CBSE Maths 12, 2007

Get latex-tikz codes from

https://github.com/PeriPriyanka/cbsemathsquestions/2007/12/problems/Questions

1 Section A

1.1. If $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$, show that $\mathbf{A}^2 - 6\mathbf{A} + 17\mathbf{I} = 0$. Hence find $\mathbf{A}^{-1}$

1.2. An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting

a) 2 red balls
b) 2 blue balls
c) one red and one blue ball

1.3. Using the properties of determinants, prove the following:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

1.4. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that it is neither a ace nor a king.

1.5. Evaluate the following integral:

$$\int \frac{1+x^2}{1+x^4}\,dx$$

1.6. Solve the following differential equation:

$$x\cos y dy = (xe^x \log x + e^x)dx$$

1.7. Form the differential equations of the family of the curves $y = A\cos 2x + B\sin 2x$, where A and B are constants.

1.8. Solve the following differential equation:

$$\frac{dy}{dx} + 2y = 6e^x$$

1.9. Evaluate:

$$\int \cos 4x \cos 3x\,dx \tag{1.9.1}$$

1.10. Using the properties of definite integrals, prove the following:

$$\int_0^{\pi} \frac{x\tan x}{\sec x\,\mathrm{cosec}\,x}\,dx = \frac{\pi^2}{4}$$

1.11. Evaluate:

$$\int \frac{\sin x}{(1-\cos x)(2-\cos x)}\,dx$$

1.12. Find the value of k if the function

$$f(x) = \begin{cases} kx^2, & x \geq 1 \\ 4, & x < 1 \end{cases}$$ is continuous at x = 1

1.13. Evalutate:

$$\lim_{x\to\frac{\pi}{4}} \left(\frac{\sin x - \cos x}{x - \frac{\pi}{4}} \right)$$

1.14. Differentiate $\sin(x^2+1)$ with respect to x from first principle.

1.15. Write the boolean expressions representing the following circuit and simplify the Boolean expression.

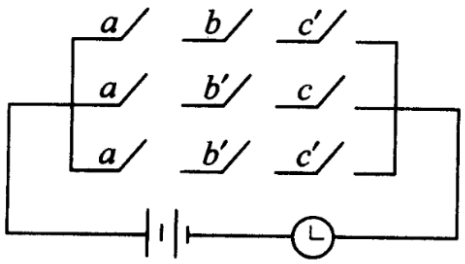

Fig. 1.15.

1.16. Show that the following argument is valid:

$$S_1 : p \vee q$$
$$S_2 :\sim q$$
$$S : p\wedge \sim q$$

1.17. If y = $\sin(\log x)$, prove that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

1.18. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 4$ on [1,4].

1.19. Using matrices, solve the following system of equation:

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

1.20. Using integration, find the are of the region enclosed between the circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$.

1.21. Evaluate $\int_0^2 (x^2 + 2x + 1)\, dx$ as limit of sum.

1.22. Find the point on the curve $x^2 = 8y$ which is nearest to the point (2,4).

1.23. Show that a right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

2 SECTION B

2.1. Find the projection of $\overrightarrow{\mathbf{b}} + \overrightarrow{\mathbf{c}}$ on $\overrightarrow{\mathbf{a}}$ where $\overrightarrow{\mathbf{a}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\overrightarrow{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{c}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

2.2. Find the value of λ which makes the vectors $\overrightarrow{\mathbf{a}}, \overrightarrow{\mathbf{b}}$ and $\overrightarrow{\mathbf{c}}$ coplanar, where $\overrightarrow{\mathbf{a}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \overrightarrow{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{c}} = 3\hat{\mathbf{i}} - \lambda\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

2.3. A particle starting with initial velocity of 30 m/sec moves with a uniform acceleration of 9 m/sec^2. Find :

a) the velocity of the particle after 6 seconds.
b) how far it will go in 9 seconds.
c) its velocity when is has traversed 150 m.

2.4. Find the resultant of two velocities 4 m/sec and 6 m/sec inclined to one another at an angle of 120°.

2.5. A ball projected with a velocity of 28 m/sec has a horizontal range 40 m. Find the two angles of projection.

2.6. A body of weight 70 N is suspended by two strings of length 27 cm and 36 cm, fastened to two points in the same horizontal line 45 cm apart and is in equilibrium. Find the tensions in the strings.

2.7. The resultant of two unlike parallel forces of 18 N and 10 N act along a line at a distance of 12 cm from the line of action of the smaller force. Find the distance between the lines of action of two forces.

2.8. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x+2y+3z-4 = 0$ and $2x+y-z+5 = 0$.

2.9. Find the equation of the sphere which passing through the points $(3, 0, 0), (0, -1, 0), (0, 0, -2)$ and and having the centre on the plane $3x + 2y + 4z = 1$.

3 SECTION C

3.1. Find the face value of a bill, discounted at 6% per annum 146 days before the legal due date, if the banker's gain is Rs. 36.

3.2. A bill for Rs. 7650 was drawn on 8th March 2005 at 7 months. It was discounted on 18 May 2005 and the holder of the bill received Rs. 7497. What rate of interest did the banker charge ?

3.3. There are two bags I and II. Bag I contains 2 white and 3 red balls and Bag II contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II.

3.4. Find the mean μ, variance σ^2 for the following probability distribution:

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

TABLE 3.4

3.5. Find the binomial distribution for which the mean is 4 and variance 3.

3.6. A, B, C entered into a partnership investing Rs. 12000, Rs. 16000 and Rs. 20000 respectively. A as working partner gets 10% of the annual profit for the same. After 5 months, B invested Rs. 2000 more while C withdrew Rs. 2000 after 8 months from the start of the business. Find the share of each in an annual profit of Rs. 97000.

3.7. Find the present value of an annuity due of Rs. 700 per annum payable at the beginning of each year for 2 years allowing interest 6% per annum, compounded annually.[Take $(1.06)^{-1} = 0.943$]

3.8. The total cost C(x), associated with the production and making x units of an item is given by

$$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$$

Find:

a) the average cost function.
b) the average cost of output of 10 units.
c) the marginal cost function.
d) the marginal cost when 3 units are produced.

3.9. If a young man rides his motorcycle at 25 km/hour, he had to spend Rs. 2 per km on

petrol. If he rides at a faster speed of 40 km/hour, the petrol cost increases at Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as an LPP and solve it graphically.