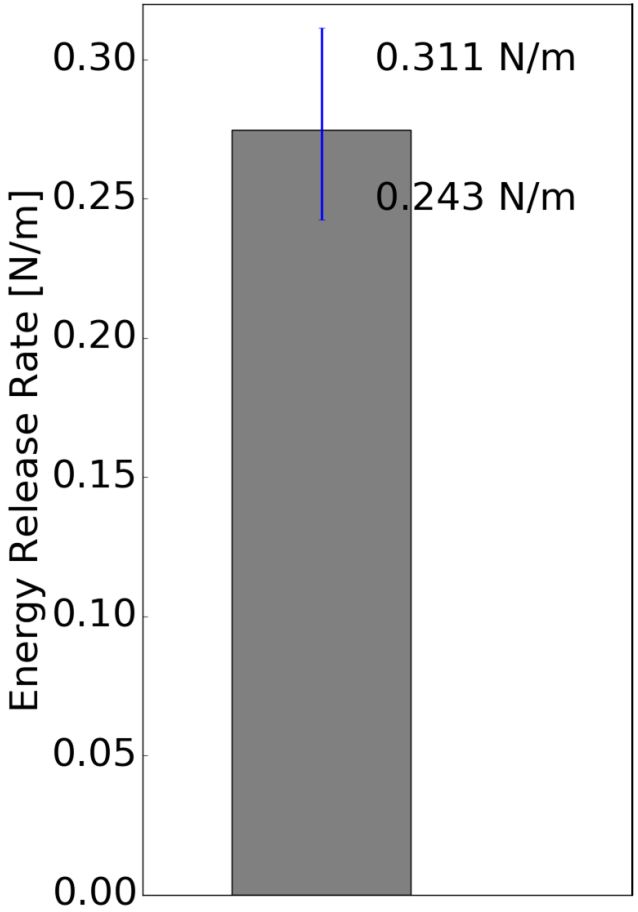
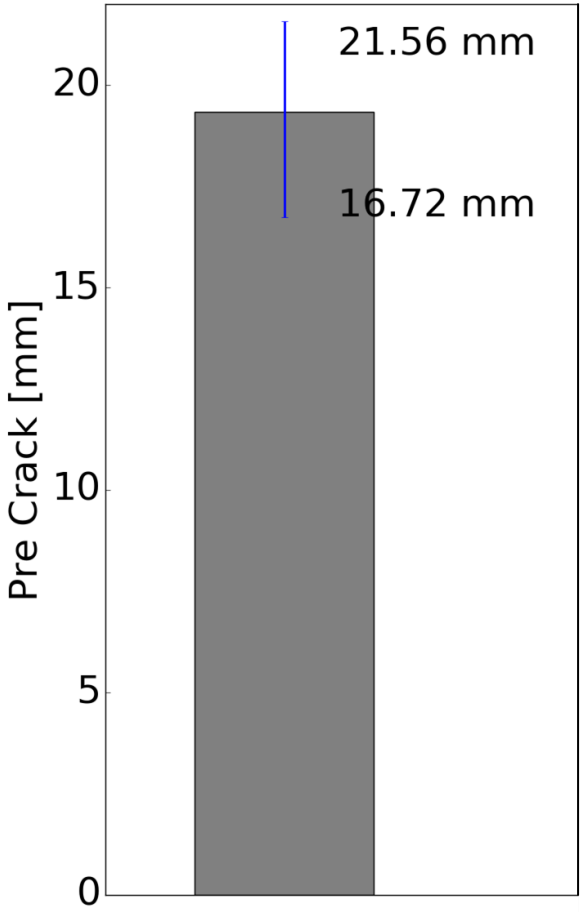
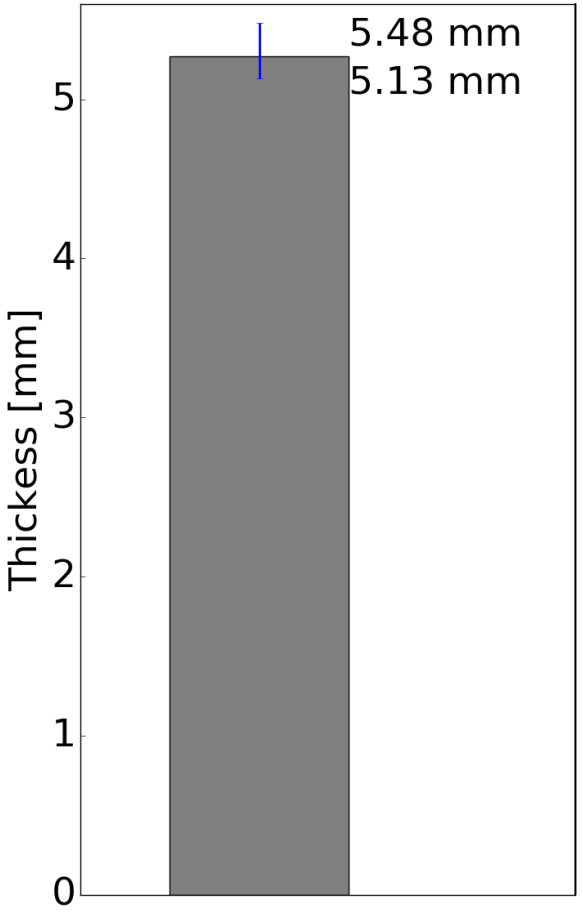

