CBSE Maths 10, 2007

Contents

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1 Discrete maths

1.1. If X +K is the GCD of $X^2-2X-15$ and X^3+27, find the value of K.
Solution: consider the given equations,

$$X^2 - 2X - 15 \tag{1.1.1}$$

$$X^3 + 27 \tag{1.1.2}$$

From reminder theorem,

$$P(x) = D(x)Q(x) + R(x) \tag{1.1.3}$$

For (1.1.1), X+K divides the equation with zero reminder.
Therefore,

$$X^2 - 2X - 15 = (X + K)Q(X) \tag{1.1.4}$$

Let X=-K,

$$K^2 + 2K - 15 = 0 \tag{1.1.5}$$

Solving (1.1.5) we get,

$$K = +3, -5 \tag{1.1.6}$$

For (1.1.2), X+K divides the equation with zero reminder.
Therefore,

$$X^3 + 27 = (X + K)Q(X) \tag{1.1.7}$$

Let X=-K,

$$-K^3 + 27 = 0 \tag{1.1.8}$$

Solving (1.1.8) we get,

$$K = 3, 3, 3 \tag{1.1.9}$$

Comparing (1.1.6) and (1.1.9), we get

$$\text{k} = 3 \tag{1.1.10}$$

1.2. Find the sum of first 25 terms of an A.P. whose n^{th} term is $1 - 4n$.
Solution: Given

$$n = 25 \tag{1.2.1}$$

$$a_n = 1 - 4n \tag{1.2.2}$$

$$s_n = \Sigma_{k=1}^{n} a_n \tag{1.2.3}$$

$$s_n = \Sigma_{k=1}^{n} (1 - 4n) \tag{1.2.4}$$

$$s_n = n - 4\frac{n(n+1)}{2} \tag{1.2.5}$$

$$s_n = -1275 \tag{1.2.6}$$

1.3. Which term of the A.P. 3, 15, 27, 39,... will be 132 more than its 54$^{\text{th}}$ term?
Solution: Given, initial term a = 3, difference d = 12.

$$a_n = a_{54} + 132 \tag{1.3.1}$$

$$a_n = a + (n-1)d \tag{1.3.2}$$

$$a_{54} = 3 + (54 - 1)12 \tag{1.3.3}$$

$$a_{54} = 639 \tag{1.3.4}$$

$$a_n = a_{54} + 132 = 771 = 3 + (n-1)12 \tag{1.3.5}$$

$$n = 65 \tag{1.3.6}$$

2 Geometry

2.1. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total

surface area of the toy.
Solution: Given,

Symbol	Description	Value
r	base radius	7 cm
h_t	height of toy	31 cm
h	height of cone	31-7 = 24 cm
l	slant height of cone	$\sqrt{h^2+r^2}$ = 25 cm
s	surface area	?

TABLE 2.1

Surface area,

$$s = 2\pi r^2 + \pi rl \tag{2.1.1}$$
$$s = 2\pi(7)^2 + \pi.7.25 \tag{2.1.2}$$
$$s = 858cm^2 \tag{2.1.3}$$

2.2. A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.
Solution: Given,

Symbol	Description	Value
d_s	diameter of sphere	12 cm
h_r	rise in water level	$3\frac{5}{9}$ cm
d_c	diameter of cylinder	?

TABLE 2.2

Rise in the volume of water in cylinder = Volume of sphere

$$\pi\frac{d_c}{2}^2 h_r = \frac{4}{3}\pi\frac{d_s}{2}^3 \tag{2.2.1}$$
$$d_c = \sqrt{\frac{2d_s^3}{3h_r}} \tag{2.2.2}$$
$$d_c = 18cm \tag{2.2.3}$$

2.3. A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere.
Solution: Given,

Symbol	Description	Value
d_c	diameter of cone	14 cm
h_c	height of cone	8cm
D	External diameter of the sphere	10cm
d	Internal diameter of the sphere	?

TABLE 2.3

Volume of the cone = volume of the hallow sphere

$$\frac{1}{3}\pi\frac{d_c}{2}^2 h_c = \frac{4}{3}\pi\left(\frac{D}{2}^3 - \frac{d}{2}^3\right) \tag{2.3.1}$$
$$d = \sqrt[3]{D^3 - d_c{}^2\frac{h_c}{2}} \tag{2.3.2}$$
$$d = 6cm \tag{2.3.3}$$

3 Algebra

3.1. A washing machine is available for Rs. 13,500 cash or Rs. 6,500 as cash down payment followed by three monthly instalments of Rs. 2,500 each. Find the rate of interest charged under instalment plan.
Solution: Cash price of washing machine = Rs. 135000
Cash down payment = Rs. 6500
Balance due = Rs(13500 − 6500) = Rs. 7000
No. of equal instalments = 3
Amount of each instalment = Rs. 2500
Amount paid in installment = Rs. 7500
Therefore, interest paid in installment scheme = Rs(7500-7000) = Rs. 500
Principal for the 1st month = Rs. 7000
Principal for the 2nd month = Rs. 4500
Principal for the 3rd month = Rs. 2000
Total = Rs. 13500
Let the rate of interest be r % per annum

$$I = \frac{p * r * 1}{1200} \quad (3.1.1)$$
$$I = \frac{13500 * r * 1}{100 * 12} \quad (3.1.2)$$
$$500 = \frac{13500 * r * 1}{100 * 12} \quad (3.1.3)$$
$$r = \frac{500 * 12}{135} \quad (3.1.4)$$
$$r = 44.4\% \quad (3.1.5)$$

3.2. Simplify:

$$\frac{X}{X-Y} - \frac{Y}{X+Y} - \frac{2XY}{X^2-Y^2}$$

Solution:
Let $\frac{X}{Y} = V$

$$= \frac{X/Y}{X/Y-1} - \frac{1}{X/Y+1} - \frac{2X/Y}{X^2/Y^2-1} \quad (3.2.1)$$
$$= \frac{X/Y}{X/Y-1} - \frac{1}{X/Y+1} - \frac{2X/Y}{X^2/Y^2-1} \quad (3.2.2)$$
$$= \frac{V}{V-1} - \frac{1}{V+1} - \frac{2V}{V^2-1} \quad (3.2.3)$$
$$= \frac{V(V+1) - 1(V-1) - 2V}{V^2-1} \quad (3.2.4)$$
$$= \frac{V^2 + V - V + 1 - 2V}{V^2-1} \quad (3.2.5)$$
$$= \frac{(V-1)^2}{V^2-1} \quad (3.2.6)$$
$$= \frac{V-1}{V+1} \quad (3.2.7)$$
$$= \frac{X/Y-1}{X/Y+1} \quad (3.2.8)$$
$$= \frac{X-Y}{X+Y} \quad (3.2.9)$$

