EE 101 Tutorial Assignment-1

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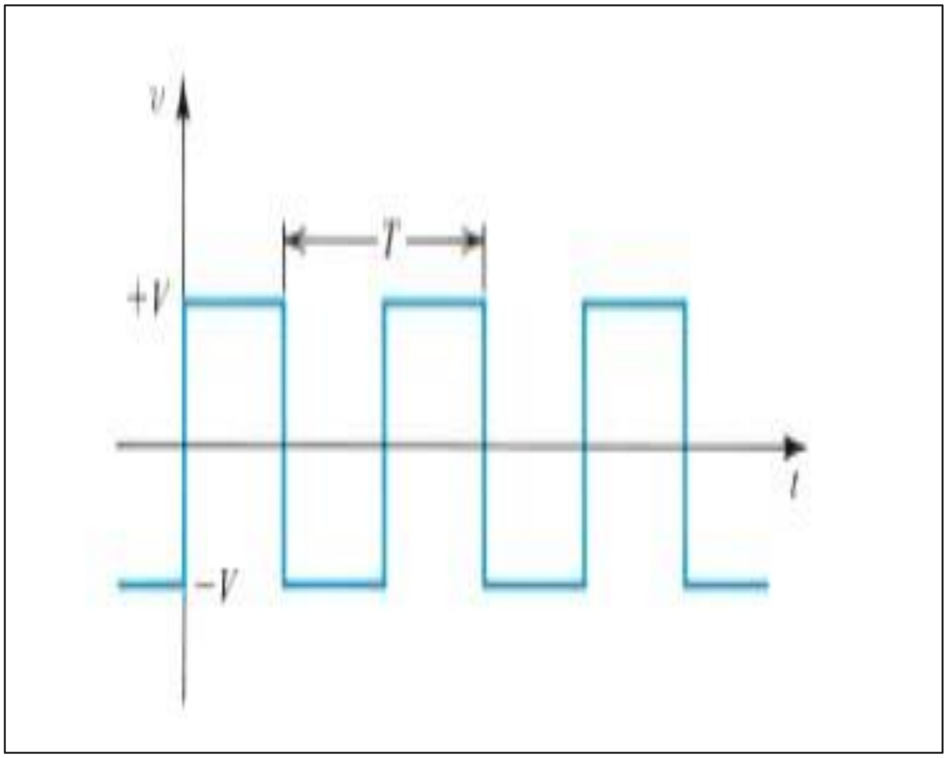
Question 1:



$$v(t) = V \quad (nT \le t \le nT + T/2)$$

$$v(t) = -V \quad (nT + T/2 \le t \le (n+1)T)$$

By fourier Transform

$$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{T} t\right) + b_n \sin\left(\frac{2\pi n}{T} t\right)$$

$$a_0 = \frac{1}{T} \int_0^T v(t) \overset{\cos}{\cancel{\sin}} \left(\frac{2\pi n}{T} t\right) dt$$

$$a_n = \frac{2}{T} \int_0^T v(t) \cos\left(\frac{2\pi n}{T} t\right) dt$$

$$b_n = \frac{2}{T} \int_0^T v(t) \sin\left(\frac{2\pi n}{T} t\right) dt$$

$$a_0 = \frac{1}{T} \int_0 v(t) \cos\left(\frac{2\pi n}{T} t\right) dt = \frac{1}{T}\left[\int_0^{T/2} V \cos \frac{2\pi n}{T} t \, dt - \int_{T/2} V \cos \frac{2\pi n t}{T} dt\right]$$

$$= \frac{V}{T} \times \frac{T}{2\pi n}\left[\left[\sin \frac{2\pi n t}{T}\right]_0^{T/2} - \left[\sin \frac{2\pi n}{T} t\right]_{T/2}^{T}\right]$$

$$= \frac{V}{2\pi}\left[\sin n\pi - \sin 0 - \sin 2n\pi + \sin n\pi\right] = 0$$

$$a_n = \frac{2}{T} \int_0^T v(t) \cos\left(\frac{2\pi n t}{T}\right) dt = \frac{2V}{T}\left[\int_0^{T/2} \cos \frac{2\pi n}{T} dt - \int_{T/2}^{T} \cos \frac{2\pi n}{T} dt\right]$$

$$= \frac{2V}{T} \times \frac{T}{2\pi n}\left[\sin n\pi - \sin 0 - \sin 2n\pi + \sin n\pi\right] = 0$$

$$b_n = \frac{2}{T} \int_0^T v(t) \sin\left(\frac{2\pi t}{T} n\right) dt = \frac{2V}{T} \times \frac{T}{2\pi n}\left[\int_0^{T/2} \sin \frac{2n\pi}{T} dt - \int_{T/2} \sin \frac{2\pi n}{T} dt\right]$$

$$= \frac{V}{n\pi} \times \left[-\cos n\pi + \cos 0 + \cos 2\pi n - \cos n\pi\right]$$

if $n = 2K$ then $b_n = \frac{V}{n\pi}\left[-1 + 1 + 1 - 1\right] = 0$

if $n = 2K+1$ then $b_n = \frac{V}{n\pi}\left[1 + 1 + 1 + 1\right] = \frac{4V}{n\pi}$

$$\therefore v(t) = \sum_{k=1}^{\infty} b_{2k+1} \sin \frac{(2k+1)2\pi}{T} t \quad \left[\text{and } b_{2k}, a_n = 0\right]$$

$$\therefore v(t) = \frac{4V}{\pi}\left(\sin \frac{2\pi}{T} t + \frac{1}{3}\sin \frac{2\pi}{T} t \times 3 + \frac{1}{5}\sin \frac{2\pi}{T} t \times 5 \cdots\right)$$

Let $\frac{2\pi}{T} = \omega_0 \quad \therefore \omega_0 = \frac{2\pi}{2\pi/10} = 10$

$$\therefore v(t) = \frac{4V}{\pi}\left(\sin \omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + \frac{1}{7}\sin 7\omega_0 t \cdots\right)$$

Fundamental frequency $= \omega_0$ (First harmonic)

Fourth ~~Third~~ Second ~~First~~ harmonic has 0 amplitude

Third ~~Second~~ harmonic has $\frac{4V}{3\pi}$ Amplitude

and Fifth ~~Fourth~~ harmonic has $\frac{4V}{5\pi}$ Amplitude

a) Identify the first five harmonics of the square wave $v(t)$ expressed in the Fig 1. Consider V = +2 Volt, T = 628 msec

a) $\therefore$ First five harmonics are

$$\frac{4V}{\pi}\left(\sin \omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t\right)$$

b)What fraction of energy of $v(t)$ in its fundamental

(b) We Know $E \propto \int_0^T V^2 dt$ (sine Waves are periodic with Time Period)

Let nth harmonic (if n is odd) $= \frac{4V}{n\pi} \sin(n\omega_0 t)$

$$E_n = \frac{16V^2}{n^2\pi^2} \int_0^T \sin^2(n\omega_0 t)\, dt$$

$$\frac{16V^2}{n^2\pi^2} \times \frac{1}{2} \int_0^T 1 - \cos 2n\omega_0 t \; dt$$

$$= \frac{8V^2}{n^2\pi^2}\left(T - \frac{\sin 2n\omega_0 T}{2n\omega_0}\right) \quad \left[\text{Since } \omega_0 T = 2\pi \;\; \sin 2n\omega_0 T = 0\right]$$

$$= \frac{8V^2}{n^2\pi^2} \times T$$

$$\therefore E_n \propto \frac{1}{n^2} \Rightarrow E_n = \frac{E_0}{n^2}$$

fraction in fundamental $\left.\right\} = \frac{E_1}{E_1 + E_3 + E_5 + \cdots} = \frac{E_0}{E_0 + \frac{E_0}{9} + \frac{E_0}{25} + \frac{E_0}{49} \cdots} = \frac{1}{1 + \frac{1}{9} + \frac{1}{25} + \cdots} = \frac{1}{\pi^2/8} = \frac{8}{\pi^2} \approx 0.81$

c) What fraction of energy of $v(t)$ in its first five harmonics?

(c) Energy Possessed in first five harmonics

$$= \frac{E_0 + \frac{E_0}{9} + \frac{E_0}{25}}{E_0 + \frac{E_0}{9} + \frac{E_0}{25} + \frac{E_0}{49} + \cdots} = \frac{1 + \frac{1}{9} + \frac{1}{25}}{\pi^2/8} = 0.81\left(1 + \frac{1}{9} + \frac{1}{25}\right)$$

$$= 0.81 + 0.09 + 0.032$$

$$= 0.932$$

d) Till what number of harmonics is 90% of the energy?

(d) First harmonic has 81% of total energy

Second harmonic has 0% of total energy

Third harmonic contributes to 9% of total energy

$\therefore$ Number of harmonics which contributes to 90% energy = 3

QUESTION 2

a)Find the time period. and the cyclic and radian frequencies for each of the following sinusoids

$v1\ (t\ )$= 17 cos(2000t − 30 °) $v2\ (t)$ = 12 cos(2000t + 30 °)

$v_1(t) = 17\cos(2000t - 30^\circ)$

It is of the form $A\cos(\omega t + \phi)$ where $\omega = 2000\ \text{rad/s}$

We Know Time period is $\frac{2\pi}{\omega} = \frac{2\pi}{2000} = \frac{\pi}{1000} = 3.14$ msec

Radian frequency $= \omega = 2000$ rad/s

Cyclic frequency $= \frac{\omega}{2\pi} = \frac{2000}{2\pi} \approx \frac{1000}{3.14} \approx 318.3$ Hz

$v_2(t) = 12\cos(2000t - 30^\circ)$

Radian frequency $(\omega) = 2000\ \text{rad}\ s^{-1}$

Cyclic frequency $(f) = \frac{\omega}{2\pi} = \frac{2000}{2\pi} \approx 318.3$ Hz

TimePeriod $(T) = 2\pi$ $= 3.14$msec

b) Derive the expression and sketch the graph of $v3\ (t)$ = $v1\ (t)$ + $v2$(t)

(b) $v_3(t) = v_1(t) + v_2(t)$

$= 17\cos(2000t - 30^\circ) + 12\cos(2000t + 30^\circ)$

$= 17(\cos 2000t \cos 30 + \sin 2000t \sin 30) + 12(\cos 2000t \cos 30 - \sin 2000t \sin 30t)$

$= 29\cos 2000t \cos 30 + 5\sin 2000t \sin 30$

$= \frac{29\sqrt{3}}{2}\cos 2000t + \frac{5}{2}\sin 2000t$

Let $\cot\phi = \left(\frac{29\sqrt{3}}{5}\right)$ $\Rightarrow \phi = \cot^{-1}\left(\frac{29\sqrt{3}}{5}\right) = 5.69^\circ$

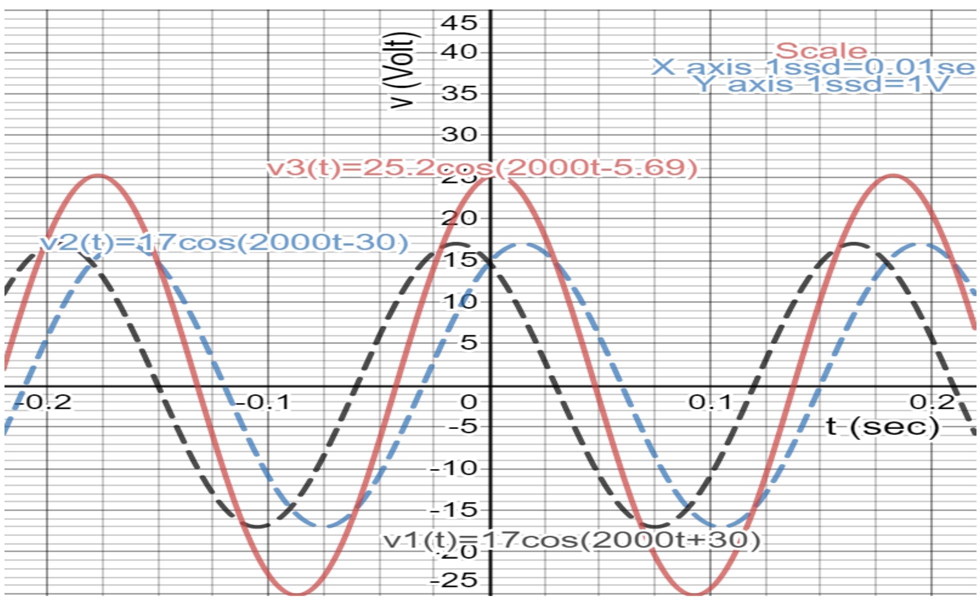
$$\text{Let}(v_3 t) = A\cos\phi\cos(2000t) + A\sin\phi\sin 2000t$$

$$= A \qquad \text{We Know } \phi = 5.69^\circ$$

$$A\cos\phi = \frac{29\sqrt{3}}{2}$$

$$A = \frac{29\sqrt{3}}{2\cos(5.69)} = 25.2$$

$$\therefore v_3 t = 25.2\left(\cos(2000t - \phi)\right) = 25.2\cos(2000t - 5.69)$$



