Sequencing in the Run-Time Event Calculus
Technical Appendix

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This document is structured as follows. First, we provide proofs for the propositions of the paper. Second, we outline the syntax of simple fluent-value pair (FVP) definitions.

1 Proofs of Propositions

Proposition 2 ($[\![\alpha_1 \,;\alpha_2]\!]^{rt}_S$ consists of MDIs)**.** Consider a stream S and activities α_1 and α_2. The intervals in $[\![\alpha_1 \,;\alpha_2]\!]^{rt}_S$ are MDIs. ♦

Proof. Suppose that the intervals in $[\![\alpha_1 \,;\alpha_2]\!]^{rt}_S$ are not MDIs. In other words, there exist $i_a, i_b \in [\![\alpha_1 \,;\alpha_2]\!]^{rt}_S$, such that $i_a \not\prec_{rt} i_b$ and $i_a \not\prec_{rt} i_b$, i.e., i_a and i_b are overlapping.

Since $i_a, i_b \in [\![\alpha_1 \,;\alpha_2]\!]^{rt}_S$, it holds that:

$$\exists i_{1a} \in [\![\alpha_1]\!]^{rt}_S, i_{2a} \in [\![\alpha_2]\!]^{rt}_S : i_a = i_{1a} \otimes_{rt} i_{2a}$$
$$\exists i_{1b} \in [\![\alpha_1]\!]^{rt}_S, i_{2b} \in [\![\alpha_2]\!]^{rt}_S : i_b = i_{1b} \otimes_{rt} i_{2b}$$

We may infer the following properties for these intervals:

$$\text{Def of } \otimes_{rt} \Rightarrow i_a = (s(i_{1a}), e(i_{2a})) \wedge i_b = (s(i_{1b}), e(i_{2b}))$$
$$i_a \not\prec_{rt} i_b \Rightarrow e(i_{2a}) \geq s(i_{1b}) \tag{1}$$
$$i_b \not\prec_{rt} i_a \Rightarrow e(i_{2b}) \geq s(i_{1a}) \tag{2}$$
$$i_{1a} \prec_{rt} i_{2a} \Rightarrow e(i_{1a}) < s(i_{2a}) \tag{3}$$
$$i_{1b} \prec_{rt} i_{2b} \Rightarrow e(i_{1b}) < s(i_{2b}) \tag{4}$$

Based on the right-hand side of expressions (1), (2) and (3), the temporal ordering of intervals i_{1a}, i_{1b}, i_{2a} and i_{2b} needs to abide by the constraints shown in Figure 1.

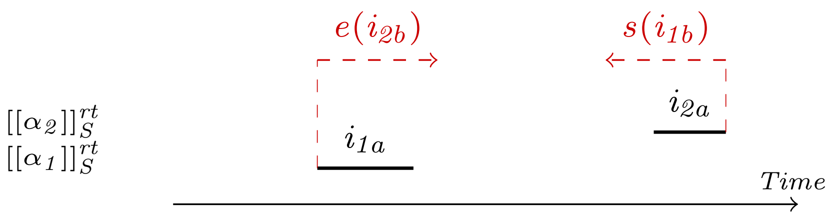

Figure 1: Temporal ordering and constraints based on expressions (1), (2) and (3).

Based on Figure 1, there are four ways of placing intervals i_{1b} and i_{2b}, while respecting expression (4), which we oultine below:

1. $i_{1a} \prec_{rt} i_{1b} \prec_{rt} i_{2b} \prec_{rt} i_{2a}$
2. $i_{1a} \prec_{rt} i_{1b} \prec_{rt} i_{2a} \prec_{rt} i_{2b}$
3. $i_{1b} \prec_{rt} i_{1a} \prec_{rt} i_{2b} \prec_{rt} i_{2a}$
4. $i_{1b} \prec_{rt} i_{1a} \prec_{rt} i_{2a} \prec_{rt} i_{2b}$

