

Neural Networks for Computer Vision



(Supervised) Machine Learning (ML)

- Start with past data
 - For 1,000,000 images, what's in the image?
 - For 1,000,000 past customers (and their consumer behaviour), did they repay their loans?
 - For 1,000,000 patients admitted to the hospital, what was the patient outcome?
- Predict the outcome for new data
 - What's in the new image, based on the raw pixel data?
 - What should the credit rating be for the new customer, based on their demographics and past behavior?
 - What is the prognosis for a new patient, given their vitals, lab data, etc. so far
- ML algorithms are often treated as “black boxes”
 - Goal: understand *how* ML-based systems produce the outputs they produce

Machine Learning vs. Intro to Programming

- Intro to Programming

- Write code that processes inputs in a specific way to produce the desired output, for any provided input
- Sample inputs and outputs are sometimes provided
 - Write a function f that does [...]. For example, $f(x^{(1)})$ should output $y^{(1)}$, and $f(x^{(2)})$ should output $y^{(2)}$

- Machine Learning (ML)

- A **Training Set** contains sample inputs and sample outputs
 - Training set: $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(N)}, y^{(N)})$
- The ML algorithm uses the **Training Set** to (hopefully) produce the desired outputs
- The ML algorithm does that by using the **Training Set** to find the parameters θ such that $h_{\theta}(x^{(i)}) \approx y^{(i)}$

e.g.: set $h_{(\theta_1, \theta_2, \theta_3)}(x) = \theta_1 + \theta_2 x + \theta_3 x^2$, try to find the best $\theta = (\theta_1, \theta_2, \theta_3)$

Shotgun debugging

Machine Learning vs. Intro to Programming

- Intro to Programming ***done badly***

```
def double_list(L):  
    for e in L:  
        e *= 2  
    return L  
  
>>> double_list([0, 0])  
[0, 0]  
>>> double_list([1, 2])  
[1, 2]
```

- Machine Learning ***done right***

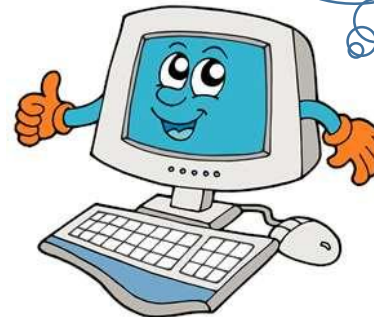
```
>>> h(0,1.2,0.1)([0, 0])  
[0, 0]  
>>> h(0,1.2,0.1)([1, 2])  
[1.3, 2.8]
```

$$h_{(\theta_1, \theta_2, \theta_3)}(x) = \theta_1 + \theta_2 x + \theta_3 x^2$$

Change the
for to while?



Change θ_2 to 1.3?



Machine
learning (kind
of)

Supervised Machine Learning

- Training set:

- Training example 1: $x^{(1)} = (x_1^{(1)}, x_2^{(1)}, \dots, x_m^{(1)})$ output: $y^{(1)}$

- Training example 2: $x^{(2)} = (x_1^{(2)}, x_2^{(2)}, \dots, x_m^{(2)})$ output: $y^{(2)}$

...

- Training example N: $x^{(N)} = (x_1^{(N)}, x_2^{(N)}, \dots, x_m^{(N)})$ output: $y^{(N)}$

- Test set:

- Test Example 1: $x^{(N+1)} = (x_1^{(N+1)}, x_2^{(N+1)}, \dots, x_m^{(N+1)})$ output: $y^{(N+1)}$

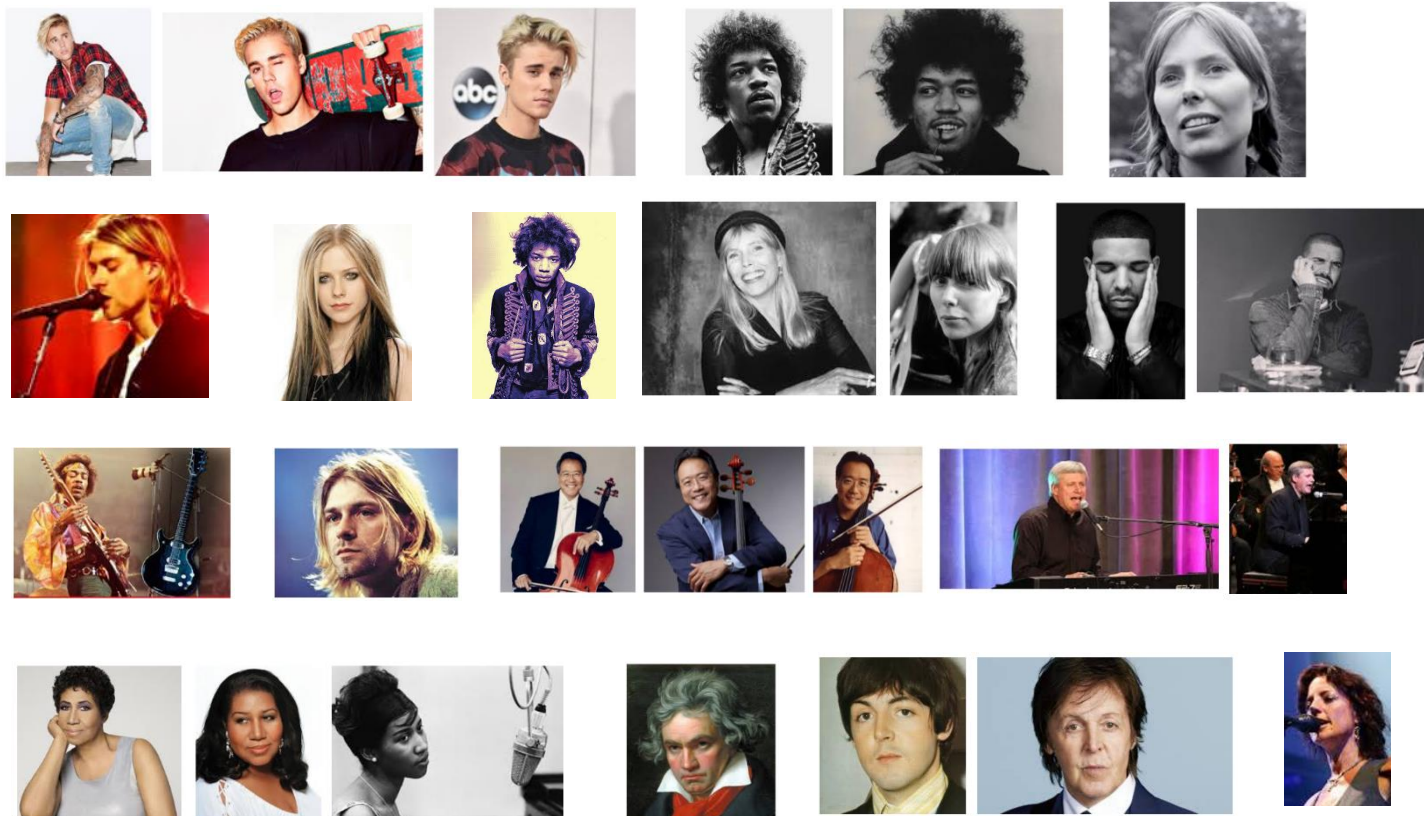
- Test Example 2: $x^{(N+2)} = (x_1^{(N+2)}, x_2^{(N+2)}, \dots, x_m^{(N+2)})$ output: $y^{(N+2)}$

- ...

- Test Example K: $x^{(N+K)} = (x_1^{(N+K)}, x_2^{(N+K)}, \dots, x_m^{(N+K)})$ output: $y^{(N+K)}$

- Goal: Find a θ such that $h_{\theta}(x^{(i)}) \approx y^{(i)}$ for $i \in 1, \dots, N$
- Hope: $h_{\theta}(x^{(i)}) \approx y^{(i)}$ for any i
- For new input x , predict $h_{\theta}(x)$

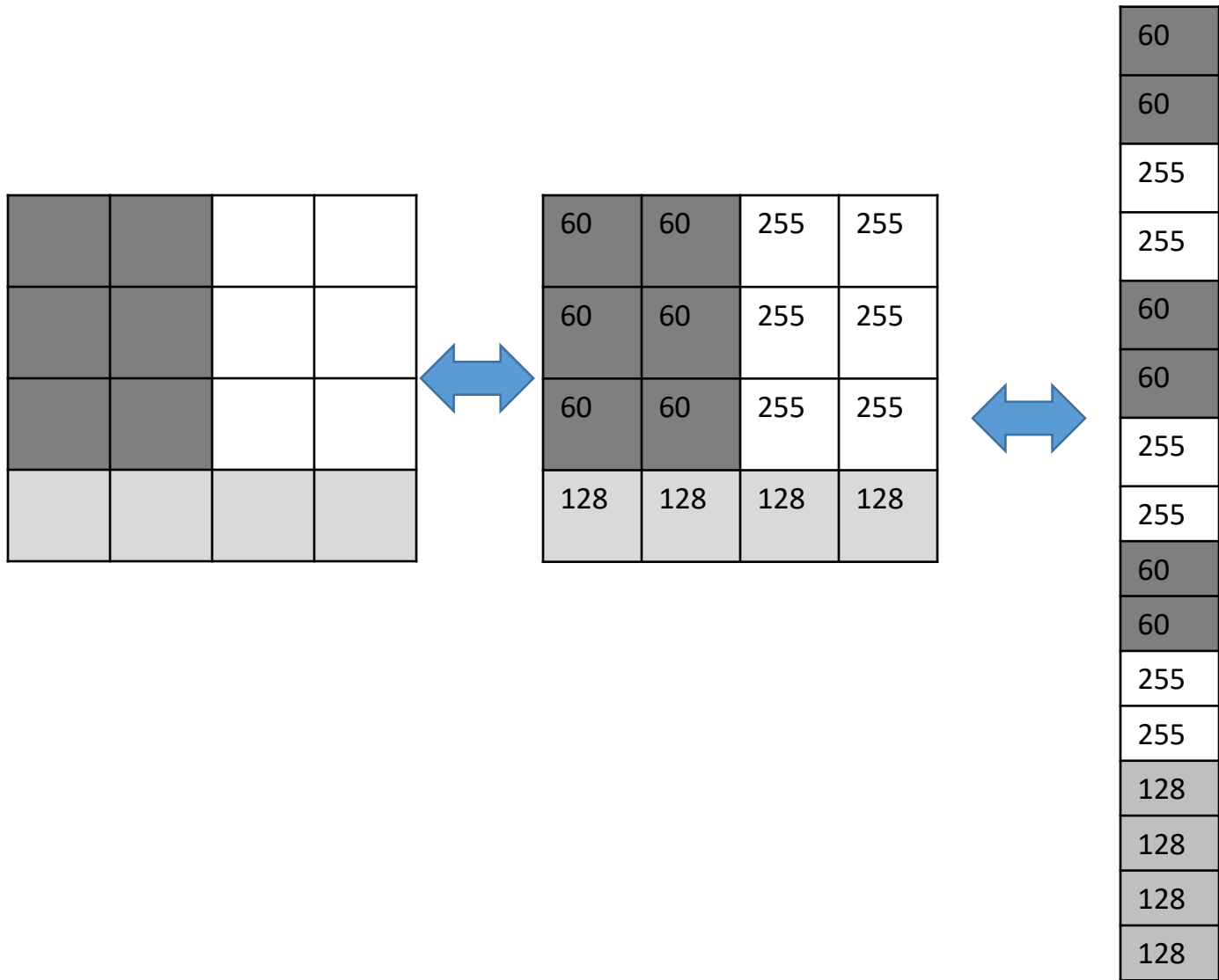
Sample ML task: Recognizing Justin Bieber



