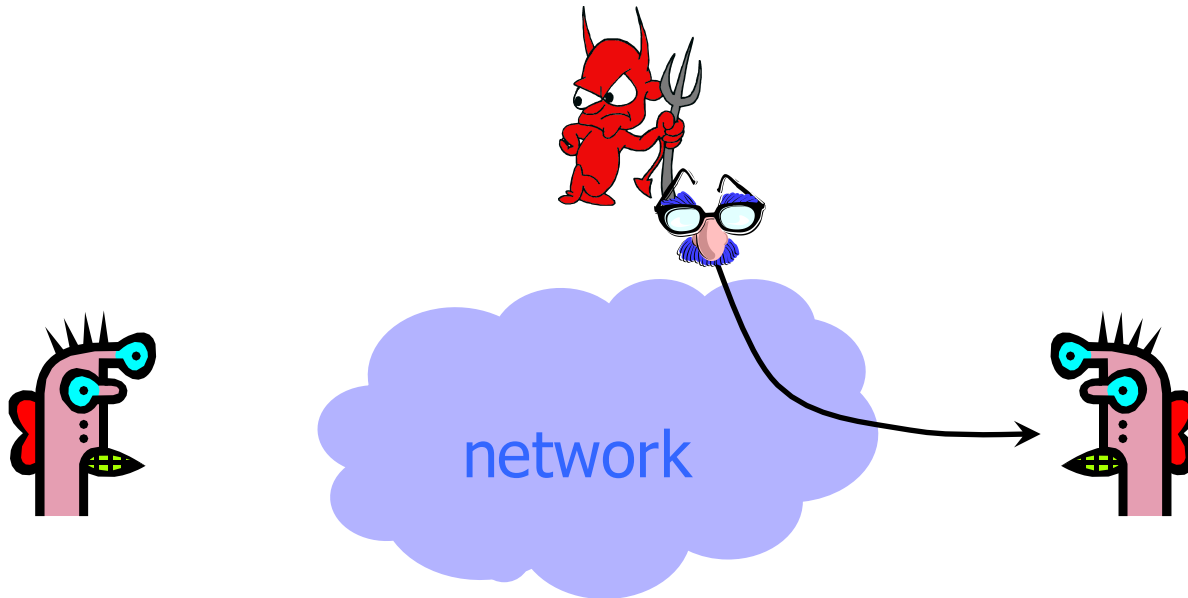
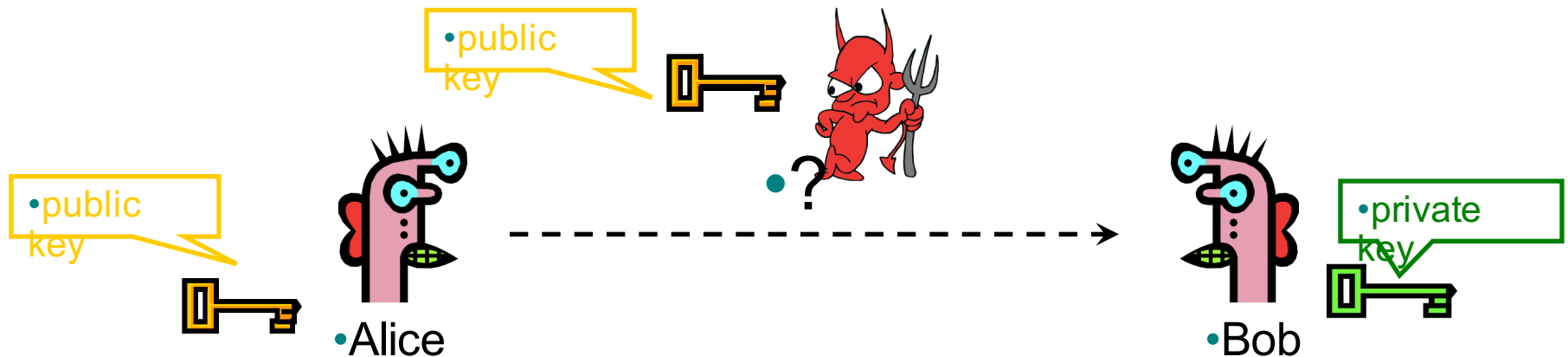


