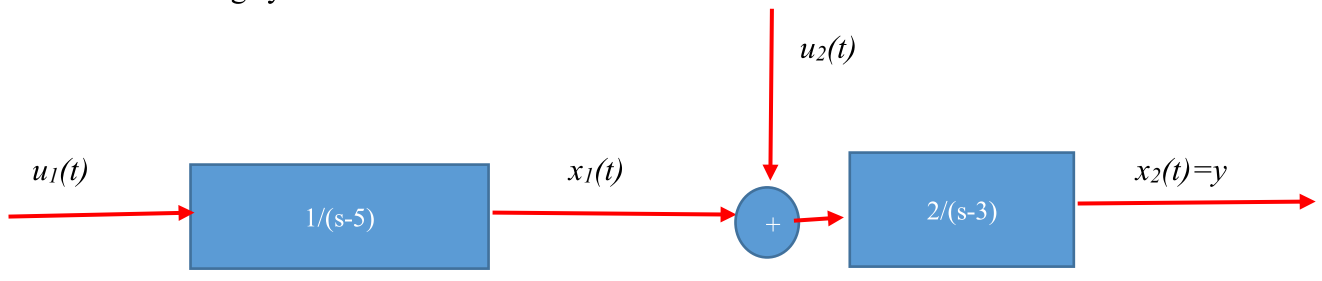
The computer exercise.

Consider the following system



1) Write the state equations of the system
2) Determine the stationary state-feedback control law having structure

$$\underline{\boldsymbol{u}}(\boldsymbol{t}) = -\boldsymbol{L}\underline{\boldsymbol{x}}(\boldsymbol{t})$$

optimizing the infinite horizon cost function

$$\int_{t_0}^{\infty}\left[\underline{\boldsymbol{x}}^T(\boldsymbol{t})\boldsymbol{V}\underline{\boldsymbol{x}}(\boldsymbol{t}) + \underline{\boldsymbol{u}}^T(\boldsymbol{t})\boldsymbol{P}\underline{\boldsymbol{u}}(\boldsymbol{t})\right] \boldsymbol{dt}$$

where

$$\boldsymbol{Q} = \begin{bmatrix} \mathbf{10} & \mathbf{0} \\ \mathbf{0} & \mathbf{10} \end{bmatrix}$$

and

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} \end{bmatrix}$$

3) Check whether this control law ensures that the closed-loop system is asymptotically stable

4) Assume now that the state of the system is not fully accessible (but only the output $\boldsymbol{y}$ is). Design an asymptotic observer of the state choosing the poles of the observer so that the observer dynamics is considerably “faster” than the dynamics of the original closed-loop system (e.g., ensuring that the time constants of the observer are an order of magnitude smaller than those of that system).

5) Build the overall model (by SIMULINK) of the overall system (the original open-loop system + the feedback controller + the state observer) and analyze the behavior of the system when the input is given by

$$\underline{\boldsymbol{u}}(\boldsymbol{t}) = -\boldsymbol{L}\underline{\boldsymbol{x}}(\boldsymbol{t}) + \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix} \boldsymbol{r}(\boldsymbol{t})$$

being ***r(t)*** an external sinusoidal input.

6) Determine the closed-loop transfer function $\boldsymbol{T}(\boldsymbol{s}) = \boldsymbol{Y}(\boldsymbol{s})/\boldsymbol{R}(\boldsymbol{s})$ (assuming a very fast convergence of the observed system state to the true system state) and analyze the frequency response (for different frequencies of the sinusoidal input).