3.3. A man borrows money from a finance company and has to pay it back in two equal half-yearly instalments of Rs. 7,396 each. If the interest is charged by the finance company at the rate of 15 % per annum, compounded semi-annually, find the principal and the total interest paid.
Solution:
Given,

Symbol	Description	Value
P	Principal amount	?
R	Rate of interest	15% per annum
n	Period of interest	0.5 year
I	Interest paid	?
T	Total amount paid	14792

TABLE 3.3

$$T = P\left(1 + \frac{R}{100}\right)^n \quad (3.3.1)$$
$$14792 = P\left(1 + \frac{15}{100}\right)^{0.5}\left(1 + \frac{15}{100}\right)^{0.5} \quad (3.3.2)$$
$$T = P\left(1 + \frac{15}{100}\right)^1 \quad (3.3.3)$$
$$P = \frac{14792}{1.15} \quad (3.3.4)$$
$$P = 12862 \quad (3.3.5)$$
$$I = T - P = 14792 - 12862 = 1930 \quad (3.3.6)$$

3.4. By increasing the list price of a book by Rs. 10 a person can buy 10 less books for Rs. 1,200. Find the original list price of the book.
Solution: Given,

Symbol	Description	Value
x	list price of a book	?
y	no. of books	?
P	Total price	1200

TABLE 3.4

$$y = \frac{1200}{x} \quad (3.4.1)$$
$$y - 10 = \frac{1200}{x+10} \quad (3.4.2)$$
$$y = \frac{1200}{x+10} + 10 \quad (3.4.3)$$

comparing equations (3.4.2) and (3.4.3)

$$\frac{1200}{x} = \frac{1200}{x+10} + 10 \tag{3.4.4}$$

$$x^2 + 10x - 1200 = 0 \tag{3.4.5}$$

$$(x-30)(x+40) = 0 \tag{3.4.6}$$

$$x = 30, -40 \tag{3.4.7}$$

Therefore, the original price list is Rs. 30

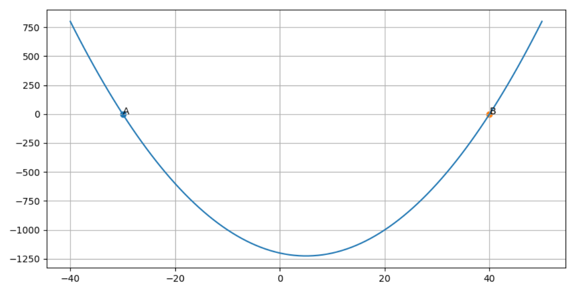

Fig. 3.4

3.5. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

Solution:

$$x - y = 5 \tag{3.5.1}$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{10} \tag{3.5.2}$$

Modifying equation (3.5.2), we get

$$\frac{1}{x} - \frac{1}{x-5} = \frac{1}{10} \tag{3.5.3}$$

$$x^2 - 5x - 50 = 0 \tag{3.5.4}$$

$$x = 10, -5 \tag{3.5.5}$$

$$y = 5, -10 \tag{3.5.6}$$

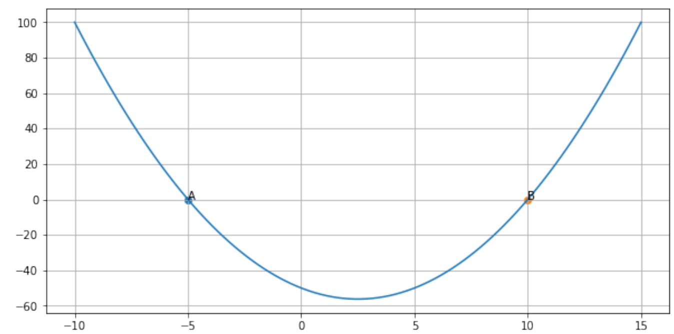

Fig. 3.5

3.6. Ms. Shahnaz earns Rs. 35,000 per month (excluding HRA). She donates Rs. 30,000 to Prime Minister Relief Fund (100% exemption) and Rs. 40,000 to a Charitable Hospital (50% exemption). She contributes Rs. 5,000 per month to Provident Fund and Rs. 25,000 per annum towards LIC premium. She purchases NSC worth Rs. 20,000. She pays Rs. 2,300 per month towards income tax for 11 month. Find the amount of income tax she has to pay in 12th month of the year.
Use the following to calculate income tax :

(a) Saving:	100 % exemption for permissible savings upto Rs. 1,00,000
(b) Rates of income tax for ladies:	
Slab	**Income tax**
(i) Upto Rs. 1,35,000	No tax
(ii) From Rs. 1,35,001 to Rs. 1,50,000	10% of taxable income exceeding Rs. 1,35,000
(iii) From Rs. 1,50,001 to Rs. 2,50,000	Rs. 1,500 + 20% of the amount excedding Rs. 1,50,000
(iv) From Rs. 2,50,001 and above	Rs. 21,500 + 30% of the amount exceeding Rs. 2,50,000
Education Cess :	2% of Income tax payable

Solution:
Annual Income = 35000 * 12 = Rs 420000
PF = 5000 * 12 = Rs 60000
LIC Premium = Rs 25000

NSC = Rs 20000
Saving Under 80C = 60000 + 25000 + 20000 = Rs 105000
Prime minister relief fund exemption = Rs 30000
Charitable hospital exemption = (50/100) * 40000 = Rs 20000
Total Donation exemption = 30000 + 20000 = Rs 50000
Taxable income = 420000 - 105000 - 50000 = Rs 265000
Tax Slab = 0-250000 = 0 %
250000 - 50000 = 5 %
265000 - 250000 = Rs 15000
5% of Rs 15000 = Rs 750
Income tax already paid in 11 months = 2300 * 12 = Rs 27600

4 Trigonometry

4.1. Prove that

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\sin A+\cos A$$

Solution:

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\sin A+\cos A \quad (4.1.1)$$

$$=\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A} \quad (4.1.2)$$

$$=\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-1/\tan A} \quad (4.1.3)$$

$$=\frac{\cos A}{1-\tan A}+\frac{\sin A\tan A}{\tan A-1} \quad (4.1.4)$$

$$=\frac{\cos A}{1-\tan A}-\frac{\sin A\tan A}{1-\tan A} \quad (4.1.5)$$

$$=\frac{\cos A-\sin A\tan A}{1-\tan A} \quad (4.1.6)$$

$$=\frac{\cos^2 A-\sin^2 A}{\cos A-\sin A} \quad (4.1.7)$$

$$=\cos A+\sin A \quad (4.1.8)$$

4.2. Evaluate without using trigonometric tables :

$$\frac{3\cos 55^\circ}{7\sin 35^\circ}+\frac{4(\cos 70^\circ.\operatorname{cosec} 20^\circ)}{7(\tan 5^\circ.\tan 25^\circ.\tan 45^\circ.\tan 65^\circ.\tan 85^\circ)}$$

