Chapter Summary

1. Meaning of identities

(a) Concept of identities

(i) An equation that can be satisfied by ALL values of the unknown(s) is called an identity.

(ii) We use the symbol '≡' instead of '=' to represent an identity.

Example 1

Determine whether each of the following equations is an identity.

(a) $2x(x+2)-(2x^2+3)=4x-3$

(b) $3(x+1)-2(3x-1)=1-3x$

Solution

(a)
$$\begin{aligned}\text{L.H.S.} &= 2x(x+2)-(2x^2+3)\\ &= 2x^2+4x-2x^2-3\\ &= 4x-3\end{aligned}$$

R.H.S. $=4x-3$

$\because$ L.H.S. = R.H.S.

$\therefore$ $2x(x+2)-(2x^2+3)\equiv 4x-3$

(b)
$$\begin{aligned}\text{L.H.S.} &= 3(x+1)-2(3x-1)\\ &= 3x+3-6x+2\\ &= -3x+5\end{aligned}$$

R.H.S. $=1-3x$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ $3(x+1)-2(3x-1)=1-3x$ is not an identity.

(b) Finding unknown constants in an identity

We can make use of the following two properties to find the unknown constants in an identity.

Property 1: For an identity involving polynomials only, the terms on both sides after expansion and simplification are the same.

Property 2: An identity can be satisfied by all values of the unknown(s).

Example 2

If $2(5x+2)\equiv Ax+B$, where A and B are constants, find the values of A and B.

Solution

Method 1

$$\begin{aligned}\text{L.H.S.} &= 2(5x+2)\\ &= 10x+4\end{aligned}$$

R.H.S. $=Ax+B$

By comparing the like terms, we have

$A=\underline{\underline{10}}$ and $B=\underline{\underline{4}}$

Note: This method is applicable to identities involving polynomials only.

Method 2

When $x=0$,

$$\begin{aligned}2[5(0)+2] &= A(0)+B\\ B &= \underline{\underline{4}}\end{aligned}$$

$\therefore$ The identity becomes $2(5x+2)\equiv Ax+4$.

When $x=1$,

$$\begin{aligned}2[5(1)+2] &= A(1)+4\\ A &= \underline{\underline{10}}\end{aligned}$$

2. Some important algebraic identities

(a) Difference of two squares

$$a^2 - b^2 \equiv (a+b)(a-b)$$

> **Example 3**
>
> Expand $(x+6)(x-6)$.
>
> *Solution*
>
> $$\begin{aligned}(x+6)(x-6) &= x^2 - 6^2 \\ &= \underline{\underline{x^2 - 36}}\end{aligned}$$

(b) Perfect square

(i) $(a+b)^2 \equiv a^2 + 2ab + b^2$

(ii) $(a-b)^2 \equiv a^2 - 2ab + b^2$

> **Example 4**
>
> Expand the following expressions.
>
> **(a)** $(x+7)^2$ **(b)** $(2x-5)^2$
>
> *Solution*
>
> **(a)** $$\begin{aligned}(x+7)^2 &= x^2 + 2(x)(7) + 7^2 \\ &= \underline{\underline{x^2 + 14x + 49}}\end{aligned}$$
>
> **(b)** $$\begin{aligned}(2x-5)^2 &= (2x)^2 - 2(2x)(5) + 5^2 \\ &= \underline{\underline{4x^2 - 20x + 25}}\end{aligned}$$

3. Factorization of simple algebraic expressions

(a) The process of expressing an algebraic expression as a product of its factors is called factorization.

(b) Factorization is the reverse process of expansion.

$$a(x+y) \underset{\text{Factorization}}{\overset{\text{Expansion}}{\rightleftarrows}} ax + ay$$

4. Methods of factorization

(a) By taking out common factors

(b) By grouping terms method

(c) By using identities

(i) Difference of two squares

$$a^2 - b^2 \equiv (a+b)(a-b)$$

(ii) Perfect square

$$(a+b)^2 \equiv a^2 + 2ab + b^2$$
$$(a-b)^2 \equiv a^2 - 2ab + b^2$$

> **Example 5**
>
> Factorize the following expressions.
>
> **(a)** $4x + 6xy$
>
> **(b)** $3x + 3y - bx - by$
>
> **(c)** **(i)** $3x^2 - 48$
>
> **(ii)** $2k^2 + 12k + 18$
>
> *Solution*
>
> **(a)** $$\begin{aligned}4x + 6xy &= \mathbf{2x}(2) + \mathbf{2x}(3y) \\ &= \underline{\underline{\mathbf{2x}(2+3y)}}\end{aligned}$$
>
> **(b)** $$\begin{aligned}3x + 3y - bx - by &= 3(x+y) - b(x+y) \\ &= \underline{\underline{(x+y)(3-b)}}\end{aligned}$$
>
> **(c)** **(i)** $$\begin{aligned}3x^2 - 48 &= 3(x^2 - 16) \\ &= 3(x^2 - 4^2) \\ &= \underline{\underline{3(x+4)(x-4)}}\end{aligned}$$
>
> **(ii)** $$\begin{aligned}2k^2 + 12k + 18 &= 2(k^2 + 6k + 9) \\ &= 2[\mathbf{k^2} + 2(k)(3) + \mathbf{3^2}] \\ &= \underline{\underline{2(k+3)^2}}\end{aligned}$$

