Chapter Summary

1. Meaning of identities

(a) Concept of identities

(i) An equation that can be satisfied by ALL values of the unknown(s) is called an identity.

(ii) We use the symbol '$\equiv$' instead of '$=$' to represent an identity.

Example 1

Determine whether each of the following equations is an identity.

(a) $2x(x+2)-(2x^2+3)=4x-3$

(b) $3(x+1)-2(3x-1)=1-3x$

Solution

(a) $$\begin{aligned}\text{L.H.S.} &= 2x(x+2)-(2x^2+3)\\ &= 2x^2+4x-2x^2-3\\ &= 4x-3\end{aligned}$$

R.H.S. $= 4x-3$

$\because$ L.H.S. $=$ R.H.S.

$\therefore$ $2x(x+2)-(2x^2+3)\equiv 4x-3$

(b) $$\begin{aligned}\text{L.H.S.} &= 3(x+1)-2(3x-1)\\ &= 3x+3-6x+2\\ &= -3x+5\end{aligned}$$

R.H.S. $= 1-3x$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ $3(x+1)-2(3x-1)=1-3x$ is not an identity.

(b) Finding unknown constants in an identity

We can make use of the following two properties to find the unknown constants in an identity.

Property 1: For an identity involving polynomials only, the terms on both sides after expansion and simplification are the same.

Property 2: An identity can be satisfied by all values of the unknown(s).

Example 2

If $2(5x+2)\equiv Ax+B$, where A and B are constants, find the values of A and B.

Solution

Method 1

$$\begin{aligned}\text{L.H.S.} &= 2(5x+2)\\ &= 10x+4\end{aligned}$$

R.H.S. $= Ax+B$

By comparing the like terms, we have

$A = \underline{\underline{10}}$ and $B = \underline{\underline{4}}$

Note: This method is applicable to identities involving polynomials only.

Method 2

When $x=0$,

$$\begin{aligned}2[5(0)+2] &= A(0)+B\\ B &= \underline{\underline{4}}\end{aligned}$$

$\therefore$ The identity becomes $2(5x+2)\equiv Ax+4$.

When $x=1$,

$$\begin{aligned}2[5(1)+2] &= A(1)+4\\ A &= \underline{\underline{10}}\end{aligned}$$

2. Some important algebraic identities

(a) Difference of two squares

$$a^2 - b^2 \equiv (a+b)(a-b)$$

Example 3

Expand $(x+6)(x-6)$.

Solution

$$\begin{aligned}(x+6)(x-6) &= x^2 - 6^2\\ &= \underline{\underline{x^2 - 36}}\end{aligned}$$

(b) Perfect square

(i) $(a+b)^2 \equiv a^2 + 2ab + b^2$

(ii) $(a-b)^2 \equiv a^2 - 2ab + b^2$

Example 4

Expand the following expressions.

(a) $(x+7)^2$ **(b)** $(2x-5)^2$

Solution

(a) $$\begin{aligned}(x+7)^2 &= x^2 + 2(x)(7) + 7^2\\ &= \underline{\underline{x^2 + 14x + 49}}\end{aligned}$$

(b) $$\begin{aligned}(2x-5)^2 &= (2x)^2 - 2(2x)(5) + 5^2\\ &= \underline{\underline{4x^2 - 20x + 25}}\end{aligned}$$

3. Factorization of simple algebraic expressions

(a) The process of expressing an algebraic expression as a product of its factors is called factorization.

(b) Factorization is the reverse process of expansion.

$$a(x+y) \underset{\text{Factorization}}{\overset{\text{Expansion}}{\rightleftarrows}} ax + ay$$

4. Methods of factorization

(a) By taking out common factors

(b) By grouping terms method

(c) By using identities

(i) Difference of two squares

$$a^2 - b^2 \equiv (a+b)(a-b)$$

(ii) Perfect square

$$(a+b)^2 \equiv a^2 + 2ab + b^2$$

$$(a-b)^2 \equiv a^2 - 2ab + b^2$$

Example 5

Factorize the following expressions.

