# Assignment 4

Priyanka - EE21MTECH12002

Download all python codes from

https://github.com/PeriPriyanka/Quadratic_forms_assign/Assignment4/code

and latex-tikz codes from

https://github.com/PeriPriyanka/Quadratic_forms_assign/Assignment4

## 1 Problem

(Quadratic forms 2.4) Find the points on the curve given at which tangents are parallel to x-axis

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} - 3 = 0 \tag{1.0.1}$$

## 2 Solution

Consider the equation (1.0.1)

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} - 3 = 0 \tag{2.0.1}$$

where,

$$\mathbf{X} = \begin{pmatrix}x\\y\end{pmatrix} \tag{2.0.2}$$

Comparing the equation with standard form of circle

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{O}^T\mathbf{x} + ||O||^2 - \mathbf{r}^2 = 0 \tag{2.0.3}$$

$$\tag{2.0.4}$$

where O is the origin and r is the radius of the circle we get,

$$\mathbf{O} = -\mathbf{u} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.5}$$

$$||O||^2 = 1 \tag{2.0.6}$$

$$||O||^2 - \mathbf{r}^2 = \mathbf{f} = -3 \tag{2.0.7}$$

$$\mathbf{r}^2 = 1 + 3 = 4 \tag{2.0.8}$$

$$\mathbf{r} = 2 \tag{2.0.9}$$

Tangent to circle parallel to x-axis will have a normal of the form

$$\mathbf{n} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.10}$$

$$\mathbf{m} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.11}$$

For the given circle the tangent parallel to x-axis with normal n is

$$\mathbf{P}_i = (\mathbf{k}_i\mathbf{n} - \mathbf{u}) \tag{2.0.12}$$

where i=1,2,3.....

$$\mathbf{k}_i = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{u} - \mathbf{f}}{\mathbf{n}^T\mathbf{n}}} \tag{2.0.13}$$

$$\mathbf{k}_i = \pm 2 \tag{2.0.14}$$

The point on the circle where tangent is parallel to x-axis is given by

$$\mathbf{k}_i = 2; \mathbf{P_1} = \begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.15}$$

$$\mathbf{k}_i = -2; \mathbf{P_2} = \begin{pmatrix}1\\-2\end{pmatrix} \tag{2.0.16}$$

Plot of tangent and circle are shown in the Fig0

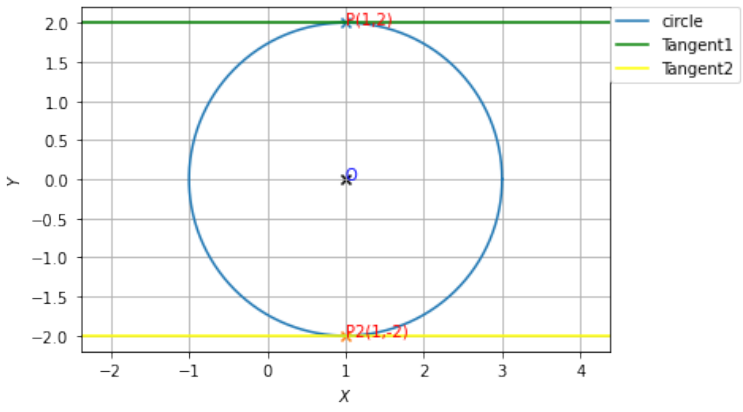


Fig. 0: Plot of circle.