# Authentication

- Authenticity is identification and assurance of origin of information
  - We'll see many specific examples in different scenarios



# Basic Public Key Cryptography



- Given: Everybody knows Bob's **public key**
  - - How is this achieved in practice?
- Only Bob knows the corresponding **private key**
- Goals: 1. Alice wants to send a secret message to Bob
  - 2. Bob wants to authenticate himself

# Requirements for Public-Key Crypto

- **Key generation:** computationally easy to generate a pair (public key PK, private key SK)
  - Computationally infeasible to determine private key SK given only public key PK
- **Encryption:** given plaintext M and public key PK, easy to compute ciphertext  $C = E_{PK}(M)$
- **Decryption:** given ciphertext  $C = E_{PK}(M)$  and private key SK, easy to compute plaintext M
  - Infeasible to compute M from C without SK
  - $\text{Decrypt}(\text{SK}, \text{Encrypt}(\text{PK}, M)) = M$

# Requirements for Public-Key Cryptography

1. Computationally easy for a party B to generate a pair (public key  $KU_b$ , private key  $KR_b$ )
2. Easy for sender to generate ciphertext:

$$C = E_{KU_b}(M)$$

3. Easy for the receiver to decrypt ciphertext using private key:

$$M = D_{KR_b}(C) = D_{KR_b}[E_{KU_b}(M)]$$

# Requirements for Public-Key Cryptography

4. Computationally infeasible to determine private key ( $KR_b$ ) knowing public key ( $KU_b$ )
5. Computationally infeasible to recover message  $M$ , knowing  $KU_b$  and ciphertext  $C$
6. Either of the two keys can be used for encryption, with the other used for decryption:

$$M = D_{KRb}[E_{KU_b}(M)] = D_{KU_b}[E_{KRb}(M)]$$

# Public-Key Cryptographic Algorithms

---

- RSA and Diffie-Hellman
- **RSA** - Ron Rivest, Adi Shamir and Len Adleman at MIT, in 1977.
  - RSA is a block cipher
  - The most widely implemented
- **Diffie-Hellman**
  - Exchange a secret key securely
  - Compute discrete logarithms

# Rivest, Shamir and Adleman (1977)

## Key Generation

Select  $p, q$

$p$  and  $q$  both prime,  $p \neq q$

Calculate  $n = p \times q$

Calculate  $\phi(n) = (p - 1)(q - 1)$

Select integer  $e$

$\gcd(\phi(n), e) = 1; 1 < e < \phi(n)$

Calculate  $d$

$de \bmod \phi(n) = 1$

Public key

$KU = \{e, n\}$

Private key

$KR = \{d, n\}$

# RSA en/decryption

Calculate  $d$

$$de \bmod \phi(n) = 1$$

Public key

$$KU = \{e, n\}$$

Private key

$$KR = \{d, n\}$$

## Encryption

Plaintext:

$$M < n$$

Ciphertext:

$$C = M^e \bmod n$$

## Decryption

Ciphertext:

$$C$$

Plaintext:

$$M = C^d \bmod n$$



# Example of RSA Algorithm

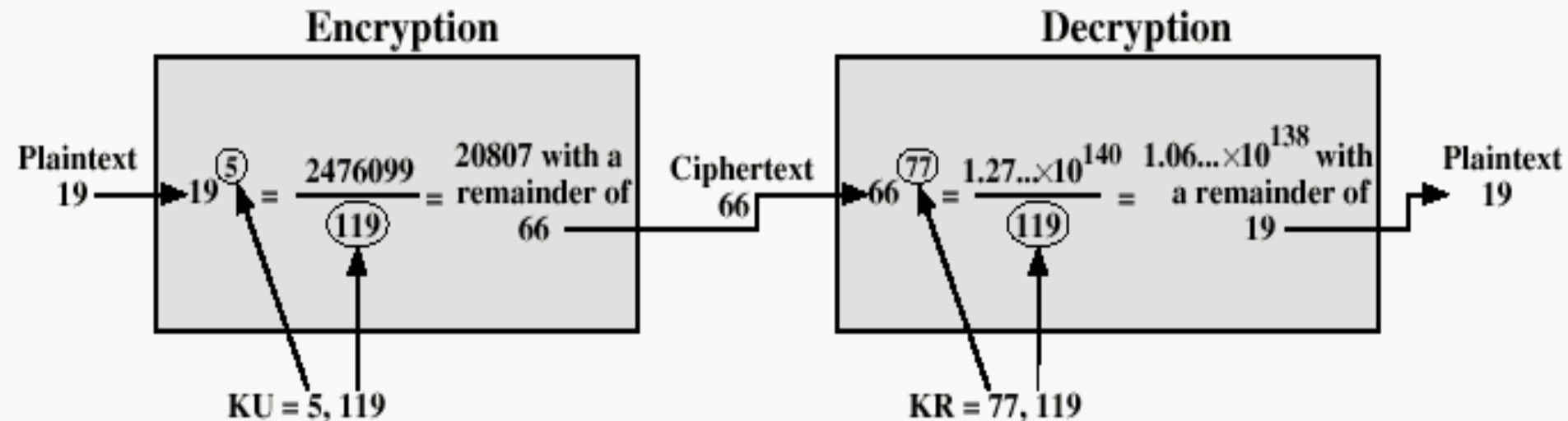


Figure 3.9 Example of RSA Algorithm

# Why Is RSA Secure?

- **RSA problem:** given  $n=pq$ ,  $e$  such that  $\gcd(e, (p-1)(q-1))=1$  and  $c$ , find  $m$  such that  $m^e = c \pmod n$ 
  - i.e., recover  $m$  from ciphertext  $c$  and public key  $(n, e)$  by taking  $e^{\text{th}}$  root of  $c$
  - There is no known efficient algorithm for doing this
- **Factoring** problem: given positive integer  $n$ , find primes  $p_1, \dots, p_k$  such that  $n = p_1^{e_1} p_2^{e_2} \dots p_k^{e_k}$
- If factoring is easy, then RSA problem is easy, but there is no known reduction from factoring to RSA
  - It may be possible to break RSA without factoring  $n$

# Other Public-Key Cryptographic Algorithms

---

- Digital Signature Standard (DSS)
  - Makes use of the SHA-1
  - Not for encryption or key exchange
- Elliptic-Curve Cryptography (ECC)
  - Good for smaller bit size
  - Low confidence level, compared with RSA
  - Very complex

# Applications of Public-Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (maybe)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can “sign” a message with your private key
- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