(a) $4x + 6xy$

(b) $3x + 3y - bx - by$

(c) **(i)** $3x^2 - 48$

(ii) $2k^2 + 12k + 18$

Solution

(a) $$\begin{aligned}4x + 6xy &= \mathbf{2x}(2) + \mathbf{2x}(3y)\\ &= \underline{\underline{\mathbf{2x}(2+3y)}}\end{aligned}$$

(b) $$\begin{aligned}3x + 3y - bx - by &= 3(x+y) - b(x+y)\\ &= \underline{\underline{(x+y)(3-b)}}\end{aligned}$$

(c) **(i)** $$\begin{aligned}3x^2 - 48 &= 3(x^2 - 16)\\ &= 3(x^2 - \mathbf{4^2})\\ &= \underline{\underline{3(x+\mathbf{4})(x-\mathbf{4})}}\end{aligned}$$

(ii) $$\begin{aligned}2k^2 + 12k + 18 &= 2(k^2 + 6k + 9)\\ &= 2[\mathbf{k^2} + 2(k)(3) + \mathbf{3^2}]\\ &= \underline{\underline{2(k+3)^2}}\end{aligned}$$

Chapter Summary

1. Algebraic fractions

(a) If both A and B of an algebraic expression $\frac{A}{B}$ are polynomials, and B is not a constant number, this algebraic expression is called an algebraic fraction.

For example, $\frac{1}{a}$, $\frac{x+1}{x+2}$ and $\frac{p}{q+2r}$

(b) Like fractions, an algebraic fraction can be reduced to its simplest form by cancelling out the common factor(s) of its numerator and denominator.

Example 1

Simplify the following algebraic fractions.

(a) $\frac{6ab^3}{2a^2b^2}$ **(b)** $\frac{y-2xy}{12x^2-6x}$

Solution

(a) $\frac{6ab^3}{2a^2b^2} = \frac{^{3}\cancel{6} \times \cancel{a} \times \cancel{b^3}^{\,b}}{\cancel{2} \times \cancel{a^2}_{a} \times \cancel{b^2}}$

$= \frac{3b}{a}$

◀ Cancel out the common factors 2, a and b^2.

(b) $\frac{y-2xy}{12x^2-6x} = \frac{y(1-2x)}{6x(2x-1)}$

$= \frac{y\cancel{(1-2x)}}{-6x\cancel{(1-2x)}}$

$= -\frac{y}{6x}$

◀ Cancel out the common factor $1-2x$.

2. Manipulation of algebraic fractions

(a) Multiplication and division

The principles are the same as those of fractions.

Example 2

Simplify the following expressions.

(a) $\frac{a^2}{5b^3} \times \frac{10b}{3a}$ **(b)** $\frac{3}{2a+4} \div \frac{6a}{5a+10}$

Solution

(a) $\frac{a^2}{5b^3} \times \frac{10b}{3a}$

$= \frac{\cancel{a^2}^{\,a}}{\cancel{5}\cancel{b^3}_{b^2}} \times \frac{^{2}\cancel{10}\cancel{b}}{3\cancel{a}}$

$= \frac{2a}{3b^2}$

◀ Cancel out the common factors 5, a and b.

(b) $\frac{3}{2a+4} \div \frac{6a}{5a+10}$

$= \frac{\cancel{3}}{2\cancel{(a+2)}} \times \frac{5\cancel{(a+2)}}{_{2}\cancel{6}a}$

$= \frac{5}{4a}$

◀ Cancel out the common factors 3 and $a+2$.

(b) Addition and subtraction

(i) If the denominators of the algebraic fractions involved are <u>the same</u>, we can directly perform addition or subtraction on their numerators and keep the denominator unchanged.

(ii) If the denominators are <u>different</u>, we should expand the algebraic fractions in order to make the denominators the same.

Example 3

Simplify the following expressions.

(a) $\frac{2}{3b}+\frac{4}{3b}$ **(b)** $\frac{1}{x-y}-\frac{1}{x+y}$

Solution

(a)
$$\begin{aligned}&\frac{2}{3b}+\frac{4}{3b}\\&=\frac{2+4}{3b}\\&=\frac{\cancel{6}^{\,2}}{\cancel{3}b}\\&=\underline{\underline{\frac{2}{b}}}\end{aligned}$$

(b)
$$\begin{aligned}&\frac{1}{x-y}-\frac{1}{x+y}\\&=\frac{x+y}{(x-y)(x+y)}-\frac{x-y}{(x-y)(x+y)}\\&=\frac{x+y-(x-y)}{(x-y)(x+y)}\\&=\frac{x+y-x+y}{(x-y)(x+y)}\\&=\underline{\underline{\frac{2y}{(x-y)(x+y)}}}\end{aligned}$$

◀ Expand both fractions.

$$\frac{1}{x-y}=\frac{1\times(x+y)}{(x-y)\times(x+y)}=\frac{x+y}{(x-y)(x+y)}$$
$$\frac{1}{x+y}=\frac{1\times(x-y)}{(x+y)\times(x-y)}=\frac{x-y}{(x-y)(x+y)}$$

3. Formulas

(a) Formulas and method of substitution

A formula is an equality relating two or more variables. By substitution, we can find the value of a variable in a formula when the values of other variables are known.