Solution:

$$=\frac{3\cos 55^\circ}{7\sin 35^\circ}+\frac{4(\cos 70^\circ.\operatorname{cosec} 20^\circ)}{7(\tan 5^\circ.\tan 25^\circ.\tan 45^\circ.\tan 65^\circ.\tan 85^\circ)}$$

$$=\frac{3\cos(90-35)^\circ}{7\sin 35^\circ}+\frac{4(\cos 70^\circ.\operatorname{cosec}(90-70)^\circ)}{7(\tan 5^\circ.\tan 25^\circ.\tan 45^\circ.\tan(90-25)^\circ.\tan(90-5)^\circ)}$$

$$=\frac{3\sin 35^\circ}{7\sin 35^\circ}+\frac{4(\cos 70^\circ.\sec 70^\circ)}{7(\tan 5^\circ.\tan 25^\circ.\tan 45^\circ.\cot 25^\circ.\cot 5^\circ)}$$

$$=1$$

4.3. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite sides of the bird. Find the distance of bird from the girl.

Solution:
Given,

Symbol	Description	Value
d_b	distance from bird to boy	100m
θ_b	elevation of bird from boy	30°
h_g	Height of girl from ground	20m
θ_g	elevation of bird from girl	45°
d_g	distance of girl from bird	?
h	height of bird from ground	?

TABLE 4.3

$$h=d_b\tan\theta_b \quad (4.3.1)$$

$$h=100 \text{ x } 0.577=57.7 \quad (4.3.2)$$

$$d_g=(h-h_b)/\tan\theta_g \quad (4.3.3)$$

$$d_g=(57.7-20) \text{ x } 1=37.7 \quad (4.3.4)$$

5 Linear Algebra

5.1. solve the values of x and y.

$$x+\frac{6}{y}=6 \quad (5.1.1)$$

$$3x-\frac{8}{y}=5 \quad (5.1.2)$$

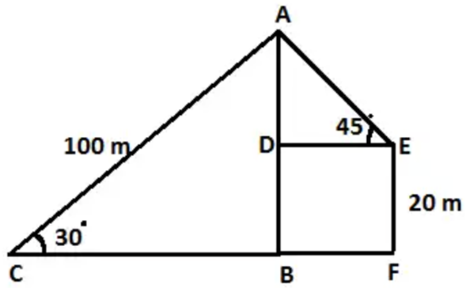

Fig. 4.3

Solution: Consider the equations given in the problem statement.

$$x + \frac{6}{y} = 6 \tag{5.1.3}$$

$$3x - \frac{8}{y} = 5 \tag{5.1.4}$$

The solution can be found by solving the above system of linear equations.
System of linear equations are defined as

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{5.1.5}$$

From the equations (5.1.3) and (5.1.4),

$$\mathbf{A} = \begin{pmatrix} 1 & 6 \\ 3 & -8 \end{pmatrix} \tag{5.1.6}$$

$$\mathbf{x} = \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} \tag{5.1.7}$$

$$\mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{5.1.8}$$

Substituting the values of **A**, **x** and **B** in the equation (5.1.5) We get,

$$\begin{pmatrix} 1 & 6 \\ 3 & -8 \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{5.1.9}$$

Considering the augmented matrix

$$\begin{pmatrix} 1 & 6 & 6 \\ 3 & -8 & 5 \end{pmatrix} \tag{5.1.10}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & 6 & 6 \\ 0 & -26 & -13 \end{pmatrix} \tag{5.1.11}$$

$$\xleftrightarrow{R_1 \leftarrow 13R_1 + 3R_2} \begin{pmatrix} 13 & 0 & 39 \\ 0 & -26 & -13 \end{pmatrix} \tag{5.1.12}$$

$$\xleftrightarrow{R_1 \leftarrow R_1/3, R_2 \leftarrow R_2/-26} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0.5 \end{pmatrix} \tag{5.1.13}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} \tag{5.1.14}$$

From the above equation (5.1.14) we get,

$$x = 3 \tag{5.1.15}$$

$$y = 2 \tag{5.1.16}$$

Therefore, x=3 and y= 2 are solutions to the given equations.

5.2. solve the values of x and y

$$\frac{x+1}{2} + \frac{y-1}{3} = 8 \tag{5.2.1}$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9 \tag{5.2.2}$$

Solution: Consider the equations given in the problem statement.

$$\frac{x+1}{2} + \frac{y-1}{3} = 8 \tag{5.2.3}$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9 \tag{5.2.4}$$

The above equations can be rearranged as the following equations

$$3x + 2y = 47 \tag{5.2.5}$$

$$2x + 3y = 53 \tag{5.2.6}$$

The solution can be found by solving the above system of linear equations.
System of linear equations are defined as

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{5.2.7}$$

From the equations (5.2.5) and (5.2.6),

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \tag{5.2.8}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{5.2.9}$$

$$\mathbf{B} = \begin{pmatrix} 47 \\ 53 \end{pmatrix} \tag{5.2.10}$$

Substituting the values of **A**, **x** and **B** in the equation (5.2.7) We get,

$$\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 47 \\ 53 \end{pmatrix} \tag{5.2.11}$$

Considering the augmented matrix

$$\begin{pmatrix} 3 & 2 & 47 \\ 2 & 3 & 53 \end{pmatrix} \tag{5.2.12}$$

$$\xleftrightarrow{R_2 \leftarrow 3R_2 - 2R_1} \begin{pmatrix} 3 & 2 & 47 \\ 0 & 5 & 65 \end{pmatrix} \tag{5.2.13}$$

$$\xleftrightarrow{R_1 \leftarrow 5R_1 - 2R_2} \begin{pmatrix} 15 & 0 & 105 \\ 0 & 5 & 65 \end{pmatrix} \tag{5.2.14}$$

$$\xleftrightarrow{R_1 \leftarrow R_1/15, R_2 \leftarrow R_2/5} \begin{pmatrix} 1 & 0 & 7 \\ 0 & 1 & 13 \end{pmatrix} \tag{5.2.15}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 13 \end{pmatrix} \tag{5.2.16}$$

By solving equation (5.2.16) we get,

$$x = 7 \tag{5.2.17}$$

$$y = 13 \tag{5.2.18}$$

Therefore, x=7 and y= 13 are solutions to the given equations.