All cases lead to a contradiction. In cases 1 and 2, we have $\mathsf{A}(i_{1a}, [[\alpha_1]]_S^{rt}, [[\alpha_2]]_S^{rt}) = \emptyset$, and not i_{2a}, implying that $i_a \notin [[\alpha_1 ; \alpha_2]]_S^{rt}$ (Definition 7), which is a contradiction. Cases 3 and 4 also lead to a contradiction; we have $\mathsf{A}(i_{1b}, [[\alpha_1]]_S^{rt}, [[\alpha_2]]_S^{rt}) = \emptyset$, and not i_{2b}, implying that $i_b \notin [[\alpha_1 ; \alpha_2]]_S^{rt}$.

Therefore, there are no intervals i_a, i_b in $[[\alpha_1 ; \alpha_2]]_S^{rt}$, such that $i_a \not\prec_{rt} i_b$ and $i_b \not\prec_{rt} i_a$. Thus, the intervals in $[[\alpha_1 ; \alpha_2]]_S^{rt}$ are MDIs. □

Proposition 3 ($[[(\alpha_1 ; \alpha_2) ; \alpha_3]]_S^{rt} = [[\alpha_1 ;(\alpha_2 ; \alpha_3)]]_S^{rt}$)**.** Consider a stream S and activities α_1, α_2 and α_3. It holds that

$$i \in [[(\alpha_1 ; \alpha_2) ; \alpha_3]]_S^{rt} \text{ iff } i \in [[\alpha_1 ;(\alpha_2 ; \alpha_3)]]_S^{rt} \qquad \blacklozenge$$

Proof. The following equivalence holds:

$$\begin{aligned}
& i \in [[(\alpha_1 ; \alpha_2) ; \alpha_3]]_S^{rt} \Leftrightarrow \\
& \exists i_{12} \in [[\alpha_1 ; \alpha_2]]_S^{rt}, i_3 \in [[\alpha_3]]_S^{rt} : i = (s(i_{12}), e(i_3)) \wedge \\
& \exists i_1 \in [[\alpha_1]]_S^{rt}, i_2 \in [[\alpha_2]]_S^{rt} : i_{12} = (s(i_1), e(i_2)) \wedge \\
& \nexists i'_{12} \in [[\alpha_1 ; \alpha_2]]_S^{rt} : i_{12} \prec_{rt} i'_{12} \prec_{rt} i_3 \wedge && (5) \\
& \nexists i'_3 \in [[\alpha_3]]_S^{rt} : i_{12} \prec_{rt} i'_3 \prec_{rt} i_3 \wedge && (6) \\
& \nexists i'_1 \in [[\alpha_1]]_S^{rt} : i_1 \prec_{rt} i'_1 \prec_{rt} i_2 \wedge && (7) \\
& \nexists i'_2 \in [[\alpha_2]]_S^{rt} : i_1 \prec_{rt} i'_2 \prec_{rt} i_2 && (8)
\end{aligned}$$

The above equivalence imposes constraints on the temporal ordering of intervals i_1, i_2 and i_3. Conditions (5) and (6) impose that there are no intervals for $[[\alpha_1 ; \alpha_2]]_S^{rt}$ and $[[\alpha_3]]_S^{rt}$ between the ending point of i_2 and the starting point of i_3. Conditions (7) and (8) impose that there are no intervals for $[[\alpha_1]]_S^{rt}$ and $[[\alpha_2]]_S^{rt}$ between the ending point of i_1 and the starting point of i_2. Moreover, since there is no interval of $[[\alpha_1 ; \alpha_2]]_S^{rt}$ between i_2 and i_3, if $\exists i'_1 \in [[\alpha_1]]_S^{rt}$: $i_2 \prec_{rt} i'_1 \prec_{rt} i_3$, then $\nexists i'_2 \in [[\alpha_2]]_S^{rt}$: $i'_1 \prec i'_2 \prec i_3$, i.e., an interval in $[[\alpha_1]]_S^{rt}$ that is after i_1 may only be situated after the last interval of $[[\alpha_2]]_S^{rt}$ that is before i_3. Figure 2 depicts these temporal constraints.

