Robot Dynamics and Control Assignment 2

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Recursive Newton-Euler algorithm for inverse dynamics

This assignment is focused on the recursive Newton-Euler algorithm for inverse dynamics.

The Inverse Dynamics Problem (**IDP**), also called Control Problem, consists of determining the actuation torques based on the kinematics of a body and the body's inertial properties (such as mass and moment of inertia). To solve this problem, we make use of the recursive N-E algorithm where the inputs and the outputs of the algorithm are:

INPUTS:

- $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$, respectively the joint position, velocity and acceleration.
- $\{\mathbf{F^{ext}_{C_i}}, \mathbf{M^{ext}_{C_i}}\}_{1 \div n}$, respectively the external Forces and Torques acting on point C_i.
- $\{{}^{i}I_{C_i}\}_{1 \div n}$, MoI tensors with respect to point C_i in i-th reference frame.
- $\mathbf{g}$, gravity acceleration vector.

OUTPUTS:

- τ, actuation torques vector.
- $\phi_{\mathbf{i/i-1}}$, reaction forces vector.
- $\psi_{\mathbf{i/i-1}}$, reaction torques vector.

This algorithm is logically divided in two steps, which are the **Forward** and **Backward Recursion**. The former, for each joint i and starting from 1 to n, is used to achieve (assuming that P_i is a point in link "i" which corresponds to the pivot for R-joints and to the center of mass for P-joints in their initial configuration):

- Position/Orientation computation:
  - $\Rightarrow \mathbf{r_{i/i-1}}$ (position of P_i with respect to P_{i-1});
  - $\Rightarrow {}^{i-1}_{i}\mathcal{R}$ (Rotation matrix of frame <i> with respect to frame <$i-1$>).
- Velocity computations:
  - $\Rightarrow \omega_{\mathbf{i/0}}$ (angular velocity of P_i with respect to base frame);
  - $\Rightarrow v_{\mathbf{i/0}}$ (linear velocity of P_i with respect to base frame).
- Acceleration computations:
  - $\Rightarrow \dot{\omega}_{\mathbf{i/0}}$ (angular acceleration of P_i with respect to base frame);
  - $\Rightarrow \dot{v}_{\mathbf{i/0}}$ (linear acceleration of P_i with respect to base frame).

Then, after the forward iterations are completed, we have the configuration, twist, and acceleration of each link.

The Backward Recursion is the second step of the algorithm. For each i and starting from n to 1, the just computed values of the Forward Recursion are used to obtain the actuation torques vector τ. Please note that there are other optional outputs which can be useful in some cases (not in ours). These outputs are the reaction forces $\phi_{\mathbf{i/i-1}}$ and the reaction torques $\psi_{\mathbf{i/i-1}}$.

Hence, at the end of the backward iterations, all the joint forces and torques needed to create the desired joint accelerations at the current joint positions and velocities have been computed.

Notes about the implementation of the algorithm

The implementation of the algorithm has been done on MATLAB and it keeps into account the requested criteria. In particular, the data of the Robot have been represented by a unique structure, which is divided into two structs.

The first one is simply called "**Data**" and contains, for each link:

- Mass [Kg]

- Position with respect to the previous frame [m]
- Center of Mass (CoM) [m]
- MoI tensor [Kg*m^2]
- R_i, rotation matrix which represents the fixed rotation between the i-th frame and the (i-1)-th one
- Type of joint (”*revolute*” or “*prismatic*”).

The second struct, “**Config**” contains infos related to a specific configuration of the robot. To be more precise, it contains the values of position, velocity and acceleration of the joints, referred to a certain motion snapshot.

The whole struct is passed as an argument to the function dedicated to solving the Inverse Dynamic Problem. This function also takes as arguments the external forces and torques (if any) and the gravity acceleration vector (if gravity is enabled). To omit one of these terms, it is sufficient to pass as argument a null *3 x n* matrix for the first two and a null *3 x 1* vector for the gravity.

In the algorithm, all values are expressed with respect to the base frame for convenience.