Probe_1
Probe_2
Probe_3
Probe_4
Probe_5
Probe_6
Probe_7
Force [N]
Displacement [mm]
0
20
40
60
80
100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

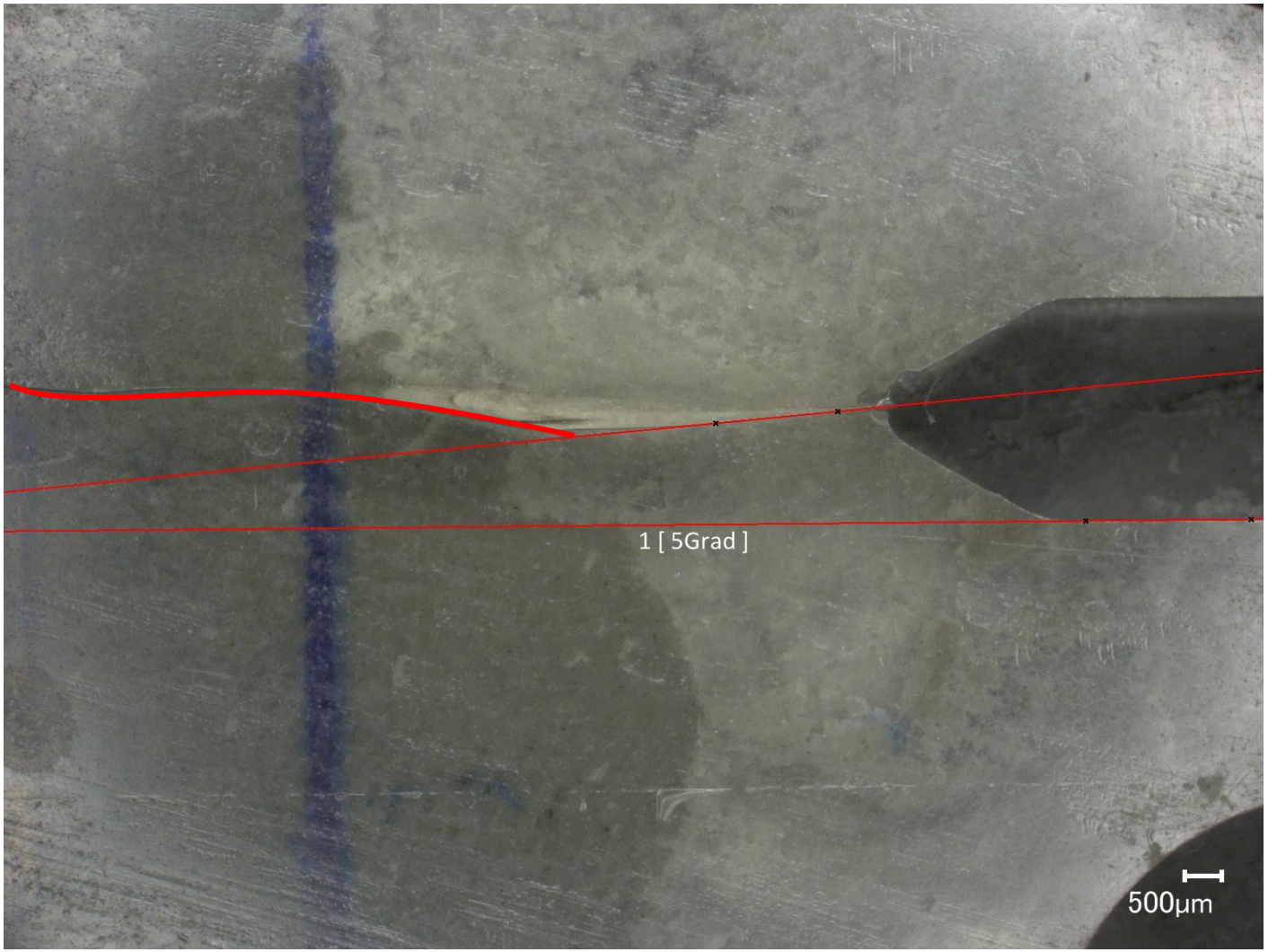
Range of geometrical data and energy release rate