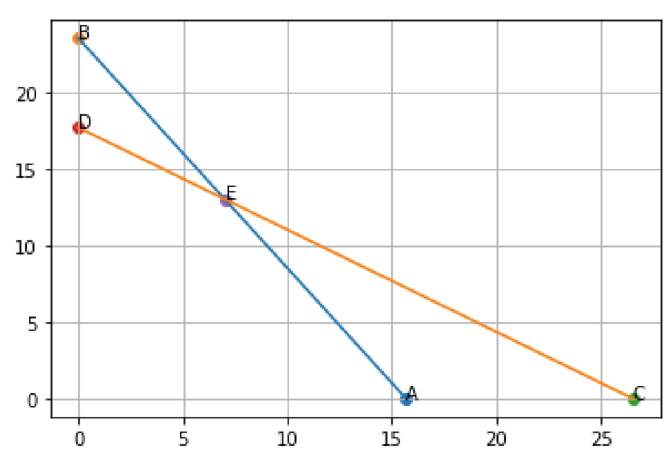

Fig. 5.2

5.3. Show that the points given below are vertices of an isosceles right angle triangle.

$$\begin{pmatrix} 7 \\ 10 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \end{pmatrix} \text{and} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{5.3.1}$$

Solution: Consider the given points as vectors,

$$\mathbf{A} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} \tag{5.3.2}$$

$$\mathbf{B} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \tag{5.3.3}$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{5.3.4}$$

For a triangle to be an isosceles, any two sides of the triangle should be equal. For finding a triangle to be isosceles and right angle, we consider,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} \tag{5.3.5}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 9 \end{pmatrix} \tag{5.3.6}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 7 \\ 10 \end{pmatrix} = \begin{pmatrix} -4 \\ -14 \end{pmatrix} \tag{5.3.7}$$

$$(\mathbf{A} - \mathbf{B})^T (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} 9 & 5 \end{pmatrix} \begin{pmatrix} -5 \\ 9 \end{pmatrix} \tag{5.3.8}$$

$$= -45 + 45 = 0 \tag{5.3.9}$$

$$(\mathbf{C} - \mathbf{A})^T (\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -4 & -14 \end{pmatrix} \begin{pmatrix} 9 \\ 5 \end{pmatrix} \tag{5.3.10}$$

$$= -36 - 70 = -106 \tag{5.3.11}$$

$$(\mathbf{B} - \mathbf{C})^T (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} -5 & 9 \end{pmatrix} \begin{pmatrix} -4 \\ -14 \end{pmatrix} \tag{5.3.12}$$

$$= 20 - 126 = -106 \tag{5.3.13}$$

From the equation (5.3.9)

$$\mathbf{A} - \mathbf{B} \perp \mathbf{B} - \mathbf{C} \tag{5.3.14}$$

Therefore $\measuredangle B = 90°$. From the equations (5.3.11) and (5.3.13) $\measuredangle CAB = \measuredangle BCA$. Therefore, $\triangle ABC$ is an isosceles right angle triangle with sides AB=BC and right angle at B.

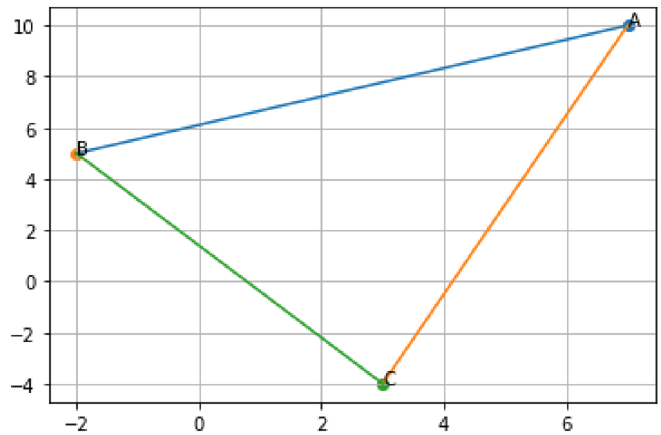

Fig. 5.3

5.4. In what ratio does the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 2 \tag{5.4.1}$$

divides the line segment joining

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{and} \begin{pmatrix} 8 \\ 9 \end{pmatrix} \tag{5.4.2}$$

Solution: Consider the line

$$\mathbf{n}^T \mathbf{x} = c \tag{5.4.3}$$

divides the line segment **A** and **B** in $k : 1$ ratio.
p is point of intersection of two lines.
From the section formula we can write,

$$\mathbf{p} = \frac{1}{k+1} [\mathbf{A} + k\mathbf{B}] \tag{5.4.4}$$

$$\tag{5.4.5}$$

The point **p** passes through the line $\mathbf{n}^T \mathbf{x} = c$, therefore,

$$\mathbf{n}^T \mathbf{p} = c \tag{5.4.6}$$

$$\mathbf{n}^T \left(\frac{\mathbf{A} + k\mathbf{B}}{k+1} \right) = c \tag{5.4.7}$$

Solving for k, we get,

$$k = \frac{c - \mathbf{n}^T \mathbf{A}}{\mathbf{n}^T \mathbf{B} - c} \tag{5.4.8}$$

From the equations (5.4.1) and (5.4.2),

$$\mathbf{n}^T = \begin{pmatrix} 1 & -1 \end{pmatrix} \tag{5.4.9}$$

$$\mathbf{A} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{5.4.10}$$

$$\mathbf{B} = \begin{pmatrix} 8 \\ 9 \end{pmatrix} \tag{5.4.11}$$

Substituting the above values in the equation (5.4.8), we get,

$$k = \frac{2 - \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix}}{\begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 8 \\ 9 \end{pmatrix} - 2} \tag{5.4.12}$$

$$k = \frac{2}{3} \tag{5.4.13}$$

Therefore, the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 2 \tag{5.4.14}$$

divides the line segment joining

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{and} \begin{pmatrix} 8 \\ 9 \end{pmatrix} \tag{5.4.15}$$

in 2:3 ratio.

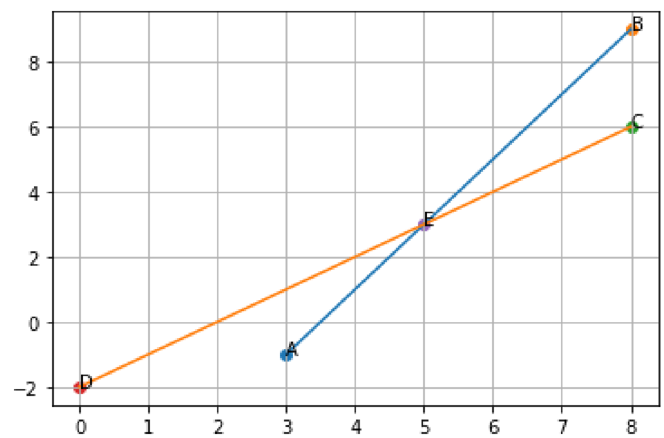

Fig. 5.4

5.5. **P** and **Q** are points on the sides **CA** and **CB** respectively of $\triangle ABC$, right angled at C. Prove that

$$\mathbf{AQ}^2 + \mathbf{BP}^2 = \mathbf{AB}^2 + \mathbf{PQ}^2 \tag{5.5.1}$$

Solution: construction of figure.
Input parameters and output parameters are shown in the table 5.5.

