Lineær Algebra - Exercises

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1 Exercises 1

Question 1

What does the following matrix multiplication give? Add your own calculations:

$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 3 & 1 \\ 0 & 4 \end{pmatrix} = \quad \dots$$

Question 2

What does the following matrix multiplication give? Add your own calculations:

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \quad \dots$$

Question 3

What did the matrix in Question 2 do to the vector? If it is not clear, then try multiplying another vector with this matrix. Discuss.

Question 4

From your answers to the first two questions can you see a general rule for matrix multiplication? I.e. what determines whether two matrices/vectors can be multiplied and what determines the resulting dimensions? Discuss.

Question 5

Now we want to combine operations just to make sure that we have understood the concepts so far. So, calculate the following:

$$\frac{1}{2}\begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix} \cdot \left(\begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right) = \dots$$

Question 6

We have seen how the matrix holding the basis vectors of our normal 2-D coordinate system looks. How do you think the matrix holding the basis vectors of a 3-D coordinate system should look (i.e. adding the z-direction)?

2 Exercises 2

We now return to the data that we discussed earlier. Recall that we had the matrix with 1 as intercept and x-values:

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}.$$

and we had the outcome vector:

$$y = \begin{pmatrix} 1 \\ 4 \\ 13 \\ 18 \end{pmatrix}.$$

Q1: Calculate Least Squares

NB: you can use a calculator for this if you want.

1. find $A_{new} = A^T \cdot A$

2. find $y_{new} = A^T \cdot y$

3. now we have this equation $A_{new} \cdot x = y_{new}$ (called the "normal" equation in danish)

4. with Gauss-Jordan elimination find the vector x that solves this equation

5. what are the squared errors of this model?

6. is the model any good? draw the line in R or by hand.

Q2: calculating R^2

NB: You should use a calculator for this as you will run into some non-integer numbers for the grand mean model.

To know how good the model is we would like some notion of how much variance is explained by the model. This has to be compared to something. To find the R^2 of a model we compare the sum of squared errors (SSE) to the SSE of the simplest model: the mean. The mean simply predicts the mean value of the y-vector for each y-data point. This highlights that to obtain R^2 we need to do some kind of model comparison - it is a value that has to be relative to something.

1. calculate R^2 of your linear model. Do you think that the linear fit is significant? Try to draw the two models (grand mean vs. regression).

Q3: check your results in R

Check your results in R and see whether you calculated everything correctly, and whether the model is significant. See the R-markdown file called "instructions.Rmd", and the first task (Exercises 2 $\rightarrow$ Regression 1).

Q4: run more complicated models

In the same R-markdown document do the tasks under the heading (Exercises 2 $\rightarrow$ Regression 2).

3 Exercise 3

See R under ”Exercise 3 (more data)”.

4 Exercises 4

See R under ”Exercise 4 (embeddings)”.

5 Exercises 5

See R under ”Exercise 5 (PCA)”