Peridynamics: Convergence & Influence of Propabilistic Material Distribution on Crack Initiation

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Outline

Motivation

Peridynamics

Problem

Model

Results

Conclusion

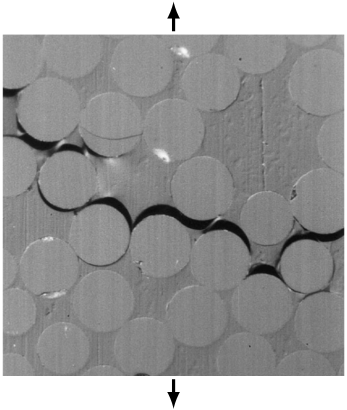
The German Aerospace Center (DLR)

Quick facts

- National aeronautics and space research centre & space agency
- Research focus: aeronautics, space, energy, transport & security
- 16 locations + 4 new in 2017
- 33 institutes
- 8000 employees
- International offices: Brussels, Paris, Tokyo & Washington D.C.

Motivation

- Challenges:
  - Exploitation of fiber reinforced plastics (FRP) lightweight potential limited
  - Missing reliability of failure predictions
- Goals:
  - Increase understanding of failure mechanisms
  - Derive improved failure criteria for preliminary design
- Use case:
  - Matrix failure in FRP
  - Origin of other failure phenomena



Matrix failure in FRP [1]

Continuum mechanics (CM) & FEM

- Assumptions [2]:
  1. Continuous medium
  2. Internal force = Local contact force
  3. $\mathbf{u}$ 2x continuously differentiable
  4. Conservation equations satisfied
- Linear momentum conservation:
  - $\text{div}(\boldsymbol{\sigma}) + \mathbf{b} = \rho\ddot{\mathbf{u}}$
  - Not defined @ discontinuities
  - 1. & 3. violated
- CM & FEM unable to capture damage

Peridynamics (PD)

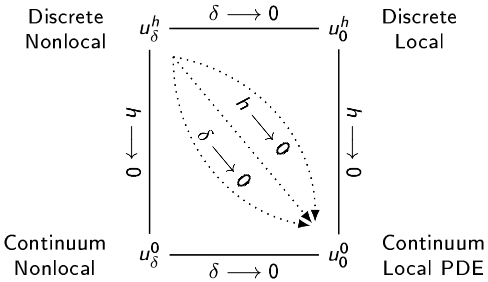
- Assumptions:
  - Conservation equations satisfied
- Linear momentum conservation:

$$\int_{\delta} (\underline{\mathbf{T}}(\mathbf{x}, t)\langle\mathbf{q}-\mathbf{x}\rangle - \underline{\mathbf{T}}(\mathbf{q}, t)\langle\mathbf{x}-\mathbf{q}\rangle) dV_{\mathbf{q}} + \mathbf{b} = \rho\ddot{\mathbf{u}}$$

- Non-local
- Defined @ discontinuities: "*[...] cracks are part of the solution, not part of the problem*" F. Bobaru
- Converges to CM solution for $\delta \to 0$
- Damage intrinsic in material models

Peridynamics
Horizon & convergence

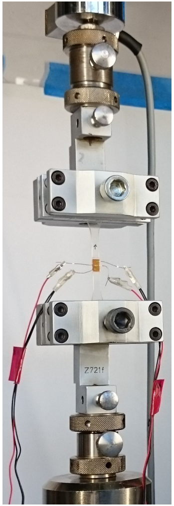
- Ideally continuous & homogeneous structure
- Standard local theory applies until failure
- PD as way to model damage and fracture
- Questions:
  - Choice of discretization?
  - Choice of horizon?
  - Convergence?
  - Identical for different base discretizations?



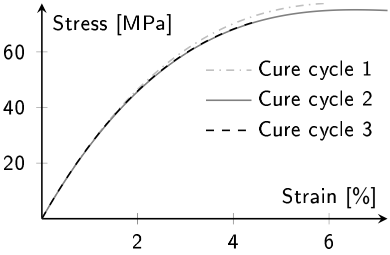
Types of convergence in PD [2]

Problem

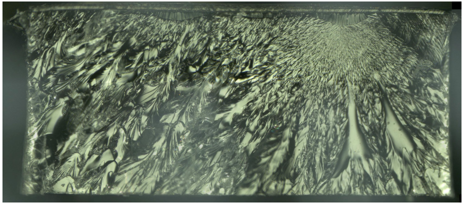
- Matrix identification
- Goal: Description of individual component material properties & failure patterns before application in complex FRP structure
- LY564 bulk resin tensile tests ISO 527-2
- 1st step: Discretization & convergence study
    - LPS material model
    - No surface correction



