

CBSE Maths Questions 2007

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Download all python codes from

<https://github.com/PeriPriyanka/cbsemathquestions/2007/10/matrices/codes/solutions>

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- 1) (CBSE 2007-Question 2) solve the values of x and y.

$$x + \frac{6}{y} = 6 \quad (0.0.1)$$

$$3x - \frac{8}{y} = 5 \quad (0.0.2)$$

Solution: Consider the equations given in the problem statement.

$$x + \frac{6}{y} = 6 \quad (0.0.3)$$

$$3x - \frac{8}{y} = 5 \quad (0.0.4)$$

The solution can be found by solving the above system of linear equations.

System of linear equations are defined as

$$\mathbf{Ax} = \mathbf{B} \quad (0.0.5)$$

From the equations (0.0.3) and (0.0.4),

$$\mathbf{A} = \begin{pmatrix} 1 & 6 \\ 3 & -8 \end{pmatrix} \quad (0.0.6)$$

$$\mathbf{x} = \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} \quad (0.0.7)$$

$$\mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \quad (0.0.8)$$

Substituting the values of \mathbf{A} , \mathbf{x} and \mathbf{B} in the equation (0.0.5) We get,

$$\begin{pmatrix} 1 & 6 \\ 3 & -8 \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \quad (0.0.9)$$

Considering the augmented matrix

$$\begin{pmatrix} 1 & 6 & 6 \\ 3 & -8 & 5 \end{pmatrix} \quad (0.0.10)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & 6 & 6 \\ 0 & -26 & -13 \end{pmatrix} \quad (0.0.11)$$

$$\xrightarrow{R_1 \leftarrow 13R_1 + 3R_2} \begin{pmatrix} 13 & 0 & 39 \\ 0 & -26 & -13 \end{pmatrix} \quad (0.0.12)$$

$$\xrightarrow{R_1 \leftarrow R_1/3, R_2 \leftarrow R_2/-26} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0.5 \end{pmatrix} \quad (0.0.13)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 3 \\ 0.5 \end{pmatrix} \quad (0.0.14)$$

From the above equation (0.0.14) we get,

$$x = 3 \quad (0.0.15)$$

$$y = 2 \quad (0.0.16)$$

Therefore, $x=3$ and $y=2$ are solutions to the given equations.

- 2) (CBSE 2007-Question 3) solve the values of x and y

$$\frac{x+1}{2} + \frac{y-1}{3} = 8 \quad (0.0.17)$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9 \quad (0.0.18)$$

Solution: Consider the equations given in the problem statement.

$$\frac{x+1}{2} + \frac{y-1}{3} = 8 \quad (0.0.19)$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9 \quad (0.0.20)$$

The above equations can be rearranged as the following equations

$$3x + 2y = 47 \quad (0.0.21)$$

$$2x + 3y = 53 \quad (0.0.22)$$

The solution can be found by solving the above system of linear equations.

System of linear equations are defined as

$$\mathbf{Ax} = \mathbf{B} \quad (0.0.23)$$

From the equations (0.0.21) and (0.0.22),

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \quad (0.0.24)$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (0.0.25)$$

$$\mathbf{B} = \begin{pmatrix} 47 \\ 53 \end{pmatrix} \quad (0.0.26)$$

Substituting the values of \mathbf{A} , \mathbf{x} and \mathbf{B} in the equation (0.0.23) We get,

$$\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 47 \\ 53 \end{pmatrix} \quad (0.0.27)$$

Considering the augmented matrix

$$\begin{pmatrix} 3 & 2 & 47 \\ 2 & 3 & 53 \end{pmatrix} \quad (0.0.28)$$

$$\xleftrightarrow{R_2 \leftarrow 3R_2 - 2R_1} \begin{pmatrix} 3 & 2 & 47 \\ 0 & 5 & 65 \end{pmatrix} \quad (0.0.29)$$

$$\xleftrightarrow{R_1 \leftarrow 5R_1 - 2R_2} \begin{pmatrix} 15 & 0 & 105 \\ 0 & 5 & 65 \end{pmatrix} \quad (0.0.30)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 / 15, R_2 \leftarrow R_2 / 5} \begin{pmatrix} 1 & 0 & 7 \\ 0 & 1 & 13 \end{pmatrix} \quad (0.0.31)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 13 \end{pmatrix} \quad (0.0.32)$$

By solving equation (0.0.32) we get,

$$x = 7 \quad (0.0.33)$$

$$y = 13 \quad (0.0.34)$$

Therefore, $x=7$ and $y=13$ are solutions to the given equations.