Chapter Summary

1. Algebraic fractions

(a) If both A and B of an algebraic expression $\frac{A}{B}$ are polynomials, and B is not a constant number, this algebraic expression is called an algebraic fraction.

For example, $\frac{1}{a}$, $\frac{x+1}{x+2}$ and $\frac{p}{q+2r}$

(b) Like fractions, an algebraic fraction can be reduced to its simplest form by cancelling out the common factor(s) of its numerator and denominator.

Example 1

Simplify the following algebraic fractions.

(a) $\frac{6ab^3}{2a^2b^2}$ **(b)** $\frac{y-2xy}{12x^2-6x}$

Solution

(a)
$$\frac{6ab^3}{2a^2b^2} = \frac{\overset{3}{\cancel{6}} \times \cancel{a} \times \overset{b}{\cancel{b^3}}}{\cancel{2} \times \underset{a}{\cancel{a^2}} \times \cancel{b^2}}$$
$$= \underline{\underline{\frac{3b}{a}}}$$

◀ Cancel out the common factors 2, a and b^2.

(b)
$$\frac{y-2xy}{12x^2-6x} = \frac{y(1-2x)}{6x(2x-1)}$$
$$= \frac{y\cancel{(1-2x)}}{-6x\cancel{(1-2x)}}$$
$$= \underline{\underline{-\frac{y}{6x}}}$$

◀ Cancel out the common factor $1-2x$.

2. Manipulation of algebraic fractions

(a) Multiplication and division

The principles are the same as those of fractions.

Example 2

Simplify the following expressions.

(a) $\frac{a^2}{5b^3} \times \frac{10b}{3a}$ **(b)** $\frac{3}{2a+4} \div \frac{6a}{5a+10}$

Solution

(a)
$$\frac{a^2}{5b^3} \times \frac{10b}{3a}$$
$$= \frac{\overset{a}{\cancel{a^2}}}{\cancel{5}\underset{b^2}{\cancel{b^3}}} \times \frac{\overset{2}{\cancel{10}}\cancel{b}}{3\cancel{a}}$$
$$= \underline{\underline{\frac{2a}{3b^2}}}$$

◀ Cancel out the common factors 5, a and b.

(b)
$$\frac{3}{2a+4} \div \frac{6a}{5a+10}$$
$$= \frac{\cancel{3}}{2\cancel{(a+2)}} \times \frac{5\cancel{(a+2)}}{\underset{2}{\cancel{6}}a}$$
$$= \underline{\underline{\frac{5}{4a}}}$$

◀ Cancel out the common factors 3 and $a+2$.

(b) Addition and subtraction

(i) If the denominators of the algebraic fractions involved are <u>the same</u>, we can directly perform addition or subtraction on their numerators and keep the denominator unchanged.

(ii) If the denominators are <u>different</u>, we should expand the algebraic fractions in order to make the denominators the same.

Example 3

Simplify the following expressions.

(a) $\dfrac{2}{3b}+\dfrac{4}{3b}$ **(b)** $\dfrac{1}{x-y}-\dfrac{1}{x+y}$

Solution

(a)
$$\begin{aligned}&\frac{2}{3b}+\frac{4}{3b}\\ &=\frac{2+4}{3b}\\ &=\frac{\overset{2}{\cancel{6}}}{\cancel{3}b}\\ &=\underline{\underline{\frac{2}{b}}}\end{aligned}$$

(b)
$$\begin{aligned}&\frac{1}{x-y}-\frac{1}{x+y}\\ &=\frac{x+y}{(x-y)(x+y)}-\frac{x-y}{(x-y)(x+y)}\\ &=\frac{x+y-(x-y)}{(x-y)(x+y)}\\ &=\frac{x+y-x+y}{(x-y)(x+y)}\\ &=\underline{\underline{\frac{2y}{(x-y)(x+y)}}}\end{aligned}$$

◀ Expand both fractions.

$$\frac{1}{x-y}=\frac{1\times(x+y)}{(x-y)\times(x+y)}=\frac{x+y}{(x-y)(x+y)}$$
$$\frac{1}{x+y}=\frac{1\times(x-y)}{(x+y)\times(x-y)}=\frac{x-y}{(x-y)(x+y)}$$

3. Formulas

(a) Formulas and method of substitution

A formula is an equality relating two or more variables. By substitution, we can find the value of a variable in a formula when the values of other variables are known.

