CBSE Maths Questions 2007

Priyanka - EE21MTECH12002

Get latex-tikz codes from

https://github.com/PeriPriyanka/cbsemathsquestions/2007_questions

1 Problem

(CBSE 2007-Question 2) solve the values of x and y.

$$x + \frac{6}{y} = 6 \tag{1.0.1}$$

$$3x - \frac{8}{y} = 5 \tag{1.0.2}$$

2 Solution

Consider the equations 1.0.1 and 1.0.2 given in the problem statement.

$$x + \frac{6}{y} = 6 \tag{2.0.1}$$

$$3x - \frac{8}{y} = 5 \tag{2.0.2}$$

The solution can be found by solving the above system of linear equations.
System of linear equations are defined as

$$\mathbf{AX} = \mathbf{B} \tag{2.0.3}$$

From the equations 2.0.1 and 2.0.2,

$$\mathbf{A} = \begin{pmatrix} 1 & 6 \\ 3 & -8 \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{X} = \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{2.0.6}$$

Substituting the values of **A**, **X** and **B** in the equation 2.0.3 We get,

$$\begin{pmatrix} 1 & 6 \\ 3 & -8 \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{2.0.7}$$

Considering the augmented matrix

$$\begin{pmatrix} 1 & 6 & 6 \\ 3 & -8 & 5 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & 6 & 6 \\ 0 & -26 & -13 \end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix} 1 & 6 \\ 0 & -26 \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 6 \\ -13 \end{pmatrix} \tag{2.0.10}$$

$$x + \frac{6}{y} = 6 \tag{2.0.11}$$

$$\frac{-26}{y} = -13 \tag{2.0.12}$$

By solving equations 2.0.12 we get,

$$y = 2 \tag{2.0.13}$$

and by solving equation 2.0.11 we get ,

$$x = 3 \tag{2.0.14}$$

Therefore, x=3 and y= 2 are solutions to the given equations 1.0.1 and 1.0.2

3 Problem

(CBSE 2007-Question 3) solve the values of x and Yy

$$\frac{x+1}{2} + \frac{y-1}{3} = 8 \tag{3.0.1}$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9 \tag{3.0.2}$$

4 Solution

Consider the equations 3.0.1 and 3.0.2 given in the problem statement.

$$\frac{x+1}{2} + \frac{y-1}{3} = 8 \tag{4.0.1}$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9 \tag{4.0.2}$$

The above equations 4.0.1 and 4.0.2 can be rearranged as the following equations

$$3x + 2y = 47 \tag{4.0.3}$$
$$2x + 3y = 53 \tag{4.0.4}$$

The solution can be found by solving the above system of linear equations.
System of linear equations are defined as

$$\mathbf{AX} = \mathbf{B} \tag{4.0.5}$$

From the equations 4.0.3 and 4.0.4,

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \tag{4.0.6}$$
$$\mathbf{X} = \begin{pmatrix} x \; y \end{pmatrix} \tag{4.0.7}$$
$$\mathbf{B} = \begin{pmatrix} 47 \\ 53 \end{pmatrix} \tag{4.0.8}$$

Substituting the values of **A**, **X** and **B** in the equation 4.0.5 We get,

$$\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 47 \\ 53 \end{pmatrix} \tag{4.0.9}$$

Considering the augmented matrix

$$\begin{pmatrix} 3 & 2 & 47 \\ 2 & 3 & 53 \end{pmatrix} \tag{4.0.10}$$
$$\xleftrightarrow{R_2 \leftarrow 3R_2 - 2R_1} \begin{pmatrix} 3 & 2 & 47 \\ 0 & 5 & 65 \end{pmatrix} \tag{4.0.11}$$

$$\begin{pmatrix} 3 & 2 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 47 \\ 65 \end{pmatrix} \tag{4.0.12}$$
$$3x + 2y = 47 \tag{4.0.13}$$
$$5y = 65 \tag{4.0.14}$$

