Chapter-2 Functions

04 January 2021 08:30

Functions or Mappings:

Let A be the set of 30 students and B be the set of 40 chairs in a class room. The correspondence between the set A and set B is "student sitting on a chair." Then the correspondence f from A to B is a function or a mapping if and only if
(1) Every student is sitting on a chair.
(2) No student is sitting on two different chairs.

If these conditions are satisfied, f is called a function or mapping and it is denoted by f: A → B

Remarks: (1) If one student is standing, then *f* cannot be a function.
(2) If one student is occupying two different chairs still *f* cannot be a function. i.e., *f* cannot be of the type one element is corresponding to many elements.

- ❖ Function deals with linking pair of elements from two sets and then introduce relations between the two elements in the pair.
- ❖ The function is a special relation from one set to another set, in which every element of first set is in relation (uniquely) with the elements of another set.
- ❖ Practically in every day of our lives, we pair the members of two sets of numbers. For example,
  - Each hour of the day is paired with the local temperature reading by T.V. Station's weatherman,
  - A teacher often pairs each set of score with the number of students receiving that score to see more clearly how well the class has understood the lesson.

- ❖ **Definition (Functions):**

  Let A and B be two non-empty sets. Then a function or mapping *f* from the set A to the set B is a rule which assigns to each element $a \in A$ to unique element $b \in B$.
  We say that f maps element a of set A to element b of set B and that f maps set A to set B. The notation denote that *f* maps a to b is $f(a) = b$ or $(a, b) \in f$.

Remarks: *f* is well defined if $f(a_1) = b$ and $f(a_1) = c \Rightarrow b = c$

Note: If $\mathbf{n(A) = m}$ & $\mathbf{n(B) = n}$, then we can create **nm** different functions from A to B.

⊙ Let A = {1, 2, 3} and B = {a, b, c, d, e}, then
- $f = \{(1, a), (2, b), (3, c)\}$ is function from A to B.(OR $f\colon A \to B, f(1) = a, f(2) = b, f(3) = c$).
- $f = \{(1, a), (2, b), (3, b)\}$ is function from A to B.(OR $f\colon A \to B, f(1) = a, f(2) = b, f(3) = b$).
- $f = \{(1, a), (2, a), (3, a)\}$ is function from A to B.(OR $f\colon A \to B, f(1) = a, f(2) = a, f(3) = a$),

- ❖ **Representation by Diagram:**

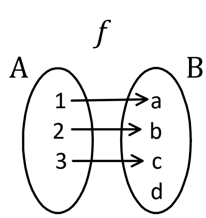
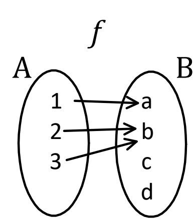
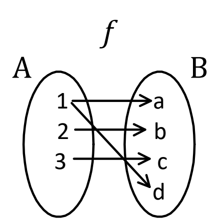


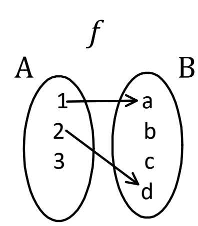
Figure (1)



Figure(2)



Figure(3)



Figure(4)

Figure (1) | Figure(2) | Figure(3) | Figure(4)

Let the interior of the two closed areas represented the sets A and B. The mapping of function $f: A \to B$ is represented by means of arc of lines joining the points representing the elements of A to the elements of B.

(1) Every $x \in A$ is joined to some $y \in B$. (Figure (1))
(2) Two or more points in A may be joined to the same point B. (Figure (2))
(3) For mapping, two or more points of B cannot be joined to the same point in A.

Here, Figure(3) is not a function as, $f(1) = a$ and $f(1) = d$ but $a \neq d$.
Also, Figure(4) is not a function as 3 from set A is not in correspondence with any element of set B.

Note: (1) Any function from $\mathbb{R}$ to $\mathbb{R}$ is called real function.
(2)A program written in a high-level language is mapped into a machine language by a compiler. Similarly, the output from a computer is a function of its input.

❖ **Definition (Image, Domain, Co-domain and Range of a Function):**

If $f: A \to B$ is a function from A to B, then
(1) for $f(a) = b$, element *b* of B is called f image of element a of A and elememt a is called preimage of *b.*
(2) Set A is known a domain of the function *f.*
(3) Set B is known as co-domain of the function *f.*
(4) Range of $(f) = \{b; b \in$ B and $f(a) = b$, for some $a \in A\}$. In other words, range of f is the set of all images of the elements of set A under *f.*

Example: If $A = \{x, y, z\}$, $B = \{a, b, c, d\}$, decide whether or not the following are functions from A to B. If they are functions, give the range of each; if not, tell why?