(b) Subject of a formula

In a formula, if a variable is a single variable on one side and it is expressed in terms of other variables, the variable is called the subject of the formula.

For example,

$$P = x + y + z$$

P is the subject

Example 4

Consider the formula $A=\dfrac{2h}{k}-1$.

(a) Make k the subject of the formula.

(b) If $A = 5$ and $h = 6$, find the value of k.

Solution

(a)
$$\begin{aligned}A&=\frac{2h}{k}-1\\ A+1&=\frac{2h}{k}\\ k&=\frac{2h}{A+1}\end{aligned}$$

(b) When $A = 5$ and $h = 6$,
$$\begin{aligned}k&=\frac{2\times 6}{5+1}\\ &=\underline{\underline{2}}\end{aligned}$$

Chapter Summary

1. Concept of errors in measurement

(a) The actual value of a measure (like length, weight, capacity, etc) in all real measurements are unknown.

(b) No matter how precise the measuring tools are, the measured values obtained are regarded as the approximate values of the measures. In other words, errors are unavoidable in measurements.

2. Maximum absolute errors

(a) The absolute error is the difference between the actual value and the measured value. It is always positive.

(b) In measurement, the actual value and the absolute error cannot be found. However, the largest possible error of the measured value, which is called the maximum absolute error, can be determined.

(i) Maximum absolute error

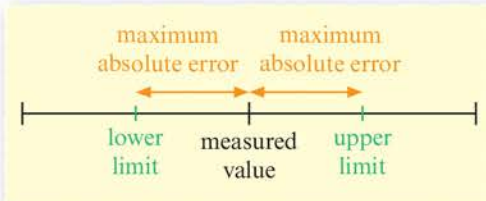
$= \frac{1}{2} \times$ scale interval of the measuring tool

(ii) Lower limit of the actual value

= measured value − maximum absolute error

(iii) Upper limit of the actual value

= measured value + maximum absolute error



Note: The range of the actual value is:

Lower limit $\leq$ the actual value $<$ upper limit

Example 1

The weight of a bar of chocolate is measured as 25.5 g and the scale interval of the measuring tool is 0.1 g.

(a) Find the maximum absolute error of the measurement.

(b) Find the range of the actual weight of the bar of chocolate.

Solution

(a) Maximum absolute error $= \frac{1}{2} \times 0.1\text{ g}$

$= \underline{\underline{0.05\text{ g}}}$

(b) Lower limit of the actual weight $= (25.5 - 0.05)$ g

$= 25.45$ g

Upper limit of the actual weight $= (25.5 + 0.05)$ g

$= 25.55$ g

$\therefore$ The range of the actual weight of the bar of chocolate is:

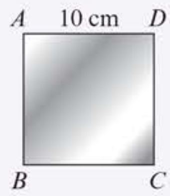
$25.45\text{ g} \leq \text{the actual weight} < 25.55\text{ g}$

Example 2

In the figure, $ABCD$ is a square metal sheet. The length of a side of the metal sheet is measured as 10 cm, correct to the nearest cm.

(a) Find the maximum absolute error of the measurement.

(b) Find the least possible area of the metal sheet.



Solution

(a) Maximum absolute error $= \frac{1}{2} \times 1\text{ cm}$

$= \underline{\underline{0.5\text{ cm}}}$

(b) Lower limit of the actual length $= (10 - 0.5)\text{ cm}$

$= 9.5\text{ cm}$

$\therefore$ Least possible area $= 9.5 \times 9.5\text{ cm}^2$

$= \underline{\underline{90.25\text{ cm}^2}}$

3. Relative errors and percentage errors

(a) Relative error $= \dfrac{\text{maximum absolute error}}{\text{measured value}}$

(b) Percentage error = relative error $\times$ 100%

Note: The smaller the relative error (or percentage error) is, the more accurate the measured value will be.

Example 3

The weight of a girl is measured as 42 kg, correct to the nearest 2 kg.

(a) Find the relative error of the measured weight.

(b) Find the percentage error of the measured weight.

(Give your answers correct to 3 significant figures.)

Solution

(a) Maximum absolute error $= \frac{1}{2} \times 2\text{ kg}$

$= 1\text{ kg}$

Relative error $= \dfrac{1\text{ kg}}{42\text{ kg}}$

$= \underline{\underline{0.0238}}$ (cor. to 3 sig. fig.)