What Justin Bieber looks like to a computer

33 99 74 70 72 68 80 89 84 84 89 81 61 49 50 39 44 55 31 34 161 186 174 155 150 187 182 195 190 162 151 144 139 128 128 126 125 123 120 120 123 119 168 231 212 130 85 8
5 44 43 62 27 26 34 34 53 39 59 64 25 32 54 32 85 68 82 88 53 77 55 77 74 82 81 89 86 77 73 64 52 51 51 33 59 83 76 63 147 148 122 141 166 188 202 194 169 150 146 140 1
29 127 126 128 127 124 123 125 126 141 215 217 137 82 69 33 34 49 28 19 32 30 28 29 40 39 31 24 33 33 43 36 63 58 71 54 68 77 65 79 72 84 75 64 70 68 54 49 57 56 72 89
96 76 77 132 113 151 172 184 194 193 175 150 147 142 90 96 100 101 100 98 98 103 107 104 181 195 130 90 79 61 46 29 25 17 27 37 28 45 42 28 32 36 18 36 32 34 59 58 72 6
2 64 78 76 91 93 95 83 79 71 61 66 59 59 58 61 91 108 78 174 164 156 164 181 190 202 194 163 155 151 149 33 38 41 44 46 46 45 47 49 50 73 93 131 128 85 74 65 36 17 10 1
8 40 46 29 33 59 54 44 41 65 65 64 72 77 68 80 83 72 87 93 101 106 95 89 83 72 71 68 63 51 63 92 47 165 189 174 174 172 188 201 199 180 154 149 151 151 26 28 27 28 30 3
0 27 27 28 26 23 35 96 126 98 98 72 70 63 57 50 35 21 22 65 61 76 109 102 102 105 93 110 92 87 89 90 97 110 116 104 96 98 107 94 68 59 56 58 61 64 64 137 200 189 186 18
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0 46 74 75 54 42 38 56 69 74 127 175 182 188 182 194 183 194 202 182 165 160 153 146 142 33 40 47 52 58 63 65 67 65 82 87 78 83 100 74 78 43 52 34 23 6 13 59 70 74 42 3
0 19 40 63 73 105 122 102 122 136 145 147 97 52 35 23 11 13 36 62 66 60 84 109 126 132 178 167 186 180 187 190 196 188 164 158 163 155 150 146 104 115 125 132 138 141 1
41 142 126 153 172 141 80 82 70 75 74 59 61 42 24 16 15 52 21 12 67 88 106 123 128 153 121 118 114 150 127 70 63 29 11 36 32 17 28 33 45 89 90 115 114 101 139 154 180 1
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126 118 48 20 61 72 53 54 42 29 57 95 99 100 96 122 119 154 174 178 189 174 159 152 144 149 155 154 153 122 125 123 118 119 121 122 123 126 144 196 179 84 103 91 34 45
58 65 55 45 47 48 52 65 64 82 69 75 67 68 74 85 110 112 111 127 139 117 73 5 53 84 68 60 61 50 52 91 52 80 92 118 127 141 171 177 178 153 151 163 153 152 162 158 156 12
5 121 120 123 125 123 121 120 123 155 199 159 104 118 57 48 40 48 47 63 53 51 54 66 66 88 75 82 97 103 108 106 105 128 110 115 118 138 129 63 12 67 67 37 47 47 54 73 77
86 68 91 131 109 143 170 175 148 136 152 143 154 151 161 170 181 122 120 121 122 122 122 119 184 202 137 138 127 106 93 62 50 39 53 69 46 46 64 69 90 67 66 52
53 52 51 110 128 93 94 92 132 123 37 34 86 50 40 58 53 81 99 95 107 75 145 112 149 159 177 163 131 143 145 174 156 165 157 172 177 120 120 120 120 120 121 121 122 131 1
95 187 107 156 92 80 68 60 42 43 57 51 58 72 60 66 85 80 60 51 47 64 59 89 116 85 124 125 135 100 12 65 73 43 58 64 51 79 94 130 132 105 159 138 162 183 150 154 152 140
158 163 187 182 182 185 180 118 120 120 119 119 121 120 118 147 209 174 148 94 90 64 75 73 66 62 81 96 80 58 45 77 89 70 72 56 82 85 79 87 94 94 131 124 118 39 32 96 7
5 44 69 80 75 74 120 136 162 148 180 181 153 185 147 149 156 159 177 184 189 192 202 192 178 118 118 117 117 119 120 118 115 160 208 177 97 95 62 73 59 63 79 80 94 113
97 69 55 77 72 73 71 77 87 114 123 110 109 85 135 92 35 20 86 97 47 41 62 72 74 90 98 152 152 132 134 149 107 149 165 141 168 189 181 198 194 209 199 205 192 117 116 11
6 117 119 118 117 117 176 196 106 79 75 73 82 87 71 73 98 80 129 106 97 41 91 77 66 89 80 97 113 147 163 132 40 19 11 61 96 118 97 44 66 50 66 122 93 110 142 108 104 13
2 135 113 115 169 163 188 193 210 202 203 201 197 197 195 115 113 114 116 116 114 118 124 182 115 78 89 101 86 114 84 95 106 80 101 115 116 88 94 93 73 64 73 90 99 106
118 164 147 69 92 95 94 116 106 61 43 75 65 100 101 155 97 113 124 102 121 90 150 139 174 175 198 202 202 194 188 187 160 173 183 113 111 112 115 113 111 119 130 130 88
90 100 113 103 102 104 115 97 105 87 117 101 123 103 76 90 92 89 81 97 99 113 154 139 70 61 89 111 108 89 66 82 82 123 88 140 154 132 156 137 97 112 119 129 146 162 18
7 193 184 174 159 150 159 138 139 155 99 106 100 100 111 99 101 154 100 104 107 115 115 107 123 111 119 109 109 100 99 121 121 103 82 103 77 81 78 96 103 106 107 113 62
43 82 108 104 82 64 114 114 121 108 143 174 157 153 142 120 111 107 123 158 177 137 138 129 142 134 145 131 141 51 41 47 48 41 49 141 108 97 107 118 117 125 12
7 120 132 123 113 114 82 96 109 115 115 102 70 105 86 93 100 118 133 119 136 84 34 69 115 110 100 128 125 82 116 110 137 170 206 179 126 112 122 137 128 153 168 132
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5 84 92 87 99 95 79 140 150 190 182 167 150 145 104 142 108 155 154 149 147 133 125 147 121 122 140 128 129 24 27 30 21 24 162 102 127 120 130 135 131 138 145 143 143 1
87 131 115 101 94 108 95 141 167 124 130 121 108 135 116 115 141 140 142 116 94 94 102 121 122 132 110 131 151 166 163 171 156 157 132 88 143 74 113 157 158 155 147 123
128 120 131 126 131 133 23 28 19 29 56 125 120 118 122 134 138 135 136 142 151 152 141 146 118 117 97 102 110 159 159 133 142 135 136 140 138 133 103 94 102 122 128 13
2 147 162 175 177 174 164 151 130 134 151 172 180 126 90 134 73 90 147 160 155 131 144 118 127 121 131 125 129 29 36 46 52 156 117 116 135 126 139 144 143 135 134 148 1
52 149 149 124 129 125 118 147 170 109 146 122 155 130 169 153 113 86 93 96 110 135 152 168 182 178 156 141 137 126 121 117 129 193 208 151 71 121 57 84 148 157 153 142
154 123 128 126 121 119 122 89 94 91 115 122 116 139 132 133 138 140 146 138 135 150 153 165 149 115 137 149 158 179 172 183 150 106 129 156 183 146 142 123 133 145 14
1 147 147 135 145 144 127 125 119 122 124 120 129 185 214 163 49 79 60 94 128 144 170 168 120 153 121 124 119 123 124 155 141 137 146 122 125 139 133 135 129 127 140 14
2 144 160 161 178 170 132 154 126 150 170 175 128 71 53 48 58 93 162 145 134 106 114 109 109 120 114 136 125 118 141 116 133 117 125 136 199 203 163 105 44 57 87 120 14
2 169 152 138 125 129 121 125 121 122 112 105 106 144 113 150 129 131 138 119 118 113 161 141 178 172 154 182 92 52 45 30 33 30 45 67 27 38 71 107 108 116 108 1
11 117 114 118 119 120 122 125 128 127 124 127 136 197 219 157 54 73 81 79 129 141 139 193 166 118 137 120 121 119 120 98 99 105 147 102 136 121 118 131 112 117 117 148
178 187 153 184 196 219 212 126 41 23 32 36 35 20 30 33 35 58 92 106 110 116 114 111 119 121 120 122 124 127 127 124 127 136 197 219 162 42 50 107 82 122 136 140 15
9 196 132 130 127 136 119 118 97 101 92 138 111 126 110 116 124 97 92 121 124 171 193 160 174 155 222 236 166 68 38 37 32 32 35 30 46 27 27 35 46 84 112 117 111 115 126
122 121 122 123 127 127 126 127 135 199 220 171 41 34 122 87 123 127 146 129 173 169 115 127 127 137 128 101 102 103 130 113 121 124 108 107 96 115 132 136 182 148 124
114 112 171 234 212 108 26 36 27 32 35 27 41 28 28 30 23 51 80 101 111 114 116 115 122 122 124 126 128 127 130 136 198 219 172 50 31 99 92 127 123 131 152 150 176 131

Images \longleftrightarrow Vectors



The Face Recognition Task

- Training set:

- $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(N)}, y^{(N)})\}$

- $x^{(i)}$ is a k-dimensional vector consisting of the intensities of all the pixels in the i-th photo (20×20 photo $\rightarrow x^{(i)}$ is 400-dimensional)

- $y^{(i)}$ is the *label* (i.e., name)

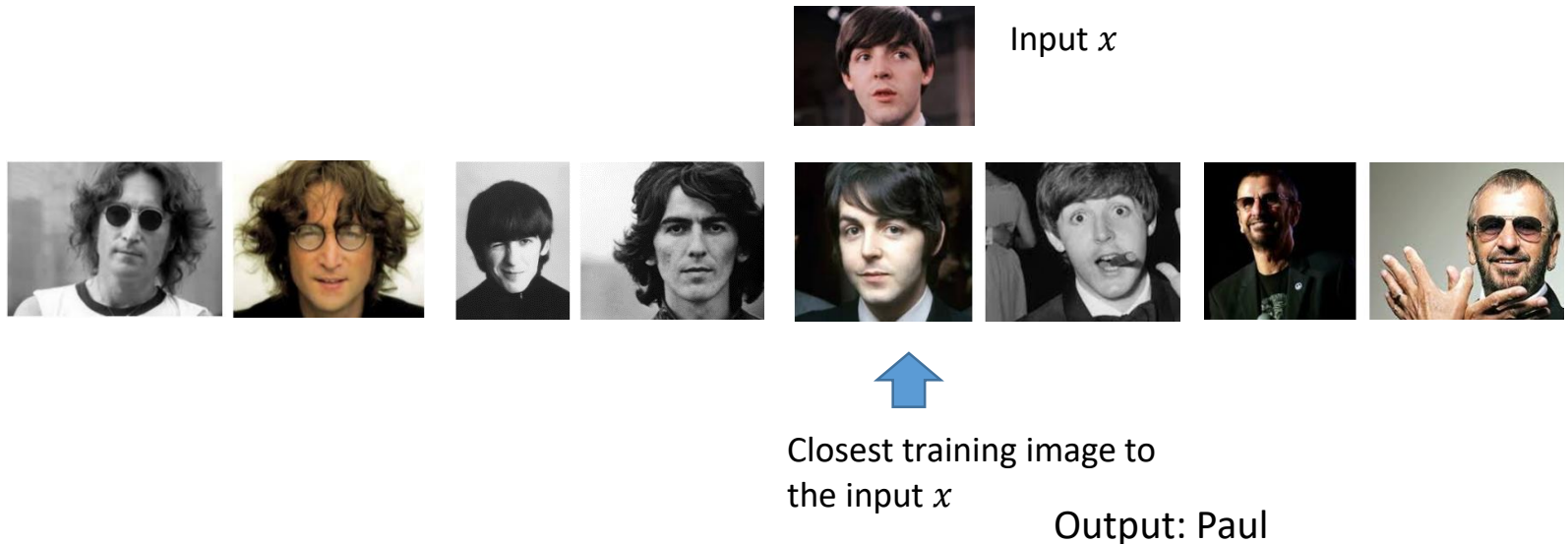
- Test phase:

- We have an input vector x , and want to assign a label y to it

- Whose photo is it?

