IE 525 – Numerical Methods in Finance - Monte Carlo, Spring 2016
Project, Due 1pm Wednesday April 27

- You must upload your report (with codes, in a single pdf or word file) to http://compass2g.illinois.edu before 1pm on Wednesday 4/27/2016. **Half or all points will be taken off of a late project**.
  - Combine your report and codes (with codes in Appendix) into a single pdf or world file and upload it online for originality screening.
  - Submit a hardcopy of your report (without codes) in class.
- This project is about pricing Asian options using Monte Carlo simulation. The project can be done in a team of two students.
- **Programming and analysis**: programming should be done in C/C++. Excel can be used for analysis and presentation of data.
- **Grading criteria**: (1) Are the results complete and correct? (2) Is the report well organized? (3) Are the results well explained using tables and plots? (4) Does the team make efforts on reducing computational time? (5) Is the report generally impressive? (6) If you work in a team, to ensure both students are fully engaged and contribute equally, **you must report precisely how you have divided the work**.
- **Monte Carlo Simulation**
  1. Develop a Monte Carlo simulation program to price Asian call options in the Black-Scholes-Merton model:
     $$\text{Asian\_Call\_MC}(K, T, S, \sigma, r, q, n, m)$$
     where n is the number of sample paths generated, m is the number of time intervals, K is the strike price, T is the maturity, S is the current asset price, σ is the volatility, r is the continuous compounding risk free interest rate, and q is the continuous yield. The length of each time interval is $\delta = T/m$. $t_i = i\delta$ for $i = 1, \cdots, m$. The average asset price is:
     $$\bar{S}_T = \frac{1}{m}\sum_{i=1}^{m} S_{t_i}.$$
     The option payoff is $(\bar{S}_T - K)^+$. Use the corresponding geometric Asian call as the control variable.

     Price a 1-year Asian call option with strike price $K = 100$ and discrete monitoring ($m = 50$). The current asset price is $S_0 = 100$. The risk free interest rate is $r = 10\%$. The volatility of the asset is 20% per year. Your result should be accurate for at least 2 digits after the decimal point. Compare the approaches with and without control variates and investigate the effectiveness of the control variate technique.
  2. Develop a program to price Asian call options in the Black-Scholes-Merton model using quasi-Monte Carlo simulation with Sobol sequences:
     $$\text{Asian\_Call\_QMC}(K, T, S, \sigma, r, q, n, L, m)$$
     Randomization with random shift should be used. n is the number of sample paths generated in each batch and L is the number of batches. Use the same parameters as in Part 1, investigate the effectiveness of randomized quasi-Monte Carlo simulation.
  3. Consider an Asian call option with payoff $(\bar{S}_T - K)^+$ with continuous average
     $$\bar{S}_T = \frac{1}{T}\int_0^T S_t dt.$$
     The continuous average asset price is approximated by the discrete average by taking a large enough m. Use your programs to price this Asian call when $T = 2$, $r = 0.05$, $\sigma = 0.5$, $S = K = 2$, $q = 0$. Compare the performance of the control variate approach and randomized quasi-Monte Carlo simulation and the impact of the dimension m.
- **Notes**: I encourage you to check with me about whether you are getting close to the correct prices.