Plot of v3(t) versus time

Question 3

Graphically sketch the waveform described by $v\ t = r\ (t\ )/Tc\ VA$ $e^{-t/T_c}\ u(\ t)\ Volt$ [Graph Paper Preferred] VA= 1 $Volt$, Tc = 1 sec

$$v(t) = \frac{r(t)}{T_C}\left[V_A\, e^{-t/T_C}\right] u(t)$$

We know $r(t) = t$ for $t>0$

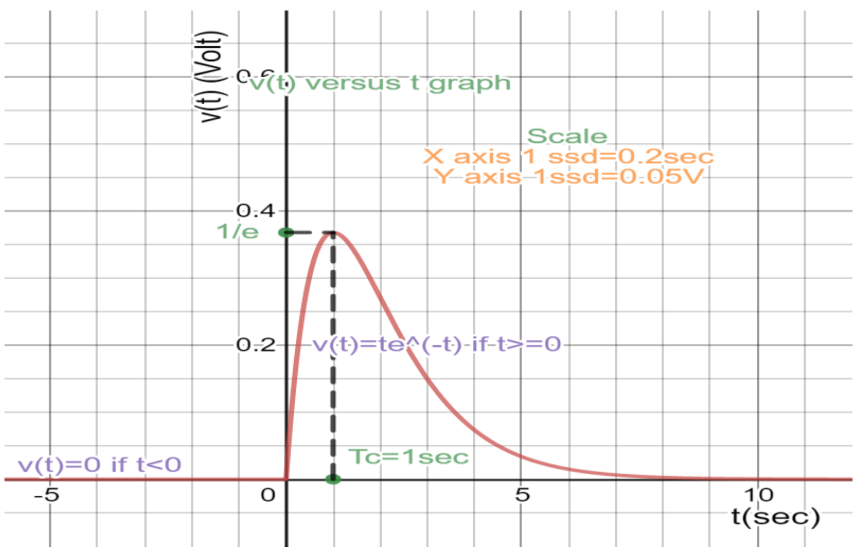
$r(t) = 0$ for $t<0$

$u(t) = 1$ for $t>0$

$u(t) = 0$ for $t<0$

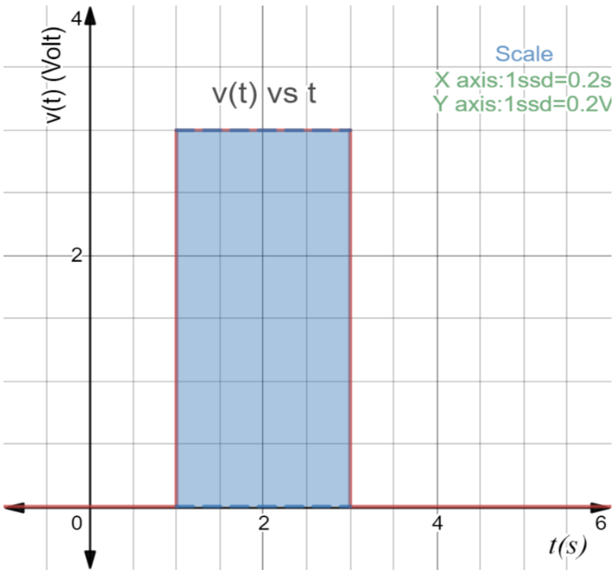
$\therefore v(t) = \frac{t}{T_C} V_A\, e^{-t/T_C}$ Volt for $t>0$ $= t e^{-t}$ [$\because$ V=1Volt Tc=1sec]

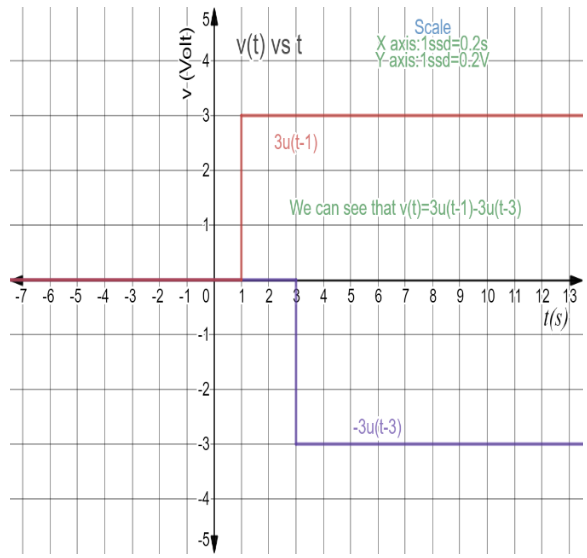
$= 0$ for $t<0$



Plot of V(t) versus time(t)

Question 4





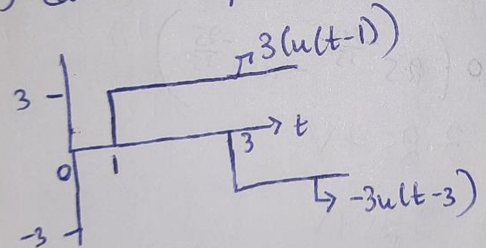
a)Express the Gated Pulse Waveform of Fig.2 in terms of Step functions u(t).

$v(t) = 0$ for $t < 1$

$= 3$ for $1 \leq t \leq 3$

$= 0$ for $t > 3$

v(t) can be expressed as sum of two waves.



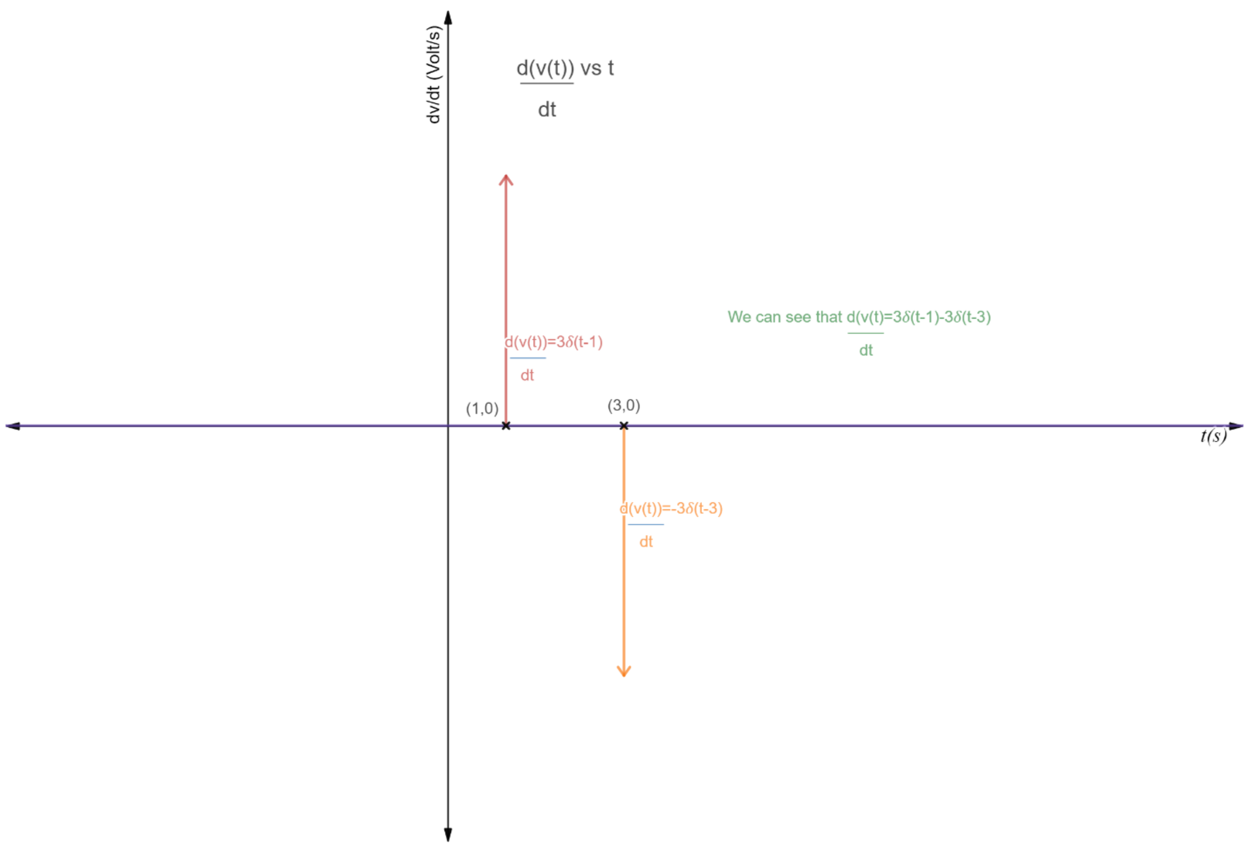
If $t > 3$ those two waves cancel each other

$\therefore v(t) = 3u(t-1) - 3u(t-3)$

b) Determine the expression and graphically sketch the derivative of the gated pulse waveform shown in Fig.2

(b) $\frac{\partial}{\partial t} v(t) = \frac{\partial}{\partial r} 3u(t-1) - 3u(t-3)$

$= 3\delta(t-1) - 3\delta(t-3)$



Plot of d(v(t))/dt versus time

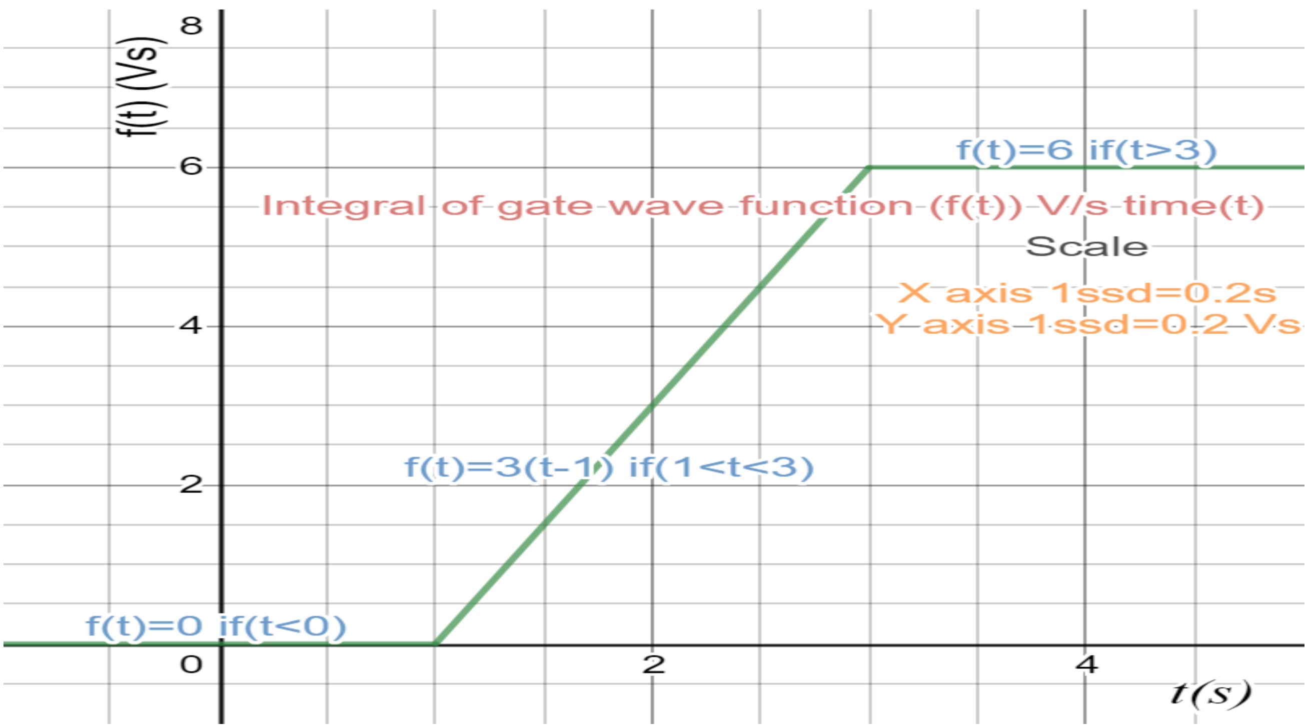
c) Determine the expression and graphically sketch the integral of the gated pulse waveform shown in Fig.2