Face Recognition using 1-Nearest Neighbors (1NN)

- Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(N)}, y^{(N)})\}$
- Input: x
- 1-Nearest Neighbor algorithm:
 - Find the training photo/vector $x^{(i)}$ that's as “close” as possible to x , and output the label $y^{(i)}$



Are the two images a and b close?

- Key idea: think of the images as *vectors*
 - Reminder: to turn an image into a vector, simply “flatten” all the pixels into a 1D vector

- Is the distance between the endpoints of vectors a and b small?

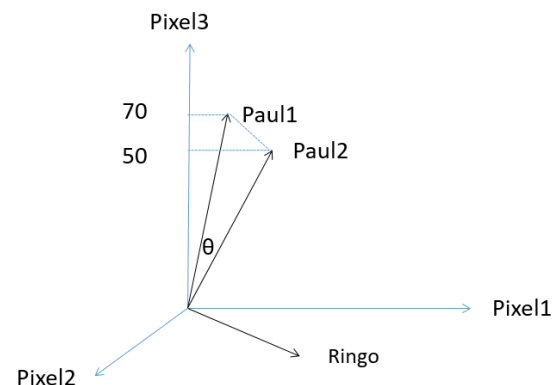
$$|a - b| = \sqrt{\sum_i (a_i - b_i)^2} \text{ small}$$

- Is the cosine of the angle between the vectors a and b large?

$$\cos \theta_{ab} = \frac{a \cdot b}{|a||b|} = \frac{\sum_i a_i b_i}{\sqrt{\sum_i a_i^2} \sqrt{\sum_i b_i^2}} \text{ large}$$

By the law of cosines

- Is $a \cdot b = \sum_i a_i b_i$ large?
 - Assume $|a| \approx |b| \approx \text{const}$



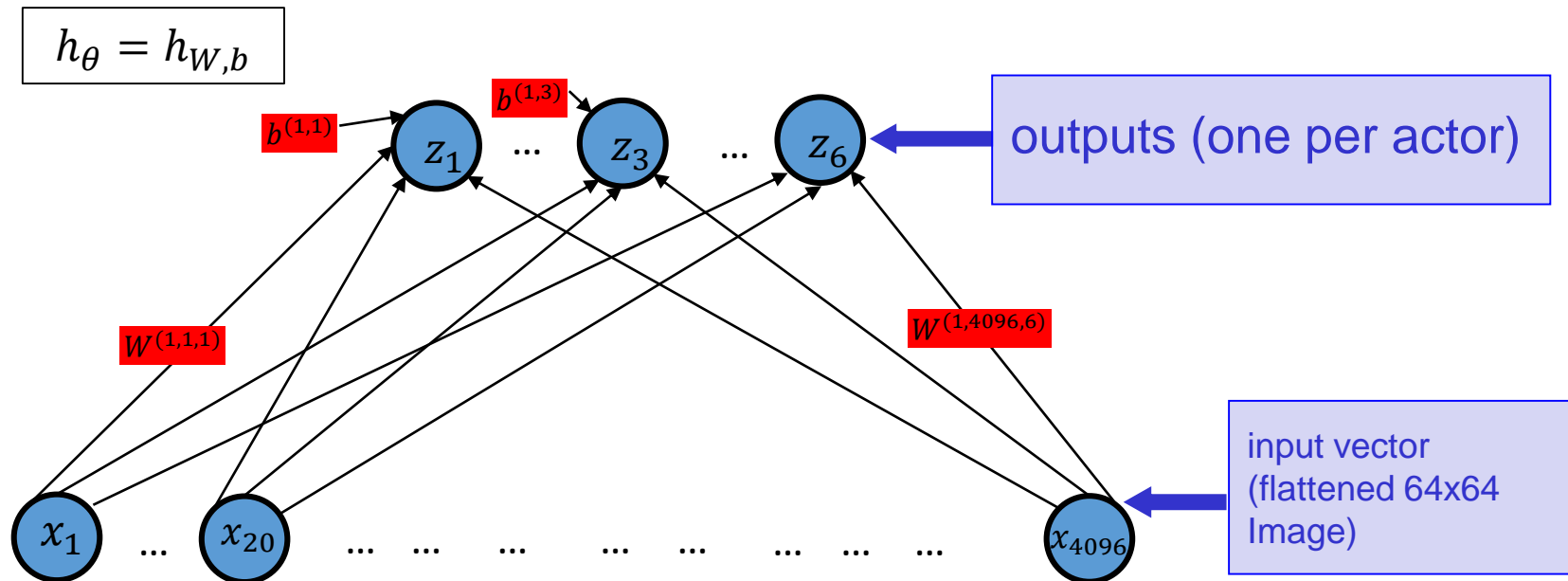
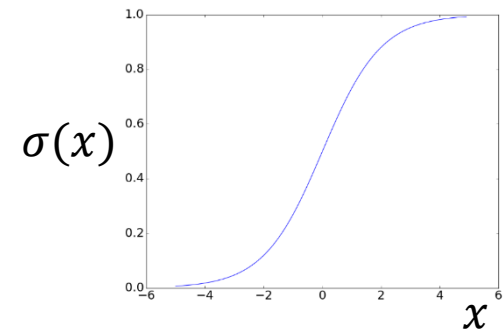
Sample task

- Training set: 6 actors, with 100 64×64 photos of faces for each
- Test set: photos of faces of the same 6 actors
- Want to classify each face as one of ['Fran Drescher', 'America Ferrera', 'Kristin Chenoweth', 'Alec Baldwin', 'Bill Hader', 'Steve Carell']



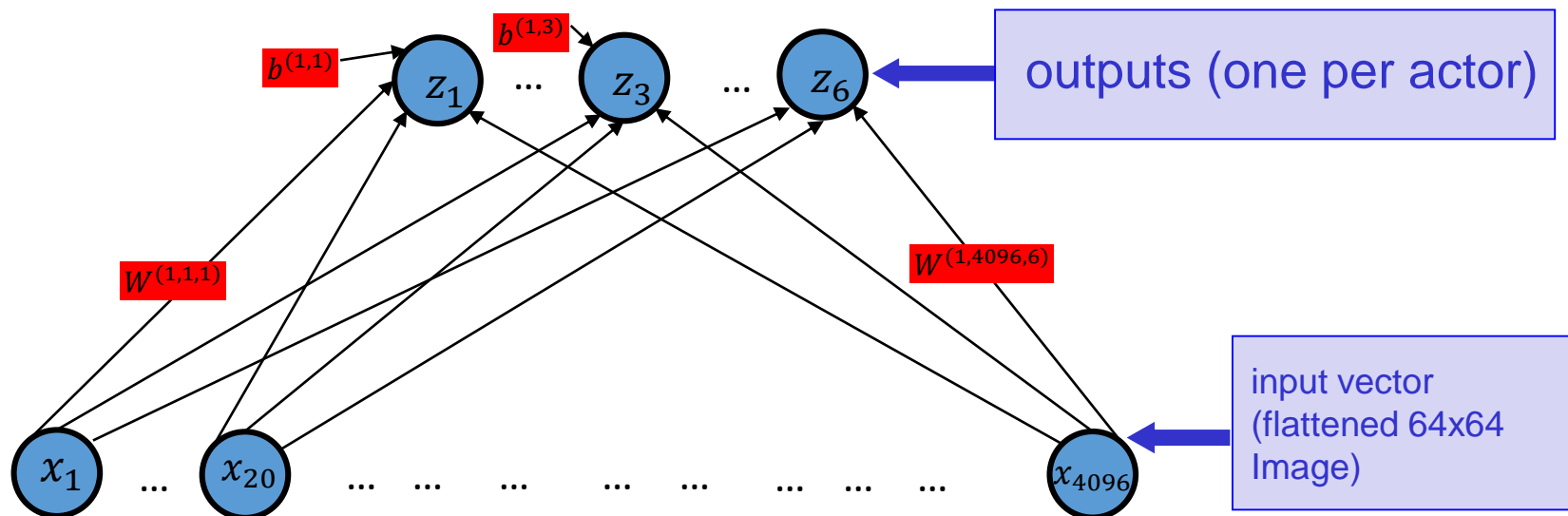
The Simplest Possible Neural Network for Face Recognition

$$z_k = \sigma \left(\sum_{j=1}^{4096} W^{(1,j,k)} x_j + b^{(1,k)} \right)$$
$$= \sigma(W^{(1,*,k)} \cdot x + b^{(1,k)})$$



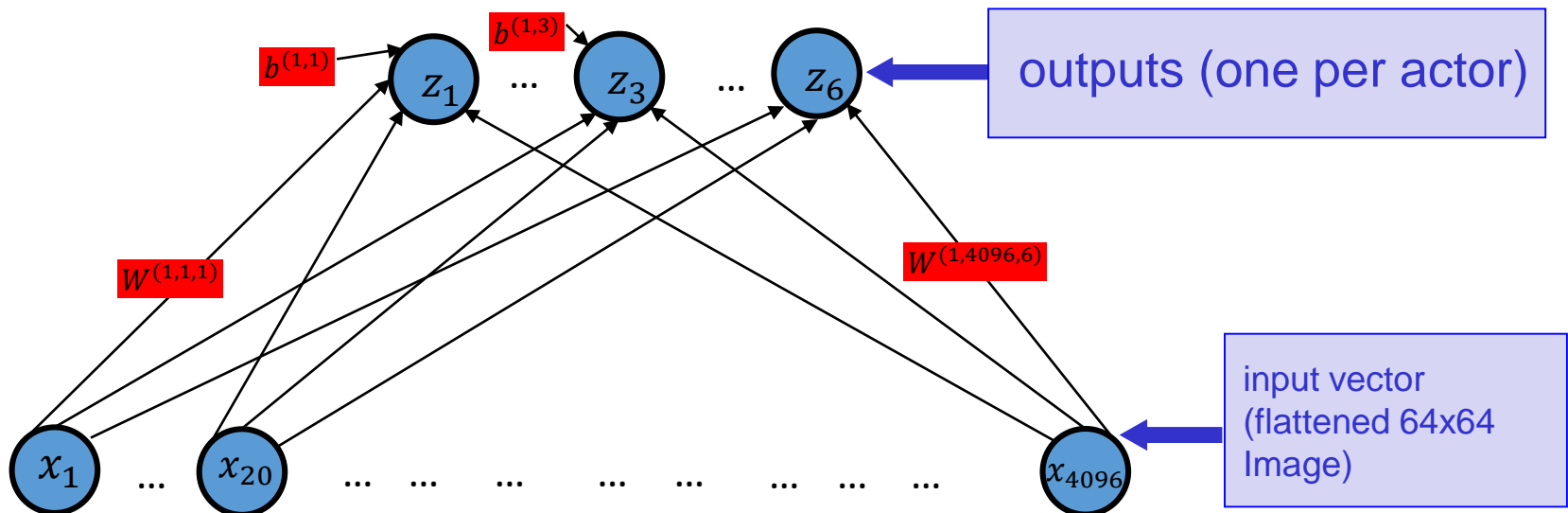
Training a neural network

- Adjust the W 's (4096×6 coefs) and b 's (6 coefs)
 - Try to make it so that if
 x is an image of actor 1, z is as close as possible to $(1, 0, 0, 0, 0, 0)$
 x is an image of actor 2, z is as close as possible to $(0, 1, 0, 0, 0, 0)$
.....