- 3) (CBSE 2007-Question 21) Show that the points given below are vertices of an isosceles right angle triangle.

$$\begin{pmatrix} 7 \\ 10 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \end{pmatrix} \text{ and } \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (0.0.35)$$

Solution: Consider the given points as vectors,

$$\mathbf{A} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} \quad (0.0.36)$$

$$\mathbf{B} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \quad (0.0.37)$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (0.0.38)$$

For a triangle to be an isosceles, any two sides of the triangle should be equal. For finding a triangle to be isosceles and right angle, we consider,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} \quad (0.0.39)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 9 \end{pmatrix} \quad (0.0.40)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 7 \\ 10 \end{pmatrix} = \begin{pmatrix} -4 \\ -14 \end{pmatrix} \quad (0.0.41)$$

$$(\mathbf{A} - \mathbf{B})^T (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} 9 & 5 \end{pmatrix} \begin{pmatrix} -5 \\ 9 \end{pmatrix} \quad (0.0.42)$$

$$= -45 + 45 = 0 \quad (0.0.43)$$

$$(\mathbf{C} - \mathbf{A})^T (\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -4 & -14 \end{pmatrix} \begin{pmatrix} 9 \\ 5 \end{pmatrix} \quad (0.0.44)$$

$$= -36 - 70 = -106 \quad (0.0.45)$$

$$(\mathbf{B} - \mathbf{C})^T (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} -5 & 9 \end{pmatrix} \begin{pmatrix} -4 \\ -14 \end{pmatrix} \quad (0.0.46)$$

$$= 20 - 126 = -106 \quad (0.0.47)$$

From the equation (0.0.43)

$$\mathbf{A} - \mathbf{B} \perp \mathbf{B} - \mathbf{C} \quad (0.0.48)$$

Therefore $\angle B = 90^\circ$. From the equations (0.0.45) and (0.0.47) $\angle CAB = \angle BCA$. Therefore, $\triangle ABC$ is an isosceles right angle triangle with sides $AB=BC$ and right angle at B.

- 4) (CBSE 2007-Question 22) In what ratio does the line

$$(1 \ -1)\mathbf{x} = 2 \quad (0.0.49)$$

divides the line segment joining

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{ and } \begin{pmatrix} 8 \\ 9 \end{pmatrix} \quad (0.0.50)$$

Solution: Consider the line

$$\mathbf{n}^T \mathbf{x} = c \quad (0.0.51)$$

divides the line segment \mathbf{A} and \mathbf{B} in $k : 1$ ratio.

\mathbf{p} is point of intersection of two lines.

From the section formula we can write,

$$\mathbf{p} = \frac{1}{k+1} [\mathbf{A} + k\mathbf{B}] \quad (0.0.52)$$

$$(0.0.53)$$

The point \mathbf{p} passes through the line $\mathbf{n}^T \mathbf{x} = c$, therefore,

$$\mathbf{n}^T \mathbf{p} = c \quad (0.0.54)$$

$$\mathbf{n}^T \left(\frac{\mathbf{A} + k\mathbf{B}}{k+1} \right) = c \quad (0.0.55)$$

Solving for k , we get,

$$k = \frac{c - \mathbf{n}^T \mathbf{A}}{\mathbf{n}^T \mathbf{B} - c} \quad (0.0.56)$$

From the equations (0.0.49) and (0.0.50),

$$\mathbf{n}^T = \begin{pmatrix} 1 & -1 \end{pmatrix} \quad (0.0.57)$$

$$\mathbf{A} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad (0.0.58)$$

$$\mathbf{B} = \begin{pmatrix} 8 \\ 9 \end{pmatrix} \quad (0.0.59)$$

Substituting the above values in the equation (0.0.56), we get,

$$k = \frac{2 - \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix}}{\begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 8 \\ 9 \end{pmatrix} - 2} \quad (0.0.60)$$

$$k = \frac{2}{3} \quad (0.0.61)$$

Therefore, the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 2 \quad (0.0.62)$$

divides the line segment joining

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{ and } \begin{pmatrix} 8 \\ 9 \end{pmatrix} \quad (0.0.63)$$

in 2:3 ratio.