(c) $v(t) = 3u(t-1) - 3u(t-3)$

$$\int_{-\infty}^{t} v(t)\,dt = 3\int_{-\infty}^{t} (u(t-1)dt) - u(t-3)\,dt$$

$$= 3[r(t-1) - r(t-3)]$$

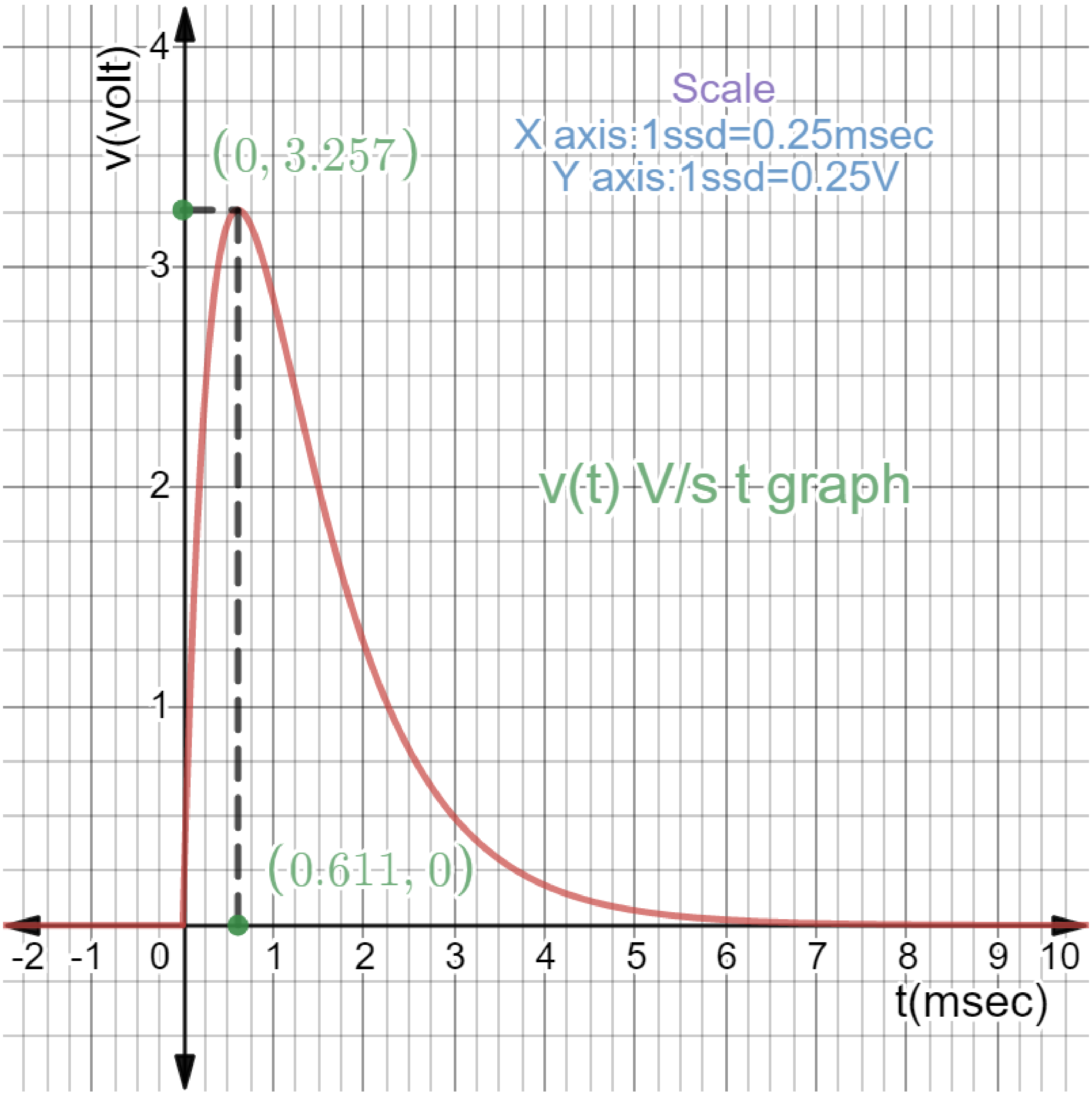
where $r(t) = \int_{-\infty}^{t} u(t)\,dt$



Plot of integral of v(t) versus time

Question 5

a)Plot $v\,(t)$ expressed by $v\,(t)$ = 10 $[e^{-1000t} - e^{-2500t}]u(t)$



b) What is the value of $v\,(t)$ at the extremum and the time when it occurs?

$$v(t) = 10\left(e^{-1000t} - e^{-2500t}\right)u(t)$$

at ~~local~~ global extrema either $\frac{\delta v(t)}{\delta t} = 0$ (or) $v(t)$ is discontinuous

$$v(t) = 10\left(e^{-1000t} - e^{-2500t}\right) \quad t \geq 0$$

$$v(t) = 0 \quad t < 0$$

$$\frac{\partial v(t)}{\partial t} = 10\left(2.5 e^{-2500t} - e^{-1000t}\right)$$

Maximum occurs when $e^{-2500t} = \frac{1}{2.5} e^{-1000t}$

$$e^{1500t} = 2.5$$

$$t = \frac{\ln 2.5}{1500} \approx 0.000611 \text{ sec} = 6.1 \times 10^{-4} \text{ sec} \approx 0.61 \text{ msec}$$

and Maximum value $= 10\left(e^{\frac{-2500 \ln 2.5}{1500}} + e^{\frac{-1000 \ln 2.5}{1500}}\right)$

$$= 10\left(2.5^{-\frac{10}{15}} - 2.5^{-\frac{25}{15}}\right)$$

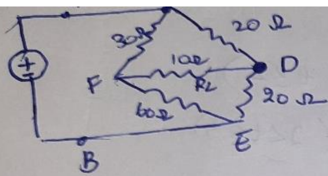
$$\simeq 3.257\,V$$

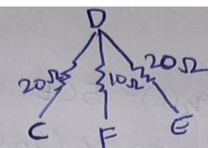
$\therefore$ Maximum value of $v(t) = 3.257\,V$

and it occurs at 0.61 msec

Question 6

Find the equivalent resistance seen from terminal A-B using Y - Δ transformation in Fig. 3 $R_L = 10\ \Omega$





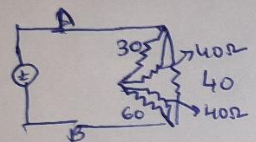
this in Starform we can convert this into Δ form

$$R'_{CD} = \frac{R_{CD}R_{ED} + R_{ED}R_{FD} + R_{FD}R_{CD}}{R_{ED}}$$

$$= \frac{20 \times 20 + 20 \times 10 + 20 \times 10}{20} = 40\ \Omega$$

$$R'_{ED} = \frac{800}{R_{ED}} = \frac{800}{20} = 40\ \Omega \text{ and } R'_{FD} = \frac{800}{R_{FD}} = \frac{800}{10} = 80\ \Omega$$

∴ Equivalent Circuit between A and B can be represented as



Between C and F we have two Resistors R_{CF} and R'_{ED} in parallel

$$R_{CFeff} = \frac{1}{\frac{1}{30}+\frac{1}{40}} = \frac{40\times30}{70} = \frac{120}{7}\ \Omega$$

Between F and E there are two Resistors R_{EF} and R'_{CD} in parallel

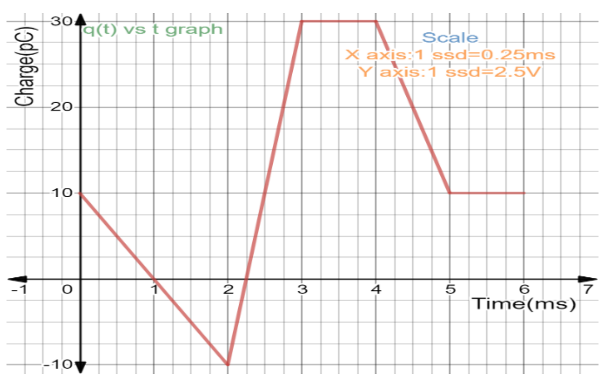
$$R_{EFeffective} = \frac{1}{\frac{1}{60}+\frac{1}{40}} = \frac{60\times40}{100} = 24\Omega$$

$$R_{AB} = R_{CE} = \frac{1}{\frac{1}{80}+\frac{1}{24+\frac{120}{7}}} = \frac{1}{\frac{1}{80}+\frac{7}{288}} = \frac{288\times80}{288+560} = \frac{288\times80}{848} = 27.169\ \Omega$$

$$\therefore R_{AB} = 27.169\Omega$$

QUESTION 7

The graph shows the charge q(t) flowing past a point in a wire as a function of time



(a) $i(t) = \frac{dq(t)}{dt}$

$q(0<t\le2) = 10-10t \Rightarrow i(0<t\le2) = -10\ pC/ms = -10nA$

$q(2<t\le3) = 40t-90 \Rightarrow i(2<t\le3) = 40nA$

$q(3<t\le4) = 30 \Rightarrow i(3<t\le4) = 0nA$

$q(4<t\le5) = 110-20t \Rightarrow i(4<t\le5) = -20nA$

$q(5<t\le6) = 10 \Rightarrow i(5<t\le6) = 0nA$

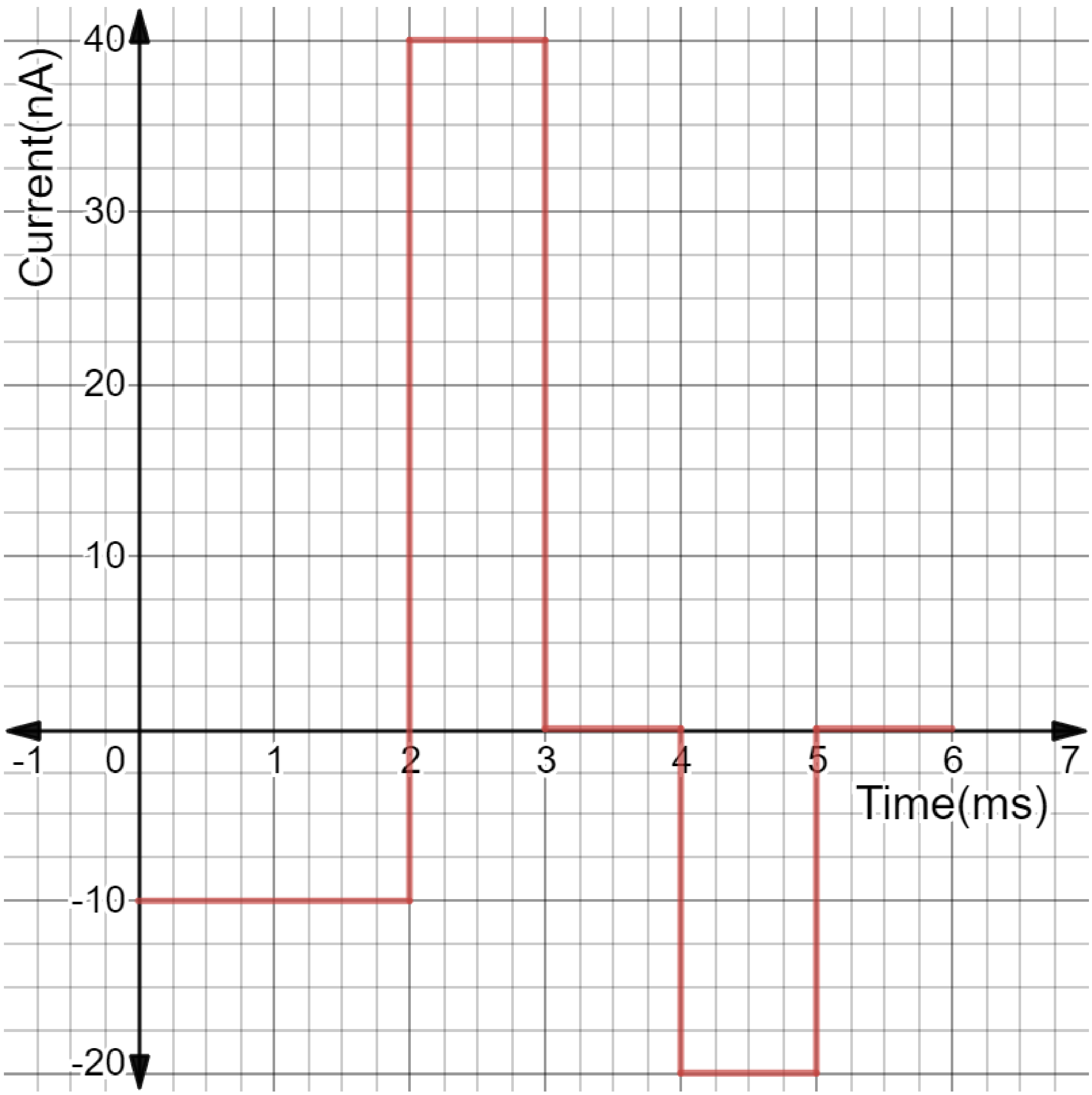
∴ The graph can be plotted now using these values a)Sketch the variation of i(t) versus time