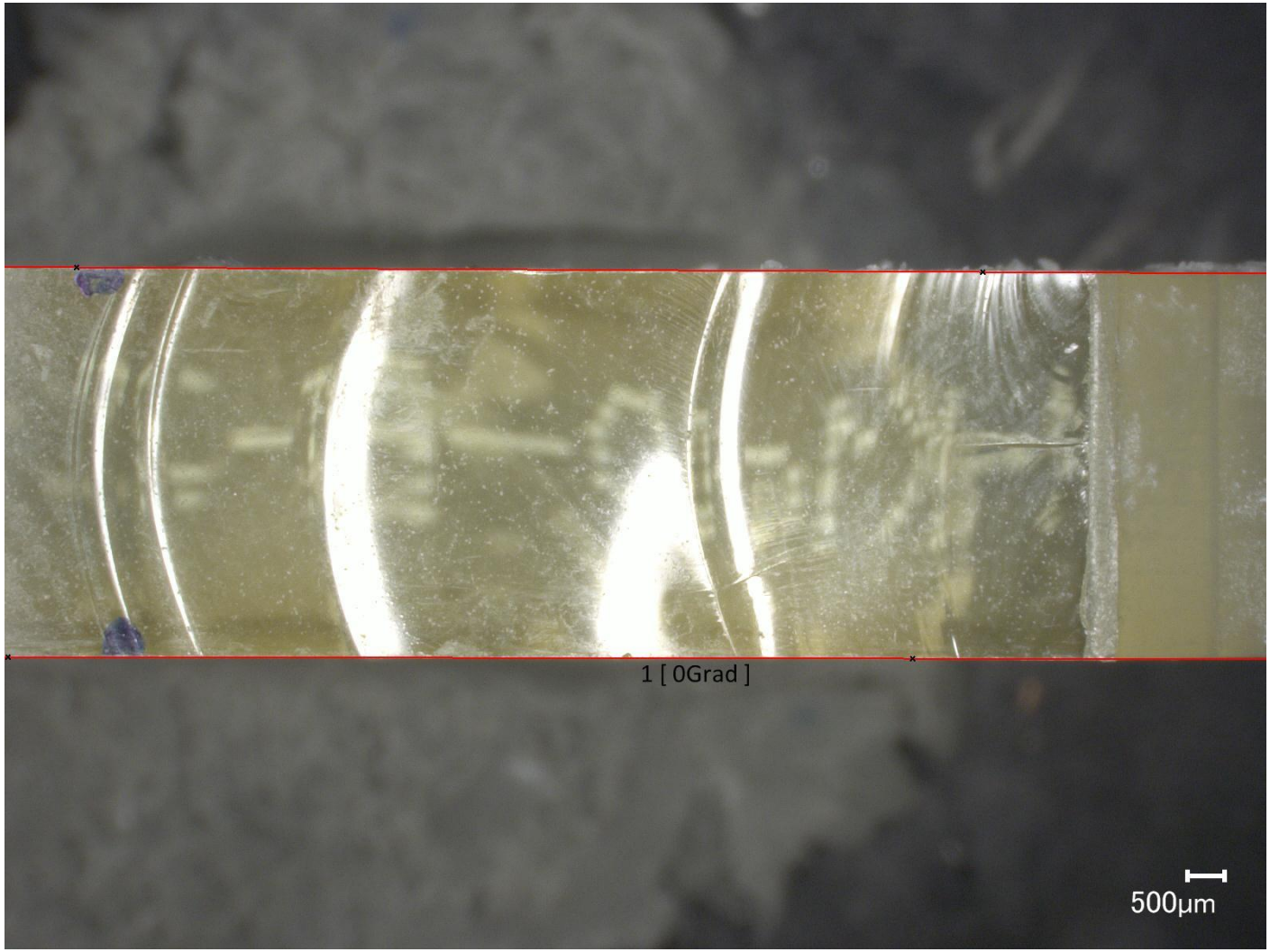


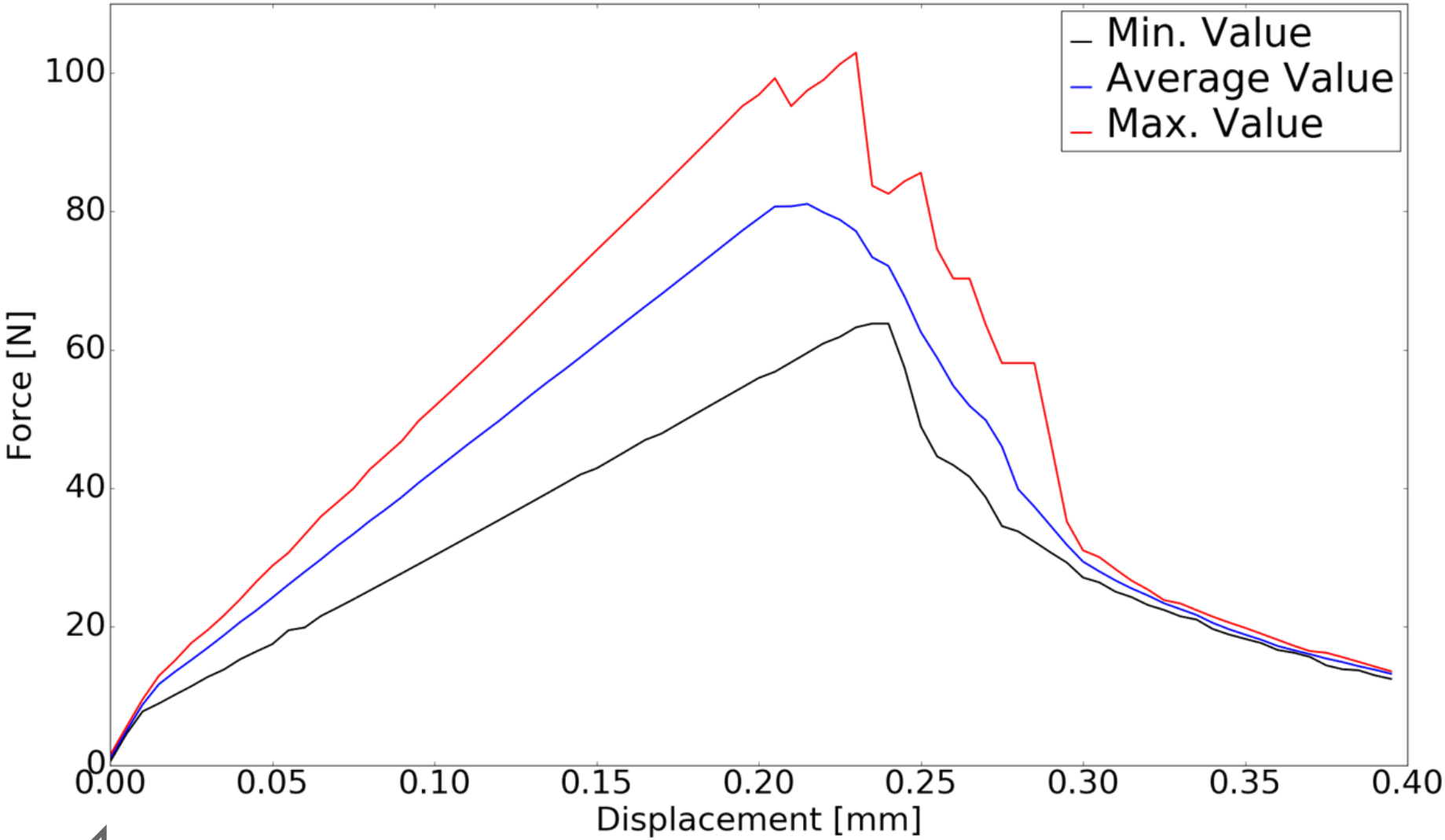


