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117366 Lógica Computacional 1 - 2016/1 Prof. Flávio L. C. de Moura

Lista de Exercícios: Cálculo de Sequentes (Gentzen)

1. Prove os sequentes a seguir:

    - $((\phi \to \bot) \to \bot) \to \bot \Leftrightarrow \phi \to \bot$
    - $((\phi \to \psi) \to \bot) \to \bot \Leftrightarrow ((\phi \to \bot)\ \bot) \to ((\psi \to \bot) \to \bot)$
    - $((\phi \wedge \psi) \to \bot) \to \bot \Leftrightarrow ((\phi \to \bot) \to \bot) \wedge ((\psi \to \bot) \to \bot)$
    - $(\phi \vee \psi) \to \bot \Leftrightarrow (\phi \to \bot) \wedge (\psi \to \bot)$
    - $(\phi \wedge \psi) \wedge \varphi \Leftrightarrow \phi \wedge (\psi \wedge \varphi)$
    - $(\phi \vee \psi) \vee \varphi \Leftrightarrow \phi \vee (\psi \vee \varphi)$
    - $\Rightarrow \phi \vee (\phi \to \bot)$
    - $(\phi \to \bot) \to \bot \Rightarrow \phi$
    - $\phi \Rightarrow (\phi \to \bot) \to \bot$
    - $\phi \to \psi, \psi \to \bot \Rightarrow \phi \to \bot$
    - $(\psi \to \bot) \to \phi \Leftrightarrow (\phi \to \bot) \to \psi$
    - $\psi \to (\phi \to \bot) \Leftrightarrow \phi \to (\psi \to \bot)$
    - $(p \to r) \wedge (q \to r) \Rightarrow (p \wedge q) \to r$
    - $q \to r \Rightarrow (p \to q) \to (p \to r)$
    - $p \to (q \to r), p \to q \Rightarrow p \to r$
    - $p \to q, r \to s \Rightarrow p \vee r \to q \vee s$
    - $p \vee q \Rightarrow r \to (p \vee q) \wedge r$
    - $(p \vee (q \to p)) \wedge q \Rightarrow p$
    - $p \to q, r \to s \Rightarrow p \wedge r \to q \wedge s$
    - $p \to q \Rightarrow ((p \wedge q) \to p) \wedge (p \to (p \wedge q))$
    - $\Rightarrow q \to (p \to (p \to (q \to p)))$
    - $p \to (q \wedge r) \Rightarrow (p \to q) \wedge (p \to r)$
    - $(p \to q) \wedge (p \to r) \Rightarrow p \to (q \wedge r)$
    - $\Rightarrow (p \to q) \to ((r \to s) \to (p \wedge r \to q \wedge s))$

- $(p \land q) \land r, s \land t \ \Rightarrow q \land s$
- $p \land q \ \Rightarrow q \land p$
- $p \to (p \to q), p \ \Rightarrow q$
- $q \to (p \to r), r \to \perp, q \ \Rightarrow p \to \perp$
- $\Rightarrow (p \land q) \to p$
- $p \ \Rightarrow q \to (p \land q)$
- $p \ \Rightarrow (p \to q) \to q$
- $p \to q \ \Rightarrow (q \to \perp) \to (p \to \perp)$
- $p \lor (p \land q) \ \Rightarrow p$
- $r, p \to (r \to q) \ \Rightarrow p \to (q \land r)$
- $p \to (q \lor r), q \to s, r \to s \ \Rightarrow p \to s$
- $(p \land q) \lor (p \land r) \ \Rightarrow p \land (q \lor r)$
- $\phi \lor \psi \ \Leftrightarrow ((\phi \to \perp) \land (\psi \to \perp)) \to \perp$
- Lei de Peirce: $\Rightarrow ((\phi \to \psi) \to \phi) \to \phi$

2. Com derivações no Cálculo de Gentzen, prove as seguintes equivalências entre os quantificadores universal e existencial:

   (a) $\neg \forall x\, \phi \ \dashv\vdash \ \exists x\, \neg \phi$

   (b) $\neg \exists x\, \phi \ \dashv\vdash \ \forall x\, \neg \phi$

   (c) $\forall x\, \phi \ \dashv\vdash \ \neg \exists x\, \neg \phi$

   (d) $\exists x\, \phi \vdash \neg \forall x\, \neg \phi$

3. Apresente derivações no Cálculo de Gentzen para os sequentes a seguir assumindo que x não ocorre livre em ψ nos itens (a), (b), (c) e (d). Para o item (g), assuma que x não ocorre livre em ϕ.

   (a) $(\forall x\, \phi) \lor \psi \vdash \forall x\, (\phi \lor \psi)$

   (b) $(\exists x\, \phi) \lor \psi \vdash \exists x\, (\phi \lor \psi)$

   (c) $\exists x\, (\phi \to \psi) \vdash (\forall x\, \phi) \to \psi$

   (d) $\exists x\, (\psi \to \phi) \vdash \psi \to \exists x\, \phi$

   (e) $(\forall x\, \phi) \land (\forall x\, \psi) \ \dashv\vdash \ \forall x (\phi \land \psi)$

   (f) $\exists x\, \phi \lor \exists x\, \psi \ \dashv\vdash \ \exists x\, (\phi \lor \psi)$

   (g) $\exists x\, (\phi \to q(x)) \ \dashv\vdash \ \phi \to \exists x\, q(x)$

   (h) $\exists x\, (\neg p(x) \land \neg q(x)) \vdash \exists x\, (\neg (p(x) \land q(x)))$

   (i) $\exists x\, (\neg p(x) \lor q(x)) \vdash \exists x\, (\neg (p(x) \land \neg q(x)))$

(j) $\forall x\, (p(x) \wedge q(x)) \vdash (\forall x\, p(x)) \wedge (\forall x\, q(x))$

(k) $(\forall x\, p(x)) \vee (\forall x\, q(x)) \vdash \forall x\, (p(x) \vee q(x))$

(l) $\exists x\, (p(x) \wedge q(x)) \vdash \exists x\, p(x) \wedge \exists x\, q(x)$

(m) $\exists x\, p(x) \vee \exists x\, q(x) \vdash \exists x\, (p(x) \vee q(x))$

(n) $\forall x\, \forall y\, (p(y) \rightarrow q(x)) \vdash \exists y\, p(y) \rightarrow \forall x\, q(x)$