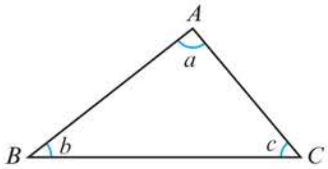
(b) Percentage error $= \dfrac{1\text{ kg}}{42\text{ kg}} \times 100\%$

$= \underline{\underline{2.38\%}}$ (cor. to 3 sig. fig.)

Chapter Summary

1. Angles of a triangle

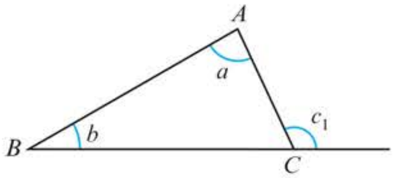
(a) Angle sum of a triangle



$a + b + c = 180°$

[Abbreviation: *∠ sum of △*]

(b) Exterior angle of a triangle

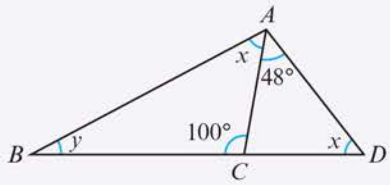


$c_1 = a + b$

[Abbreviation: *ext. ∠ of △*]

Example 1

In the figure, BCD is a straight line. Find x and y.



Solution

In $\triangle ACD$,

$x + 48° = 100°$ (ext. ∠ of △)

$x = \underline{\underline{52°}}$

In $\triangle ABC$,

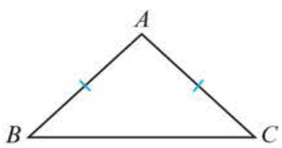
$x + y + 100° = 180°$ (∠ sum of △)

$52° + y + 100° = 180°$

$y = \underline{\underline{28°}}$

2. Isosceles triangles

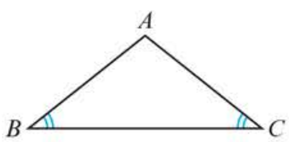
(a) Property of an isosceles triangle



If $AB = AC$, then $\angle B = \angle C$.

[Abbreviation: *base ∠s, isos. △*]

(b) Condition for an isosceles triangle

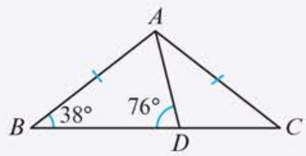


If $\angle B = \angle C$, then $AB = AC$.

[Abbreviation: *sides opp. equal ∠s*]

Example 2

In the figure, BDC is a straight line and $AB = AC$.



(a) Find $\angle ACD$.

(b) Determine whether $\triangle ACD$ is an isosceles triangle.

Solution

(a) $\because \; AB = AC$

$\therefore \; \angle ACD = \angle ABD$ (base ∠s, isos. △)

$= \underline{\underline{38°}}$

(b) In $\triangle ADC$,

$\angle DAC + \angle ACD = 76°$ (ext. ∠ of △)

$\angle DAC + 38° = 76°$

$\angle DAC = 38°$

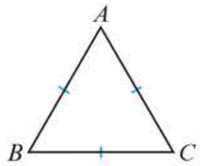
$\because \; \angle DAC = \angle ACD$

$\therefore \; AD = DC$ (sides opp. equal ∠s)

i.e. $\triangle ACD$ is an isosceles triangle.

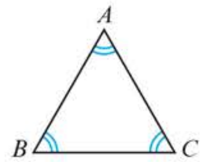
3. Equilateral triangles

(a)



If $AB = BC = AC$,
then $\angle A = \angle B = \angle C = 60°$.
[Abbreviation: *prop. of equil.* $\triangle$]

(b)



If $\angle A = \angle B = \angle C$ (= 60°),
then $\triangle ABC$ is an equilateral triangle.

4. Angles of a polygon

(a) The sum of all the interior angles of an n-sided polygon is $(n-2) \times 180°$.
[Abbreviation: $\angle$ *sum of polygon*]

(b) The sum of all the exterior angles of a convex polygon is 360°.
[Abbreviation: *sum of ext.* $\angle$*s of polygon*]

Example 3

It is given that the sum of all the interior angles of a regular polygon is 1440°.

(a) Find the number of sides of the polygon.

(b) Find the size of each exterior angle of the polygon.

Solution

(a) Let n be the number of sides of the polygon.

$$(n-2) \times 180° = 1440° \quad (\angle \text{ sum of polygon})$$
$$n - 2 = 8$$
$$n = 10$$

$\therefore$ The polygon has 10 sides.