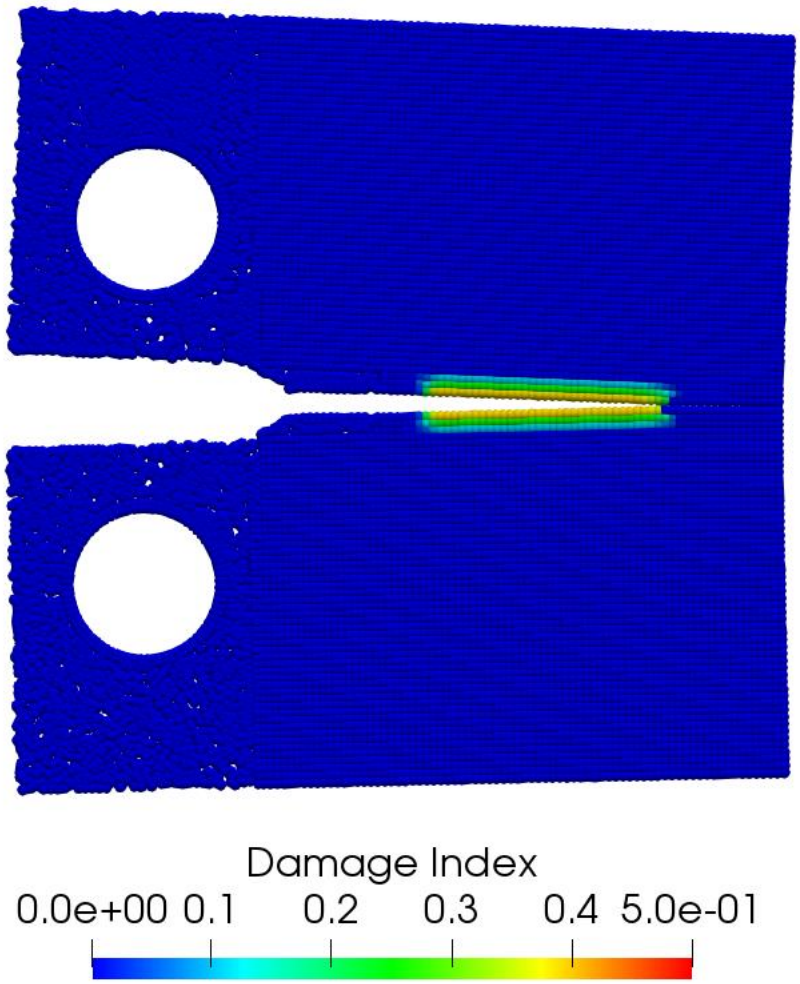
Crack propagation

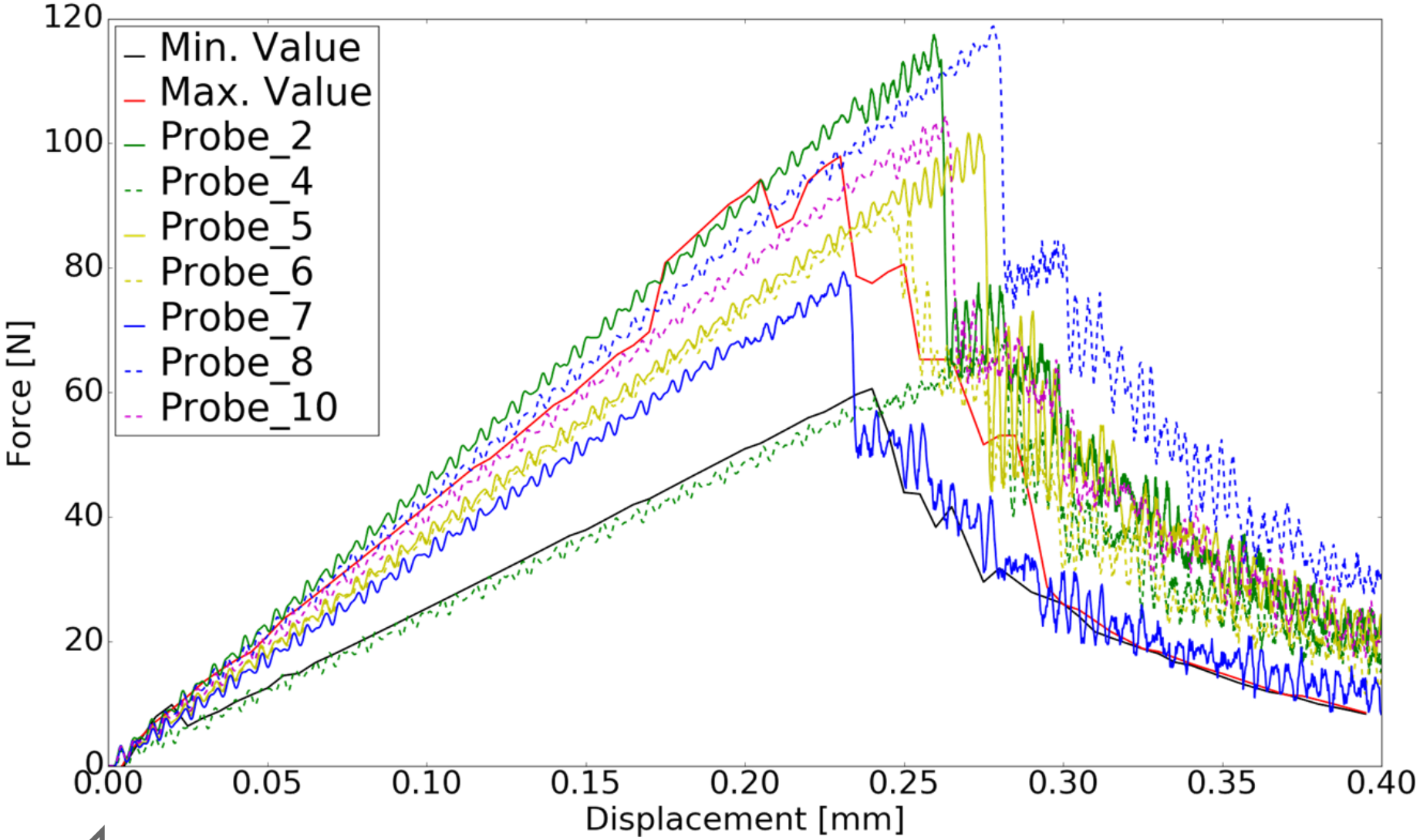


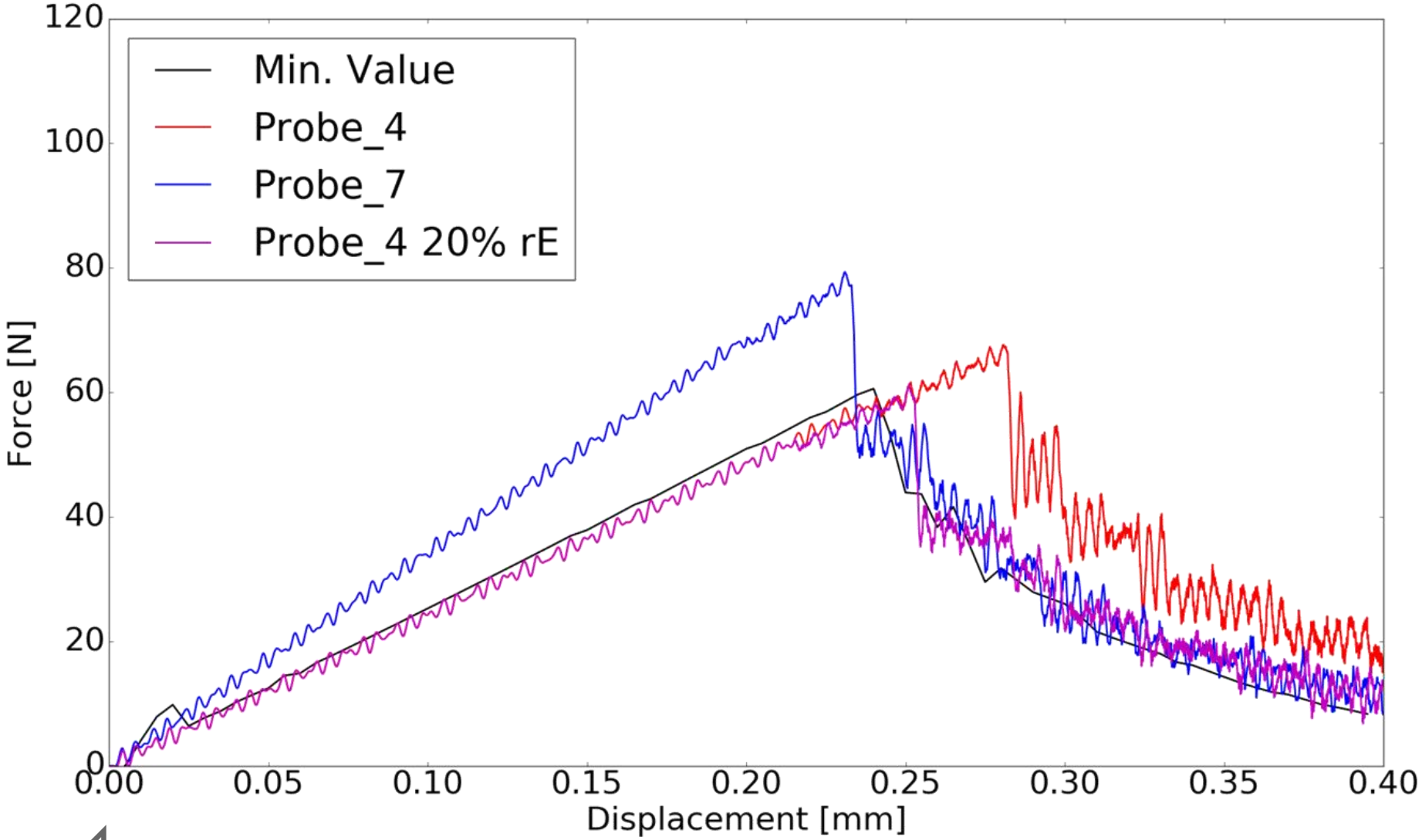