Symbol	Description	Value
$\mathbf{B}$	vertex B	$\begin{pmatrix}0\\0\end{pmatrix}$
c	length of the side opposite to vertex C	5cm
θ_B	angle at vertex B	30°
k_1	Ratio of point Q diving the line CB	2
K_2	Ratio of point P diving the line CA	3

TABLE 5.5: Input Parameters

Symbol	Description	Value
$\mathbf{A}$	vertex A	$\begin{pmatrix}b\cos B\\b\sin B\end{pmatrix}$
$\mathbf{C}$	vertex C	$\begin{pmatrix}b\cos B\\0\end{pmatrix}$
$\mathbf{P}$	vertex P	$\begin{pmatrix}b\cos B\\K_2\end{pmatrix}$
$\mathbf{Q}$	vertex Q	$\begin{pmatrix}K_1\\0\end{pmatrix}$

TABLE 5.5: Output Parameters

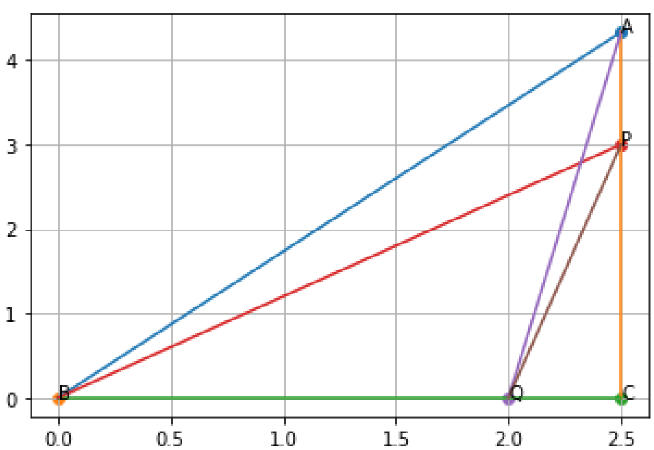

Fig. 5.5

From the python code, (5.5.1) is proved.

5.6. In figure 5.6 , $\mathbf{DE} \parallel \mathbf{AB}$ and $\mathbf{FE} \parallel \mathbf{DB}$. Prove that

$$\mathbf{DC}^2 = \mathbf{CF}.\mathbf{AC} \tag{5.6.1}$$

Solution: Construction of the figure,
Input parameters and output parameters are shown in the table 5.10

Symbol	Description	Value
$\mathbf{A}$	vertex $\mathbf{A}$	$\begin{pmatrix}0\\0\end{pmatrix}$
c	length of the side opposite to vertex C	3cm
b	length of the side opposite to vertex B	5cm
θ_A	angle at vertex $\mathbf{A}$	30°
k	Ratio of point D diving the line AC	2

TABLE 5.6: Input Parameters

Symbol	Description	Value
$\mathbf{C}$	vertex C	$\begin{pmatrix}b\cos B\\b\sin B\end{pmatrix}$
$\mathbf{B}$	vertex B	$\begin{pmatrix}c\\0\end{pmatrix}$
$\mathbf{D}$	vertex D	$\dfrac{K\mathbf{A}+\mathbf{C}}{k+1}$
$\mathbf{E}$	vertex E	$\dfrac{K\mathbf{B}+\mathbf{C}}{k+1}$
$\mathbf{F}$	vertex F	$\dfrac{K\mathbf{D}+\mathbf{C}}{k+1}$

TABLE 5.6: Output Parameters

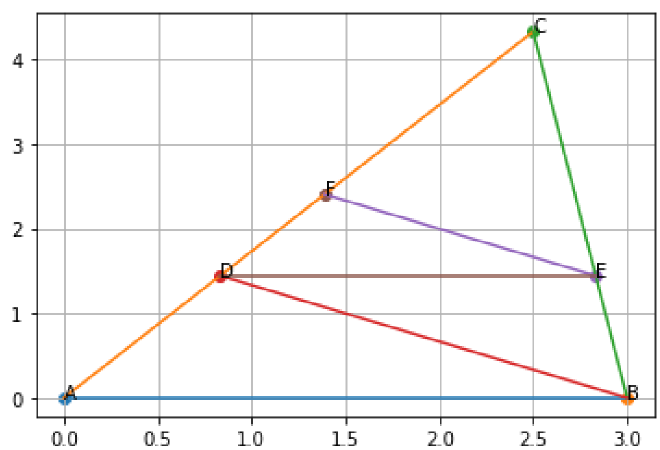

Fig. 5.6

From the python code, (5.6.1) is proved.

5.7. Solve the following system of equations graphically :

$$2X + 3Y = 8$$
$$X + 4Y = 9$$

Solution:

Consider the equations,

$$2X + 3Y = 8 \tag{5.7.1}$$
$$X + 4Y = 9 \tag{5.7.2}$$

The solution can be found by solving the above system of linear equations.
System of linear equations are defined as

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{5.7.3}$$

From the equations (5.7.1) and (5.7.2),

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \tag{5.7.4}$$
$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{5.7.5}$$
$$\mathbf{B} = \begin{pmatrix} 8 \\ 9 \end{pmatrix} \tag{5.7.6}$$

Substituting the values of **A**, **x** and **B** in the equation (5.7.3) We get,

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 9 \end{pmatrix} \tag{5.7.7}$$

Considering the augmented matrix

$$\begin{pmatrix} 2 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} \tag{5.7.8}$$
$$\xleftrightarrow{R_2 \leftarrow 2R_2 - R_1} \begin{pmatrix} 2 & 3 & 8 \\ 0 & 5 & 10 \end{pmatrix} \tag{5.7.9}$$
$$\xleftrightarrow{R_1 \leftarrow 5R_1 - 3R_2} \begin{pmatrix} 10 & 0 & 10 \\ 0 & 5 & 10 \end{pmatrix} \tag{5.7.10}$$
$$\xleftrightarrow{R_1 \leftarrow R_1/10, R_2 \leftarrow R_2/5} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{5.7.11}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{5.7.12}$$

By solving equation (5.7.12) we get,

$$x = 1 \tag{5.7.13}$$
$$y = 2 \tag{5.7.14}$$

Therefore, x=1 and y= 2 are solutions to the given equations.