(b) Size of each exterior angle $= \dfrac{360°}{10}$
$= \underline{\underline{36°}}$

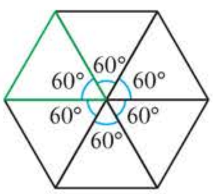
5. Tessellation

(a) If geometric figures are put together side by side to cover a plane without gaps or overlaps, the pattern formed is called a tessellation.

(b) All triangles and quadrilaterals can tessellate a plane.

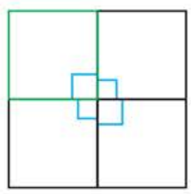
(c) Three types of regular polygons which can form tessellations on their own are:

(i) equilateral triangles



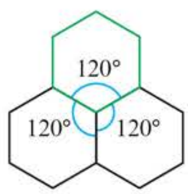
60 is a factor of 360.

(ii) squares



90 is a factor of 360.

(iii) regular hexagons



120 is a factor of 360.

Chapter Summary

1. Grouping of continuous data

We can group a large number of data into several class intervals.

For example, the following is the frequency distribution table for the weights of 30 S2 students:

Weight (kg)	Class boundaries (kg)	Class mark (kg)	Frequency
31 − 35	30.5 − 35.5	33	4
36 − 40	35.5 − 40.5	38	7
41 − 45	40.5 − 45.5	43	13
46 − 50	45.5 − 50.5	48	6
		Total	30

For the class interval 36 kg − 40 kg:

(i) The class limits are 36 kg and 40 kg.

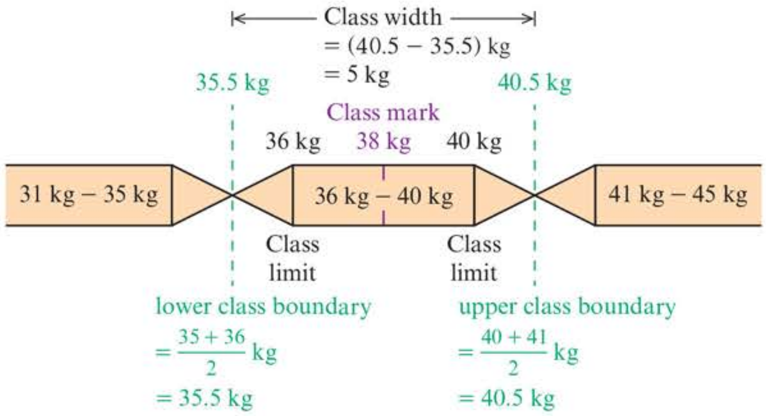
(ii) Class mark $= \dfrac{36+40}{2}$ kg

$= 38$ kg

(iii) The class boundaries are 35.5 kg and 40.5 kg.

(iv) Class width $= (40.5 - 35.5)$ kg

$= 5$ kg



Note: Class mark can also be calculated by using

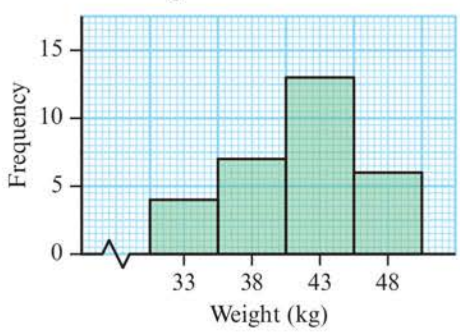
$$\text{class mark} = \frac{\text{lower class boundary} + \text{upper class boundary}}{2}.$$

2. Histogram

A histogram is often used to present grouped continuous data. It consists of rectangular bars with no spaces between the bars.

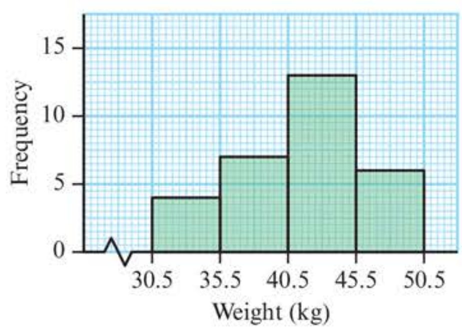
e.g. The following histograms show the data represented by the above frequency distribution table.

Weights of 30 S2 students



Class marks are labelled along the horizontal axis.

Weights of 30 S2 students



Class boundaries are labelled along the horizontal axis.

3. Frequency polygon and frequency curve

We can draw a frequency polygon from the frequency distribution table as follows:

Step 1 Add two class marks with zero frequencies at the two ends.