Crack front



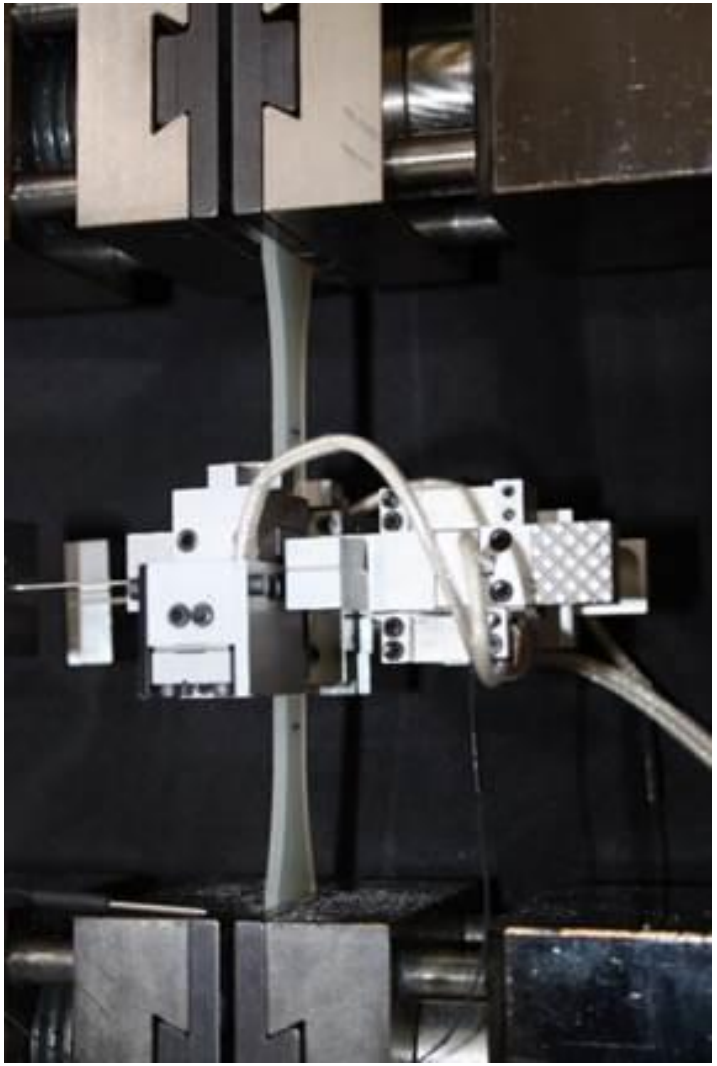

Min. Value
Average Value
Max. Value
Force [N]
0
20
40
60
80
100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
Displacement [mm]


