CBSE Maths Questions 2007

Priyanka - EE21MTECH12002

Get latex-tikz codes from

https://github.com/PeriPriyanka/cbsemathsquestions/2007_questions

1) (CBSE 2007-Question 2) solve the values of x and y.

$$x + \frac{6}{y} = 6 \tag{0.0.1}$$
$$3x - \frac{8}{y} = 5 \tag{0.0.2}$$

Solution: Consider the equations given in the problem statement.

$$x + \frac{6}{y} = 6 \tag{0.0.3}$$
$$3x - \frac{8}{y} = 5 \tag{0.0.4}$$

The solution can be found by solving the above system of linear equations.
System of linear equations are defined as

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{0.0.5}$$

From the equations (0.0.3) and (0.0.4),

$$\mathbf{A} = \begin{pmatrix} 1 & 6 \\ 3 & -8 \end{pmatrix} \tag{0.0.6}$$
$$\mathbf{x} = \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} \tag{0.0.7}$$
$$\mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{0.0.8}$$

Substituting the values of **A**, **x** and **B** in the equation (0.0.5) We get,

$$\begin{pmatrix} 1 & 6 \\ 3 & -8 \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{0.0.9}$$

Considering the augmented matrix

$$\begin{pmatrix} 1 & 6 & 6 \\ 3 & -8 & 5 \end{pmatrix} \tag{0.0.10}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & 6 & 6 \\ 0 & -26 & -13 \end{pmatrix} \tag{0.0.11}$$
$$\xleftrightarrow{R_1 \leftarrow 13R_1 + 3R_2} \begin{pmatrix} 13 & 0 & 39 \\ 0 & -26 & -13 \end{pmatrix} \tag{0.0.12}$$
$$\xleftrightarrow{R_1 \leftarrow R_1/3, R_2 \leftarrow R_2/-26} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0.5 \end{pmatrix} \tag{0.0.13}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} \tag{0.0.14}$$

From the above equation (0.0.14) we get,

$$x = 3 \tag{0.0.15}$$
$$y = 2 \tag{0.0.16}$$

Therefore, x=3 and y= 2 are solutions to the given equations.

2) (CBSE 2007-Question 3) solve the values of x and y

$$\frac{x+1}{2} + \frac{y-1}{3} = 8 \tag{0.0.17}$$
$$\frac{x-1}{3} + \frac{y+1}{2} = 9 \tag{0.0.18}$$

Solution: Consider the equations given in the problem statement.

$$\frac{x+1}{2} + \frac{y-1}{3} = 8 \tag{0.0.19}$$
$$\frac{x-1}{3} + \frac{y+1}{2} = 9 \tag{0.0.20}$$

The above equations can be rearranged as the following equations

$$3x + 2y = 47 \tag{0.0.21}$$
$$2x + 3y = 53 \tag{0.0.22}$$

The solution can be found by solving the above system of linear equations.

System of linear equations are defined as

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{0.0.23}$$

From the equations (0.0.21) and (0.0.22),

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \tag{0.0.24}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.0.25}$$

$$\mathbf{B} = \begin{pmatrix} 47 \\ 53 \end{pmatrix} \tag{0.0.26}$$

Substituting the values of **A**, **x** and **B** in the equation (0.0.23) We get,

$$\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 47 \\ 53 \end{pmatrix} \tag{0.0.27}$$

Considering the augmented matrix

$$\begin{pmatrix} 3 & 2 & 47 \\ 2 & 3 & 53 \end{pmatrix} \tag{0.0.28}$$

$$\xleftrightarrow{R_2 \leftarrow 3R_2 - 2R_1} \begin{pmatrix} 3 & 2 & 47 \\ 0 & 5 & 65 \end{pmatrix} \tag{0.0.29}$$

$$\xleftrightarrow{R_1 \leftarrow 5R_1 - 2R_2} \begin{pmatrix} 15 & 0 & 105 \\ 0 & 5 & 65 \end{pmatrix} \tag{0.0.30}$$

$$\xleftrightarrow{R_1 \leftarrow R_1/15, R_2 \leftarrow R_2/5} \begin{pmatrix} 1 & 0 & 7 \\ 0 & 1 & 13 \end{pmatrix} \tag{0.0.31}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 13 \end{pmatrix} \tag{0.0.32}$$

By solving equation (0.0.32) we get,

$$x = 7 \tag{0.0.33}$$

$$y = 13 \tag{0.0.34}$$

Therefore, x=7 and y= 13 are solutions to the given equations.

