Quest 3

i) $\lim_{n\to\infty} \frac{\ln(n)+4}{5n^4+7n^2+6}$

$\lim_{n\to\infty} \ln(n)+4 \cdot \lim_{n\to\infty} \frac{1}{5n^4+7n^2+6}$

$\lim_{n\to\infty} \ln(n) + \lim_{n\to\infty} 4 \cdot \lim_{n\to\infty} \frac{1}{5n^4+7n^2+6}$

$\lim_{n\to\infty} \ln(\infty)=\infty$, $\lim_{n\to\infty} 4 = 4$

$\infty+4 \cdot \lim_{n\to\infty} \frac{1}{5n^4+7n^2+6}$

$\infty+4 \cdot \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} 5n^4+7n^2+6}$

$\infty \cdot \frac{1}{\infty+c}$

$\cancel{\infty} \cdot \frac{1}{\cancel{\infty}}$

$=1$

(ii) $\lim_{n\to\infty} \left(\frac{2^n}{\log_2 n}\right)$

$\lim_{n\to\infty} \left(\frac{e^{\ln(2)n}}{\log_2 n}\right)$

$\lim_{n\to\infty} \left(\frac{2^n \ln 2}{\frac{1}{n\ln 2}}\right)$ l'hopital's rule

$\lim_{n\to\infty} \left(2^n n (\ln 2)^2\right)$

$\lim_{n\to\infty} \left(2^n n (\ln 2)^2\right)$

$(\ln 2)^2 \cdot \lim_{n\to\infty} (2^n n)$

$(\ln 2)^2 \cdot \lim_{n\to\infty} 2^n \cdot \lim_{n\to\infty} n$

$(\ln 2)^2 \cdot 2^\infty \cdot \infty$

$(\ln 2)^2 \cdot \infty$

$= \infty$

Approximation

i) $\sum_{k=0}^{30} k^2 \approx \frac{30^{2+1}}{2+1}$

$\approx \frac{30^3}{3}$

≈ 9000

(ii) $\sum_{k=0}^{100} k^3 \approx \frac{100^{3+1}}{3+1}$

$\approx \frac{100^4}{4}$

$\approx 25 \times 10^6$

PROOF BY INDUCTION

(i)

Quest 4

i) $T(n) = 7T(\frac{n}{2}) + n^2$

Taking the base relation

$T(n) = aT(n/b) + O(n^d)$

$a = 7,\ b = 2,\ d = 2$

$a > d$

$\therefore T(n) = O(n^{\log_2 7})$

$T(n) = O(n^{2.81})$

(ii) $T(n) = 5T(\frac{n}{3}) + O(n)$

Taking the base relation

$a = 5,\ b = 3,\ d = 1$

$a > d$

$\therefore T(n) = O(n^{\log_3 5})$

$= O(n^{1.46})$

(iii) $T(n) = 3T(\frac{n}{2}) + \frac{3}{4}n + 1$

Taking the base relation,

$a = 3,\ b = 2,\ d = 1$

$a > d$

$\therefore T(n) = O(n^{\log_2 3})$

$= O(n^{1.58})$