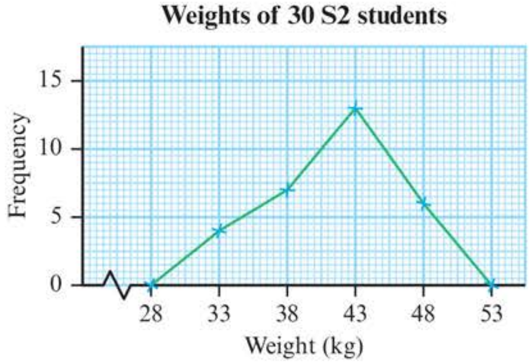
Step 2 Use the class marks and the frequencies to form ordered pairs, and plot all the ordered pairs formed.

Step 3 Join the adjacent points with line segments to obtain a frequency polygon.

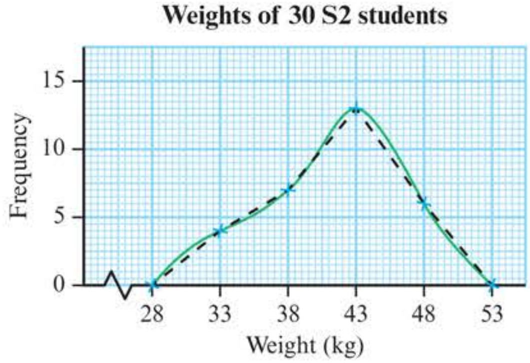
Class mark (kg)	Frequency
28	0
33	4
38	7
43	13
48	6
53	0

Also, by smoothing a frequency polygon, we can obtain its corresponding frequency curve.

For example:



Frequency polygon



Frequency curve

4. Cumulative frequency polygon and cumulative frequency curve

Before drawing a cumulative frequency polygon or curve, we have to construct a cumulative frequency table.

For example:

Weight (kg)	Class boundaries (kg)	Frequency
31 − 35	30.5 − 35.5	4
36 − 40	35.5 − 40.5	7
41 − 45	40.5 − 45.5	13
46 − 50	45.5 − 50.5	6
	Total	30

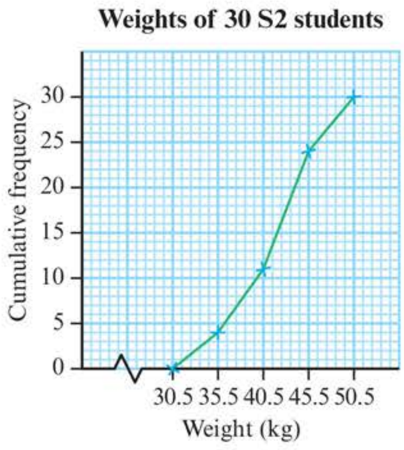
Frequency distribution table

Weight less than (kg)	Cumulative frequency
30.5	0
35.5	4
40.5	11
45.5	24
50.5	30

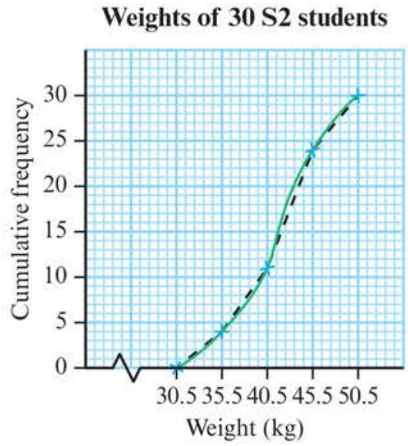
Cumulative frequency table

To obtain a cumulative frequency polygon, we can plot the points for cumulative frequencies against class boundaries, and then join the adjacent points with line segments. Also, by smoothing a cumulative frequency polygon, we can obtain its corresponding cumulative frequency curve.

For example:



Cumulative frequency polygon



Cumulative frequency curve

5. Percentiles, quartiles and median

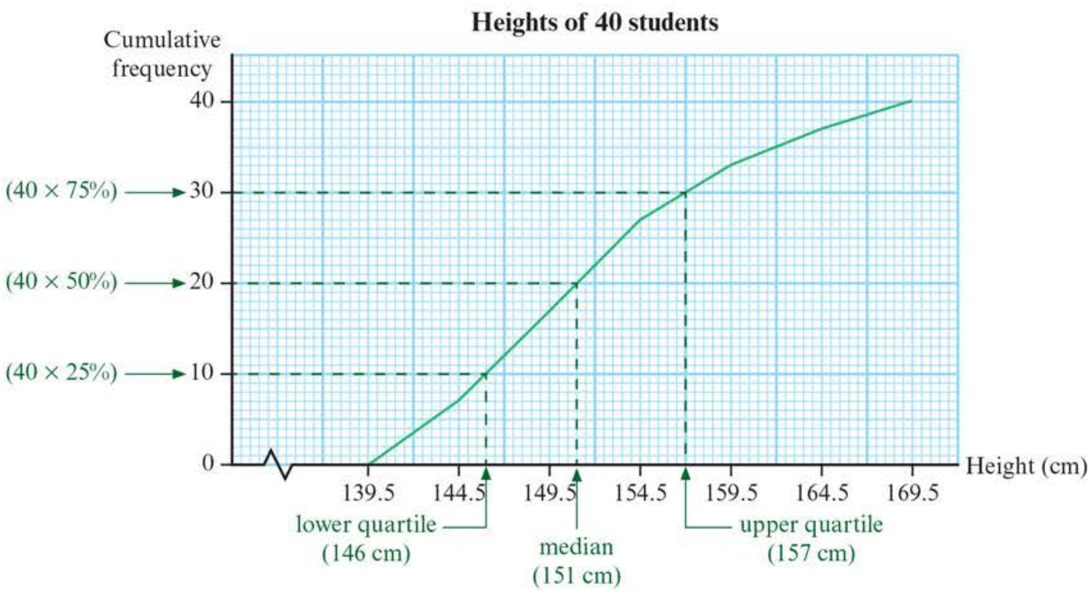
The xth percentile of a distribution is the value such that x% of the data are less than that value. In other words, x% of the data in a distribution have values below the xth percentile. We can find percentiles from the cumulative frequency polygon or curve.

In particular,

the 25th percentile = lower quartile,

the 50th percentile = median,

the 75th percentile = upper quartile.



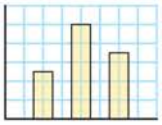
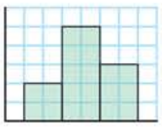
In the above distribution,

the 25th percentile is 146 cm, i.e. 25% of the data is below 146 cm,

the 50th percentile is 151 cm, i.e. 50% of the data is below 151 cm,

the 75th percentile is 157 cm, i.e. 75% of the data is below 157 cm.

6. Characteristic(s) of different statistical charts

Statistical chart		Characteristic(s)
	Bar chart	• shows the actual frequency of each item
	Broken line graph	• shows how data change over a period of time • predicts the trend of data
	Pie chart	• shows the portion or the percentage of each item compared with the whole set of data
Stem | Leaf 3 | 0 1 3 4 | 2 2 5 8 5 | 1 7 7	Stem-and-leaf diagram	• presents a small amount of data exactly
	Histogram	• shows the frequency distribution of a set of continuous data

7. Abuses of statistical charts

Some statistical charts may be misleading. We should pay attention to the following when reading statistical charts.

(i) Whether the vertical axis starts from zero.

(ii) Whether the bars used are of appropriate proportion.

Chapter Summary

1. **Rate**

A rate is a comparison of two quantities of different kinds by division. It represents the relationship of the amount of one quantity per unit of another quantity. In general, rates have units.

For example, if a factory makes 1000 bottles in 4 hours, then

$$\text{the production rate} = \frac{1000 \text{ bottles}}{4 \text{ h}} = 250 \text{ bottles/h}$$

2. **Ratio**

(a) A ratio is a comparison of quantities of the same kind by division. Ratios have no units.

(b) For two quantities a and b, the ratio of a to b can be denoted as $a:b$ or $\frac{a}{b}$, where $a \neq 0$ and $b \neq 0$.

For example, if a drink is made by mixing 3 L of orange juice with 2 L of apple juice, then volume of orange juice : volume of apple juice = 3 : 2.

(c) The ratio of three or more quantities is called a continued ratio.

If two related ratios, e.g. $a:b$ and $b:c$ are given, we can combine the two ratios into a continued ratio $a:b:c$.

For example, if $a:b = 5:6$ and $b:c = 6:1$, then $a:b:c = 5:6:1$.

Example 1

Yuki, Ray and Eric share a profit of \$2000 in the ratio 3 : 5 : 2. How much does Yuki get?

Solution

$$\begin{aligned}\text{Yuki's share} &= \$2000 \times \frac{3}{3+5+2} \\ &= \underline{\underline{\$600}}\end{aligned}$$

◀ Divide the total profit into (3 + 5 + 2) parts and Yuki gets 3 parts.

(d) When each quantity of a ratio is multiplied or divided by the same non-zero number k, the ratio remains unchanged.

