Assignment 5

Priyanka - EE21MTECH12002

Download all python codes from

https://github.com/PeriPriyanka/Example_1/Assignment5/code

and latex-tikz codes from

https://github.com/PeriPriyanka/Example_1/Assignment5

1 Problem

(Example 1 - Question 6) Find the distance between following pair of points.

$$\begin{pmatrix} acos\alpha \\ asin\alpha \end{pmatrix}, \begin{pmatrix} acos\beta \\ asin\beta \end{pmatrix} \tag{1.0.1}$$

2 Solution

Consider the given points in the equation as vectors. (1.0.1)
The distance between two vectors **A** and **B** is given by considering the norm of difference between the vectors.

$$Distance = \|\mathbf{A}-\mathbf{B}\| \tag{2.0.1}$$

$$\|\mathbf{A}-\mathbf{B}\| = \sqrt{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B})} \tag{2.0.2}$$

$$\|\mathbf{A}-\mathbf{B}\| = \left\| \begin{pmatrix} a(cos\alpha - cos\beta) \\ a(sin\alpha - sin\beta) \end{pmatrix} \right\| \tag{2.0.3}$$

From the properties of the norm; $\|K\mathbf{A}\| = |K|\|\mathbf{A}\|$.

$$\|\mathbf{A}-\mathbf{B}\| = |a| \left\| \begin{pmatrix} cos\alpha - cos\beta \\ sin\alpha - sin\beta \end{pmatrix} \right\| \tag{2.0.4}$$

$$\|\mathbf{A}-\mathbf{B}\| = |a| \sqrt{(cos\alpha - cos\beta)^2 + (sin\alpha - sin\beta)^2} \tag{2.0.5}$$

$$= |a| \sqrt{2 - 2cos\alpha cos\beta - 2sin\alpha sin\beta} \tag{2.0.6}$$

$$= |a| \sqrt{2 - 2cos(\alpha-\beta)} \tag{2.0.7}$$

$$\|\mathbf{A}-\mathbf{B}\| = |a| \sqrt{2} \sqrt{1 - cos(\alpha-\beta)} \tag{2.0.8}$$

Therefore, the distance between the given points is $|a| \sqrt{2} \sqrt{1 - cos(\alpha-\beta)}$.