$(a) f = \{(x, a), (y, b), (z, c)\}$ $(b) f = \{(x, a), (y, c), (z, b), (x, c)\}$
$(c) f = \{(x, d), (y, b)\}$ $(d) f = \{(x, a), (y, b), (z, d)\}$
$(e) f = \{(y, a), (y, b), (y, c), (y, d)\}$ $(f) f = \{(x, b), (y, c), (z, d)\}$

Sol: $f: A \to B$

① $f(x) = a$, $f(y) = b$, $f(z) = c$
Here, f is a function from A to B
Range of $f = \{a, b, c\}$

② $f(x) = a$, $f(y) = c$, $f(z) = b$, $f(x) = c$
Here, $f(x) = a$ and $f(x) = c$ but $a \neq c$.
so, f is not a function.

③ Here, $z \in A$ does not correspond to any element of B
f is not a function

(4) Here, f is a function and range of $f = \{a, b, d\}$

(5) Here, image of y is not unique and so, f is not a function

(6) Here, f is a function and range of $f = \{b, c, d\}$

✱ Equality of two functions :-

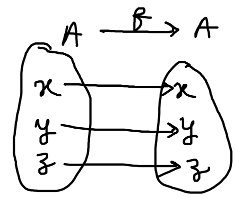
Two functions f and g from set A to set B are said to be equal functions iff $f(x) = g(x)$, $\forall x \in A$. If $\exists$ at least one element $y \in A$ s.t. $f(y) \neq g(y)$ then mapping f and g are not equal.

✱ Identity Function :-

Let A be any set and f be any function defined on A. i.e. $f : A \to A$ and $f(x) = x$, $\forall x \in A$ then f is known as identity mapping or identity function and generally denoted by I_A

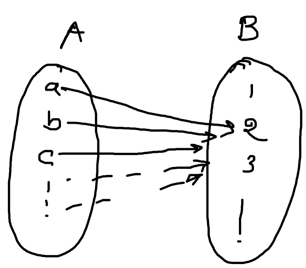
Ex: Let $A = \{x, y, z\}$

$f : A \to A$, $f(x) = x$



✱ Constant Function :-

The function defined from set A to set B s.t. $f(a) = b$, $\forall a \in A$ then f is called constant function



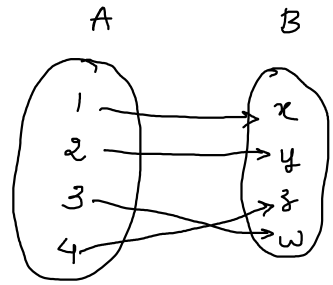
⇒ Types of Functions or Mappings :-

① One to one mapping (Injective mapping)

A function $f : A \to B$ is said to be one to one mapping or one to one correspondence or injective function or univalent function if $\forall\, a_1, a_2 \in A$

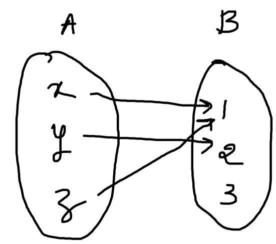
$$f(a_1) = f(a_2) \Rightarrow a_1 = a_2 \quad \text{or}$$

$$\text{if } a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$$



✱ Many to one mapping :-

Let $f : A \to B$ then f is many to one function if $a_1 \neq a_2$ but $f(a_1) = f(a_2)$. i.e. f is many to one function if two or more than two distinct elements of A have the same image in B under f

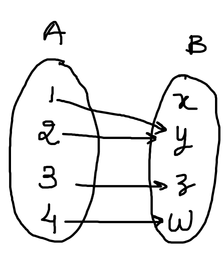


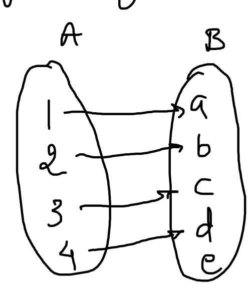
Note:- ① Identity function is one to one function

② Constant function is many to one function

✱ Into Mapping :-

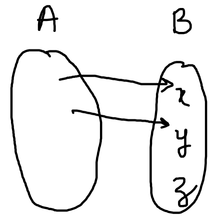
If $f : A \to B$ then f is into mapping if $\exists$ atleast one element in B which is not f-image of any element of A.





* Onto mapping or Surjective mapping :-

A function $f : A \to B$ is said to be onto mapping if each element of B is the f-image of at least one element of set A.