Damage Index
0.0e+00 0.1 0.2 0.3 0.4 5.0e-01

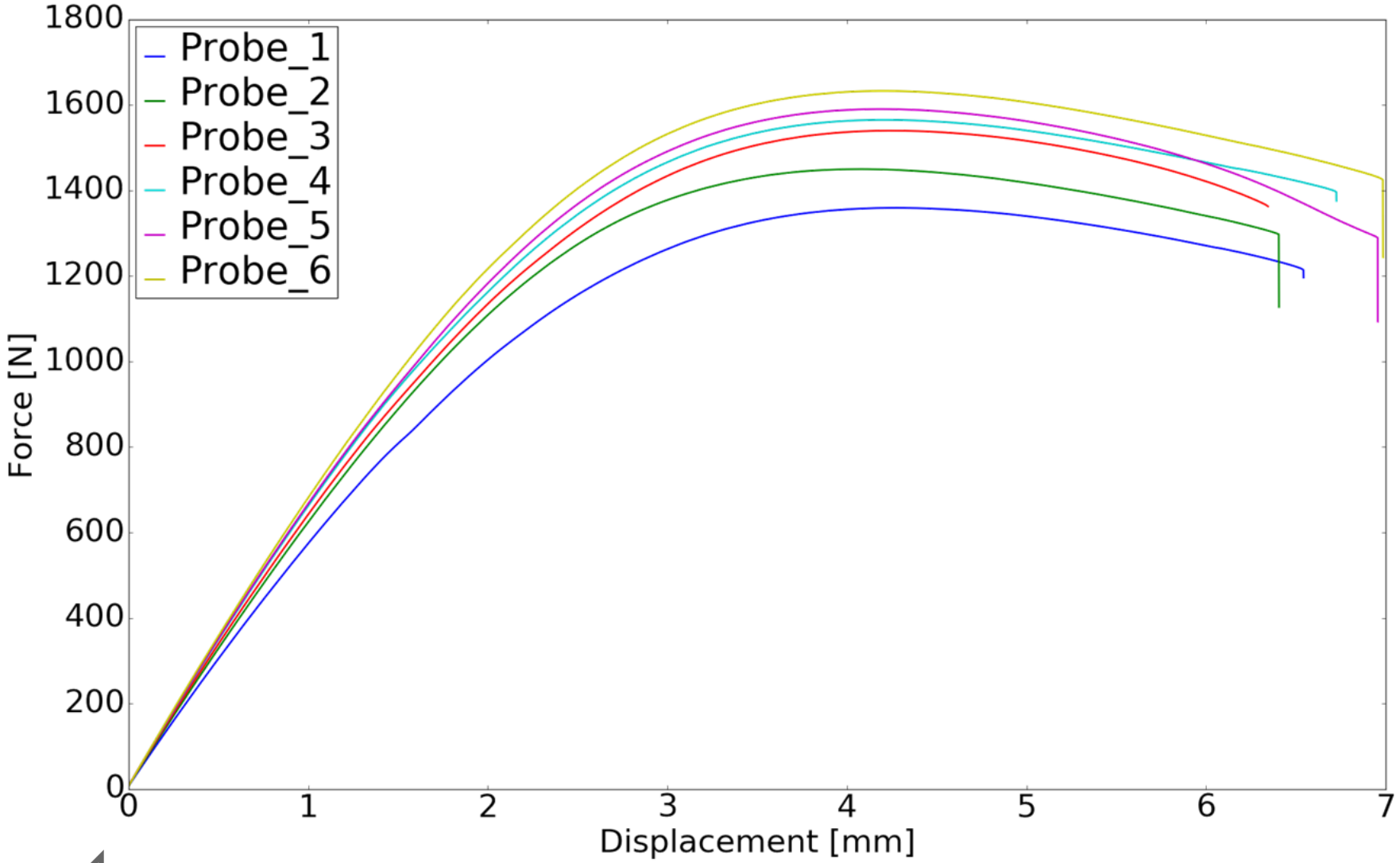

Min. Value
Max. Value
Probe_2
Probe_4
Probe_5
Probe_6
Probe_7
Probe_8
Probe_10
Force [N]
Displacement [mm]
0
20
40
60
80
100
120
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

