

Assignment 4

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Download all python codes from

https://github.com/PeriPriyanka/Quadratic_forms_assign/Assignment4/code

and latex-tikz codes from

https://github.com/PeriPriyanka/Quadratic_forms_assign/Assignment4

1 PROBLEM

(Quadratic forms 2.4) Find the points on the curve given at which tangents are parallel to x-axis

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 3 = 0 \quad (1.0.1)$$

2 SOLUTION

Consider the equation (1.0.1)

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 3 = 0 \quad (2.0.1)$$

where,

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.2)$$

Comparing the equation with standard form of circle

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{O}^T \mathbf{x} + \|\mathbf{O}\|^2 - \mathbf{r}^2 = 0 \quad (2.0.3)$$

$$(2.0.4)$$

where \mathbf{O} is the origin and \mathbf{r} is the radius of the circle we get,

$$\mathbf{O} = -\mathbf{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.5)$$

$$\|\mathbf{O}\|^2 = 1 \quad (2.0.6)$$

$$\|\mathbf{O}\|^2 - \mathbf{r}^2 = \mathbf{f} = -3 \quad (2.0.7)$$

$$\mathbf{r}^2 = 1 + 3 = 4 \quad (2.0.8)$$

$$\mathbf{r} = 2 \quad (2.0.9)$$

Tangent to circle parallel to x-axis will have a normal of the form

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.10)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.11)$$

For the given circle the tangent parallel to x-axis with normal " \mathbf{n} " is

$$\mathbf{P}_i = (\mathbf{k}_i \mathbf{n} - \mathbf{u}) \quad (2.0.12)$$

where $i=1,2,3,\dots$

$$\mathbf{k}_i = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{u} - \mathbf{f}}{\mathbf{n}^T \mathbf{n}}} \quad (2.0.13)$$

$$\mathbf{k}_i = \pm 2 \quad (2.0.14)$$

The point on the circle where tangent is parallel to x-axis is given by

$$\mathbf{k}_i = 2; \mathbf{P}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.15)$$

$$\mathbf{k}_i = -2; \mathbf{P}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad (2.0.16)$$

Plot of tangent and circle are shown in the Fig0

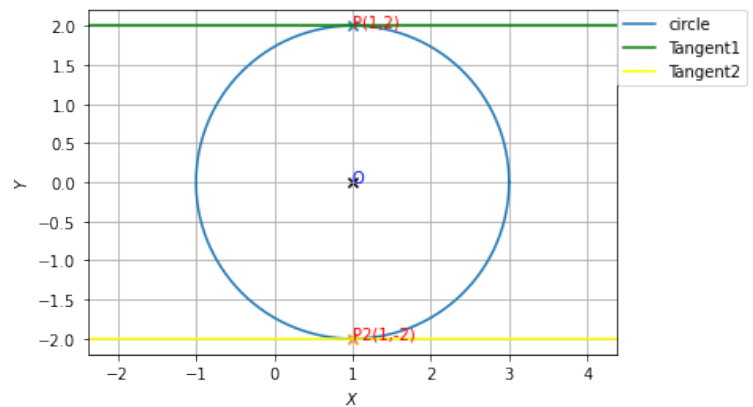


Fig. 0: Plot of circle.