Face recognition

- Compute the z for a new image x
- If z_k is the largest output, output name k



An interpretation

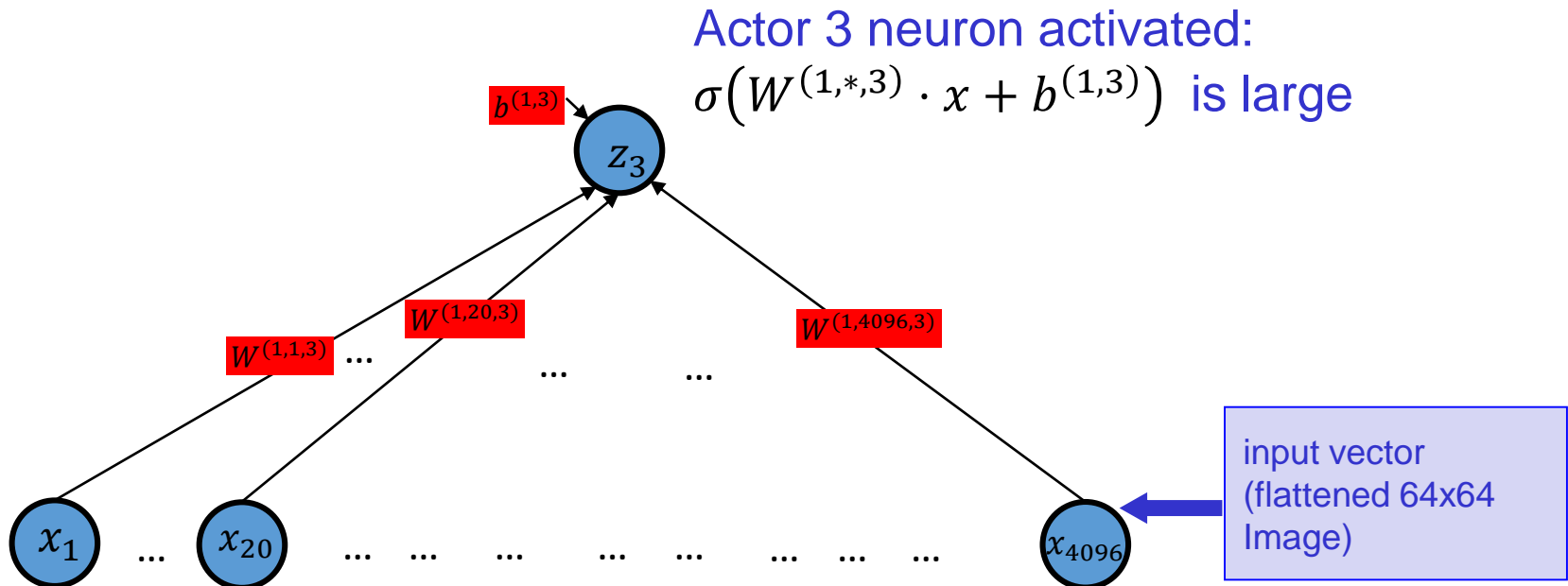
z_1 is large if $W^{(1,*,1)} \cdot x$ is large

z_2 is large if $W^{(1,*,2)} \cdot x$ is large

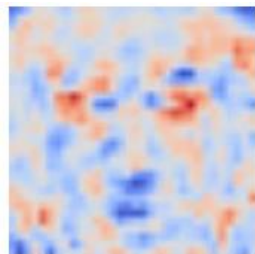
z_3 is large if $W^{(1,*,3)} \cdot x$ is large

....

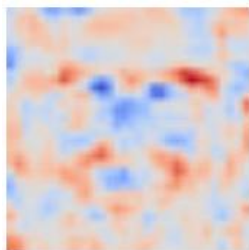
$W^{(1,*,1)}, W^{(1,*,2)}, \dots, W^{(1,*,6)}$ are *templates* for the faces of actor 1, actor 2, ..., actor 6