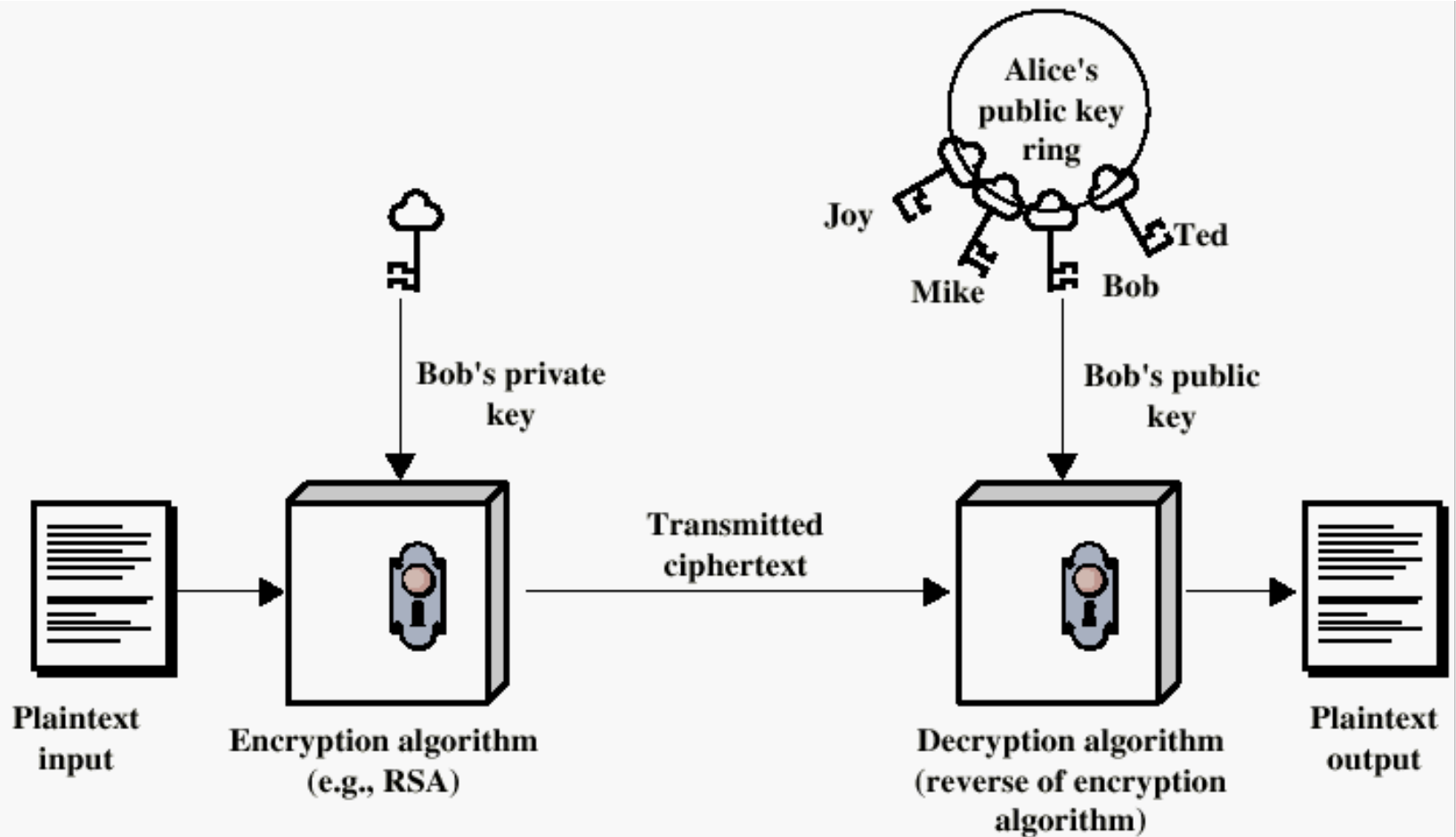
# Advantages of Public-Key Crypto

- Confidentiality without shared secrets
  - Very useful in open environments
  - No “chicken-and-egg” key establishment problem
    - With symmetric crypto, two parties must share a secret before they can exchange secret messages
- Authentication without shared secrets
  - Use digital signatures to prove the origin of messages
- Reduce protection of information to protection of authenticity of public keys
  - No need to keep public keys secret, but must be sure that Alice’s public key is really her true public key