Plot of i(t) versus time(t)

Scale:

X-axis:1ssd=0.25ms

Y-axis:1ssd=2.5microampere



b) Find the current i(t) at 1; 2.5; 3.5; 4.5, and 5.5 msec

(b) $\therefore\ i(t=1\text{ms}) = -10\text{nA} \Rightarrow |i(t)| = 10\text{nA}$

$i(t=2.5\text{ms}) = 40\text{nA} \Rightarrow |i(t)| = 40\text{nA}$

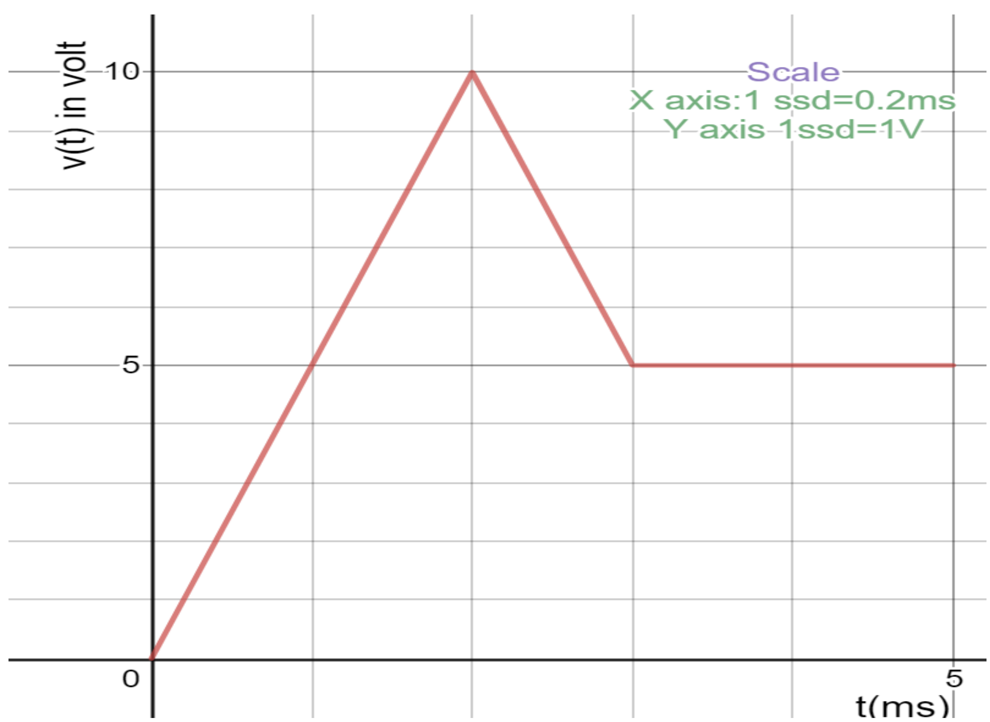
$i(t=3.5\text{ms}) = 0\text{nA} \Rightarrow |i(t)| = 0\text{nA}$

$i(t=4.5\text{ms}) = -20\text{nA} \Rightarrow |i(t)| = 20\text{nA}$

$i(t=5.5\text{ns}) = 0\text{nA} \Rightarrow |i(t)| = 0\text{nA}$

Question 8

Below Figure shows the voltage across a 0.5-µF capacitor. Determine the time varying current, energy and power of the capacitor



(8) $v_c(t) = 5t \quad (0 \le t < 2) \Rightarrow q_c(t) = 5Ct \quad (0 \le t \le 2)$

$v_c(t) = 20-5t \quad (2 \le t < 3) \Rightarrow q_c(t) = C(20-5t) \quad (2 < t \le 3)$

$v_c(t) = 5 \quad (t > 3 \Rightarrow q_c(t) = 5C \quad (t > 3)$

$$i_c = \frac{dq_c(t)}{dt}$$

$= 5C \quad (0 \le t < 2)$

$= -5C \quad (2 < t < 3)$

$= 0 \quad (t > 3)$

Energy possessed by capacitor $= \frac{1}{2} Cv^2$

$E_c(t) = \frac{1}{2} C(5t)^2 \quad (0 \le t < 2)$

$= \frac{1}{2} C(20-5t)^2 \quad (2 < t < 3)$

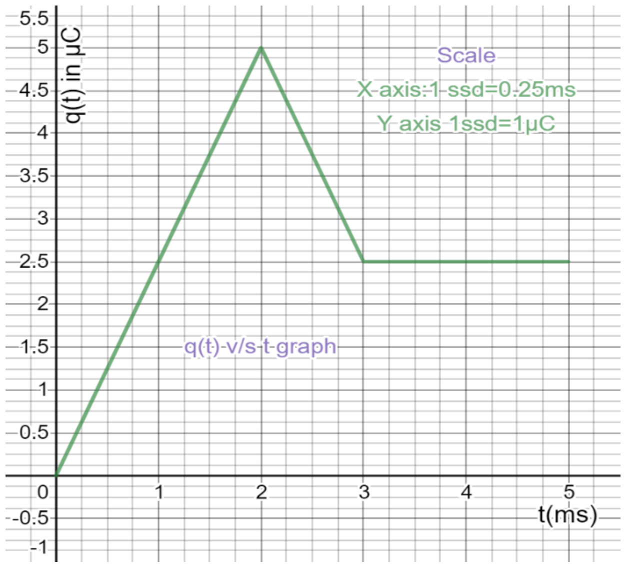
$= \frac{1}{2} C(5)^2 \quad (3 < t)$

$$P_c(t) = \frac{d(E_c)}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C}\right) = \frac{qi}{C}$$

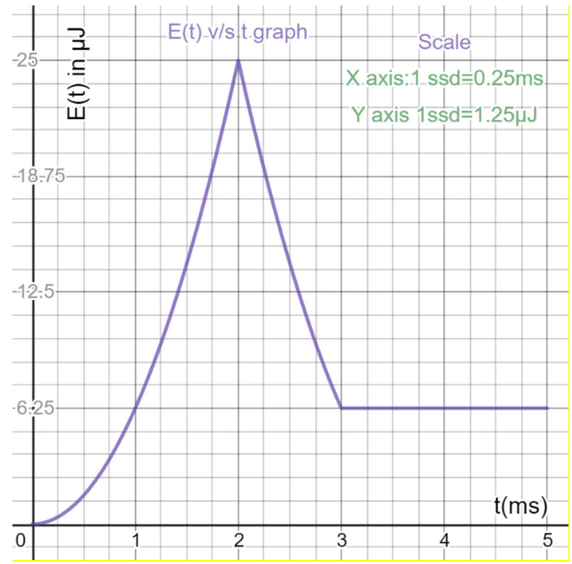
$\therefore P_c(t) = 5 \times 5Ct \quad 0 \le t < 2$

$5C(20-5t) \quad 2 < t < 3$

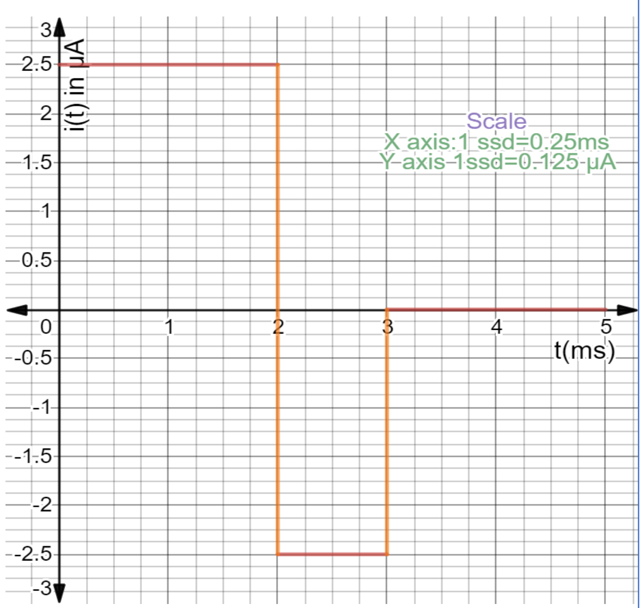
$5 \times 5C \times 0 = 0 \quad 3 < t$



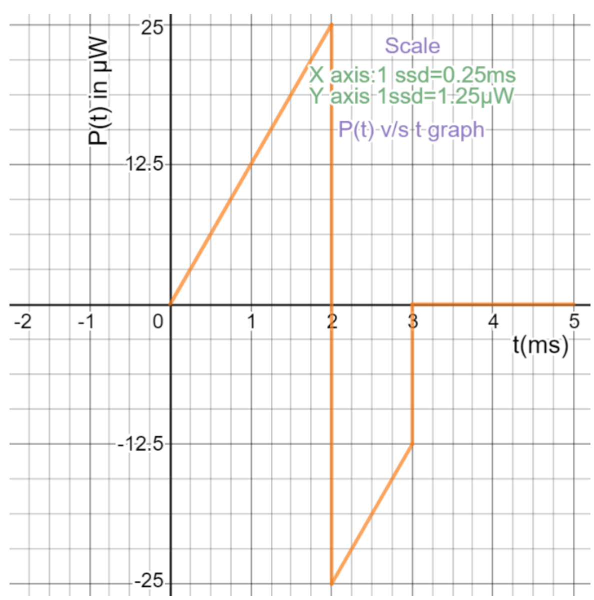
Plot of q(t) versus t



Plot of E(t) versus t



Plot of i(t) versus t



Plot of P(t) versus t

Question 9

The current through a 2.5-mH inductor $i(t)$ = 10 e^{-500t} sin(2000t). Plot the waveforms of the element current, voltage, power, and energy.

(9)

$$i(t) = 10\,e^{-500t}\sin(2000t)$$

$$L = 2.5\,mH$$

$$V = -L\frac{di}{dt}$$

$$= -2.5\times10^{-3}\times\left(\frac{d}{dt}\left(10e^{-500t}\sin 2000t\right)\right)$$

$$= -2.5\times10^{-2}\left(e^{-500t}\times 2000\cos 2000t - 500\,e^{-500t}\sin 2000t\right)$$

$$= -2.5\times10^{-2}\times 500\,e^{-500t}\left(4\cos 2000t - \sin 2000t\right)V$$