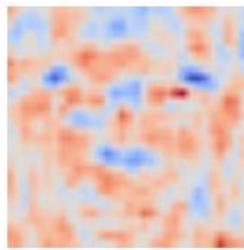
Visualizing the parameters W



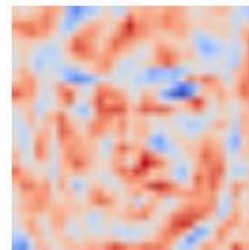
Baldwin
 $W^{(1,*,1)}$



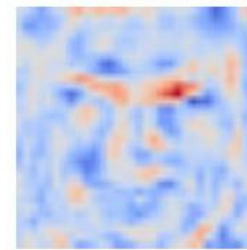
Carrel
 $W^{(1,*,2)}$



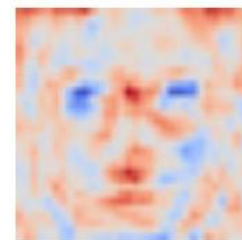
Hader
 $W^{(1,*,3)}$



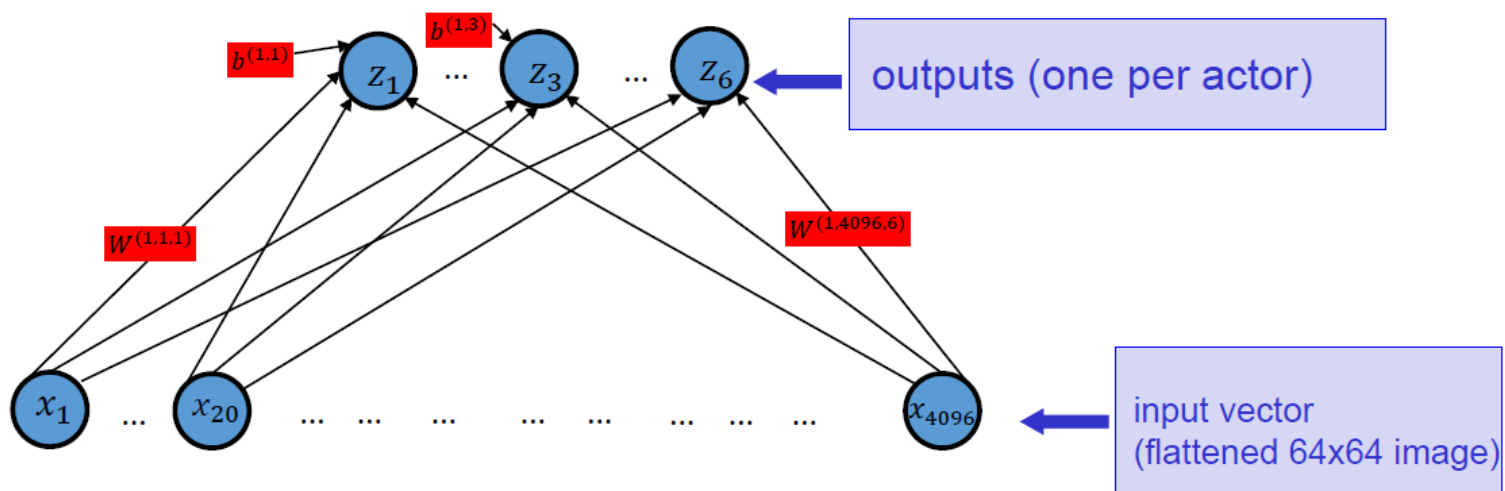
Ferrera
 $W^{(1,*,4)}$



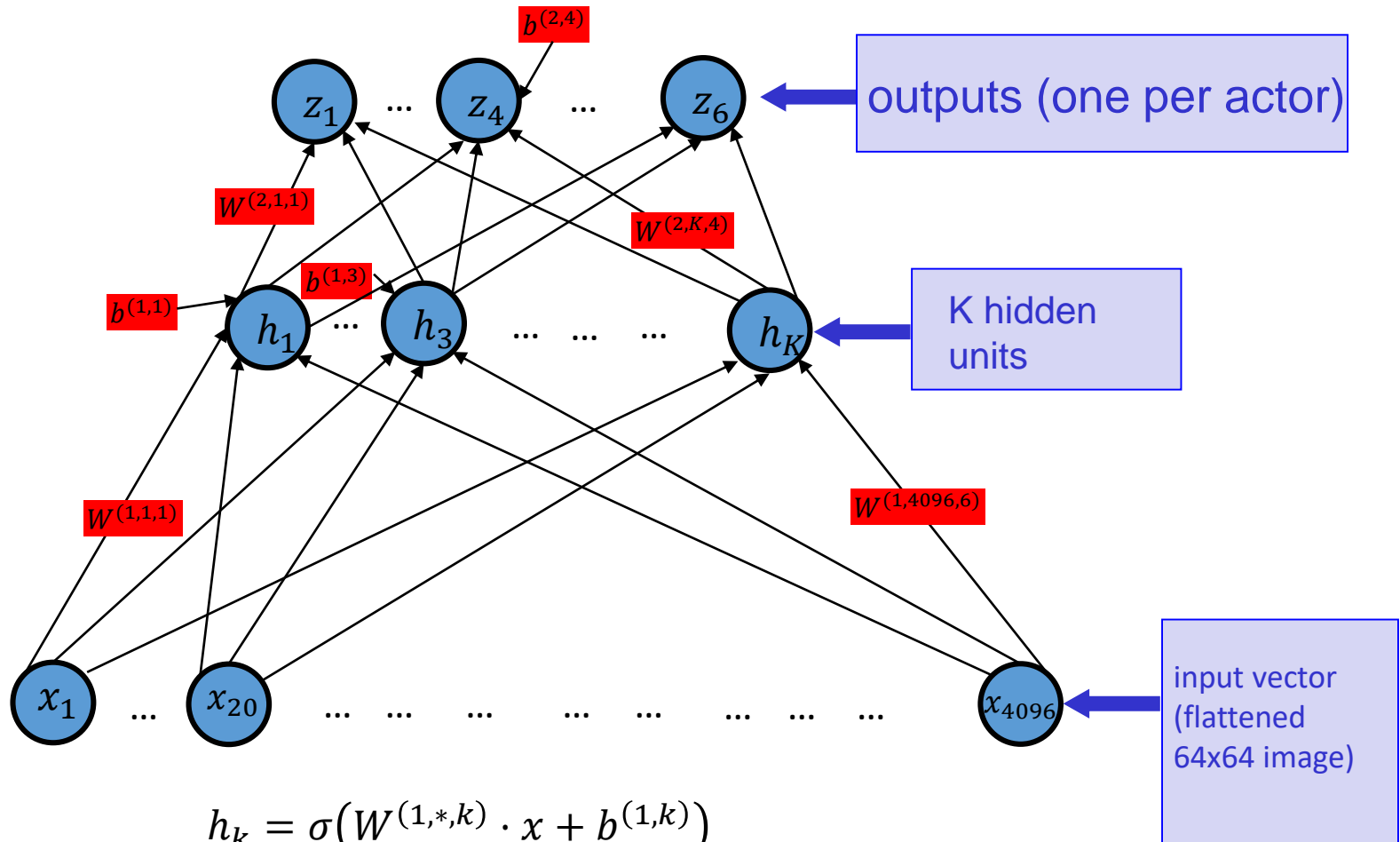
Drescher
 $W^{(1,*,5)}$



Chenoweth
 $W^{(1,*,6)}$



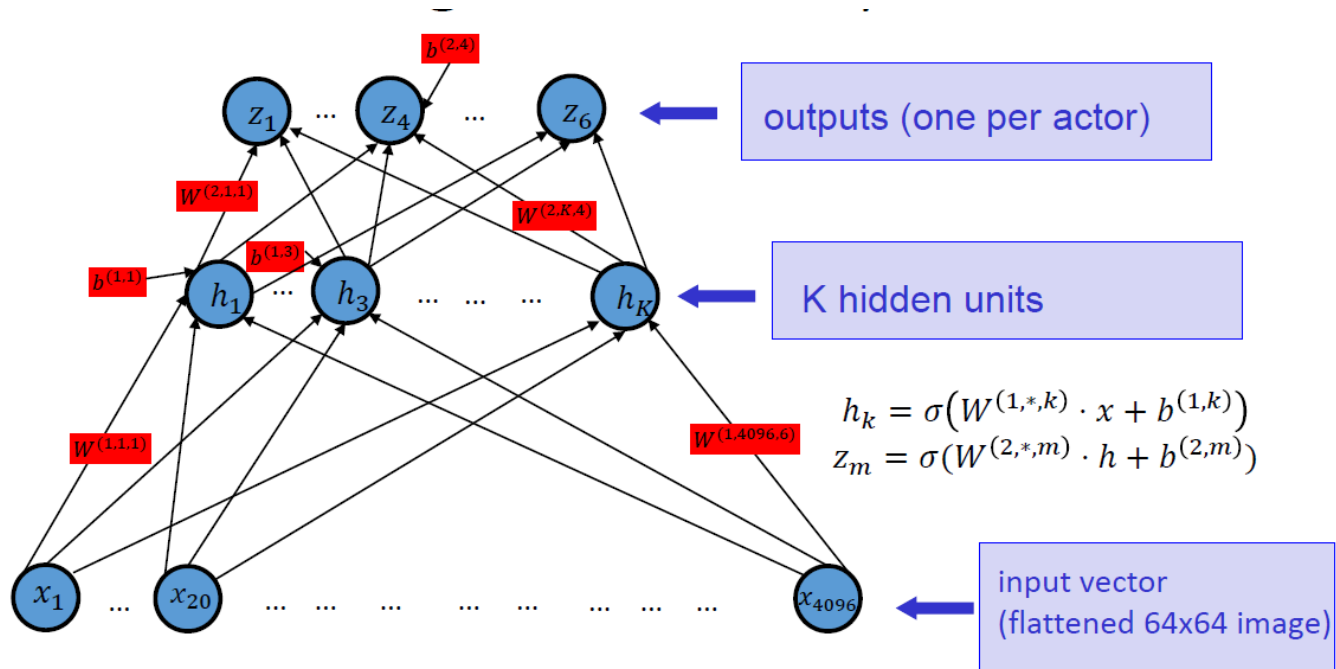
Deep Neural Networks: Introducing Hidden Layers



$$h_k = \sigma(W^{(1,*,k)} \cdot x + b^{(1,k)})$$
$$z_m = \sigma(W^{(2,*,m)} \cdot h + b^{(2,m)})$$

Why a hidden layer?

- Instead of checking whether x looks like one of 6 templates, we'll be checking whether x looks like one of K templates, for a large K
 - If template k (i.e., $W^{(1,*,k)}$) looks like actor 6, $W^{(2,k,6)}$ will be large



Recap: Face Recognition with ML

- 1-Nearest-Neighbor: match x to all the images in the training set
- 0-hidden-layer neural network*: match x to several templates, with one template per actor
 - The templates work better than any individual photo
- 1-hidden-layer neural network: match x to K templates
 - The templates work better than any individual photo
 - More templates means better accuracy on the training set

*A.K.A. multinomial logistic regression to its friends

Visualizing a One-Hidden-Layer NN



Deep Neural Networks as a Model of Computation

- Most people's first instinct a face classifier is to write a complicated computer program
- A deep neural network *is* a computer program:

$$\begin{aligned}h_1 &= f_1(x) \\h_2 &= f_2(h_1) \\h_3 &= f_3(h_2) \\&\dots \\h_9 &= f_9(h_8)\end{aligned}$$

- Can think of every layer of a neural network as one step of a parallel computation
- Features/templates are the functions that are applied to the previous layers
- Learning features \Leftrightarrow Learning what function to apply at step t of the algorithm

What are the hidden units doing?

- Find the images in the dataset that activate the units the *most*
- *Let's see some visualizations of neurons of a large deep network trained to recognize objects in images*
 - Then network classifies images as one of 1000 objects (sample objects: toy poodle, flute, forklift, goldfish...)
 - The network has 8 layers
 - Note: more tricks were used in designing the networks than we have time to mention! In particular, a convolutional architecture is crucial

Units in Layer 3



Matthew Zeiler and Rob Fergus, "Visualizing and Understanding Convolutional Networks" (ECCV 2014)

Units in Layer 4



Matthew Zeiler and Rob Fergus, "Visualizing and Understanding Convolutional Networks" (ECCV 2014)

Units in Layer 5



Which pixels are responsible for the output?

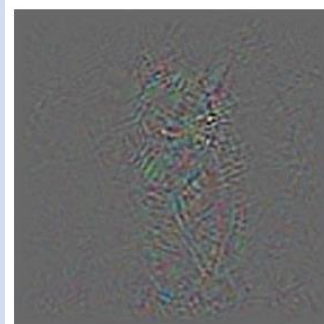
- For each pixel in a particular image ask:
 - If I changed this pixel j by a little bit, how would that influence the output i
 - Equivalent to asking: what's the gradient $\frac{\partial output_i}{\partial input_j}$
 - We can visualize why a particular output was chosen by the network by computing $\frac{\partial output_i}{\partial input_j}$ for every j , and displaying that as an image

Gradient and Guided Backpropagation

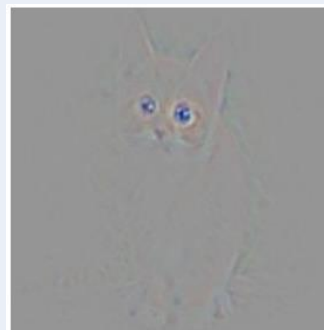
Image I



$\frac{\partial \text{Cat-Neuron}}{\partial I}$

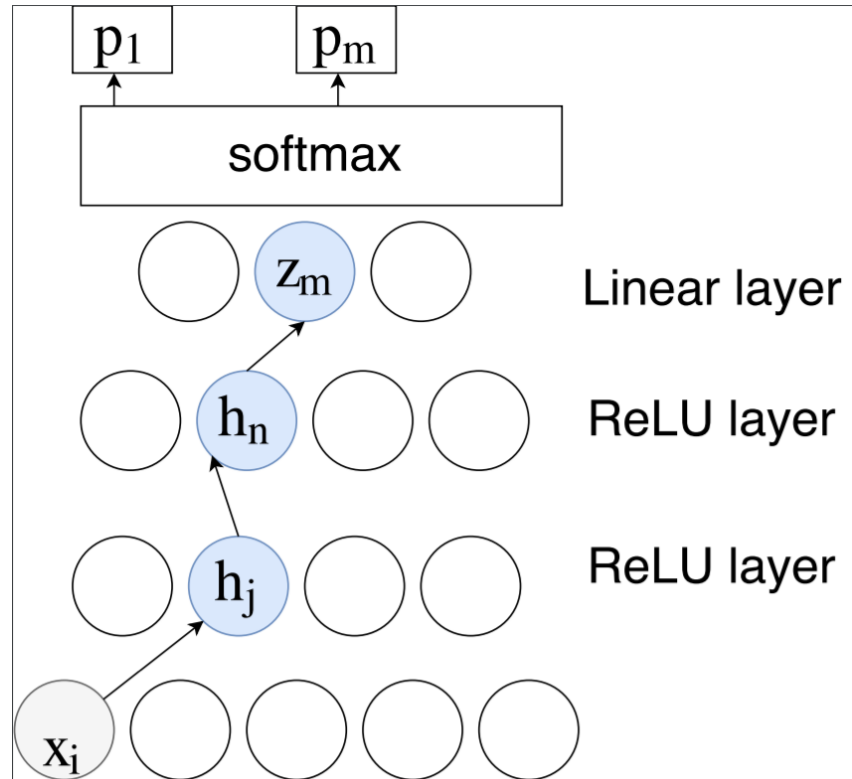


Guided Backpropagation visualization

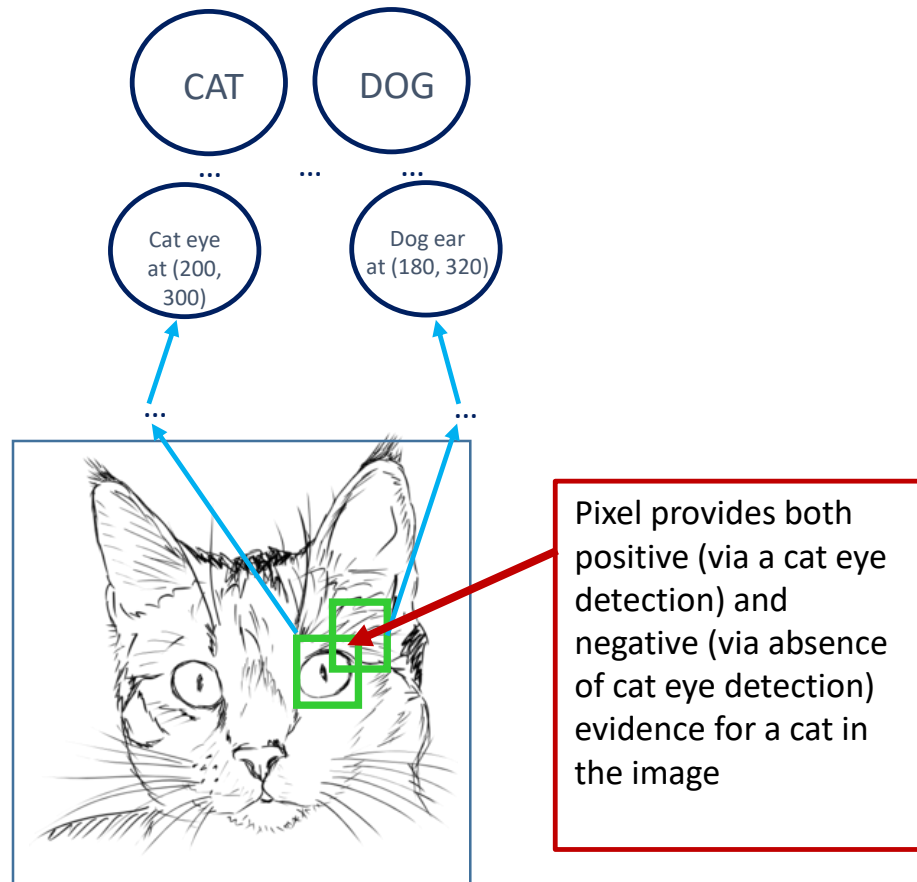


Guided backpropagation

- Instead of computing $\frac{\partial p_m}{\partial x}$, only consider paths from x to p_m where the weights are positive and all the units are positive (and greater than 0). Compute this modified version of $\frac{\partial p_m}{\partial x}$
- Only consider evidence for neurons being active, discard evidence for neurons having to be not active



Guided Backpropagation Intuition



Application: Photo Orientation

- Detect the correct orientation of a consumer photograph
- Input photo is rotated by 0° , 90° , 180° or 270°
- Help speed up the digitization of analog photos
- Need correctly oriented photos as inputs for other systems