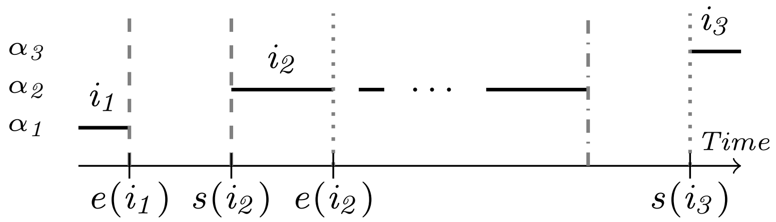

Figure 2: Constraints imposed by $i \in [[(\alpha_1 ; \alpha_2) ; \alpha_3]]_S^{rt}$, where $i = (i_1 \otimes_{rt} i_2) \otimes_{rt} i_3$. The dotted (resp. dashed) vertical lines outline a region with which no interval of $[[\alpha_1 ; \alpha_2]]_S^{rt}$ or $[[\alpha_3]]_S^{rt}$ ($[[\alpha_1]]_S^{rt}$ or $[[\alpha_2]]_S^{rt}$) may overlap. Moreover, there may be no interval of $[[\alpha_1]]_S^{rt}$ between i_1 and the dashed-dotted vertical line.

The following equivalence holds:

$$i \in [[\alpha_1 ;(\alpha_2 ; \alpha_3)]]_S^{rt} \Leftrightarrow$$

$$\exists i_1 \in [[\alpha_1]]^{rt}_S, i_{23} \in [[\alpha_2\,;\alpha_3]]^{rt}_S : i = (s(i_1), e(i_{23})) \wedge$$
$$\exists i_2 \in [[\alpha_2]]^{rt}_S, i_3 \in [[\alpha_3]]^{rt}_S : i_{23} = (s(i_2), e(i_3)) \wedge$$
$$\nexists i'_1 \in [[\alpha_1]]^{rt}_S : i_1 \prec_{rt} i'_1 \prec_{rt} i_{23} \wedge \tag{9}$$
$$\nexists i'_{23} \in [[\alpha_2\,;\alpha_3]]^{rt}_S : i_1 \prec_{rt} i'_{23} \prec_{rt} i_{23} \wedge \tag{10}$$
$$\nexists i'_2 \in [[\alpha_2]]^{rt}_S : i_2 \prec_{rt} i'_2 \prec_{rt} i_3 \wedge \tag{11}$$
$$\nexists i'_3 \in [[\alpha_3]]^{rt}_S : i_2 \prec_{rt} i'_3 \prec_{rt} i_3 \tag{12}$$

The above equivalence imposes constraints on the temporal ordering of intervals i_1, i_2 and i_3. Conditions (9) and (10) impose that there are no intervals for $[[\alpha_1]]^{rt}_S$ and $[[\alpha_2\,;\alpha_3]]^{rt}_S$ between the ending point of i_1 and the starting point of i_2. Conditions (11) and (12) impose that there are no intervals for $[[\alpha_2]]^{rt}_S$ and $[[\alpha_3]]^{rt}_S$ between the ending point of i_2 and the starting point of i_3. Moreover, since there is no interval of $[[\alpha_2\,;\alpha_3]]^{rt}_S$ between i_1 and i_2, if $\exists i'_3 \in [[\alpha_3]]^{rt}_S$: $i_1 \prec_{rt} i'_3 \prec_{rt} i_2$, then $\nexists i'_2 \in [[\alpha_2]]^{rt}_S$: $i_1 \prec_{rt} i'_2 \prec_{rt} i'_3$, i.e., an interval in $[[\alpha_3]]^{rt}_S$ that is before i_3 may only be situated before the first interval of $[[\alpha_2]]^{rt}_S$ that is after i_1. Figure 3 depicts these temporal constraints.