For $k \neq 0$,

(i) $a:b = a \times k : b \times k$

$a:b:c = a \times k : b \times k : c \times k$

(ii) $a:b = \frac{a}{k} : \frac{b}{k}$

$a:b:c = \frac{a}{k} : \frac{b}{k} : \frac{c}{k}$

Example 2

(a) $0.4:0.6$

$= 0.4 \times 5 : 0.6 \times 5$

$= \underline{\underline{2:3}}$

(b) $6:8:2$

$= \frac{6}{2} : \frac{8}{2} : \frac{2}{2}$

$= \underline{\underline{3:4:1}}$

3. Proportion

(a) A proportion is a statement that two ratios are equal. For ratios of two quantities,

if $a:b$ and $c:d$ are equal, then $a:b=c:d\left(\text{or } \dfrac{a}{b}=\dfrac{c}{d}\right)$ is a proportion.

(b) A proportion can also be used to state the equality of two ratios of three or more quantities.

e.g. $x:y:z=1:3:4$ is a proportion. From this proportion, we know that $x:y=1:3$, $y:z=3:4$ and $x:z=1:4$.

Example 3

A mixture contains pepper and tomato sauce in the ratio $2:5$ by weight. If the weight of pepper in the mixture is 6 g, find the weight of tomato sauce.

Solution

Let x g be the weight of tomato sauce.

$$6:x=2:5$$
$$\frac{6}{x}=\frac{2}{5}$$
$$2x=30 \quad \text{◀ By cross-multiplication}$$
$$x=15$$

$\therefore$ The weight of tomato sauce is 15 g.

4. Scale drawing

A scale drawing is a reduced or an enlarged drawing of a real object according to a specific ratio (called scale).

$$\text{Scale} = \frac{\text{length on the scale drawing}}{\text{actual length}}$$

A scale is usually expressed in the form $1:n$ (reduced) or $n:1$ (enlarged), where $n>1$.

Example 4

On a map with a scale $1:10\ 000$, the length of a river is 20 cm. Find the actual length (in km) of the river.

Solution

$$\frac{20\text{ cm}}{\text{Actual length}}=\frac{1}{10\ 000}$$
$$\begin{aligned}\text{Actual length} &= 20\times 10\ 000\text{ cm}\\ &= 200\ 000\text{ cm}\\ &= 2000\text{ m} && \text{◀ 1 m = 100 cm}\\ &= \underline{\underline{2\text{ km}}} && \text{◀ 1 km = 1000 m}\end{aligned}$$

5. Direct proportion

(a) For two quantities A and P, when A changes by a certain factor k, P also changes by the same factor k. Then, P and A are in direct proportion (or P is directly proportional to A).

e.g. The table on the right shows some pairs of values of A and P, which are in direct proportion.

A	1	2	3	4
P	5	10	15	20

(b) If P and A are in direct proportion,

then $p_1:p_2=a_1:a_2\quad\left(\text{or } \dfrac{p_1}{p_2}=\dfrac{a_1}{a_2}\right)$, ◀ We can also say that $p_2:p_1=a_2:a_1\left(\text{or } \dfrac{p_2}{p_1}=\dfrac{a_2}{a_1}\right)$.

where p_1 and p_2 are any two values of P; a_1 and a_2 are the corresponding values of A.

Example 5

60 invitation cards weigh 150 g. How much do 360 invitation cards weigh?

Solution

Let x g be the total weight of 360 invitation cards.

$$x : 150 = 360 : 60$$
$$\frac{x}{150} = \frac{360}{60}$$
$$x = \frac{360}{60} \times 150$$
$$= 900$$

$\therefore$ 360 invitation cards weigh 900 g.

$360 : 60$

Number of cards	60	360
Weight (g)	150	x

$x : 150$

6. Inverse proportion

(a) For two quantities A and P, when A changes by a certain factor k, P also changes by the reciprocal of the same factor, i.e. $\frac{1}{k}$. Then, P and A are in inverse proportion (or P is inversely proportional to A).

e.g. The table on the right shows some pairs of values of A and P, which are in inverse proportion.

A	1	2	3	4
P	60	30	20	15

(b) If P and A are in inverse proportion,

then $p_1 : p_2 = a_2 : a_1$ $\left(\text{or } \frac{p_1}{p_2} = \frac{a_2}{a_1}\right)$,

where p_1 and p_2 are any two values of P; a_1 and a_2 are the corresponding values of A.

We can also say that $p_2 : p_1 = a_1 : a_2$ $\left(\text{or } \frac{p_2}{p_1} = \frac{a_1}{a_2}\right)$.

Example 6

If 3 pipes can fill up a pool in 4 hours, how long will it take to fill up the pool with 5 pipes?

Solution

Let t hours be the required time.

$$3 : 5 = t : 4$$
$$\frac{3}{5} = \frac{t}{4}$$
$$t = \frac{3}{5} \times 4$$
$$= 2.4$$

$\therefore$ 5 pipes can fill up the pool in 2.4 hours.

$3 : 5$

Number of pipes	3	5
Time required (h)	4	t

$t : 4$