Assignment 4

Priyanka - EE21MTECH12002

Download all python codes from

https://github.com/PeriPriyanka/

Quadratic forms assign/Assignment4/code

and latex-tikz codes from

https://github.com/PeriPriyanka/

Quadratic forms assign/Assignment4

1 Problem

(Quadratic forms 2.4) Find the points on the curve given at which tangents are parallel to x-axis

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 3 = 0 \tag{1.0.1}$$

2 Solution

Consider the equation (1.0.1)

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 3 = 0$$
 (2.0.1)

where,

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.2}$$

Comparing the equation with standard form of circle

$$\mathbf{x}^{T}\mathbf{x} - 2\mathbf{O}^{T}\mathbf{x} + ||O||^{2} - \mathbf{r}^{2} = 0$$
 (2.0.3)

(2.0.4)

where O is the origin and r is the radius of the circle we get,

$$\mathbf{O} = \begin{pmatrix} O_x \\ O_y \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{O} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

$$||O||^2 = 1 \tag{2.0.7}$$

$$||O||^2 - \mathbf{r}^2 = -3 \tag{2.0.8}$$

$$\mathbf{r}^2 = 1 + 3 = 4 \tag{2.0.9}$$

$$\mathbf{r} = 2 \tag{2.0.10}$$

Tangent to circle parallel to x-axis will be of the form

$$\mathbf{y} = \mathbf{k} \tag{2.0.11}$$

The value of k is the height of tangent from the x-axis.

$$\mathbf{k} = \mathbf{r} + \mathbf{O}_{\mathbf{v}} \tag{2.0.12}$$

$$\mathbf{k} = 2 \tag{2.0.13}$$

(2.0.14)

Equation of tangent

$$\mathbf{y} = 2 \tag{2.0.15}$$

The point on the circle where tangent is parallel to x-axis is given by

$$\mathbf{P} = \begin{pmatrix} P_x \\ P_y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.16}$$

Plot of tangent and circle are shown in the Fig0

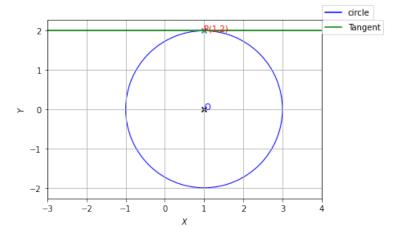


Fig. 0: Plot of circle.