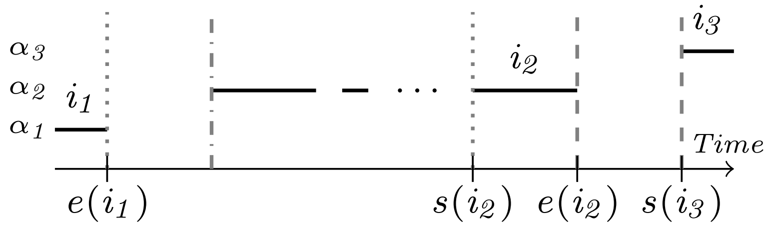

Figure 3: Constraints imposed by $i \in [[\alpha_1\,;(\alpha_2\,;\alpha_3)]]^{rt}_S$, where $i = i_1 \otimes_{rt} (i_2 \otimes_{rt} i_3)$. The dotted (resp. dashed) vertical lines outline a region with which no interval of $[[\alpha_1]]^{rt}_S$ or $[[\alpha_2\,;\alpha_3]]^{rt}_S$ ($[[\alpha_2]]^{rt}_S$ or $[[\alpha_3]]^{rt}_S$) may overlap. Moreover, there may be no interval of $[[\alpha_3]]^{rt}_S$ between the dashed-dotted vertical line and i_3.

By inspecting Figures 2 and 3, we deduce that the constraints they impose on the temporal ordering of the intervals of activities α_1, α_2 and α_3 are the same. Both cases stipulate that there may not be an interval of α_3 between the first interval of α_2 that is after i_1 and i_3, and that there may not be an interval of α_1 between i_1 and the last interval of α_2 that is before i_3. As a result, the conditions under which an interval i is included in list $[[(\alpha_1\,;\alpha_2)\,;\alpha_3]]^{rt}_S$ and the conditions under which it is included in list $[[\alpha_1\,;(\alpha_2\,;\alpha_3)]]^{rt}_S$ are the same. □

Proposition 5 (Correctness of Sequencing in RTEC$_{\mathrm{S}}$)**.** Consider activities α_1 and α_2, and a stream S. Given the sorted lists of MDIs I_1 and I_2 of α_1 and α_2 given stream S, RTEC$_{\mathrm{S}}$ computes a list of MDIs I for $\alpha_1\,;\alpha_2$ such that $i \in I$ iff $i \in [[\alpha_1\,;\alpha_2]]^{rt}_S$. ♦

Proof. RTEC$_{\mathrm{S}}$ computes a list of MDIs for $\alpha_1\,;\alpha_2$ using Algorithm 1. We prove that Algorithm 1 is sound and complete with respect to $[[\alpha_1\,;\alpha_2]]^{rt}_S$. We use the fact that $i_1 \otimes_{rt} i_2 \in [[\alpha_1\,;\alpha_2]]^{rt}_S$ iff $i_1 \prec_{rt} i_2$, $\nexists i'_1 \in I_1 :\ i_1 \prec_{rt} i'_1 \prec_{rt} i_2$ and $\nexists i'_2 \in I_2 :\ i_2 \prec_{rt} i'_2 \prec_{rt} i_2$, and the fact that Algorithm 1 may compute an interval $i_1 \otimes_{rt} i_2$ only if index j_1 points to interval i_1 and index j_2 points to interval i_2, i.e., $j_1 \mapsto i_1 \wedge j_2 \mapsto i_2$. We start with completeness because it is required in our proof of soundness.