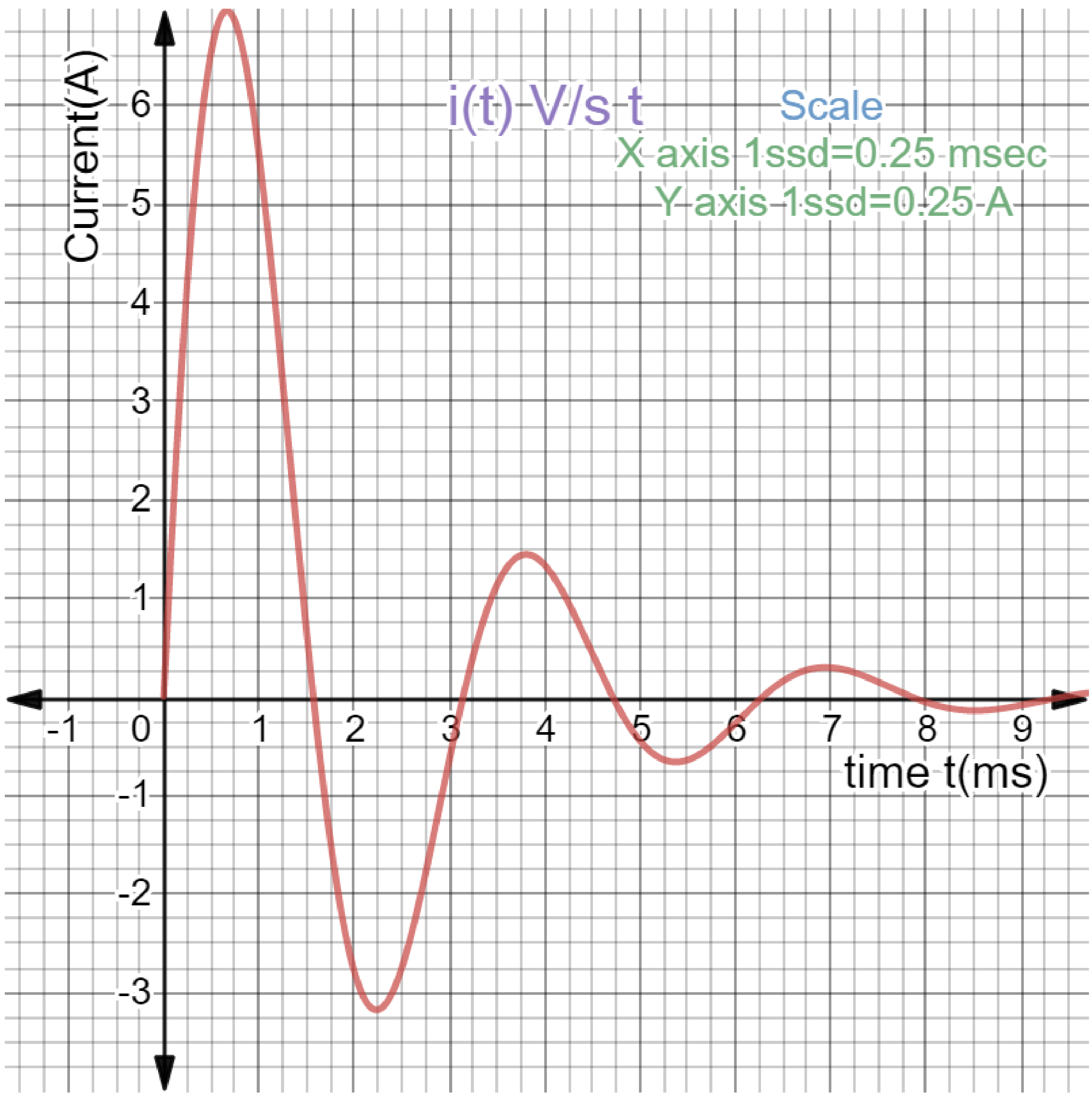
$$= -12.5\;e^{-500t}\left(4\cos 2000t - \sin 2000t\right)$$

$$\text{Power}(P) = -Li\frac{di}{dt}$$

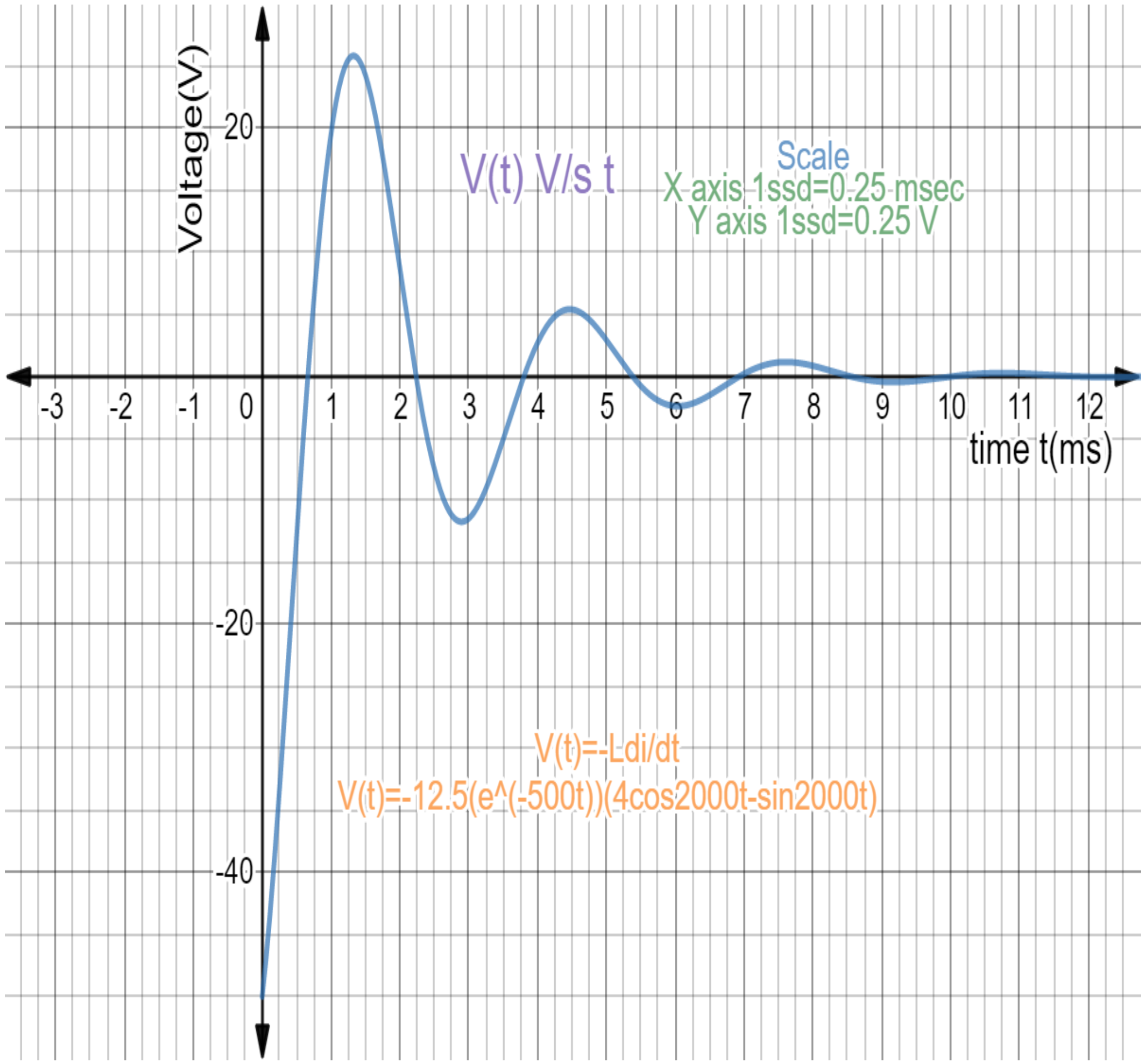
$$= 2.5\times10^{-3}\times 10\,e^{-500t}\sin(2000t)\,e^{-500t}(4\cos 2000t - \sin 2000t)12.5$$

$$E = \frac{1}{2}Li^2$$

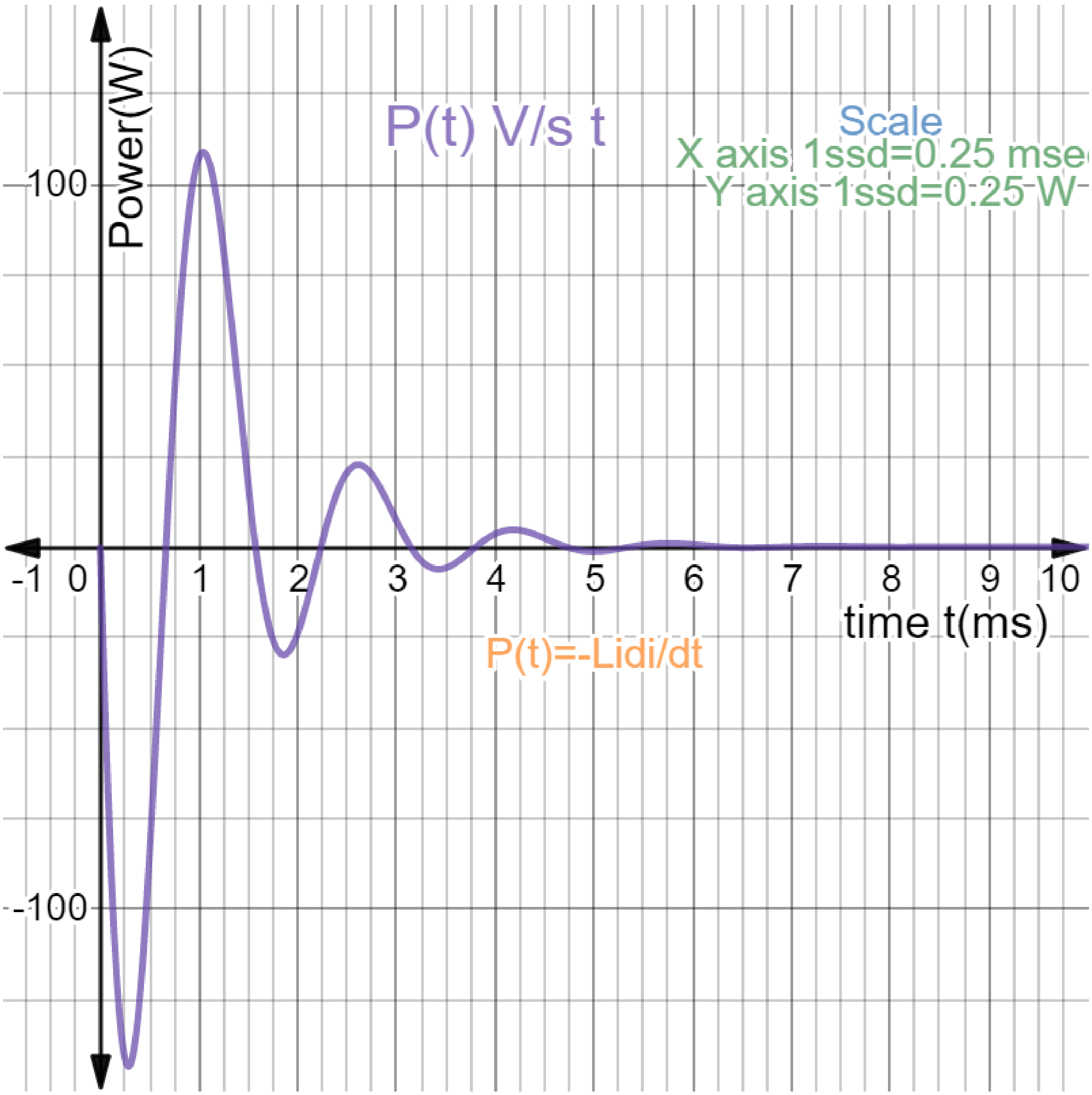
$$= \frac{1}{2}\times 2.5\times10^{-3}\times\left(10\,e^{-500t}\sin 2000t\right)^2$$