3) (CBSE 2007-Question 21) Show that the points given below are vertices of an isosceles right angle triangle.

$$\begin{pmatrix} 7 \\ 10 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \end{pmatrix} \text{and} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{0.0.35}$$

Solution: Consider the given points as vectors,

$$\mathbf{A} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} \tag{0.0.36}$$

$$\mathbf{B} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \tag{0.0.37}$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{0.0.38}$$

For a triangle to be an isosceles, any two sides of the triangle should be equal. For finding a triangle to be isosceles and right angle, we consider,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} \tag{0.0.39}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 9 \end{pmatrix} \tag{0.0.40}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 7 \\ 10 \end{pmatrix} = \begin{pmatrix} -4 \\ -14 \end{pmatrix} \tag{0.0.41}$$

$$(\mathbf{A} - \mathbf{B})^T(\mathbf{B} - \mathbf{C}) = \begin{pmatrix} 9 & 5 \end{pmatrix}\begin{pmatrix} -5 \\ 9 \end{pmatrix} \tag{0.0.42}$$

$$= -45 + 45 = 0 \tag{0.0.43}$$

$$(\mathbf{C} - \mathbf{A})^T(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -4 & -14 \end{pmatrix}\begin{pmatrix} 9 \\ 5 \end{pmatrix} \tag{0.0.44}$$

$$= -36 - 70 = -106 \tag{0.0.45}$$

$$(\mathbf{B} - \mathbf{C})^T(\mathbf{C} - \mathbf{A}) = \begin{pmatrix} -5 & 9 \end{pmatrix}\begin{pmatrix} -4 \\ -14 \end{pmatrix} \tag{0.0.46}$$

$$= 20 - 126 = -106 \tag{0.0.47}$$

From the equation (0.0.43)

$$\mathbf{A} - \mathbf{B} \perp \mathbf{B} - \mathbf{C} \tag{0.0.48}$$

Therefore $\measuredangle B = 90°$. From the equations (0.0.45) and (0.0.47) $\measuredangle CAB = \measuredangle BCA$. Therefore, $\triangle ABC$ is an isosceles right angle triangle with sides AB=BC and right angle at B.

4) (CBSE 2007-Question 22) In what ratio does the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 2 \tag{0.0.49}$$

divides the line segment joining

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{and} \begin{pmatrix} 8 \\ 9 \end{pmatrix} \tag{0.0.50}$$

Solution: Consider the line

$$\mathbf{n}^T\mathbf{x} = c \tag{0.0.51}$$

divides the line segment $\mathbf{A}$ and $\mathbf{B}$ in $k : 1$ ratio.
$\mathbf{p}$ is point of intersection of two lines.
From the section formula we can write,

$$\mathbf{p} = \frac{1}{k+1}\left[\mathbf{A} + k\mathbf{B}\right] \tag{0.0.52}$$

$$\tag{0.0.53}$$

The point $\mathbf{p}$ passes through the line $\mathbf{n}^T\mathbf{x} = c$, therefore,

$$\mathbf{n}^T\mathbf{p} = c \tag{0.0.54}$$

$$\mathbf{n}^T\left(\frac{\mathbf{A} + k\mathbf{B}}{k+1}\right) = c \tag{0.0.55}$$

Solving for k, we get,

$$k = \frac{c - \mathbf{n}^T\mathbf{A}}{\mathbf{n}^T\mathbf{B} - c} \tag{0.0.56}$$

From the equations (0.0.49) and (0.0.50),

$$\mathbf{n}^T = \begin{pmatrix} 1 & -1 \end{pmatrix} \tag{0.0.57}$$

$$\mathbf{A} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{0.0.58}$$

$$\mathbf{B} = \begin{pmatrix} 8 \\ 9 \end{pmatrix} \tag{0.0.59}$$

Substituting the above values in the equation (0.0.56), we get,

$$k = \frac{2 - \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \end{pmatrix}}{\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 8 \\ 9 \end{pmatrix} - 2} \tag{0.0.60}$$

$$k = \frac{2}{3} \tag{0.0.61}$$

Therefore, the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 2 \tag{0.0.62}$$

divides the line segment joining

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{and} \begin{pmatrix} 8 \\ 9 \end{pmatrix} \tag{0.0.63}$$

in 2:3 ratio.