0°



90°

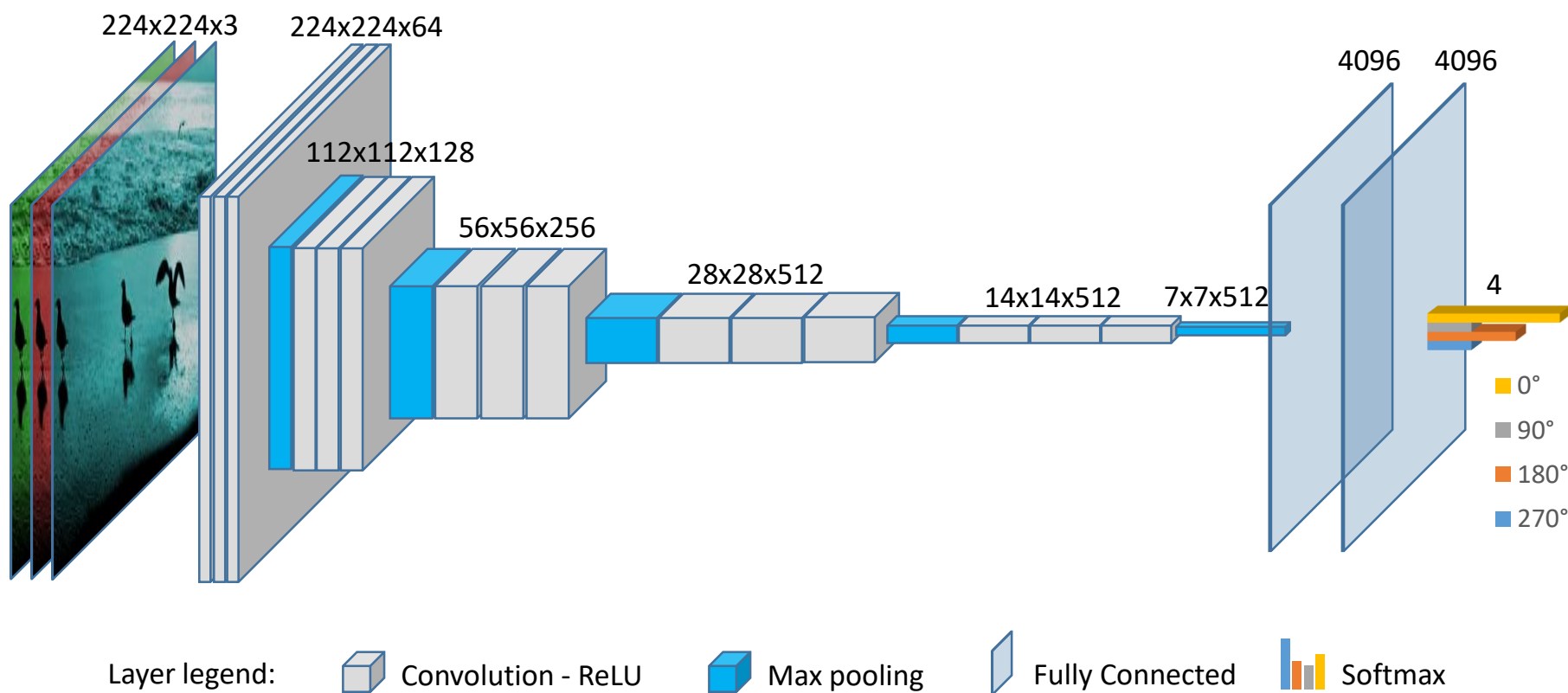


180°



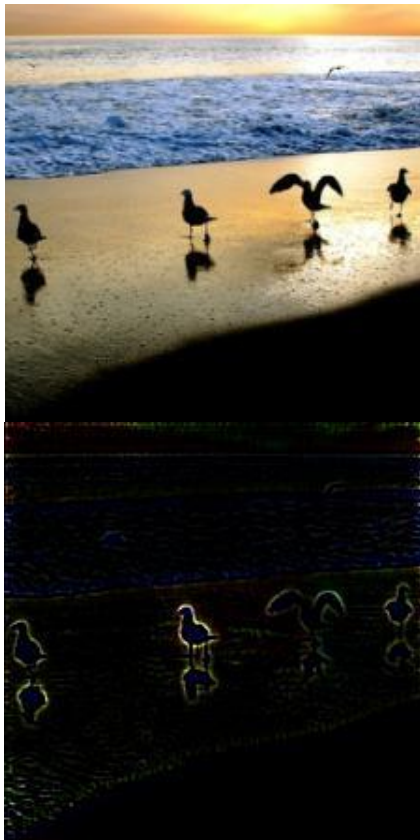
270°

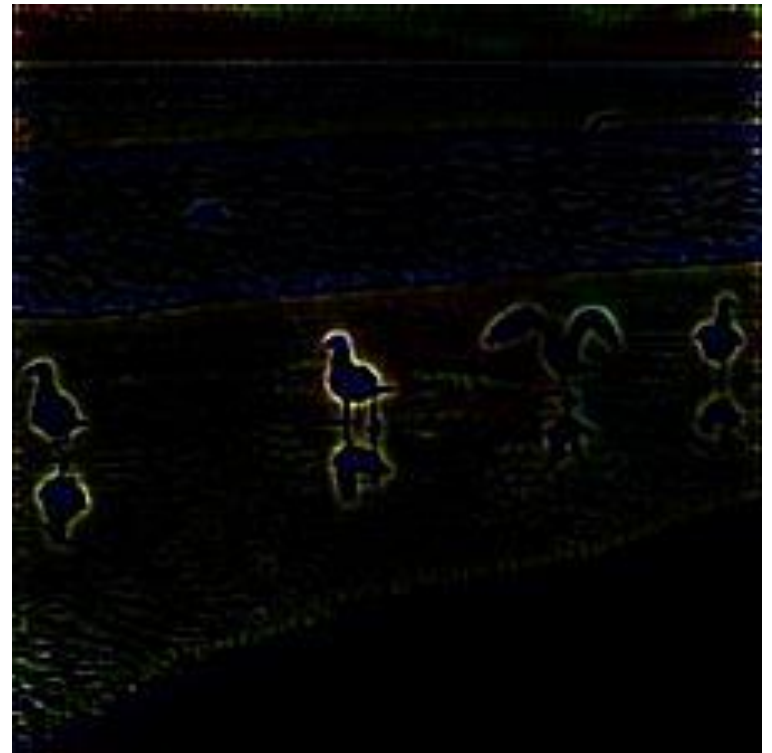
A Neural Network for Photo Orientation



Correctly Oriented Photos

- Display pixels that provide direct positive evidence for 0°



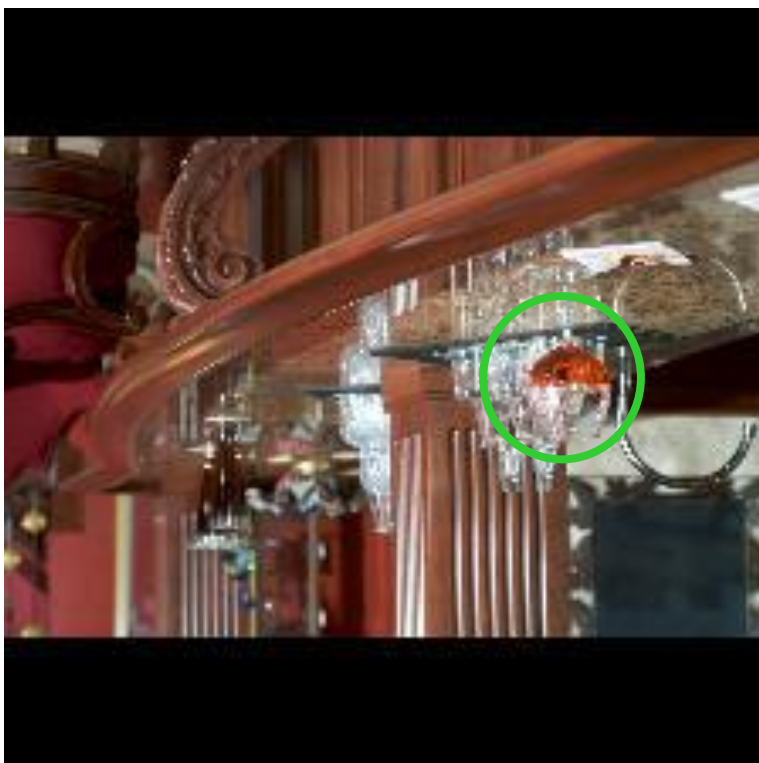


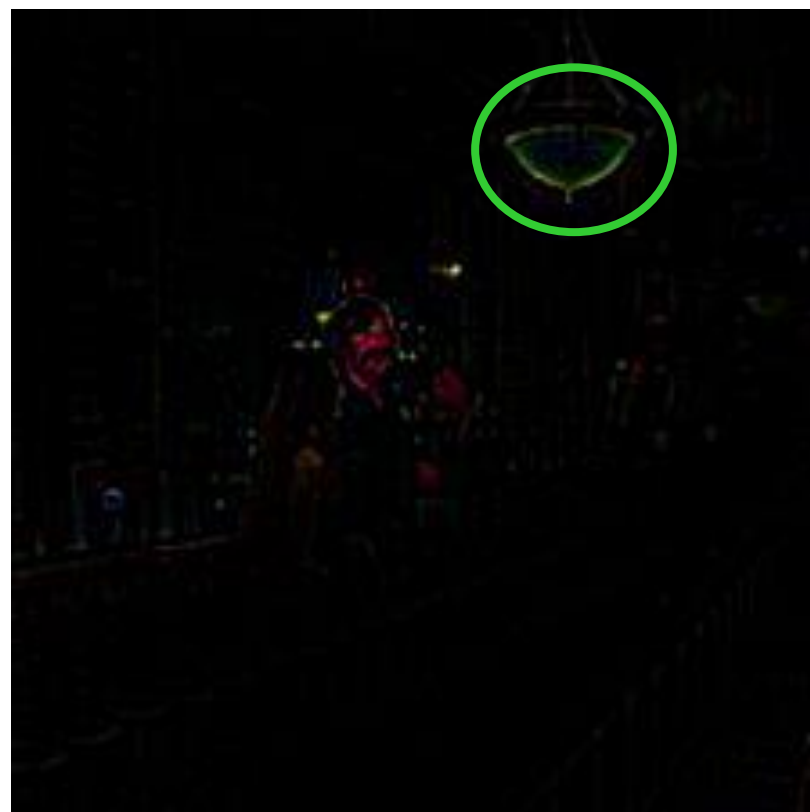


Incorrectly-oriented photos



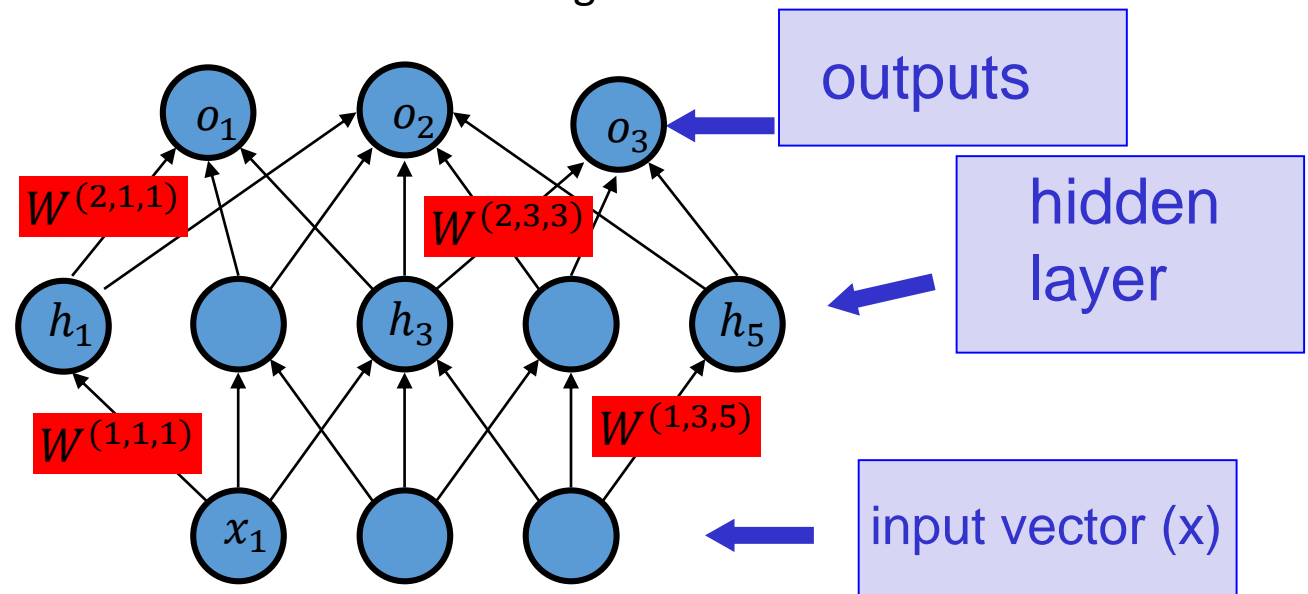






Backpropagation

o_i is large if the probability that the correct class is i is high



A possible cost function:

$$C(o, y) = \sum_{i=1}^m |y^{(i)} - o^{(i)}|^2$$

Make $y^{(i)}$'s and $o^{(i)}$ as close as possible

Partial Derivatives of the Cost Function

- We need the partial derivatives of the cost function $C(o, y)$ w.r.t all the W and b
- $o_i = \sigma(\sum_j W^{(2,j,i)} h_j + b^{(2,i)})$
- Partial derivative of $C(o, y)$ w.r.t $W^{(2,j,i)}$

$$\begin{aligned}
 \frac{\partial C}{\partial W^{(2,j,i)}}(x, y, W, b, h, o) &= \frac{\partial o_i}{\partial W^{(2,j,i)}}(x, y, W, b, h, o) \frac{\partial C}{\partial o_i}(x, y, W, b, h, o) \\
 &= \frac{\partial(\sum_j W^{(2,j,i)} h_j)}{\partial W^{(2,j,i)}}(x, y, W, b, h, o) \frac{\partial \sigma}{\partial(\sum_j W^{(2,j,i)} h_j)}(x, y, W, b, h, o) \frac{\partial C}{\partial o_i}(x, y, W, b, h, o) \\
 &= h_j \frac{\partial \sigma}{\partial \sum_j W^{(2,j,i)} h_j}(x, y, W, b, h, o) \frac{\partial C}{\partial o_i}(x, y, W, b, h, o) \\
 &= h_j \sigma' \left(\sum_j W^{(2,j,i)} h_j + b^{(2,i)} \right) \frac{\partial}{\partial o_i} C(o, y)
 \end{aligned}$$

$$h_j g' \left(\sum_j W^{(2,j,i)} h_j + b^{(2,j)} \right) \frac{\partial \mathcal{C}}{\partial o_i}(o, y)$$

$$\bullet \sigma(t) = \frac{1}{1 + \exp(-t)} \quad \rightarrow$$

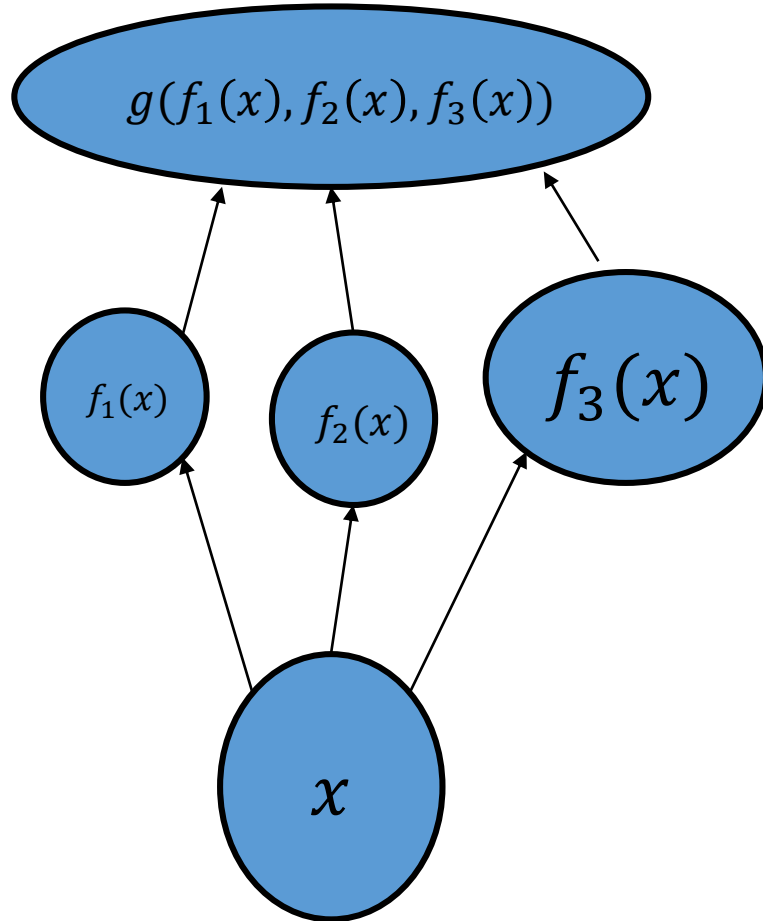
$$\sigma'(t) = \frac{\exp(-t)}{(1 + \exp(-t))^2} = \frac{1}{(1 + \exp(-t))} \frac{\exp(-x)}{(1 + \exp(-t))} = \sigma(t)(1 - \sigma(t))$$

$$\bullet \mathcal{C}(o, y) = \sum_{i=1}^N (o_i - y_i)^2 \quad \rightarrow$$

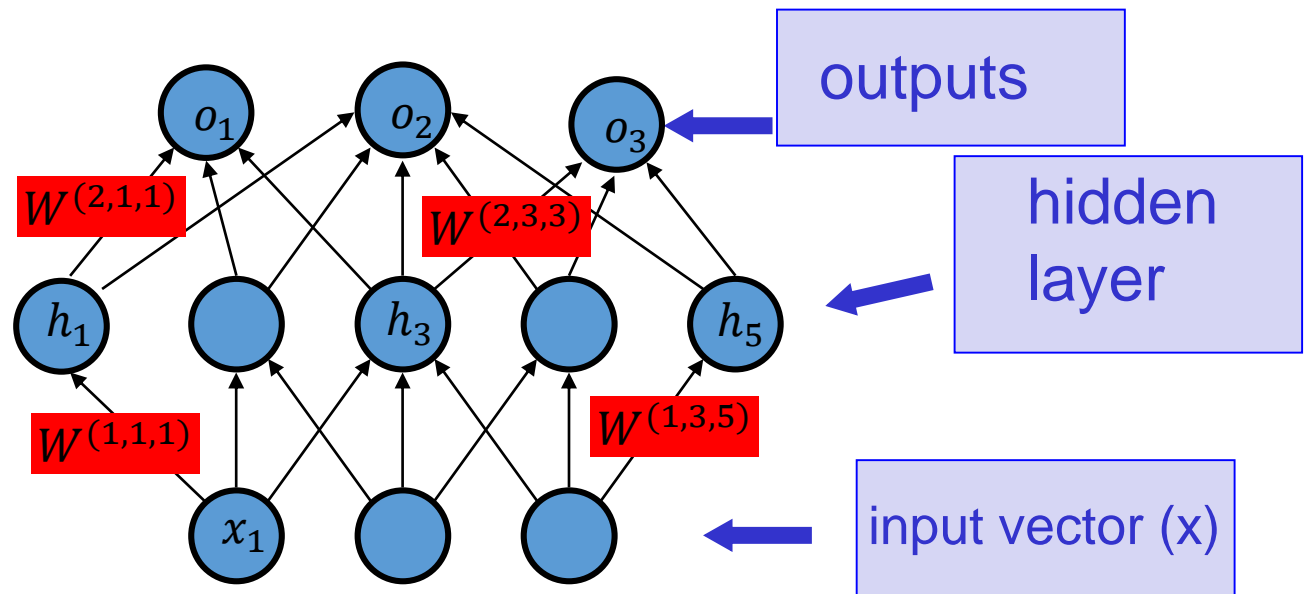
$$\frac{\partial}{\partial o_i} \sum_{i=1}^N (o_i - y_i)^2 = 2(o_i - y_i)$$

$$\begin{aligned}
\frac{\partial \mathcal{C}}{\partial W^{(2,j,i)}}(x, y, W, b, h, o) &= h_j \sigma' \left(\sum_j W^{(2,j,i)} h_j + b^{(2,j)} \right) \frac{\partial \mathcal{C}}{\partial o_i}(o, y) \\
&= 2h_j \sigma \left(\sum_j W^{(2,j,i)} h_j + b^{(2,j)} \right) \left(1 - \sigma \left(\sum_j W^{(2,j,i)} h_j + b^{(2,j)} \right) \right) (o_i - y_i)
\end{aligned}$$

Multivariate Chain Rule



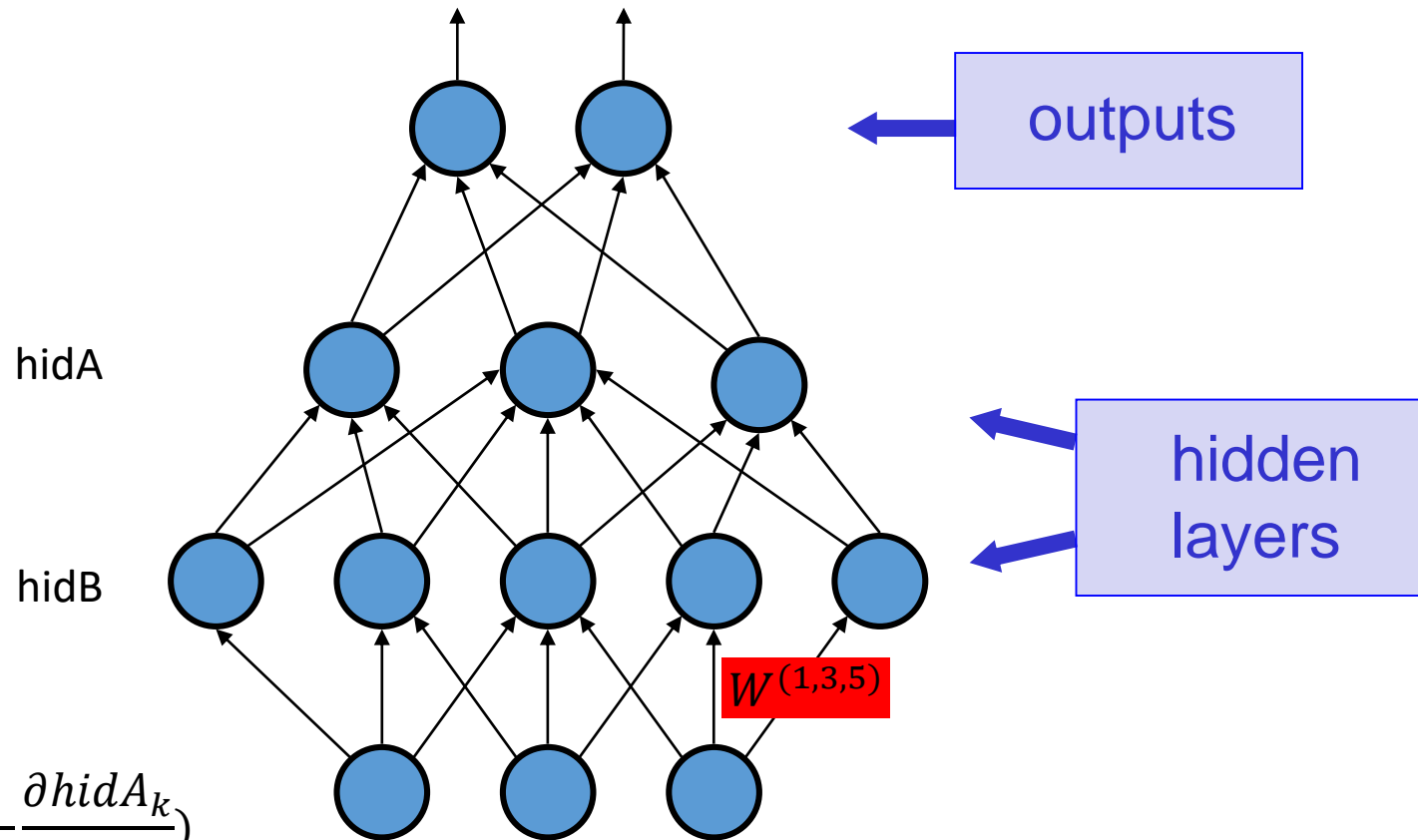
$$\frac{\partial g}{\partial x} = \sum \frac{\partial g}{\partial f_i} \frac{\partial f_i}{\partial x}$$



$$\frac{\partial C}{\partial h_i} = \sum_k \left(\frac{\partial C}{\partial o_k} \frac{\partial o_k}{\partial h_i} \right)$$

$$\frac{\partial C}{\partial W(1,j,i)} = \frac{\partial C}{\partial h_i} \frac{\partial h_i}{\partial W(1,j,i)}$$