Exercise 2 - RR Planar Manipulator

2.1

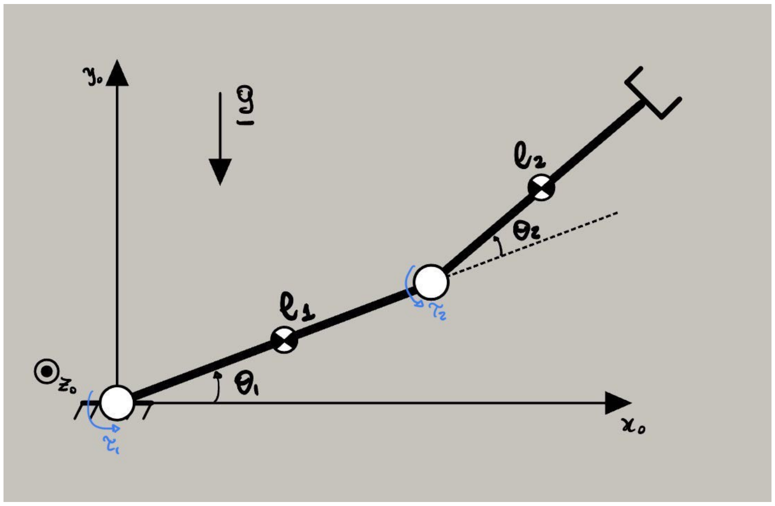

Figure 1

The motion snapshot is:

$\theta_1 = 20^\circ$	$\theta_2 = 40^\circ$
$\dot{\theta}_1 = 0.2\ rad/s$	$\dot{\theta}_2 = 0.15\ rad/s$
$\ddot{\theta}_1 = 0.1\ rad/s^2$	$\ddot{\theta}_2 = 0.085\ rad/s^2$

The values of the actuation torques vector τ in the reference frame <0>, respectively without and with gravity along $\mathbf{y_0}$, are:

	w/o $\mathbf{g}$	w/ $\mathbf{g}$
τ_1 [N*m]	4.3641	318.1937
τ_2 [N*m]	1.3955	38.6735

The torque that has to be impressed by the two actuators is positive both for the cases without and with gravity. In the latter case, gravity generates a negative torque on both links and, since both joints must have a counter-clockwise rotation about z-axis, the magnitude of the torques are higher than the former case because the torque to counter has increased.

2.2

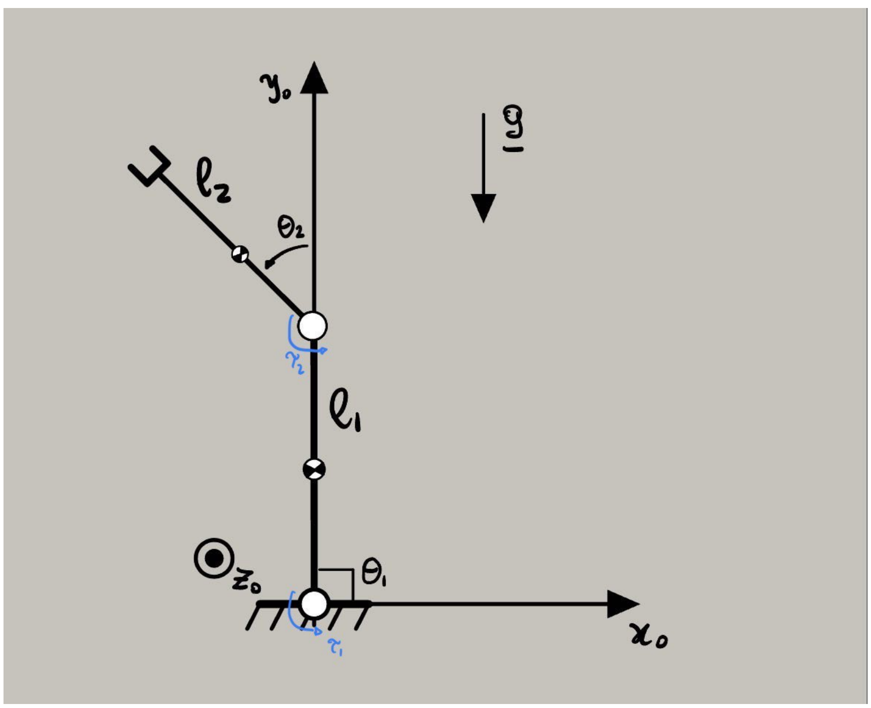

Figure 2

The motion snapshot is:

$\theta_1 = 90^\circ$	$\theta_2 = 45^\circ$
$\dot{\theta}_1 = -0.8\ rad/s$	$\dot{\theta}_2 = 0.35\ rad/s$
$\ddot{\theta}_1 = -0.4\ rad/s^2$	$\ddot{\theta}_2 = 0.1\ rad/s^2$

The values of the actuation torques vector τ in the reference frame <0>, respectively without and with gravity along $\mathbf{y_0}$, are:

	w/o $\mathbf{g}$	w/ $\mathbf{g}$
τ_1 [N*m]	-12.3727	-65.0917
τ_2 [N*m]	0.2878	-52.4313

The actuation torques should provide a torque which produces a rotation and an acceleration in the clockwise direction for the first link and a counter-clockwise ones for the second link. In the case without gravity the applied torque is slightly negative for the first motor and almost zero for the second one. When we enable it the actuators must provide a torque which generates a clockwise rotation to obtain the desired instantaneous trajectory. In this case, the second actuated torque has a bigger magnitude to balance the gravity force, which makes the second link rotate in counter-clockwise direction with an higher acceleration compared to the desired one.

Exercise 3 - RP Planar Manipulator

3.1

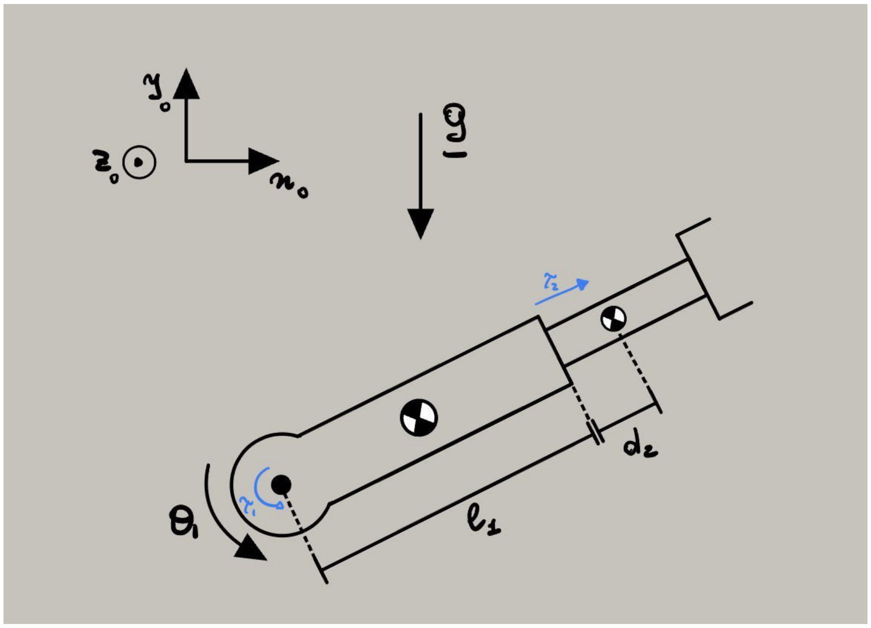

Figure 3

The motion snapshot is:

$\theta_1 = 20^\circ$	$d_2 = 0.2\ m$
$\dot{\theta}_1 = 0.08\ rad/s$	$v_2 = 0.03\ m/s$
$\ddot{\theta}_1 = 0.1\ rad/s^2$	$a_2 = 0.01\ m/s^2$

The values of the actuation torques vector τ in the reference frame <0>, respectively without and with gravity along $\mathbf{y_0}$, are:

	w/o $\mathbf{g}$	w/ $\mathbf{g}$
τ_1 [N*m]	1.2186	113.6829
τ_2 [N]	0.0139	20.1452

In this configuration, the instantaneous trajectory is given by a counter-clockwise rotation and acceleration about the z-axis for the rotational joint and a slightly positive velocity and acceleration for the prismatic one. Both for cases with and without gravity, the torque and the force provided by, respectively, the first and the second joint, are positive. Since the gravity would make the manipulator rotate in a clockwise direction, the magnitude of τ_1 is greater. Also the magnitude of the force τ_2 has increased since the component of $\mathbf{g}$ along the direction of the second link generates the trend of the prismatic joint to “stretch” itself.