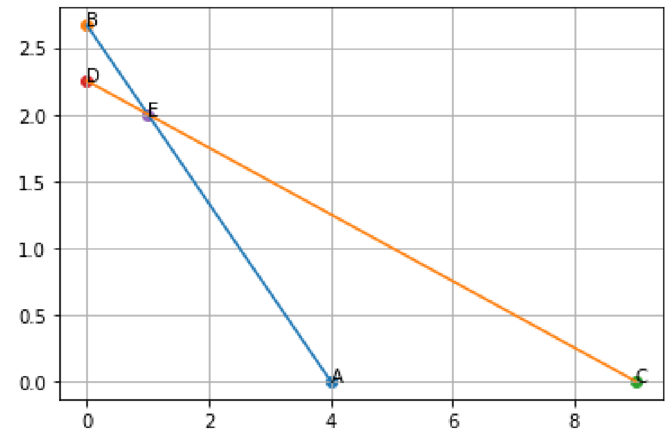

Fig. 5.7

5.8. In figure, 5.8 **TA** is a tangent to the circle from a point T and **TBC** is a secant to the circle. If **AD** is the bisector of $\measuredangle CAB$, prove that $\triangle ADT$ is isosceles.
Solution:
Input parameters,
From the figure 5.8,

Symbol	Description	Value
O	center of the circle	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
r	radius of the circle	2cm
d	distance of point **T**	5cm
β	$\measuredangle OTB$	20°
T	Point **T**	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$

TABLE 5.8: Input Parameters

$$\mathbf{T} = \left(d, 0\right) \tag{5.8.1}$$
$$\measuredangle OTA = \sin^{-1}\left(\frac{\|OA\|}{\|OT\|}\right) \tag{5.8.2}$$
$$\measuredangle OTA = \sin^{-1}\left(\frac{2}{5}\right) = 23.57° \tag{5.8.3}$$

Finding the coordinates of **A**.
Equation of line **AT** is given by,

$$\mathbf{x} = \mathbf{T} + \lambda\mathbf{m} \tag{5.8.4}$$

where m is the the directional vector, $\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix}$, where m is slope of the line.

$$m = \tan\measuredangle 180 - OTA = -0.436 \tag{5.8.5}$$

$$\mathbf{x} = \begin{pmatrix}5\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-0.436\end{pmatrix} \tag{5.8.6}$$

Point **A**is intersection of circle and tanget **AT**. Equation of the circle,

$$\mathbf{x}^T\mathbf{A}\mathbf{x} + 2\mathbf{b}^T\mathbf{x} + c = 0 \tag{5.8.7}$$

$$\mathbf{A} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{5.8.8}$$

$$\mathbf{b} = \begin{pmatrix}0\\0\end{pmatrix} \tag{5.8.9}$$

$$c = -4 \tag{5.8.10}$$

From (5.8.4) and (5.8.24),

$$\mathbf{x}^T\mathbf{A}\mathbf{x} + 2\mathbf{b}^T\mathbf{x} + c = 0 \tag{5.8.11}$$

$$\mathbf{x} = \mathbf{T} + \lambda\mathbf{m} \tag{5.8.12}$$

$$\mathbf{T} + \lambda\mathbf{m}^T\mathbf{A}\mathbf{T} + \lambda\mathbf{m} + 2\mathbf{b}^T\mathbf{T} + \lambda\mathbf{m} + c = 0 \tag{5.8.13}$$

$$\mathbf{T} + \lambda\mathbf{m}^{\mathbf{T}}\mathbf{A}\mathbf{T} + \lambda\mathbf{m} + 2\mathbf{b}^T\mathbf{T} + \lambda\mathbf{m} + c = 0 \tag{5.8.14}$$

Substituting the values of **A** and **b** we get,

$$\|\mathbf{T}\|^2 + 2\lambda(\mathbf{m}^T\mathbf{T}) + \lambda^2\|\mathbf{m}\|^2 + c = 0 \tag{5.8.15}$$

$$\lambda^2\|\mathbf{m}\|^2 + 2\lambda(\mathbf{m}^T\mathbf{T}) + \|\mathbf{T}\|^2 + c = 0 \tag{5.8.16}$$

Substituting the values of **T** and **m**,

$$1.19\lambda^2 + 10\lambda + 21 = 0 \tag{5.8.17}$$

$$\lambda = -4.2, -4.2 \tag{5.8.18}$$

Substituting the lambda value in (5.8.4) we get,

$$\mathbf{A} = \mathbf{T} + \lambda\mathbf{m} \tag{5.8.19}$$

$$\mathbf{A} = \begin{pmatrix}5\\0\end{pmatrix} - 4.2\begin{pmatrix}1\\-0.436\end{pmatrix} \tag{5.8.20}$$

$$\mathbf{A} = \begin{pmatrix}0.8\\1.831\end{pmatrix} \tag{5.8.21}$$

Finding the coordinates of **B** and **C**.
$\measuredangle OTB$ is $<$ $\measuredangle OTA$. Therefore, select $\measuredangle OTB = 20°$. Using the equations (5.8.4) and (5.8.24). Here the direction vector **m** is $\begin{pmatrix}1\\0.363\end{pmatrix}$.

$$\mathbf{x} = \begin{pmatrix}5\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\0.363\end{pmatrix} \tag{5.8.22}$$

Equation of the circle,

$$\mathbf{x}^T\mathbf{A}\mathbf{x} + 2\mathbf{b}^T\mathbf{x} + c = 0 \tag{5.8.23}$$

$$\mathbf{A} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{5.8.24}$$

$$\mathbf{b} = \begin{pmatrix}0\\0\end{pmatrix} \tag{5.8.25}$$

$$c = -4 \tag{5.8.26}$$

$$\mathbf{x}^T\mathbf{A}\mathbf{x} + 2\mathbf{b}^T\mathbf{x} + c = 0 \tag{5.8.27}$$

$$\mathbf{x} = \mathbf{T} + \lambda\mathbf{m} \tag{5.8.28}$$

$$\mathbf{T} + \lambda\mathbf{m}^T\mathbf{A}\mathbf{T} + \lambda\mathbf{m} + 2\mathbf{b}^T\mathbf{T} + \lambda\mathbf{m} + c = 0 \tag{5.8.29}$$

$$\mathbf{T} + \lambda\mathbf{m}^{\mathbf{T}}\mathbf{A}\mathbf{T} + \lambda\mathbf{m} + 2\mathbf{b}^T\mathbf{T} + \lambda\mathbf{m} + c = 0 \tag{5.8.30}$$

Substituting the values of **A** and **b** we get,

$$\|\mathbf{T}\|^2 + 2\lambda(\mathbf{m}^T\mathbf{T}) + \lambda^2\|\mathbf{m}\|^2 + c = 0 \tag{5.8.31}$$

$$\lambda^2\|\mathbf{m}\|^2 + 2\lambda(\mathbf{m}^T\mathbf{T}) + \|\mathbf{T}\|^2 + c = 0 \tag{5.8.32}$$