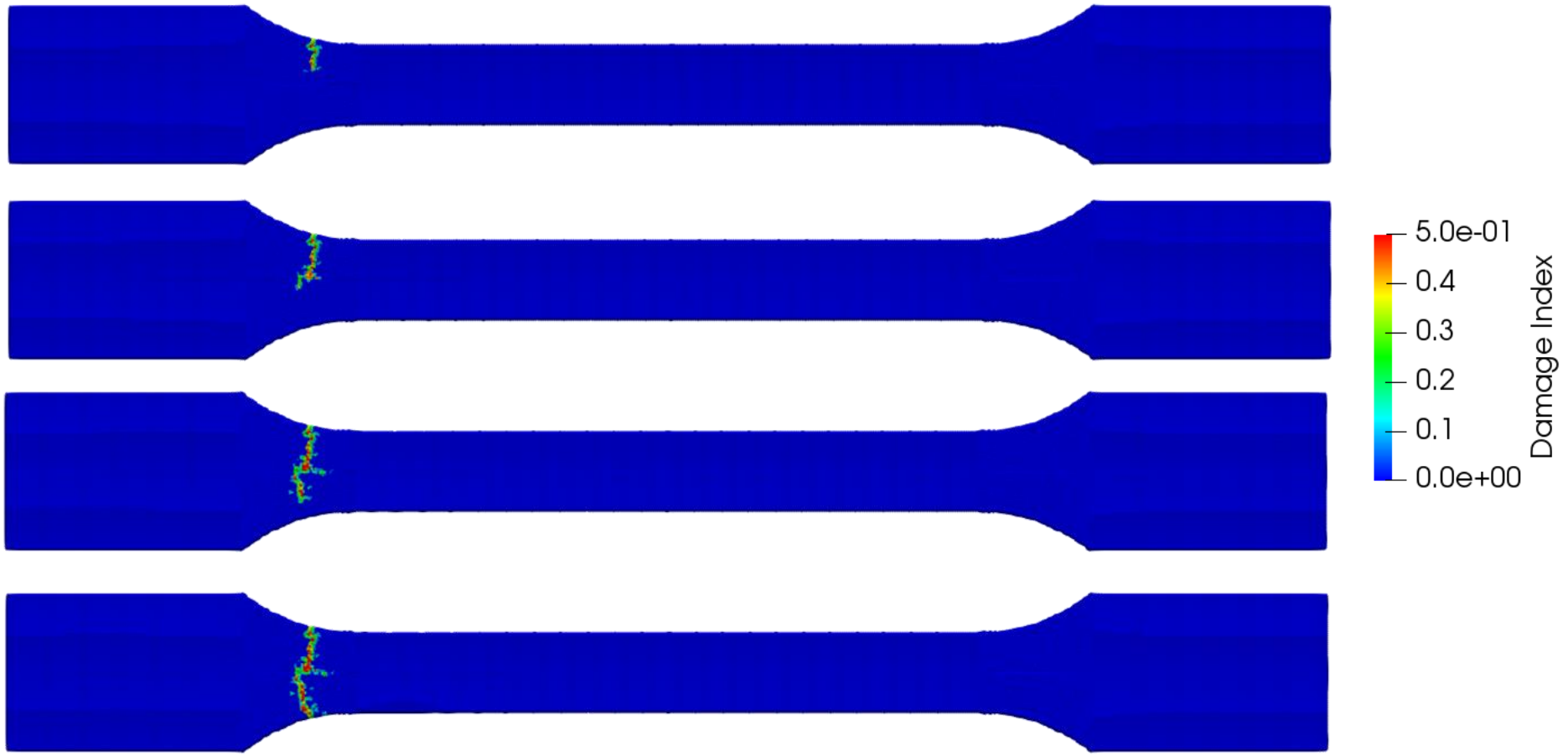

Min. Value
Probe_4
Probe_7
Probe_4 20% rE
Force [N]
Displacement [mm]
0
20
40
60
80
100
120
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

Prognosis




20 10 mm
0
10 20 30 40 50 60 70 80 90 100 10 20 30 40 50 60 70 80 90 200 10 20 30


Probe_1
Probe_2
Probe_3
Probe_4
Probe_5
Probe_6
Force [N]
Displacement [mm]
0
200
400
600
800
1000
1200
1400
1600
1800
0
1
2
3
4
5
6
7



Crack initiates by 0.85 mm

Conclusion

- 2D energy criterion has been verified
- Validation has started and the order of magnitude is reachable for KIC test
- Prognosis failed, because of missing plasticity
    - Overestimation within the KIC is maybe explainable

Acknowledgement

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Thank you!

Dr.-Ing. Christian Willberg

DLR
Institute of Composite Structures
and Adaptive Systems

Lilienthalplatz 7
38108 Braunschweig
Germany

Phone: +49 531 295 - 2232
Email: christian.willberg@dlr.de

All presented models and source code can be found here

Rädel, M. & Willberg, C. PeriDoX Repository
https://github.com/PeriDoX/PeriDoX
Doi: 10.5281/zenodo.1403015