3.2

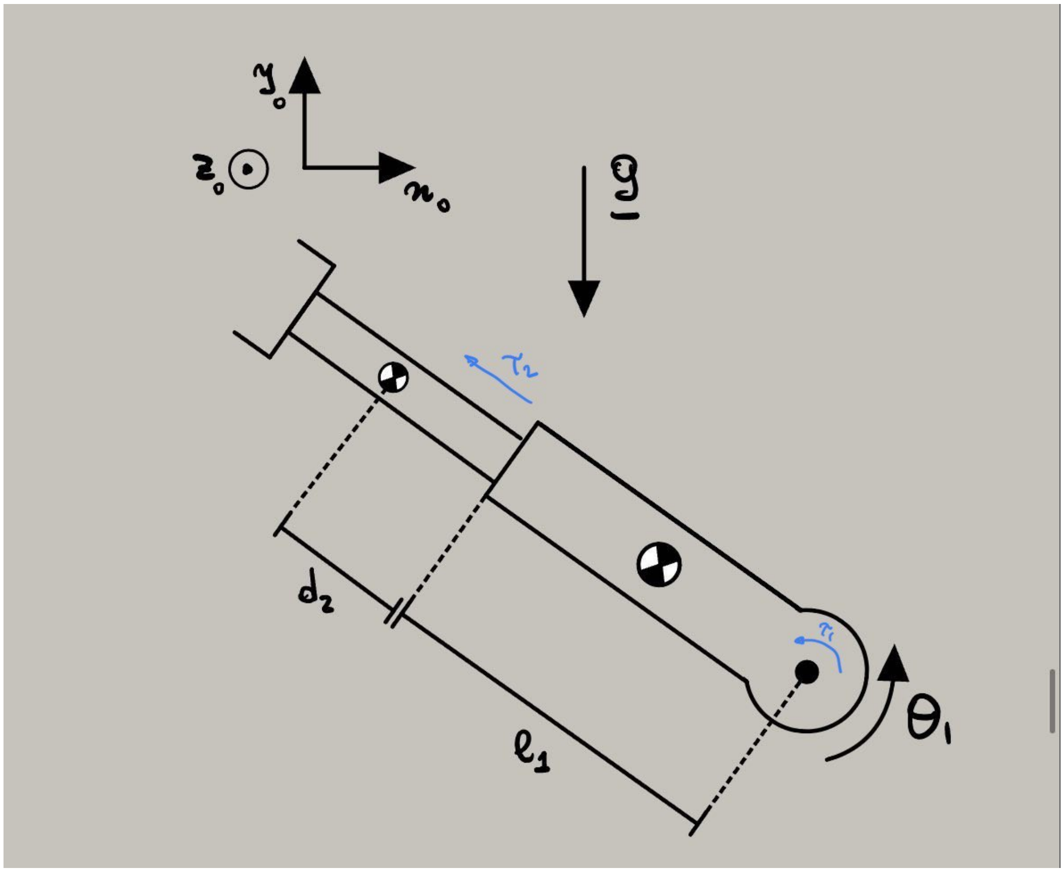

Figure 4

The motion snapshot is:

$\theta_1 = 120^\circ$	$d_2 = 0.6\ m$
$\dot{\theta}_1 = -0.4\ rad/s$	$v_2 = -0.08\ m/s$
$\ddot{\theta}_1 = -0.1\ rad/s^2$	$a_2 = -0.01\ m/s^2$

The values of the actuation torques vector τ in the reference frame <0>, respectively without and with gravity along $\mathbf{y_0}$, are:

	w/o $\mathbf{g}$	w/ $\mathbf{g}$
τ_1 [N*m]	-1.2416	-72.8546
τ_2 [N]	-1.5960	49.3783

In this configuration, the instantaneous trajectory is given by a clockwise rotation and acceleration about the z-axis for the rotational joint and a slightly negative velocity and acceleration for the prismatic one. The torque provided by the first motor is negative both for the cases with and without gravity, but for the latter the magnitude is bigger since the gravity would make the manipulator rotate in counter-clockwise direction. Concerning τ_2,the interesting fact is that if we switch on gravity the force that has to be generated by the motor changes sign. This means that the effect of gravity tends to stretch the prismatic joint too much and we need to provide a positive force to counter this trend.