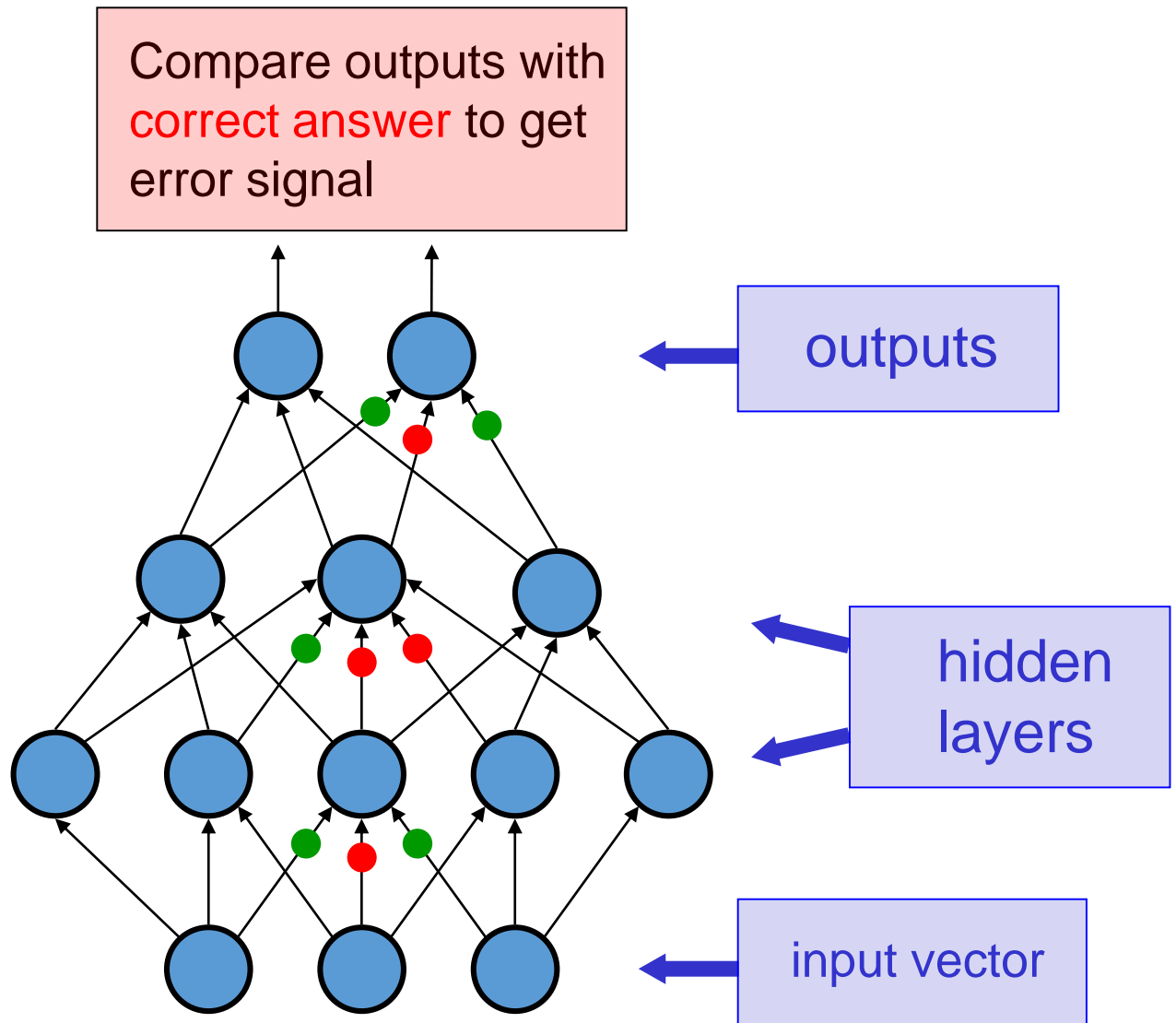
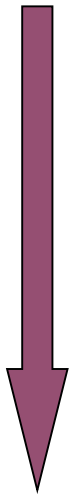
Backpropagation: dynamic programming



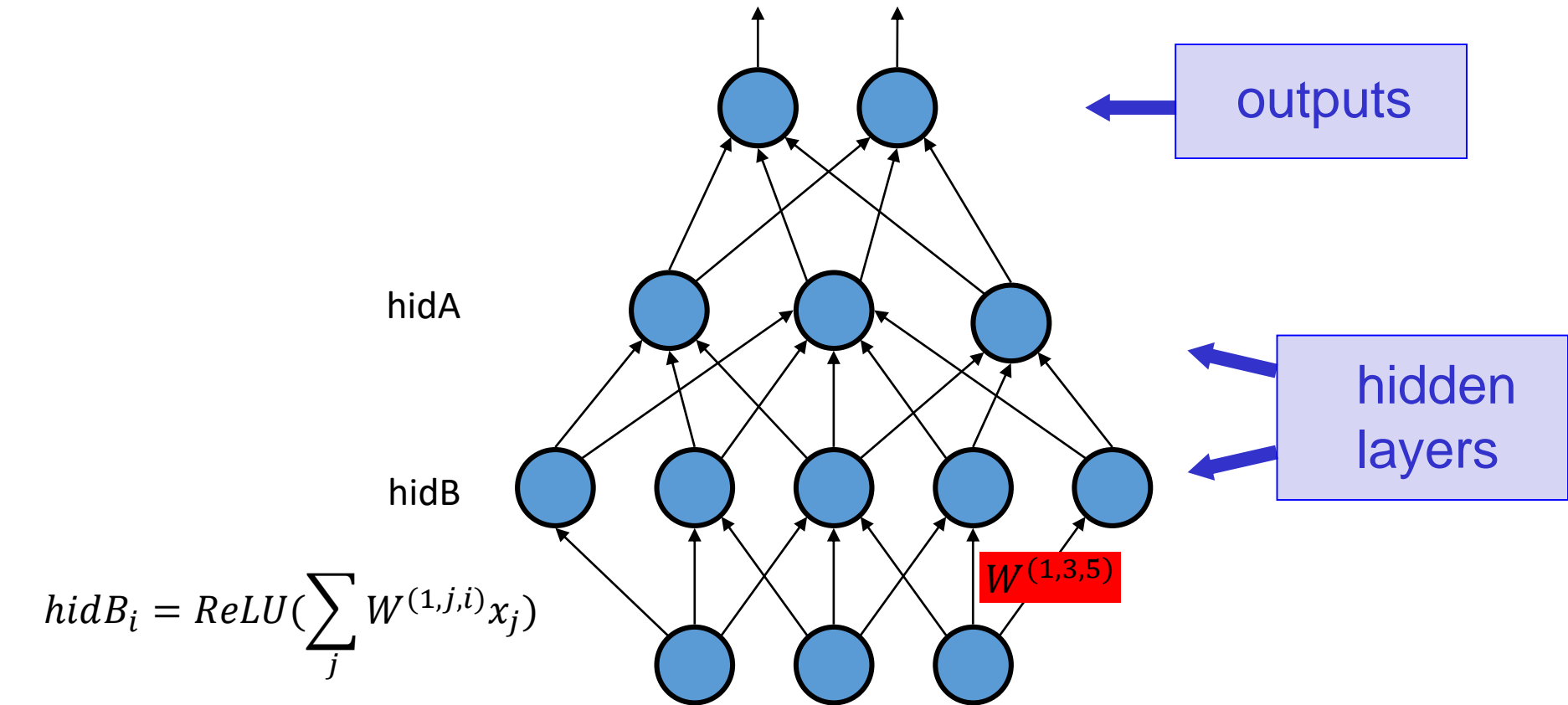
$$\frac{\partial C}{\partial \text{hid}B_i} = \sum_k \left(\frac{\partial C}{\partial \text{hid}A_k} \frac{\partial \text{hid}A_k}{\partial \text{hid}B_i} \right)$$

$$\frac{\partial C}{\partial W^{(1,j,i)}} = \frac{\partial C}{\partial \text{hid}B_i} \frac{\partial \text{hid}B_i}{\partial W^{(1,j,i)}}$$

Back-propagate
error signal to
get derivatives
for learning



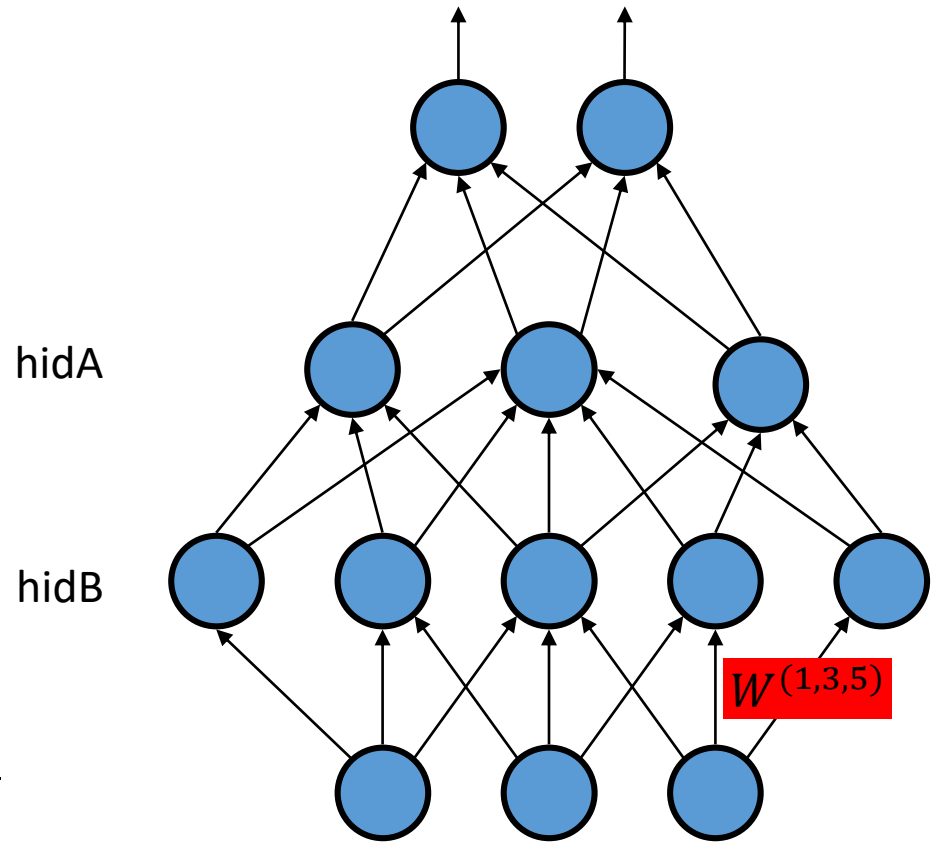
Another view of Guided Backpropagation



$$hidB_i = ReLU(\sum_j W^{(1,j,i)} x_j)$$

$$hidA_i = ReLU(\sum_j W^{(2,j,i)} hidB_j)$$

$$ReLU(z) = \begin{cases} z, & z > 0 \\ 0, & o/w \end{cases}$$



$$\frac{\partial Cost}{\partial hidB_i} = \sum_j \frac{\partial Cost}{\partial hidA_j} \frac{\partial hidA_j}{\partial hidB_i}$$

$$\frac{\partial hidA_j}{\partial hidB_i} = \frac{\partial ReLU(\sum_{j'} W^{(2,j',i)} hidB_{j'})}{\partial hidB_i} = \begin{cases} W^{(2,j,i)}, & \sum_{j'} W^{(2,j',i)} hidB_{j'} > 0 \\ 0, & o/w \end{cases}$$

Guided backprop: set $\frac{\partial hidA_j}{\partial hidB_i}$ to 0 if it's negative, and proceed with backprop computation