Static test rig



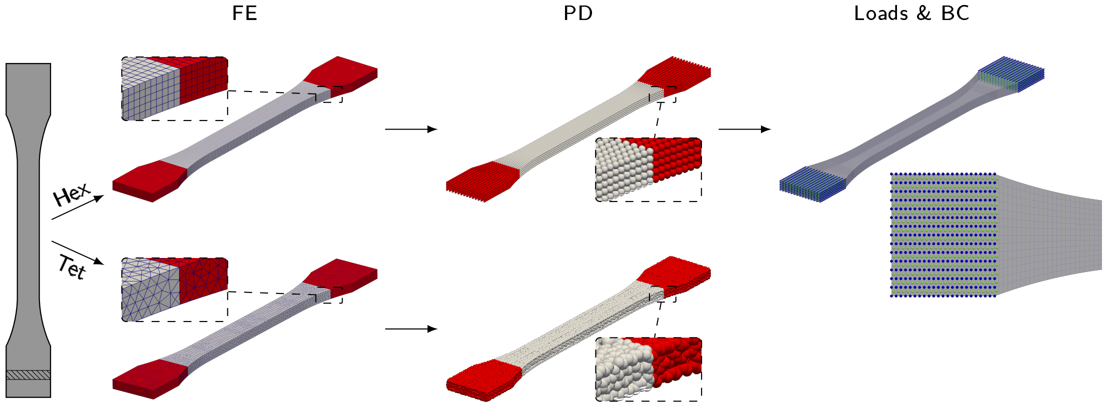
Effective LY564 stress-strain curve



Fracture plane micrograph

Model

- PD calculation via Peridigm [3] using FE input
- FE model generator including stochastic capabilities