Exercise 4 - RRR non-Planar Manipulator

4.1

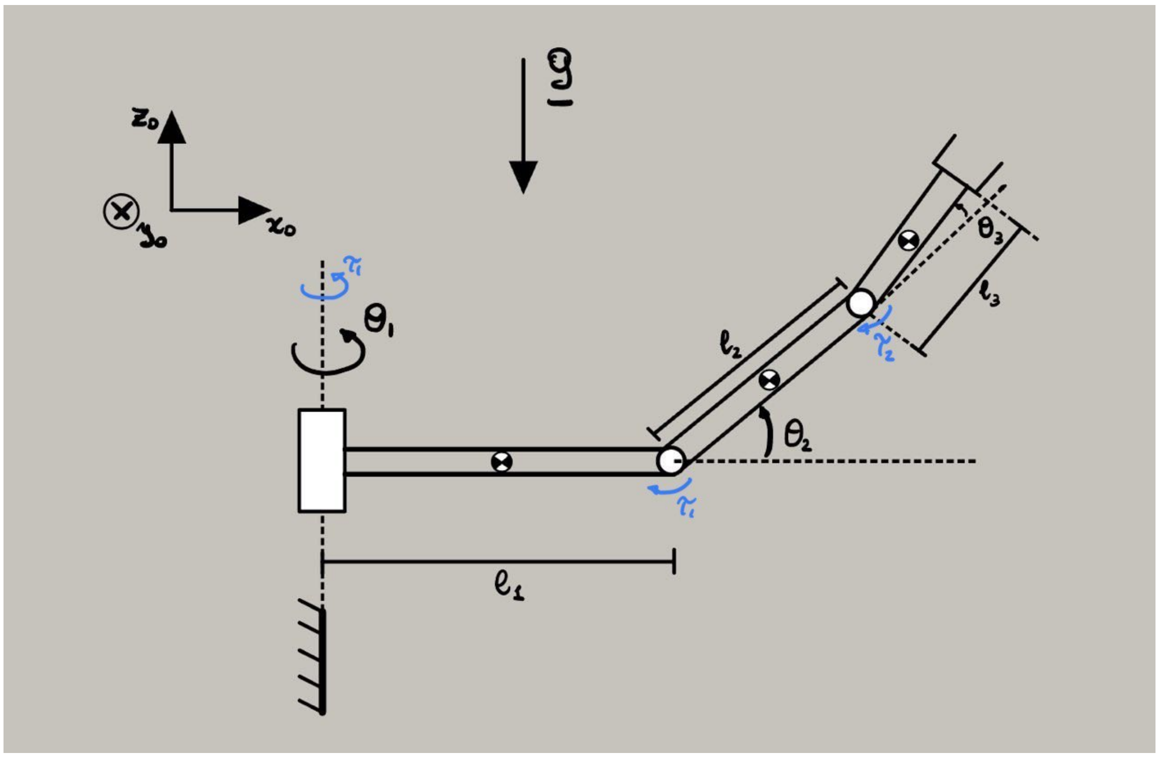

Figure 5

The motion snapshot is:

$\theta_1 = 20^\circ$	$\theta_2 = 40^\circ$	$\theta_3 = 10^\circ$
$\dot{\theta}_1 = 0.2\ rad/s$	$\dot{\theta}_2 = 0.15\ rad/s$	$\dot{\theta}_3 = -0.2\ rad/s$

$\ddot{\theta}_1 = 0.1\ rad/s^2$	$\ddot{\theta}_2 = 0.085\ rad/s^2$	$\ddot{\theta}_3 = 0\ rad/s^2$

Note: for this configuration, in order to use right-handed coordinates, the $\mathbf{y_0}$ axis is entering the sheet.

The values of the actuation torques vector τ in the reference frame <0>, respectively without and with gravity along $\mathbf{z_0}$, are:

	w/o $\mathbf{g}$	w/ $\mathbf{g}$
τ_1 [N*m]	5.1128	5.1128
τ_2 [N*m]	1.3712	104.1829
τ_3 [N*m]	0.1532	6.7743

For this non-planar manipulator, actuators must provide to R-joints 1 and 2 a torque which generates an instantaneous positive rotation and angular acceleration about their axis of rotation. The actuator of the third revolute joint must provide a torque to make it rotate in a clockwise rotation with a constant angular speed. Since the rotation axis of joint 1 is parallel to $\mathbf{g}$, the effect of this force is reciprocal to it and therefore there are no differences between the values of τ_1 in the two cases.