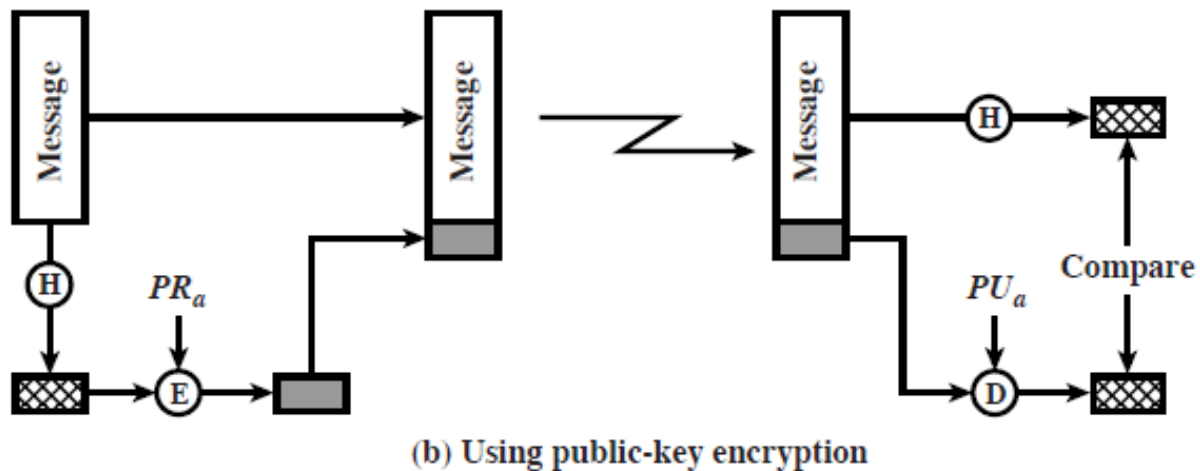
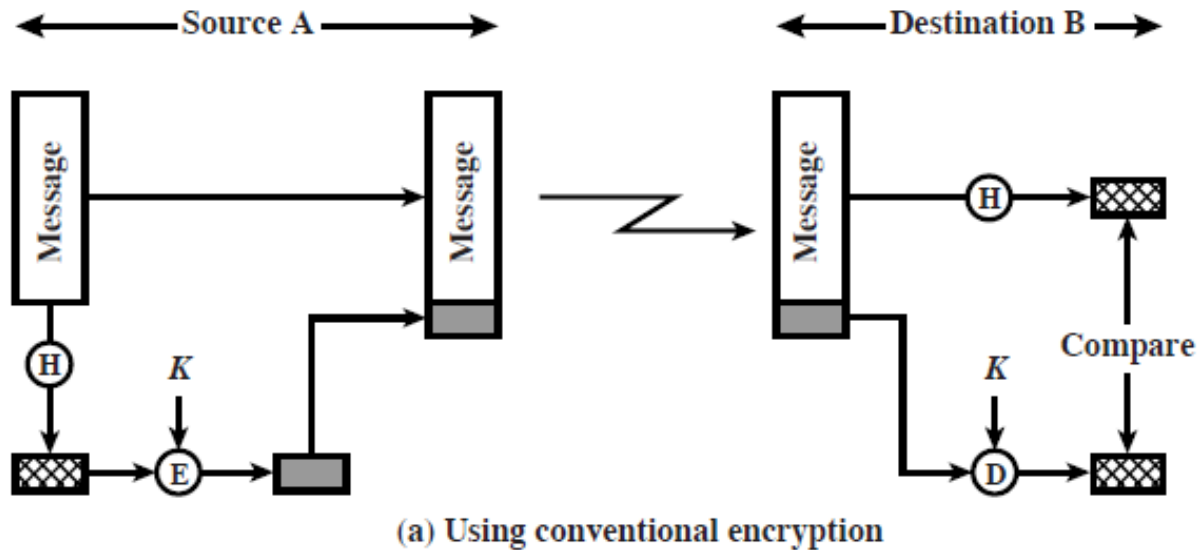
# Disadvantages of Public-Key Crypto

- Calculations are 2-3 orders of magnitude slower
  - Modular exponentiation is an expensive computation
  - Typical usage: use public-key cryptography to establish a shared secret, then switch to symmetric crypto
    - We'll see this in IPsec and SSL
- Keys are longer
  - 1024 bits (RSA) rather than 128 bits (AES)
- Relies on unproven number-theoretic assumptions
  - What if factoring is easy?
    - Factoring is believed to be neither P, nor NP-complete

# Authentication using Public-Key System



# MAC in encryptions





# Key Management

## Public-Key Certificate Use

usfCS

UNIVERSITY of SAN FRANCISCO  
department of computer sciences

Unsigned certificate:  
contains user ID,  
user's public key



Generate hash  
code of unsigned  
certificate



Encrypt hash code  
with CA's private key  
to form signature



Signed certificate:  
Recipient can verify  
signature using CA's  
public key.