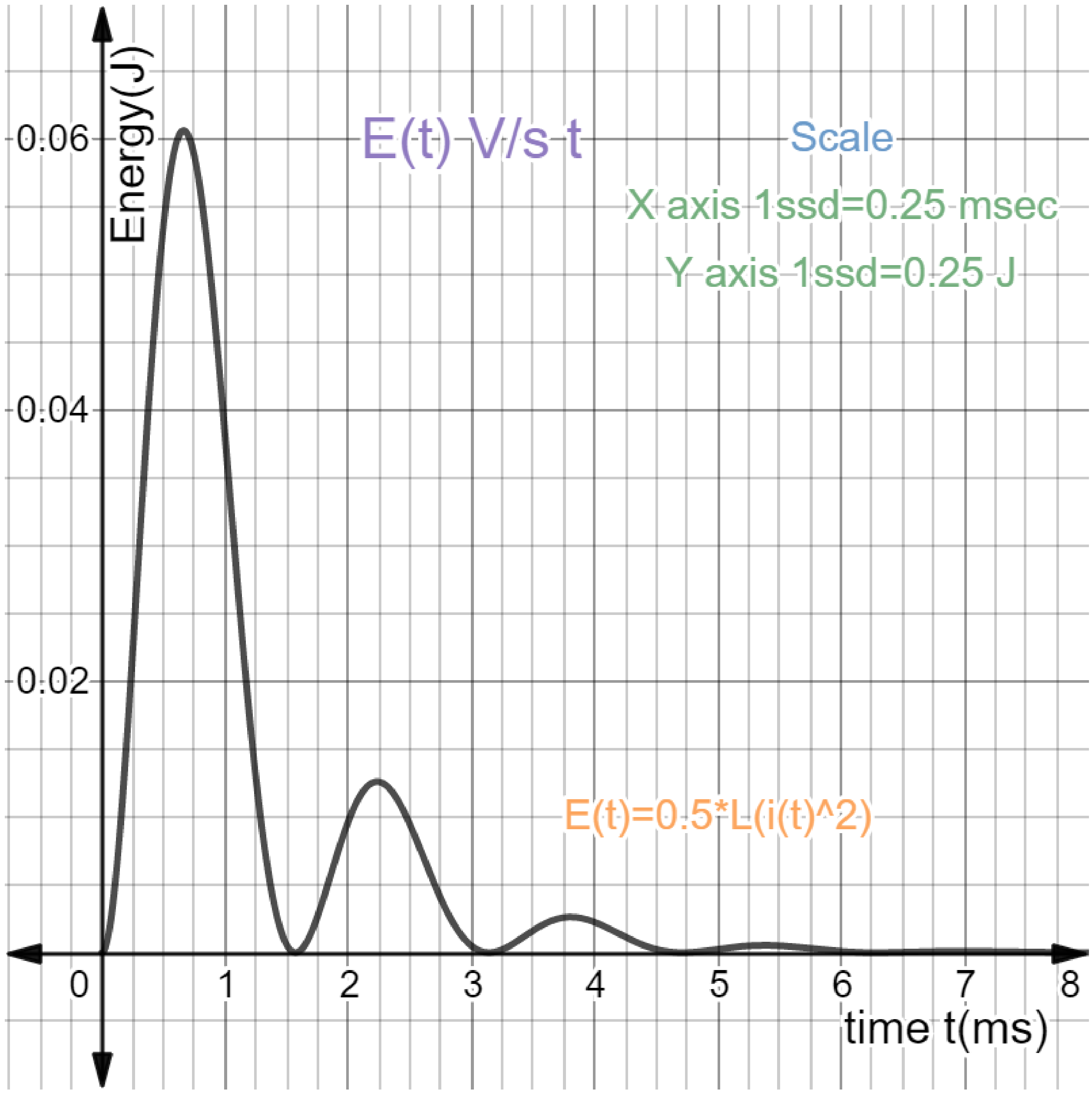
Plot of i(t) versus t



Plot of V(t) versus t



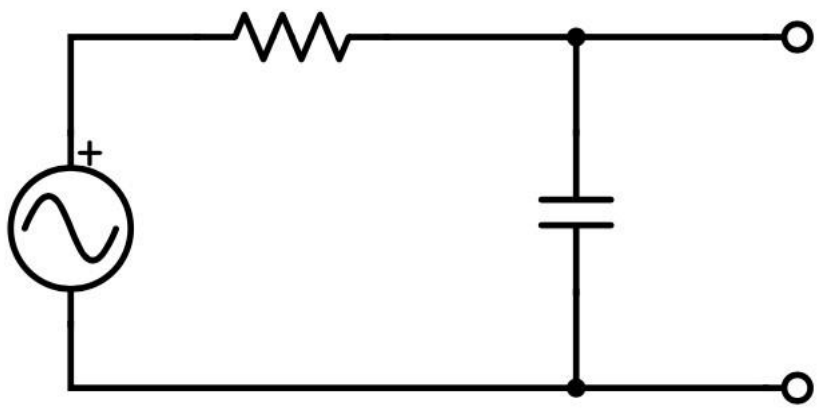
Plot of P(t) versus t



Plot of E(t) versus t

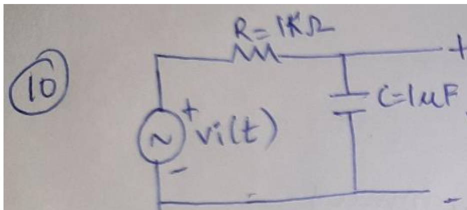
Question 10

For the first-order low pass filter as shown below,

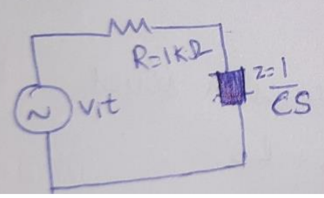


a) Draw s-domain transformed circuit for the filter.

(10)



(a) In frequency domain C is replaced by impedance of $\frac{1}{sC}$ where $s=j\omega$



$$Z_{eff} = Z_R + Z_C = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$$

b) Find transfer function in s-domain i.e $T(s) = Vo(s)/Vi(s)$.

(b) $$T(s) = \frac{Vo(t)}{Vi(t)} = \frac{Z_C}{Z_C+Z_R} = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}} = \frac{1}{RCs+1}$$

C) Hence find transfer function for physical frequencies i.e $T\ (j\omega)$ by $s = j\omega$ and derive expression for magnitude response, $|T\ j\omega\ |$

(c) $T(jw) = \dfrac{1}{Rcjw+1}$ $(s=jw)$

$|T(jw)| = \dfrac{1}{\sqrt{1+(RC)^2w^2}}$ $w_0 = \dfrac{1}{RC}$

$|T(jw)| = \dfrac{1}{\sqrt{1+(w/w_0)^2}}$

d) Also calculate the 3-dB frequency or corner frequency, ωo for the filter

(d) cutoff frequency $f_c = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi\ 10^3 \times 10^{-6}} = \dfrac{10^3}{2\pi}$ H

$w_c = 2\pi f_c = 10^3$ rads^{-1}

e) Find the transmission or gain at $\omega/\ \omega o$ = 0.1, $\omega/\ \omega o$ = 1 and at $\omega/\ \omega o$ = 10

(e) Transmission $= \dfrac{1}{\sqrt{1+\left(\frac{w}{w_0}\right)^2}}$

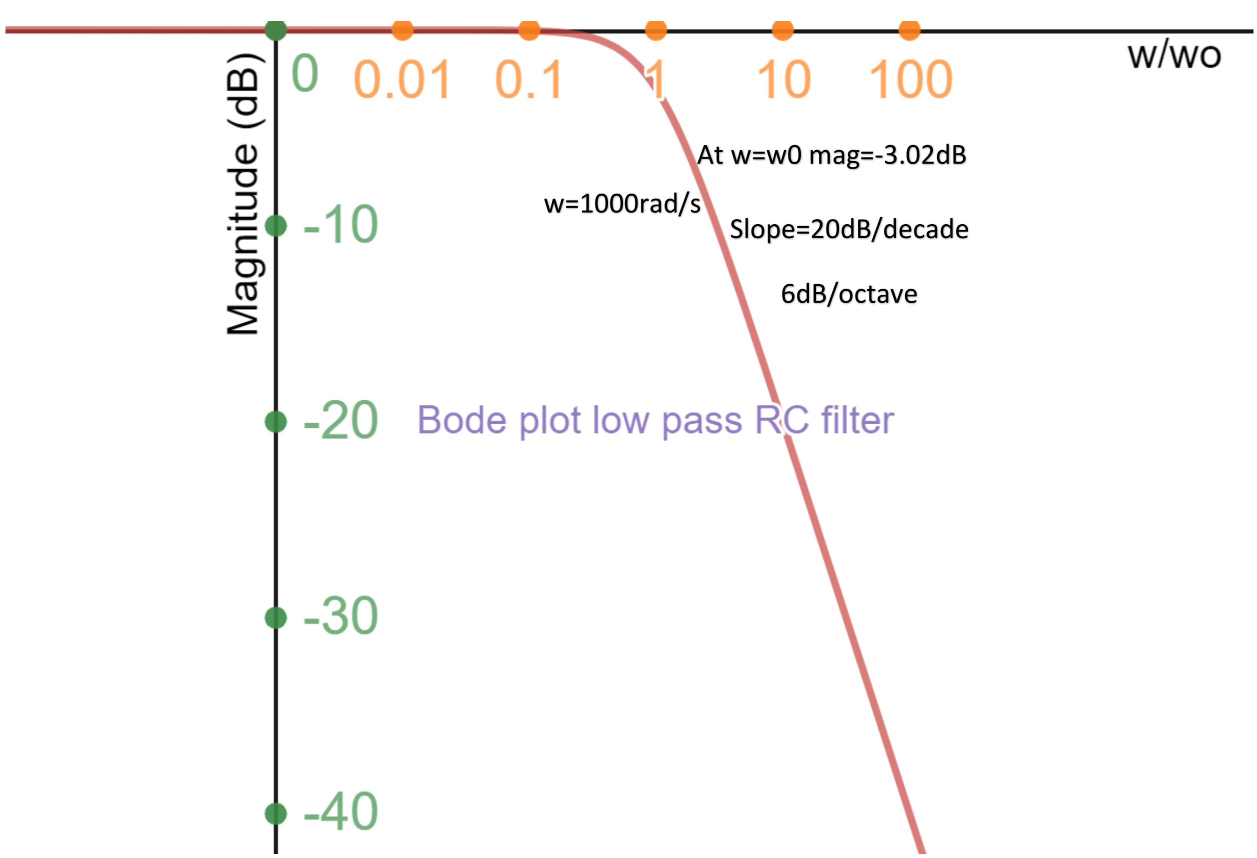
$\dfrac{w}{w_0} = 0.1 \Rightarrow T = \dfrac{1}{\sqrt{1+0.01}} = 0.9901$

$\dfrac{w}{\ } = 1 \Rightarrow T = \dfrac{1}{\sqrt{2}} = 0.707$

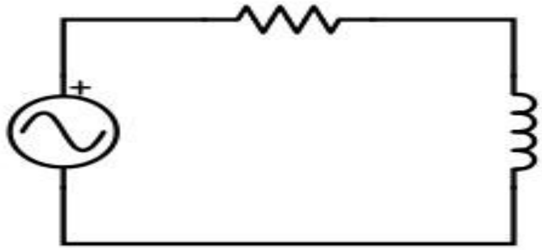
$\dfrac{w}{w_0} = 10 \Rightarrow T = \dfrac{1}{\sqrt{1+100}} = 0.099$

f)Plot magnitude response, $|T\ (j\omega)\ |$ vs. $\omega\ /\omega o$ Assume that the capacitor is at a zero state initially

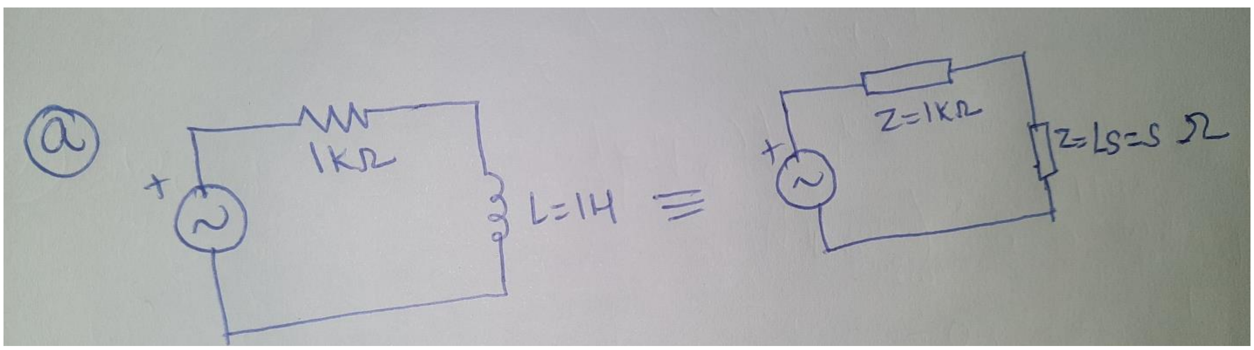
$VC(\ t$ = 0)= V (t = 0) = 0



Question 11



a)Draw s-domain transformed circuit for the filter.



b) Find transfer function in s-domain i.e $T(s) = Vo(s)/ Vi(s)$.

(b) $$T(s) = \frac{V_o}{V_i} = \frac{V_L}{V_L + V_R} = \frac{Z_L}{Z_L + Z_R} = \frac{Ls}{R + Ls}$$

c) Hence find transfer function for physical frequencies i.e $T\ j\omega$ by $s = j\omega$ and derive expression for magnitude response

(c) $$T(j\omega) = \frac{Lj\omega}{R + Lj\omega} \Rightarrow |T(j\omega)| = \frac{|Lj\omega|}{|R + Lj\omega|} = \frac{L\omega}{\sqrt{R^2 + L^2\omega^2}}$$

d) Also calculate the 3-dB frequency or corner frequency ωo for the filter.