Completeness: We prove that if $i_1 \otimes_{rt} i_2 \in [[\alpha_1\,;\alpha_2]]^{rt}_S$, then Algorithm 1 computes $i_1 \otimes_{rt} i_2$. It suffices to prove that Algorithm 1 reaches a state where $j_1 \mapsto i_1$ and $j_2 \mapsto i_2$; in such a state, since $\nexists i'_1 \in I_1 : i_1 \prec_{rt} i'_1 \prec_{rt} i_2$, Algorithm 1 would follow either lines 5–6 or lines 9–10, both of which compute $i_1 \otimes_{rt} i_2$. Since Algorithm 1 iterates over every interval of at least one of the lists I_1 and I_2, there is an iteration where $j_1 \mapsto i_1$ or an iteration where $j_2 \mapsto i_2$. Suppose that, in the first such iteration, $j_1 \mapsto i_1$. Then, we have $j_2 \mapsto i'_2$, where i'_2 is before i_2 in sorted list I_2. Since $\nexists i'_2 \in I_2 : i_2 \prec_{rt} i'_2 \prec_{rt} i_2$, i'_2 is before i_1, and thus we increment j_2 (see line 4). All intervals of I_2 that are between i'_2 and i_2, if any, are also before i_1, meaning that we keep incrementing j_2 until we have $j_1 \mapsto i_1$ and $j_2 \mapsto i_2$. Similarly, we may prove that this state is reached when starting from an iteration where $j_2 \mapsto i_2$. Therefore, if $\exists i_1 \in I_1, i_2 \in I_2$: $i_1 \otimes_{rt} i_2 \in [[\alpha_1 ; \alpha_2]]^{rt}_S$, then Algorithm 1 computes $i_1 \otimes_{rt} i_2$, proving that Algorithm 1 is complete.

Soundness: We prove that if Algorithm 1 computes $i_1 \otimes_{rt} i_2$, then $i_1 \otimes_{rt} i_2 \in [[\alpha_1 ; \alpha_2]]^{rt}_S$. Suppose that $i_1 \otimes_{rt} i_2 \notin [[\alpha_1 ; \alpha_2]]^{rt}_S$. Then, we have $i_2 \prec_{rt} i_1$, $\exists i'_1 \in I_1 : i_1 \prec_{rt} i'_1 \prec_{rt} i_2$, or $\exists i'_2 \in I_2 : i_1 \prec_{rt} i'_2 \prec_{rt} i_2$. $i_1 \otimes_{rt} i_2$ may only be computed in state $j_1 \mapsto i_1$ and $j_2 \mapsto i_2$. In this state, if $i_2 \prec_{rt} i_1$ holds, then we do not compute $i_1 \otimes_{rt} i_2$ and increment j_2 (line 4). If $\exists i'_1 \in I_1 : i_1 \prec_{rt} i'_1 \prec_{rt} i_2$, then we do not compute $i_1 \otimes_{rt} i_2$ and increment j_1 (lines 7–11). If $\exists i'_2 \in I_2 : i_1 \prec_{rt} i'_2 \prec_{rt} i_2$, then i_1 and i'_2 are adjacent, which implies that, based on the completeness proof, we reach state $j_1 \mapsto i_1 \wedge j_2 \mapsto i'_2$. In this state, since $i_1 \prec_{rt} i'_2$, we follow line 11, where j_1 is incremented beyond i_1, and thus $i_1 \prec_{rt} i_2$ may not be computed in a subsequent iteration. All cases led to a contradiction. Thus, if Algorithm 1 computes $i_1 \otimes_{rt} i_2$, then $i_1 \otimes_{rt} i_2 \in [[\alpha_1 ; \alpha_2]]^{rt}_S$, i.e., Algorithm 1 is sound. □

Proposition 6 (Complexity of Sequencing in RTEC$_\mathrm{S}$)**.** Consider activities α_1 and α_2, and a stream S. The worst-case time complexity of RTEC$_\mathrm{S}$ for computing the MDIs of $\alpha_1 ; \alpha_2$ given S is $O(|[[\alpha_1]]^{rt}_S|+|[[\alpha_2]]^{rt}_S|)$. ♦

