APPENDIX E

Primitive Variables

While CFL3D solves the governing equations in conservation law form, only the primitive variables are stored in permanent memory. This necessitates the use of the transformation matrix:

$$\mathbf{M} = \frac{\partial \mathbf{Q}}{\partial \mathbf{q}}$$

(E-1)

where

$$\mathbf{Q} = \begin{bmatrix} \rho & \rho u & \rho v & \rho w & e \end{bmatrix}^{\mathrm{T}}$$

(E-2)

$$\mathbf{q} = \begin{bmatrix} \rho & u & v & w & p \end{bmatrix}^{\mathrm{T}}$$

(E-3)

From Equation (A-14),

$$e = \frac{p}{\gamma - 1} + \frac{\rho}{2}(u^2 + v^2 + w^2)$$

(E-4)

Therefore,

$$\mathbf{M} = \begin{bmatrix} \dfrac{\partial \rho}{\partial \rho} & \dfrac{\partial \rho}{\partial u} & \dfrac{\partial \rho}{\partial v} & \dfrac{\partial \rho}{\partial w} & \dfrac{\partial \rho}{\partial p} \\[2mm] \dfrac{\partial \rho u}{\partial \rho} & \dfrac{\partial \rho u}{\partial u} & \dfrac{\partial \rho u}{\partial v} & \dfrac{\partial \rho u}{\partial w} & \dfrac{\partial \rho u}{\partial p} \\[2mm] \dfrac{\partial \rho v}{\partial \rho} & \dfrac{\partial \rho v}{\partial u} & \dfrac{\partial \rho v}{\partial v} & \dfrac{\partial \rho v}{\partial w} & \dfrac{\partial \rho v}{\partial p} \\[2mm] \dfrac{\partial \rho w}{\partial \rho} & \dfrac{\partial \rho w}{\partial u} & \dfrac{\partial \rho w}{\partial v} & \dfrac{\partial \rho w}{\partial w} & \dfrac{\partial \rho w}{\partial p} \\[2mm] \dfrac{\partial e}{\partial \rho} & \dfrac{\partial e}{\partial u} & \dfrac{\partial e}{\partial v} & \dfrac{\partial e}{\partial w} & \dfrac{\partial e}{\partial p} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ u & \rho & 0 & 0 & 0 \\ v & 0 & \rho & 0 & 0 \\ w & 0 & 0 & \rho & 0 \\ \dfrac{u^2 + v^2 + w^2}{2} & \rho u & \rho v & \rho w & \dfrac{1}{\gamma - 1} \end{bmatrix}$$

(E-5)

The inverse of $\mathbf{M}$ is

$$\mathbf{M}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\dfrac{u}{\rho} & \dfrac{1}{\rho} & 0 & 0 & 0 \\ -\dfrac{v}{\rho} & 0 & \dfrac{1}{\rho} & 0 & 0 \\ -\dfrac{w}{\rho} & 0 & 0 & \dfrac{1}{\rho} & 0 \\ \dfrac{\gamma-1}{2}(u^2+v^2+w^2) & -u(\gamma-1) & -v(\gamma-1) & -w(\gamma-1) & \gamma-1 \end{bmatrix} \tag{E-6}$$