Substituting the values of **T** and **m**,

$$1.13\lambda^2 + 10\lambda + 21 = 0 \tag{5.8.33}$$

$$\lambda = -3.42, -5.42 \tag{5.8.34}$$

Substituting the lambda value in (5.8.4) we get,

$$\mathbf{x} = \mathbf{T} + \lambda\mathbf{m} \tag{5.8.35}$$

$$\mathbf{C} = \begin{pmatrix}5\\0\end{pmatrix} - 3.42\begin{pmatrix}1\\0.363\end{pmatrix} \tag{5.8.36}$$

$$\mathbf{C} = \begin{pmatrix}1.58\\-1.241\end{pmatrix} \tag{5.8.37}$$

$$\mathbf{B} = \begin{pmatrix}5\\0\end{pmatrix} - 5.42\begin{pmatrix}1\\0.363\end{pmatrix} \tag{5.8.38}$$

$$\mathbf{B} = \begin{pmatrix}-0.42\\1.96\end{pmatrix} \tag{5.8.39}$$

Finding the coordinates of **D**.
AD is the angular bisector of $\measuredangle BAC$.
Equation of angular bisector,

$$\frac{\mathbf{n_1}^T\mathbf{x} + c_1}{\|n_1\|} = \pm\frac{\mathbf{n_2}^T\mathbf{x} + c_2}{\|n_2\|} \tag{5.8.40}$$

where, $\mathbf{n_1}^T\mathbf{x} + c_1$ is the equation of line **AB** and

$\mathbf{n_2}^T\mathbf{x} + c_2$ is the equation of line **AC**. Equation of line **AB**,

$$\mathbf{n_1}^T\mathbf{x} - \mathbf{A} = 0 \tag{5.8.41}$$

$$\begin{pmatrix} 3.99 & 1 \end{pmatrix}\mathbf{x} = 5.02 \tag{5.8.42}$$

Equation of line **AC**,

$$\mathbf{n_2}^T\mathbf{x} - \mathbf{A} = 0 \tag{5.8.43}$$

$$\begin{pmatrix} 3.21 & -1 \end{pmatrix}\mathbf{x} = 0.72 \tag{5.8.44}$$

From equation (5.8.40) of angular bisector we get,

$$\frac{\begin{pmatrix} 3.99 & 1 \end{pmatrix}\mathbf{x} - 5.02}{\left\|\begin{pmatrix} 3.99 \\ 1 \end{pmatrix}\right\|} = \pm\frac{\begin{pmatrix} 3.21 & -1 \end{pmatrix}\mathbf{x} - 0.72}{\left\|\begin{pmatrix} 3.21 \\ -1 \end{pmatrix}\right\|} \tag{5.8.45}$$

Solving equation (5.8.45), we get 2 equations of angular bisectors

$$\begin{pmatrix} 0.211 & 7.476 \end{pmatrix}\mathbf{x} - 13.916 = 0 \tag{5.8.46}$$

$$\begin{pmatrix} 26.619 & 0.751 \end{pmatrix}\mathbf{x} - 19.84 = 0 \tag{5.8.47}$$

From the equations (5.8.46) and (5.8.47), only (5.8.47) forms point **D**, intersection of **AD** and **TBC**

$$\mathbf{AD} = \begin{pmatrix} 26.619 & 0.751 \end{pmatrix}\mathbf{x} - 19.84 = 0 \tag{5.8.48}$$

$$\mathbf{TBC} = \begin{pmatrix} 0.36 & -1 \end{pmatrix}\mathbf{x} - 1.81 = 0 \tag{5.8.49}$$

Solving (5.8.48) and (5.8.49), we get

$$\mathbf{D} = \begin{pmatrix} 0.703 \\ -1.556 \end{pmatrix} \tag{5.8.50}$$

Proof of $\triangle ADT$ being an isosceles is done using python code

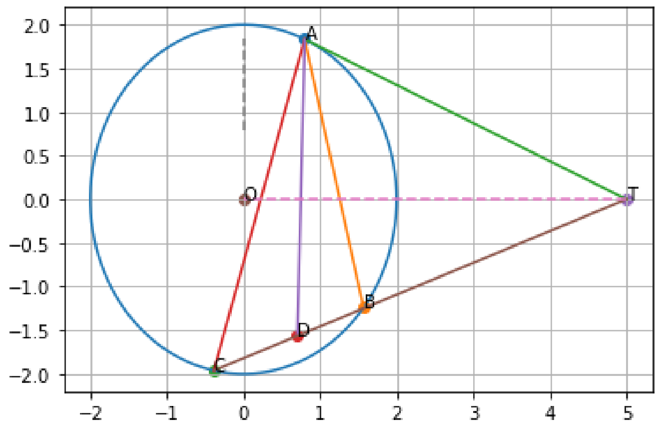

Fig. 5.8

5.9. In the $\triangle ABC$, $\mathbf{AD} \perp \mathbf{BC}$ and $\mathbf{AD^2} = \mathbf{BD.DC}$. Prove that $\measuredangle BAC$ is a right angle.
Solution:
Consider, Given,

Symbol	Description	Value
B	vertex **A**	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
c	length of the side opposite to vertex C(**AB**)	5cm
θ_B	$\measuredangle ABC$	60°

TABLE 5.9

$$\mathbf{AD}^2 = \mathbf{BDCD} \tag{5.9.1}$$

From the figure 5.9, we get

$$\|AD\| = c\sin\theta_B = 5\sin 30^\circ \tag{5.9.2}$$

$$\|BD\| = c\cos\theta_B = 5\cos 30^\circ \tag{5.9.3}$$

$$\|DC\| = \frac{\|AD\|^2}{\|BD\|} \tag{5.9.4}$$

$$\|BC\| = \|BD\| + \|DC\| \tag{5.9.5}$$

$$\mathbf{A} = \begin{pmatrix} -c \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \tag{5.9.6}$$

$$\mathbf{D} = \begin{pmatrix} -\|BD\|\cos\theta_B \\ \|BD\|\sin\theta_B \end{pmatrix} \tag{5.9.7}$$

$$\mathbf{C} = \begin{pmatrix} -\|BC\|\cos\theta_B \\ \|BC\|\sin\theta_B \end{pmatrix} \tag{5.9.8}$$

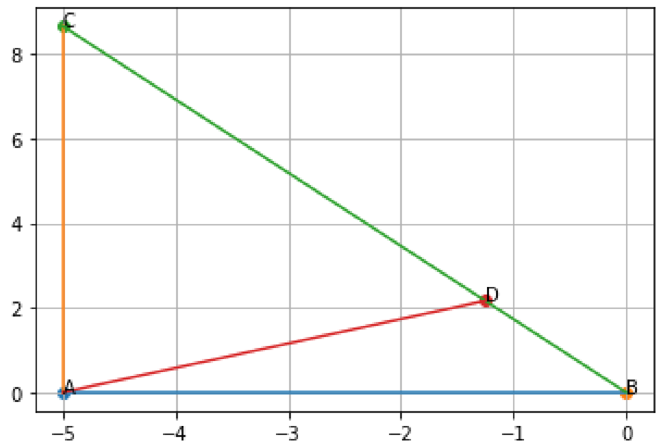

Fig. 5.9

Proof that $\measuredangle CAB$ is right angle is done in python

5.10. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Figure, 5.10 $\mathbf{DE} \| \mathbf{BC}$ and $\mathbf{BD} = \mathbf{CE}$. Prove that $\triangle ABC$ is an isosceles triangle.