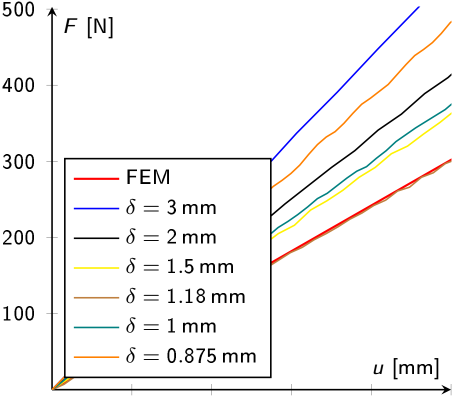


Results

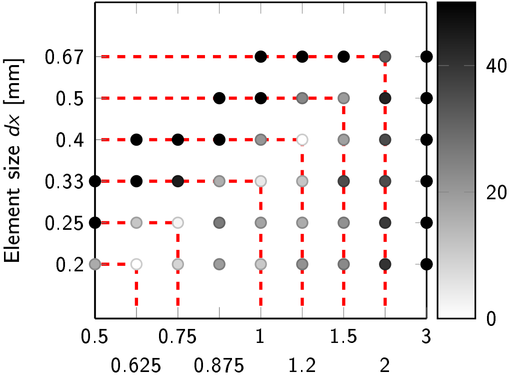
Convergence - Stiffness

- Hex: $m = \frac{\delta}{dx} \approx 3$

Element size $dx = 0.4\,\mathrm{mm}$



$|\Delta_F|$ [%] @ $u = 0.1\,\mathrm{mm}$

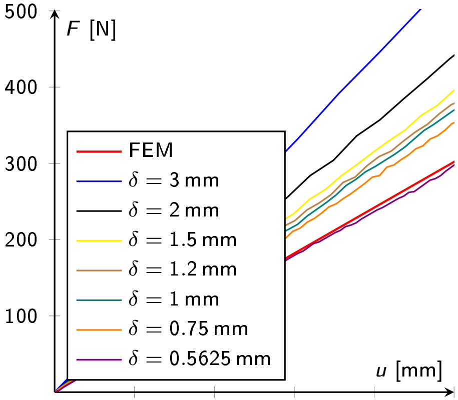


Results

Convergence - Stiffness

- Tet: $m = \frac{\delta}{dx} \approx 1.125$

Element size $dx = 0.5\,\text{mm}$



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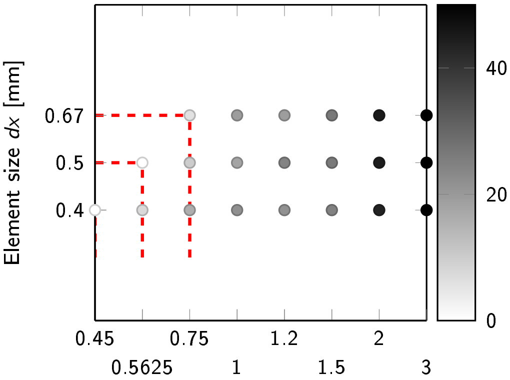
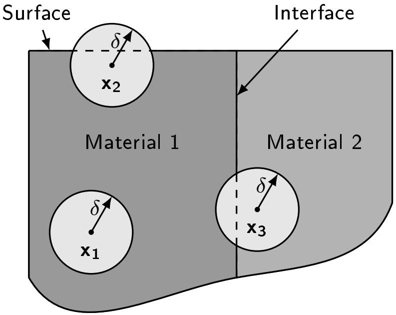


Results

Convergence - Stiffness

- Hex: $m = \frac{\delta}{dx} \approx 3$
- What about surface correction?
- PD equation assumptions:
  - Material point in single material
  - Complete family within δ
- LPS $\leftrightarrow$ PALS $\leftrightarrow$ Correspondence material models?
- LPS: No surface correction
- PALS: Influence functions
- CP: Not necessary



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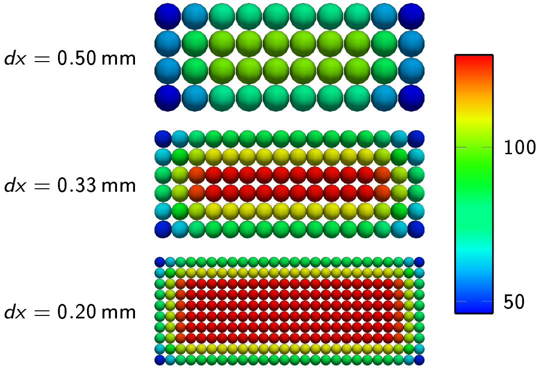

Number of Neighbors [-] for $m \approx 3$ in specimen cross-section

Results

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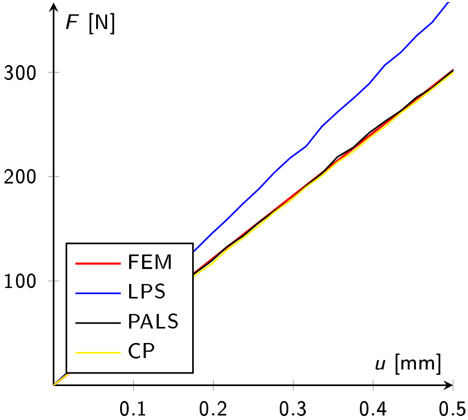

$dx = 0.5\,\text{mm}$

Results

Convergence - Stiffness

- Hex: $m = \frac{\delta}{dx} \approx 3$
- What about surface correction?
- PD equation assumptions:
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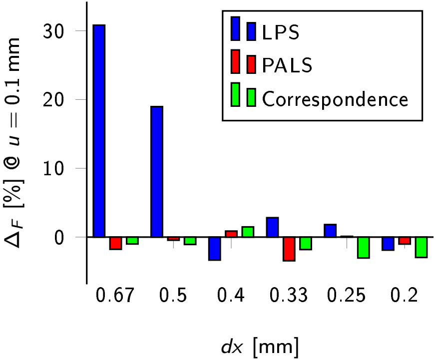


Results

Convergence - Stiffness

- Convergence $= f\,(dx, \delta, \text{Material formulation})$
- LPS material in Peridigm: no surface correction
  - Results $dx \geq \frac{t}{4}$ and $\delta > 1\,\text{mm} \rightarrow$ "all surface families"
  - No family without effect of missing surface correction $\rightarrow$ Results not viable
  - dx and δ so that majority of families without influence of surface correction $\rightarrow$ Computational expense?!
- Correspondence materials:
  - Viable option
  - Numerical problems $\rightarrow$ 3 non-collinear bonds must remain in family
  - Hourglassing problematic
- Position-aware models $\rightarrow$ promising

Results

Failure

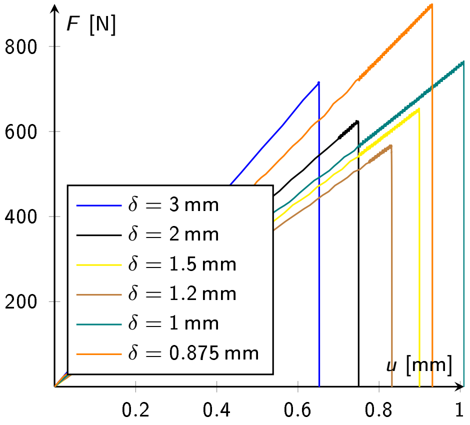
- Bond-based critical stretch models:

$$\epsilon_c = \sqrt{\frac{5\,G_c}{9K\delta}} \qquad [2, 4]$$

$$\epsilon_c = \sqrt{\frac{G_c}{\left[3G + \left(\frac{3}{4}\right)^4 \left(K - \frac{5G}{3}\right)\right]\delta}} \qquad [5]$$