(b) Subject of a formula

In a formula, if a variable is a single variable on one side and it is expressed in terms of other variables, the variable is called the subject of the formula.

For example,

$$P=x+y+z$$

P is the subject

Example 4

Consider the formula $A=\frac{2h}{k}-1$.

(a) Make k the subject of the formula.

(b) If $A=5$ and $h=6$, find the value of k.

Solution

(a)
$$\begin{aligned}A&=\frac{2h}{k}-1\\A+1&=\frac{2h}{k}\\k&=\frac{2h}{A+1}\end{aligned}$$

(b) When $A=5$ and $h=6$,
$$\begin{aligned}k&=\frac{2\times 6}{5+1}\\&=\underline{\underline{2}}\end{aligned}$$

Chapter Summary

1. Concept of errors in measurement

(a) The actual value of a measure (like length, weight, capacity, etc) in all real measurements are unknown.

(b) No matter how precise the measuring tools are, the measured values obtained are regarded as the approximate values of the measures. In other words, errors are unavoidable in measurements.

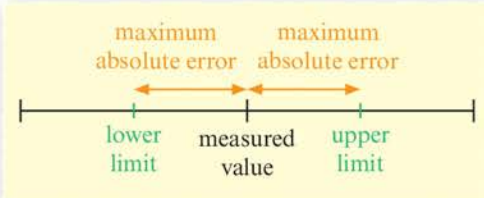
2. Maximum absolute errors

(a) The absolute error is the difference between the actual value and the measured value. It is always positive.

(b) In measurement, the actual value and the absolute error cannot be found. However, the largest possible error of the measured value, which is called the maximum absolute error, can be determined.

(i) Maximum absolute error

$= \frac{1}{2} \times$ scale interval of the measuring tool



(ii) Lower limit of the actual value

= measured value − maximum absolute error

(iii) Upper limit of the actual value

= measured value + maximum absolute error

Note: The range of the actual value is:

Lower limit $\leq$ the actual value $<$ upper limit

Example 1

The weight of a bar of chocolate is measured as 25.5 g and the scale interval of the measuring tool is 0.1 g.

(a) Find the maximum absolute error of the measurement.

(b) Find the range of the actual weight of the bar of chocolate.

Solution

(a) Maximum absolute error $= \frac{1}{2} \times 0.1\text{ g}$

$= \underline{\underline{0.05\text{ g}}}$

(b) Lower limit of the actual weight $= (25.5 - 0.05)\text{ g}$

$= 25.45\text{ g}$

Upper limit of the actual weight $= (25.5 + 0.05)\text{ g}$

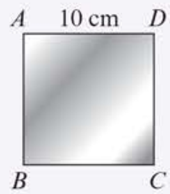
$= 25.55\text{ g}$

$\therefore$ The range of the actual weight of the bar of chocolate is:

$25.45\text{ g} \leq$ the actual weight $< 25.55\text{ g}$

Example 2

In the figure, $ABCD$ is a square metal sheet. The length of a side of the metal sheet is measured as 10 cm, correct to the nearest cm.



(a) Find the maximum absolute error of the measurement.

(b) Find the least possible area of the metal sheet.

Solution

(a) $$\text{Maximum absolute error} = \frac{1}{2} \times 1 \text{ cm} = \underline{\underline{0.5 \text{ cm}}}$$

(b) $$\text{Lower limit of the actual length} = (10 - 0.5) \text{ cm} = 9.5 \text{ cm}$$

$$\therefore \quad \text{Least possible area} = 9.5 \times 9.5 \text{ cm}^2 = \underline{\underline{90.25 \text{ cm}^2}}$$

3. Relative errors and percentage errors

(a) $$\text{Relative error} = \frac{\text{maximum absolute error}}{\text{measured value}}$$

(b) $$\text{Percentage error} = \text{relative error} \times 100\%$$

Note: The smaller the relative error (or percentage error) is, the more accurate the measured value will be.

Example 3

The weight of a girl is measured as 42 kg, correct to the nearest 2 kg.

(a) Find the relative error of the measured weight.

(b) Find the percentage error of the measured weight.

(Give your answers correct to 3 significant figures.)

Solution

(a) $$\text{Maximum absolute error} = \frac{1}{2} \times 2 \text{ kg} = 1 \text{ kg}$$

$$\text{Relative error} = \frac{1 \text{ kg}}{42 \text{ kg}} = \underline{\underline{0.0238}} \text{ (cor. to 3 sig. fig.)}$$

(b) $$\text{Percentage error} = \frac{1 \text{ kg}}{42 \text{ kg}} \times 100\% = \underline{\underline{2.38\%}} \text{ (cor. to 3 sig. fig.)}$$