(d)

$$|Tj\omega| = \frac{|L\omega|}{\sqrt{R^2+L^2\omega^2}}$$

$$20\log |Tj\omega|_{dB} = -3dB$$

$$\Rightarrow |Tj\omega| = \frac{1}{\sqrt{2}}$$

$$\frac{|L\omega|}{\sqrt{R^2+L^2\omega^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 2L^2\omega^2 = R^2+L^2\omega^2$$

$$\Rightarrow R^2 = L^2\omega^2 \Rightarrow \omega = \sqrt{\left(\frac{R}{L}\right)^2} = \frac{R}{L} = 10^3\ rads^{-1}$$

$$\therefore \omega o = 10^3\ rads^{-1}$$

e) Find the transmission or gain at $\omega\ \omega o$ = 0.1, $\omega\ \omega o$ = 1 and at $\omega\ \omega o$ = 10

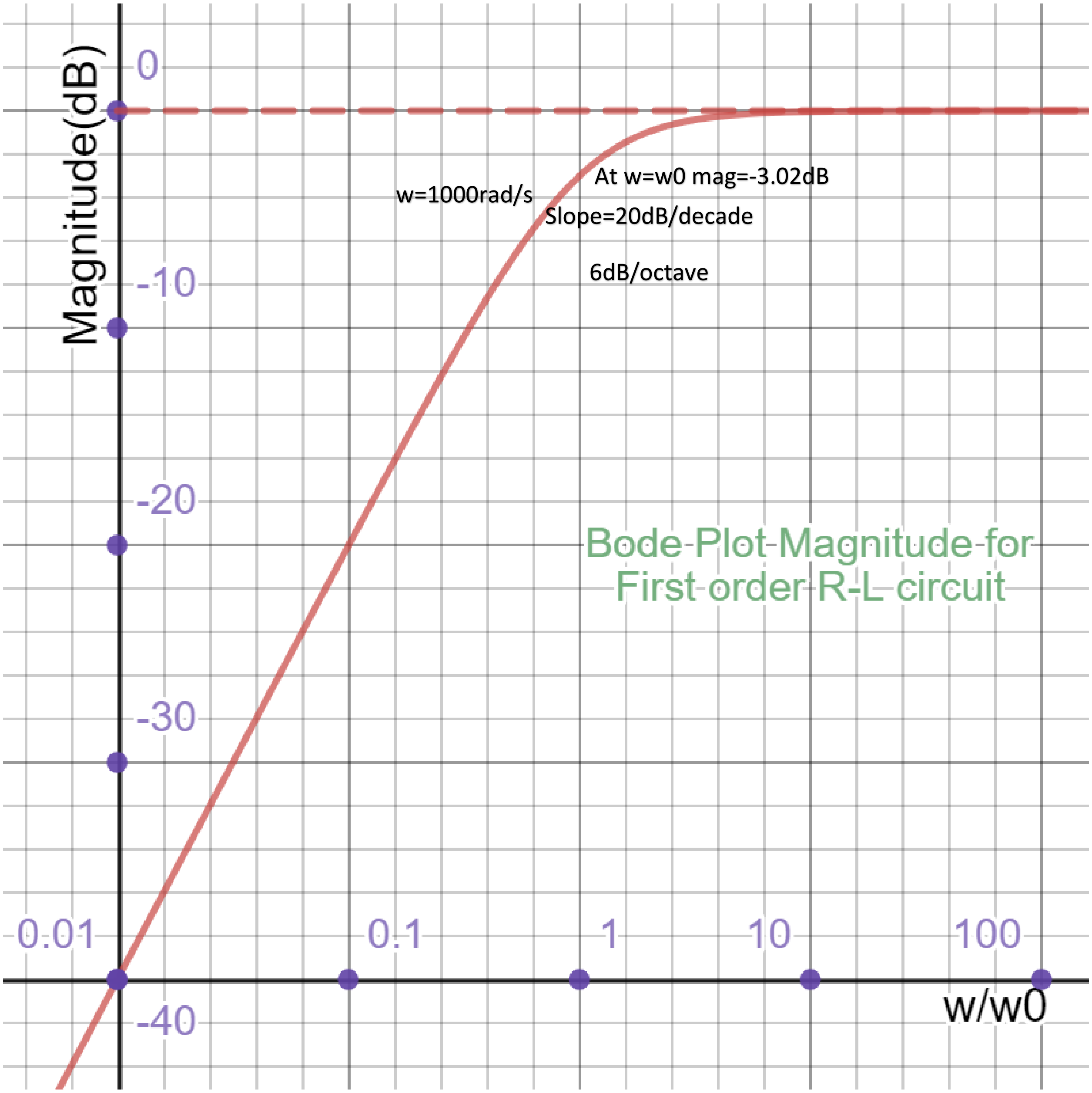
(e) $$|T(j\omega)| = \frac{|\omega|}{\sqrt{|\omega_0^2+\omega^2|}} = \frac{1}{\sqrt{\left|\left(\frac{\omega_0}{\omega}\right)^2+1\right|}}$$

if $\frac{\omega}{\omega_0} = 0.1$ then $|T(j\omega)| = \frac{1}{\sqrt{1+100}} = 0.099$

if $\frac{\omega}{\omega_0} = 1$ then $|T(j\omega)| = \frac{1}{\sqrt{2}} = 0.707$

if $\frac{\omega}{\omega_0} = 10$ then $|T(j\omega)| = \frac{1}{\sqrt{1+0.01}} = 0.9901$

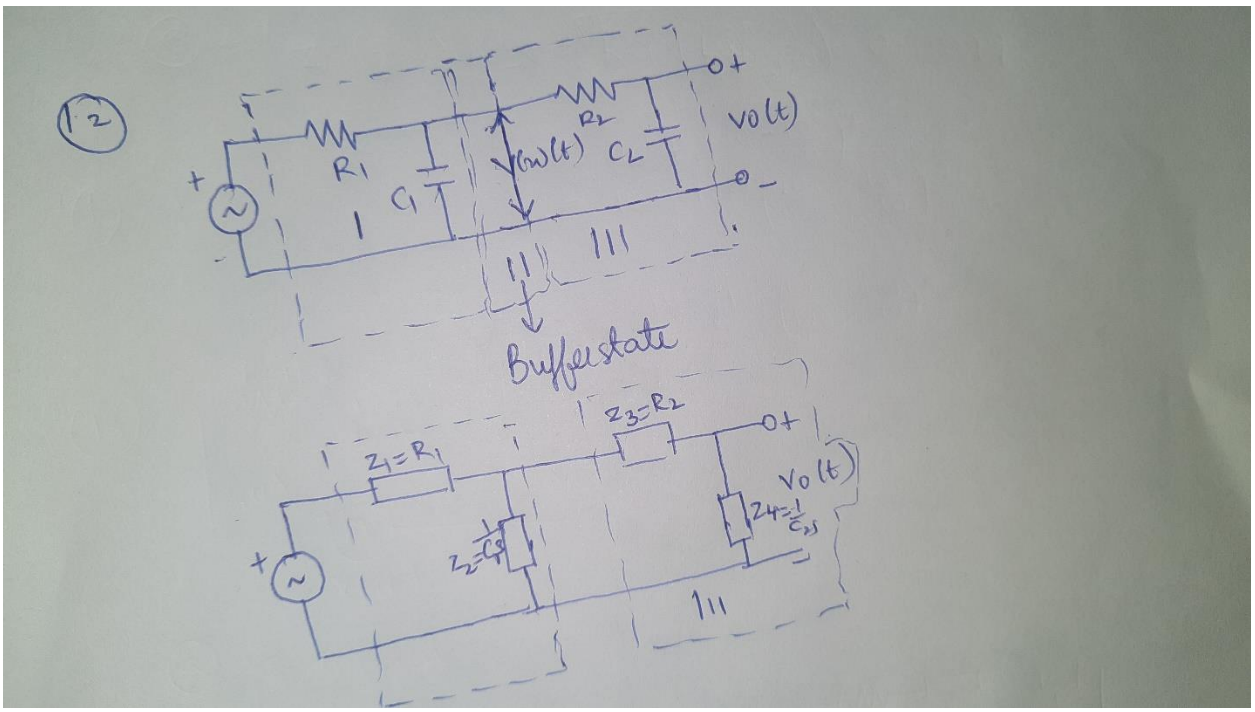
f) Plot magnitude response, $|T(j\omega)|$ vs. $\omega/\omega0$



12. For a second-order low pass filter derived by cascading two first order RC low pass filter stage connected through an isolator or buffer stage

with Gain, Av =1 as shown below. Assume that the capacitor is at a zero state initially i.e. VC t = 0 = 0 V.

(a) Draw s-domain transformed circuit for the filter.



b) Find transfer function in s-domain for Stage (I), Stage (II) and overall transfer function i.e. $T_1(s) = V_x(s)/ V_1(s)$, $T_{11}(s) = V_0(s)/ V_x(s)$ and $T(s) = Vo(s)/ V_i(s)$ respectively

(b) $V_{(x)}(t) = \dfrac{V_1(t) \times Z_{C1}}{Z_{C1} + Z_{R1}}$

$$\therefore T_1(s) = \frac{\frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} = \frac{1}{R_1 C_1 s + 1}$$

$$V_o(t) = \frac{V_x(t)\, Z_{C2}}{Z_{C2} + Z_{R2}} \Rightarrow T_2(s) = \frac{1}{R_2 C_2 s + 1}$$

$$T(s) = T_{11}(s) \times T_1(s) = \frac{1}{(R_1 C_1 s + 1)} * \frac{1}{(R_2 C_2 s + 1)}$$

c) Hence find transfer function for physical frequencies i.e $T(j\omega)$ by $s = j\omega$ and derive expression for magnitude response, $|T(j\omega)|$

(c) $$T(jw) = \frac{1}{R_1 C_1 jw + 1} * \frac{1}{1 + R_2 C_2 jw}$$

$$|T(jw)| = \frac{1}{\sqrt{1 + R_1^2 C_1^2 w^2}} \times \frac{1}{\sqrt{1 + R_2^2 C_2^2 w^2}}$$

d) Also calculate the corner frequency, ωo for the filter

Ⓓ $|T(j\omega)| = \dfrac{1}{\sqrt{1+10^{-6}\omega^2}\,\sqrt{1+10^{-6}\omega^2}} = \dfrac{1}{1+10^{-6}\omega^2}$

ω_0 is the frequency for which $|T(j\omega)|_{dB} = -6\,dB$

$\Rightarrow \quad 20\log(|T(j\omega)|) = -6.02$

$\Rightarrow \quad \log|T(j\omega)| = -0.301$

$\Rightarrow \quad |T(j\omega)| = \frac{1}{2}$

$\Rightarrow \quad 1+10^{-6}\omega^2 = 2 \Rightarrow \omega = 10^3\ \text{rads}^{-1}$

$\therefore$ corner frequency $(\omega_0) = 10^3\ \text{rad s}^{-1}$

e) Find the transmission or gain at $\omega/\omega o = 0.1$, $\omega/\omega o = 1$ and at $\omega/\omega o = 10$

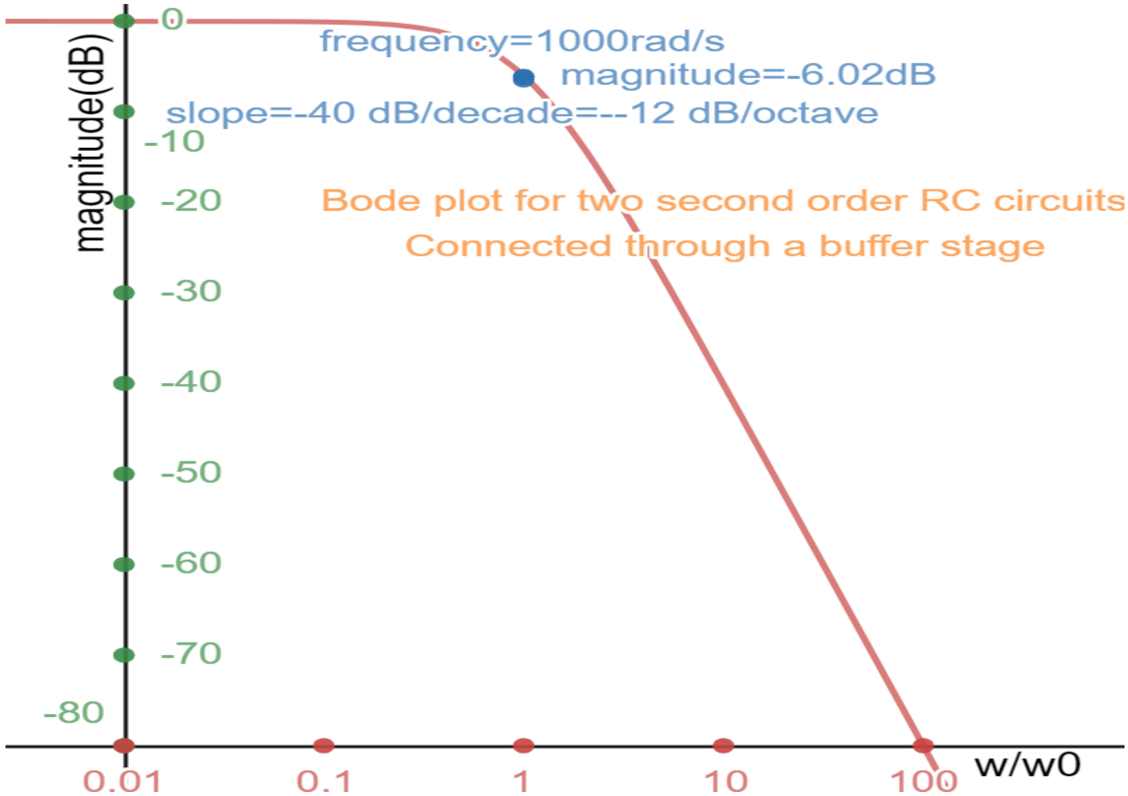
Ⓔ $\dfrac{\omega}{\omega_0} = 0.1 \Rightarrow |T(j\omega)| = \dfrac{1}{\left(\sqrt{1+\frac{\omega^2}{\omega_0^2}}\right)^2} = \dfrac{1}{1+\frac{\omega^2}{\omega_0^2}} = \dfrac{1}{1+0.01} = 0.99$