Proof. RTEC$_\mathrm{S}$ computes the MDIs of $\alpha_1 ; \alpha_2$ using Algorithm 1. In each iteration of the loop of this algorithm, we increment at least of the indices traversing the MDIs in $[[\alpha_1]]^{rt}_S$ and $[[\alpha_2]]^{rt}_S$. As a result, the number of iterations of this loop is bounded by $|[[\alpha_1]]^{rt}_S| + |[[\alpha_2]]^{rt}_S|$. Thus, the worst-case time complexity of sequencing in RTEC$_\mathrm{S}$ is $O(|[[\alpha_1]]^{rt}_S|+|[[\alpha_2]]^{rt}_S|)$. □

Proposition 7 (Correctness of Sequencing over Windows)**.** Consider a window w over a stream S, and activities α_1 and α_2. Moreover, suppose that i_f and i_l are, respectively, the earliest and the most recent interval in $[[\alpha_1]]^{rt}_{S_w} \cup [[\alpha_2]]^{rt}_{S_w}$. $[[\alpha_1 ; \alpha_2]]^{rt}_{S_w} = [[\alpha_1 ; \alpha_2]]^{rt}_S{\downarrow}w$ if $i_f \in [[\alpha_1]]^{rt}_{S_w}$ and $i_l \in [[\alpha_2]]^{rt}_{S_w}$. ♦

Proof. Suppose that the earliest interval i_f of α_1 or α_2 in a window w is an interval of α_1. Then, there is a no interval i_1 of α_1 before w that is adjacent to an interval i_2 of α_2 in w, because $i_1 \prec_{rt} i_f \prec_{rt} i_2$. Thus, there is no interval of $\alpha_1 ; \alpha_2$ that overlaps the start of w and is not included $[[\alpha_1 ; \alpha_2]]^{rt}_{S_w}$. Similarly, if the latest interval i_l of α_1 or α_2 in w is an interval of α_1, then there is no interval i_2 of α_2 that is after w and is adjacent to an interval i_1 of α_1 in w, because we have $i_1 \prec_{rt} i_l \prec_{rt} i_2$. Thus, there is no interval of $\alpha_1 ; \alpha_2$ that overlaps the end of w and is not included $[[\alpha_1 ; \alpha_2]]^{rt}_{S_w}$. Since, for an interval i that is entirely within w, we have $i \in [[\alpha_1 ; \alpha_2]]^{rt}_{S_w}$ iff $i \in [[\alpha_1 ; \alpha_2]]^{rt}_S{\downarrow}w$, it holds that $[[\alpha_1 ; \alpha_2]]^{rt}_{S_w} = [[\alpha_1 ; \alpha_2]]^{rt}_S{\downarrow}w$ if $i_f \in [[\alpha_1]]^{rt}_{S_w}$ and $i_l \in [[\alpha_2]]^{rt}_{S_w}$. □

2 Syntax of Simple FVP Definitions

Simple FVP definitions have the following syntax.

Definition 1 (Syntax of Rules Defining Simple FVPs)**.** Consider a simple FVP $F = V$. The $\mathsf{initiatedAt}(F = V, T)$ rules of the event description have the following syntax:

$$\begin{array}{l} \mathsf{initiatedAt}(F = V,\ T) \leftarrow \\ \quad \mathsf{happensAt}(E_1,\ T)[[,[\mathsf{not}]\ \mathsf{happensAt}(E_2,\ T),\ \ldots, \\ \quad [\mathsf{not}]\ \mathsf{happensAt}(E_n,\ T),[\mathsf{not}]\ \mathsf{holdsAt}(F_1 = V_1,\ T),\ \ldots, \\ \quad [\mathsf{not}]\ \mathsf{holdsAt}(F_k = V_k,\ T)]]. \end{array}$$

The first body literal of an $\mathsf{initiatedAt}$ rule is a positive $\mathsf{happensAt}$ predicate; this is followed by a possibly empty set, denoted by '[[ ]]', of positive/negative $\mathsf{happensAt}$ and $\mathsf{holdsAt}$ predicates. 'not' expresses negation-by-failure, while '[not]' denotes that 'not' is optional. All (head and body) predicates are evaluated on the same time-point T. The bodies of $\mathsf{terminatedAt}(F = V, T)$ rules have the same form. ■