- Based on Griffith for existing pre-crack
- $\epsilon_c = f\left(\delta^{-\frac{1}{2}}\right)$
- Reproducibility for various δ and adapted ϵ_c in SB-PD 1D-case for no pre-crack?

Hex, $dx = 0.4\,\mathrm{mm}$



Results

Failure

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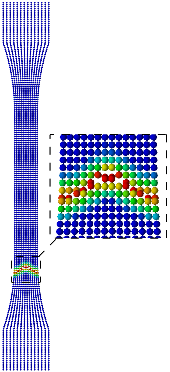
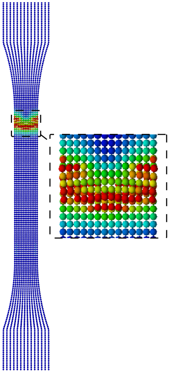


$\delta = 1.2\,\mathrm{mm}$



$\delta = 2\,\mathrm{mm}$

Results

Failure

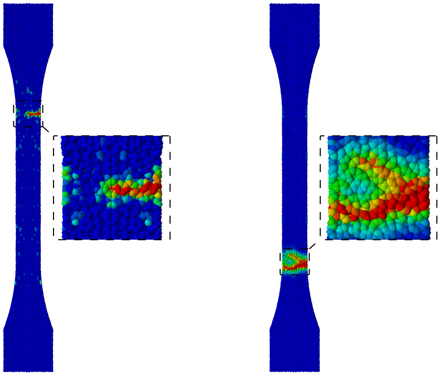
- Bond-based critical stretch models:

$$\epsilon_c = \sqrt{\frac{5G_c}{9K\delta}} \qquad [2, 4]$$

$$\epsilon_c = \sqrt{\frac{G_c}{\left[3G + \left(\frac{3}{4}\right)^4 \left(K - \frac{5G}{3}\right)\right]\delta}} \qquad [5]$$

- Based on Griffith for existing pre-crack
- $\epsilon_c = f\left(\delta^{-\frac{1}{2}}\right)$
- Reproducibility for various δ and adapted ϵ_c in SB-PD 1D-case for no pre-crack?

Tet, $dx = 0.5\,\mathrm{mm}$



$\delta = 0.56\,\mathrm{mm}$ $\qquad$ $\delta = 1.5\,\mathrm{mm}$

Results

Failure

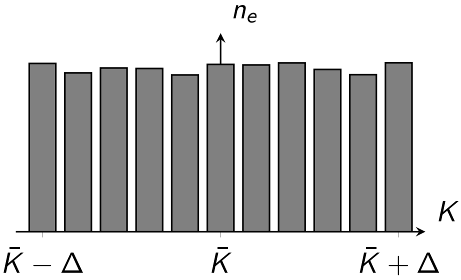
- No
- Assumption of pre-crack not met
- Equality of Griffith-based relationship between critical stretch, strain energy release rate, bulk modulus & horizon not met
- Unphysical damage shapes for larger horizons $\rightarrow$ effect of missing surface correction in material model
- Energy-based failure criterion?

Results

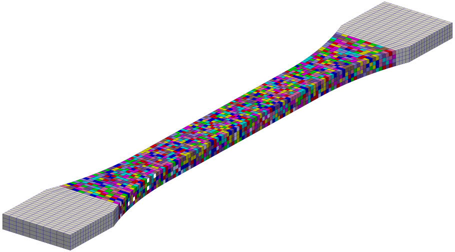
Stochastics - Why & Implementation

- Ensure effect causing failure adequately described & robust solution
- Prove influence on damage of
  - Scatter in stress-strain-curves, micro-voids & findings in micrographs
  - Varying degree of cure, disparities from machining