Solution:

Consider, Given, two sides **AB** and**AC** are

Symbol	Description	Value
A	vertex **A**	$\begin{pmatrix}0\\0\end{pmatrix}$
c	length of the side opposite to vertex C(**AB**)	5cm
a	length of the side opposite to vertex A(**BC**)	4cm
θ_B	$\measuredangle ABC$	60°
k	Ratio of point **P** and **Q** diving the line **AB** and **AC** respectively	2

TABLE 5.10

divided equally by the points **P** and **Q**. Let K is the ratio.

$$\mathbf{P} = \frac{K\mathbf{A} + \mathbf{C}}{k+1} \tag{5.10.1}$$

$$\mathbf{Q} = \frac{K\mathbf{A} + \mathbf{B}}{k+1} \tag{5.10.2}$$

Given $\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix}$, c=5, a = 4 and $\measuredangle CAB = 60°$

$$\mathbf{B} = \begin{pmatrix}a\\0\end{pmatrix} = \begin{pmatrix}4, 0\end{pmatrix} \tag{5.10.3}$$

$$\mathbf{C} = \begin{pmatrix}c\cos\measuredangle CAB\\c\sin\measuredangle CAB\end{pmatrix} = \begin{pmatrix}5\cos 60°\\5\sin 60°\end{pmatrix} \tag{5.10.4}$$

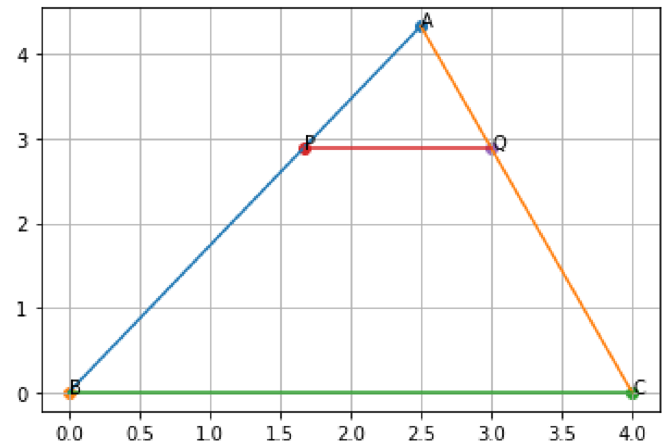

Fig. 5.10

Proof of $\triangle ABC$ is isosceles done in python code.

6 Probability and statastics

6.1. The mean of the following frequency distribution is 62.8. Find the missing frequency x.

Solution:

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	x	12	7	8

TABLE 6.1

let

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	x	12	7	8
x_i	10	30	50	70	90	110
$f_i x_i$	50	240	50x	840	630	880

TABLE 6.1

$$\mathbf{fx} = \begin{pmatrix}50\\240\\840\\630\\880\end{pmatrix} \tag{6.1.1}$$

$$\mathbf{f} = \begin{pmatrix}5\\8\\12\\7\\8\end{pmatrix} \tag{6.1.2}$$

$$E(x) = \frac{\mathbf{1}^T\mathbf{fx} + 50x}{\mathbf{1}^T\mathbf{f} + x} \quad (6.1.3)$$
$$x = \frac{E(x)(\mathbf{1}^T\mathbf{f}) - \mathbf{1}^T\mathbf{fx}}{50 - E(x)} \quad (6.1.4)$$
$$62.8 = \frac{2640 + 50x}{40 + x} \quad (6.1.5)$$
$$x = 10 \quad (6.1.6)$$

6.2. Cards marked with numbers 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that number on the drawn card is

a) divisible by 7.
b) a number which is a perfect square.

Solution:
Let A = numbers divisible by 7.

$$\{A\} = 7, 14, 21, 28, 35, 42, 49 \quad (6.2.1)$$
$$\{S\} = 3, 4, 5, ..., 50 \quad (6.2.2)$$
$$n(A) = 9 \quad (6.2.3)$$
$$n(S) = 48 \quad (6.2.4)$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{9}{48} \quad (6.2.5)$$

Let B = a number which is a perfect square.

$$\{B\} = 4, 9, 16, 25, 36, 49 \quad (6.2.6)$$
$$\{S\} = 3, 4, 5, ..., 50 \quad (6.2.7)$$
$$n(B) = 6 \quad (6.2.8)$$
$$n(S) = 48 \quad (6.2.9)$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{48} \quad (6.2.10)$$

6.3. The enrolment of a secondary school in different classes is given below. Draw a pie chart to represent the above data.
Solution:

Class	VI	VII	VIII	IX	X
Enrolment	600	500	400	700	200

TABLE 6.3

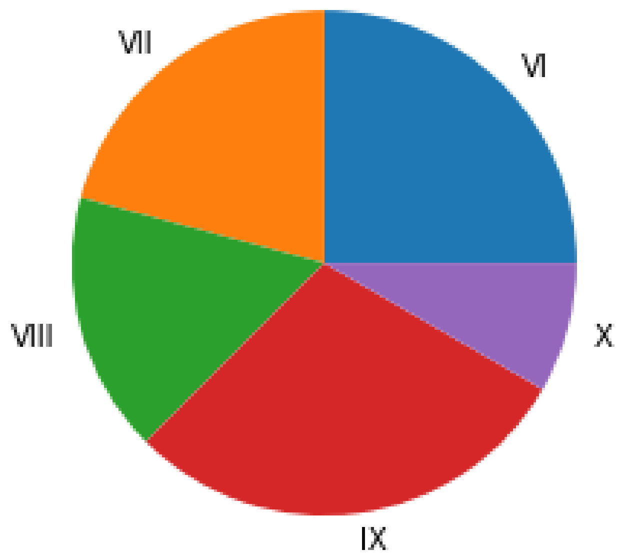

Fig. 6.3

6.4. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of a red ball, find the number of blue balls in the bag.
Solution:
Let X be a random variable, X= No. of balls
X=0 is no.of red balls
X=1 is no. of blue balls
Let s be total no.of balls.
Given,

$$P_X(x = 0) = \frac{5}{s} \quad (6.4.1)$$
$$P_X(x = 1) = \frac{s - 5}{s} \quad (6.4.2)$$
$$P_X(x = 1) = 3P_X(x = 0) \quad (6.4.3)$$
$$\frac{s - 5}{s} = \frac{15}{s} \quad (6.4.4)$$
$$s = 20 \quad (6.4.5)$$

Therefore no.of blue balls are 15.