$\dfrac{\omega}{\omega_0} = 1 \Rightarrow |T(j\omega)| = \dfrac{1}{1+1} = 0.5$

$\dfrac{\omega}{\omega_0} = 10 \Rightarrow |T(j\omega)| = \dfrac{1}{1+100} = 0.0099$

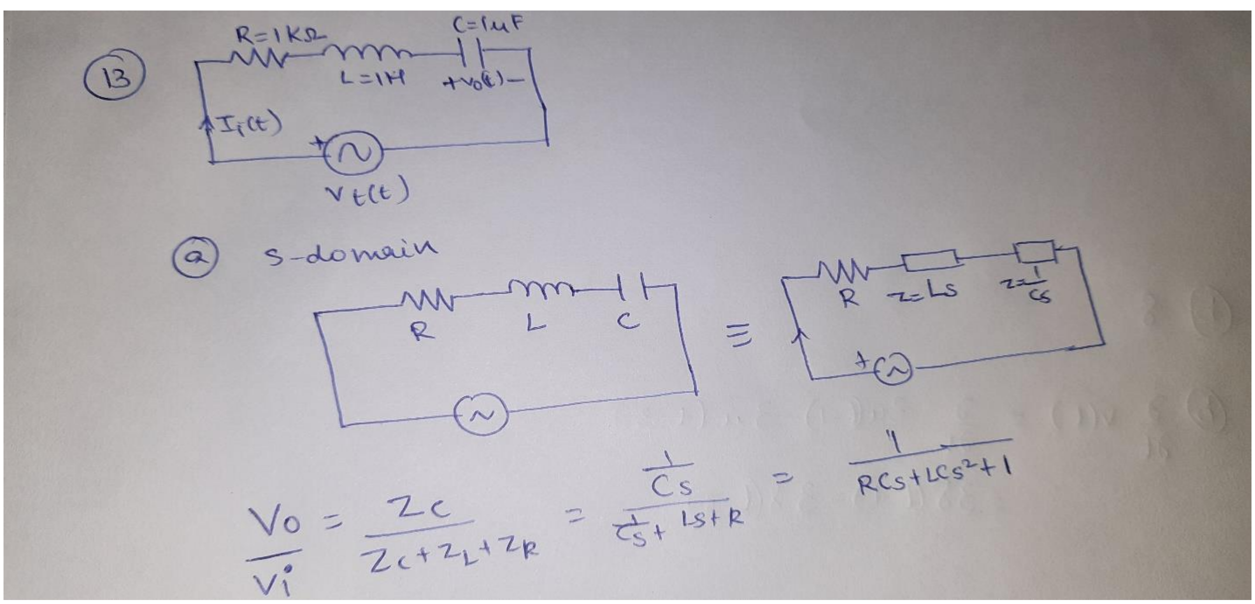
f) Plot magnitude response, $|T\ j\omega\ |$ vs. $\omega\ \omega o$ Assume that the capacitor is at a zero state initially i.e. V_{C1} (t= 0)= V_{C2} (t = 0) = 0

V, also R1 = R2 , C1= C2



Question 13

a) Draw s-domain transformed circuit for the resonator



b) Find impedance offered by the circuit in s-domain i.e. $Z(s)=V_1(s)/ I_1(s)$

(b) $Zs = Z_R + Z_L + Z_C$

$= R + Ls + \frac{1}{Cs}$

c) Hence find impedance for physical frequencies i.e $Z(j\omega)$ by considering $s = j\omega$ and derive expression for magnitude response, $|Z(j\omega)|$

(c) $|Z(j\omega)| = \left|R + L\omega j + \frac{1}{Cj\omega}\right|$

$= \left|R + L\omega j - \frac{1}{C\omega} j\right|$

$= \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$

(d) Determine the resonant frequency, ωo where the circuit offers purely resistive impedance

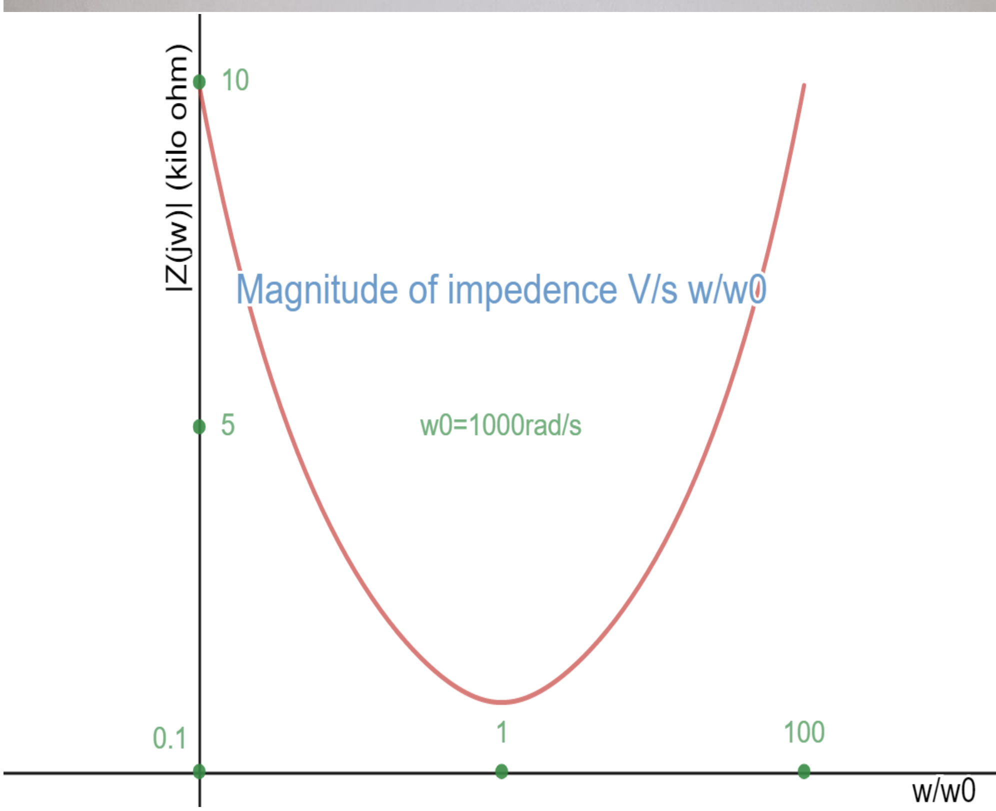
(d) ω_o is such that $|Z(j\omega)| = R$

$\Rightarrow R = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} \Rightarrow L\omega - \frac{1}{C\omega} = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

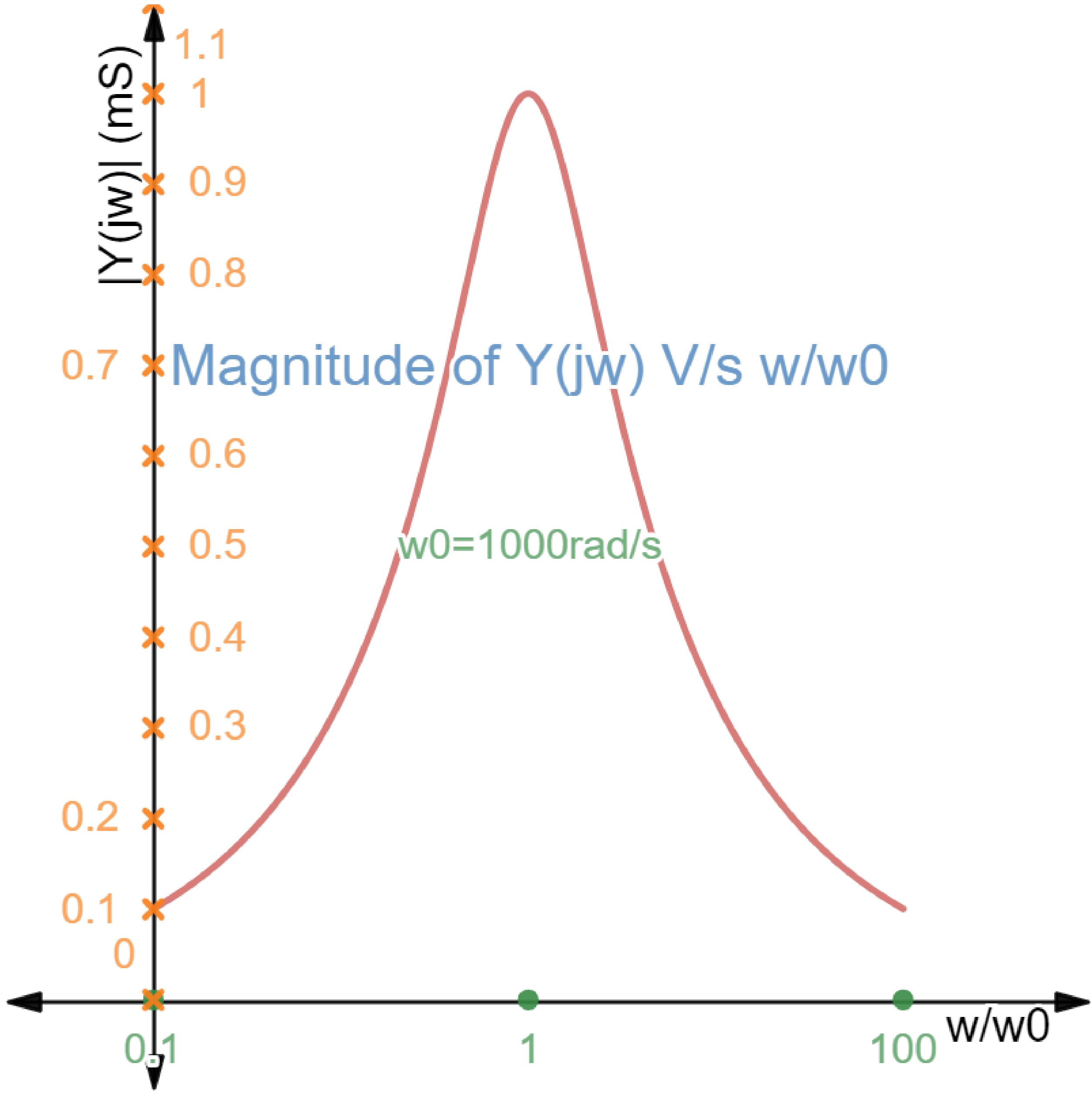
$= \frac{1}{\sqrt{10^{-6}}} = 10^3 \text{ rads}^{-1}$

(e) Plot $|Z(j\omega)|$ and $|Y(j\omega)| = 1/Z(j\omega)$ versus normalised frequency, $\omega/\omega o$ Assume that the inductor and capacitor are at zero state initially i.e. $I_L(t = 0) = 0\,A$ and $V_C(t = 0) = 0$

(e) $$|Z(j\omega)| = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$
$$= \sqrt{10^6 + \left(\omega - \frac{10^6}{\omega}\right)^2}$$
$$= 10^3\sqrt{1 + \left(\frac{\omega}{10^3} - \frac{10^3}{\omega}\right)^2}$$
$$= 10^3\sqrt{1 + \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}$$



Plot of Impedence versus w/w0



Plot of Admitance versus w/w0

Thank you!!