Stochastic model



Base FE model

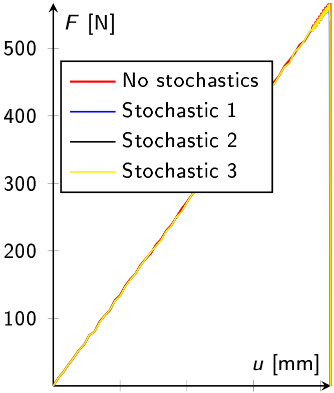


Results

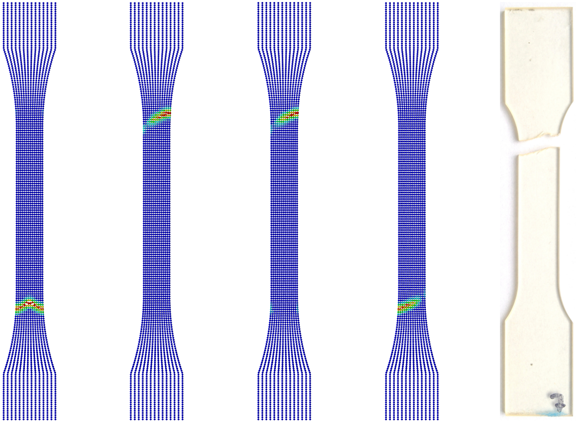
Stochastics

- Hex: $dx = 0.4\,\mathrm{mm}$, $\delta = 1.2\,\mathrm{mm}$

Force-Displacement plot



Failure patterns

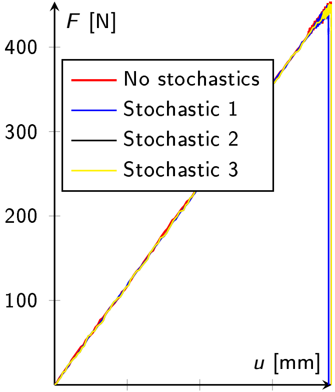


Results

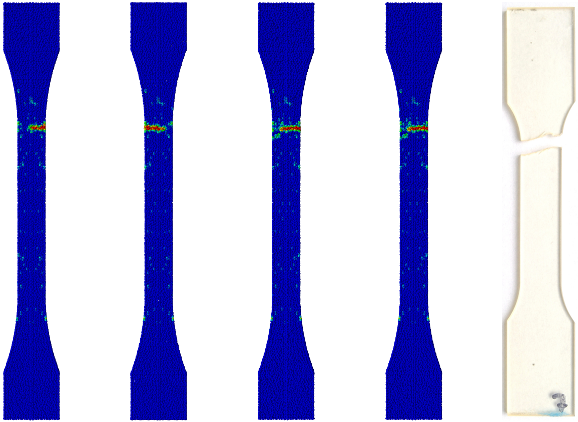
Stochastics

- Tet: $dx = 0.5\,\mathrm{mm}$, $\delta = 0.5625\,\mathrm{mm}$

Force-Displacement plot



Failure patterns

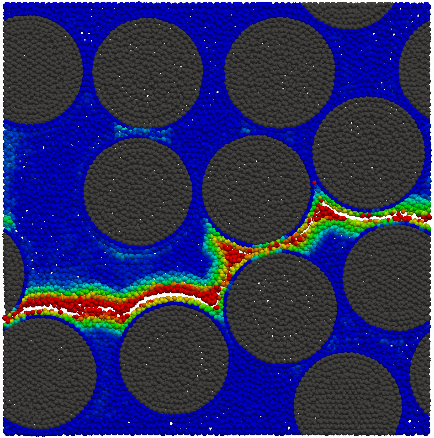


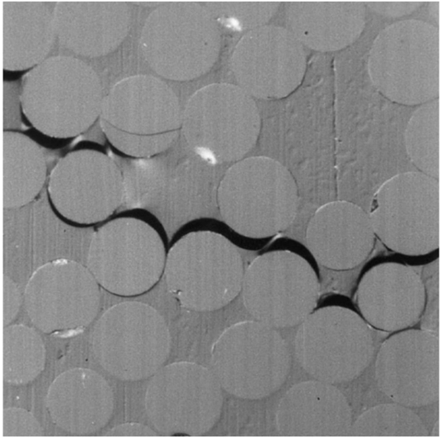
Conclusion

- Discretization:
  - Influence of base discretization
  - Entropy in tet mesh benefitial
- Convergence:
  - Discontinuous
  - $m \approx 3$ for hex & $m \approx 1.1$ for tet mesh best values $\leftrightarrow$ FEM for tension
- Failure:
  - Bond-based critical stretch criterion not suitable in state-based materials
  - Energy-based criterion required
- Stochastics:
  - Suitable method to check model robustness
  - Increases entropy in hex mesh, effect smaller for tet mesh

Conclusion

- RVE: Initial attempt with tet mesh and critical stretch model





Thank you for your attention.

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