By solving equations 4.0.14 we get,

$$y = 13 \tag{4.0.15}$$

and by solving equation 4.0.13 we get ,

$$x = 7 \tag{4.0.16}$$

Therefore, x=7 and y= 13 are solutions to the given equations 3.0.1 and 3.0.2

5 Problem

(CBSE 2007-Question 21) Show that the points given below are vertices of an isosceles right angle triangle.

$$\begin{pmatrix} 7 \\ 10 \end{pmatrix} \tag{5.0.1}$$
$$\begin{pmatrix} -2 \\ 5 \end{pmatrix} \tag{5.0.2}$$
$$\begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{5.0.3}$$

6 Solution

Consider the given points as vectors,

$$\mathbf{A} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} \tag{6.0.1}$$
$$\mathbf{B} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \tag{6.0.2}$$
$$\mathbf{C} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{6.0.3}$$

For a triangle to be an isosceles, any two sides of the triangle should be equal. For finding a triangle to be isosceles and right angle, we consider,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} \tag{6.0.4}$$
$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 9 \end{pmatrix} \tag{6.0.5}$$
$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 7 \\ 10 \end{pmatrix} = \begin{pmatrix} -4 \\ -14 \end{pmatrix} \tag{6.0.6}$$

$$(A - B)^T (B - C) = \begin{pmatrix} 9 & 5 \end{pmatrix} \begin{pmatrix} -5 \\ 9 \end{pmatrix} \tag{6.0.7}$$
$$= -45 + 45 = 0 \tag{6.0.8}$$
$$(C - A)^T (A - B) = \begin{pmatrix} -4 & -14 \end{pmatrix} \begin{pmatrix} 9 \\ 5 \end{pmatrix} \tag{6.0.9}$$
$$= -36 - 70 = -106 \tag{6.0.10}$$
$$(B - C)^T (C - A) = \begin{pmatrix} -5 & 9 \end{pmatrix} \begin{pmatrix} -4 \\ -14 \end{pmatrix} \tag{6.0.11}$$
$$= 20 - 126 = -106 \tag{6.0.12}$$

From the equation 6.0.8 $\mathbf{A} - \mathbf{B} \perp \mathbf{B} - \mathbf{C}$, Therefore $\measuredangle B = 90°$
From the equations 6.0.10 and 6.0.12 $\measuredangle CAB = \measuredangle BCA$
Therefore, $\triangle ABC$ is an isosceles right angle triangle with sides $\mathbf{AB} = \mathbf{BC}$ and right angle at **B**

7 Problem

(CBSE 2007-Question 22) In what ratio does the line x-y-2=0 divides the line segment joining $\begin{pmatrix} 3 & -1 \end{pmatrix}$ and $\begin{pmatrix} 8 & 9 \end{pmatrix}$?

8 Solution

Consider the line x-y-2=0 divides the line segment $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 9 \end{pmatrix}$ in $k : 1$ ratio.
$\mathbf{P} = \begin{pmatrix} x & y \end{pmatrix}$ is point of intersection of two lines.
From the section formula we can write,

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{k+1}\left[\begin{pmatrix} 3 \\ -1 \end{pmatrix} + k\begin{pmatrix} 8 \\ 9 \end{pmatrix}\right] \tag{8.0.1}$$

$$= \begin{pmatrix} \dfrac{3+3k}{k+1} \\ \dfrac{-1+9k}{k+1} \end{pmatrix} \tag{8.0.2}$$

The point $\mathbf{P}$ passes through the line x-y-2=0, therefore,

$$\frac{3+3k}{k+1} - \frac{-1+9k}{k+1} - 2 = 0 \tag{8.0.3}$$

$$k = \frac{2}{3} \tag{8.0.4}$$

Therefore, the line x-y-2=0 divides the line segment $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 9 \end{pmatrix}$ in $2 : 3$ ratio.