listed?

That many monsters total.

For quest card 517, what counts as different monster types?

Each monster stat card (the large square ones) depicts a different monster type. So, for instance, Inox Guards and Bandit Guards are two different monster types and each different boss is a different monster type.

Do random scenarios count for quest card 518?

Yes.

For quest card 519, can I lose progress for this if I lose check marks through events? Do I gain progress if I gain check marks through events or bonuses other than battle goals?

No, only battle goals count as progress towards this quest. It may be necessary to track this separately in the notes section.