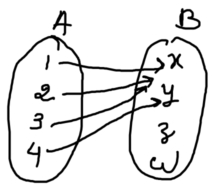


Note :- If $f : A \to B$ is onto mapping then range of f = co-domain set = B

In this case, we can write $f(A) = B$.

* Many one into mapping :-

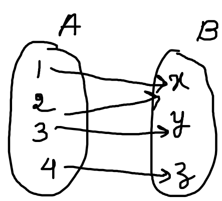
If $f : A \to B$ is many one mapping and also f is into mapping then f is said to be many one into mapping



$f(1) = x = f(2) = f(3)$

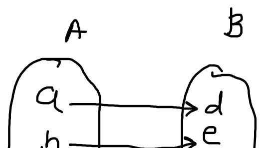
* Many one Onto mapping :-

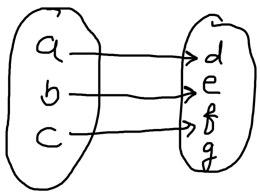
If $f : A \to B$ is many one mapping and also f is onto then f is said to be many one onto mapping



* One-one Into Mapping :-

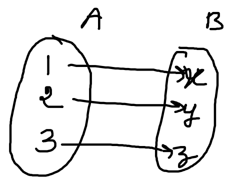
If $f : A \to B$ is one-one and also into mapping then f is said to be one-one into mapping





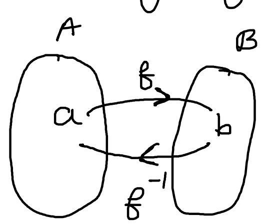
* **One-one onto mapping or Bijective mapping (Bijection) :-**

A mapping $f : A \to B$ which is one-one as well as onto is called one-one onto mapping or Bijective mapping



* **Inverse mapping :-**

If $f : A \to B$ is an one-one onto (Bijection) mapping then the mapping $f^{-1} : B \to A$, which associates to element $b \in B$, to the unique element $a \in A$ such that $f(a) = b$ is called inverse of mapping $f : A \to B$



If $f(a) = b$ then $f^{-1}(b) = a$

$\Rightarrow$ **Conditions for a function to be invertible :-**

① If $f : A \to B$ is one-one and onto then only f^{-1} exists and f^{-1} is also one-one and onto mapping

② In case of one-one into, many one into and many one onto inverse does not exist.

* **Theorem :-** If $f : A \to B$ is one-one onto function then show that inverse of f ; $f^{-1} : B \to A$ is also one-one onto function.

Proof :- Here given that f is one-one and onto

(i) $\forall a_1, a_2 \in A$
$f(a_1) = f(a_2) \Rightarrow a_1 = a_2$

(ii) $\forall b \in B \; \exists \; a \in A$
$\ni f(a) = b$

Let $f(a_1) = b_1$ and $f(a_2) = b_2$

$\Rightarrow f^{-1}(f(a_1)) = f^{-1}(b_1)$ and $f^{-1}(f(a_2)) = f^{-1}(b_2)$

$\Rightarrow a_1 = f^{-1}(b_1)$ and $a_2 = f^{-1}(b_2)$

Now, $f^{-1}(b_1) = f^{-1}(b_2) \Rightarrow a_1 = a_2 \Rightarrow f(a_1) = f(a_2)$ ($\because f$ is well defined)

$\Rightarrow b_1 = b_2$

Thus, f^{-1} is one-one function.

Again, $\forall b \in B \; \exists \; a \in A \ni f(a) = b$

$\Rightarrow f^{-1}(f(a)) = f^{-1}(b) \Rightarrow a = f^{-1}(b)$

$\Rightarrow f^{-1}(b) = a$

Hence, $\forall a \in A \; \exists \; b \in B \ni f^{-1}(b) = a$

Hence, f^{-1} is onto function.

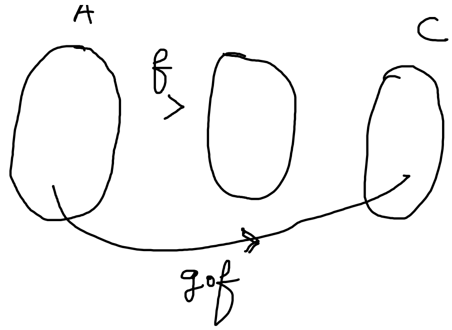
Note:- $(f^{-1})^{-1} = f$

* Composition of functions or Product of functions :-

Let A, B, C be three sets and $f : A \to B$ and $g : B \to C$ then the composition of two functions f and g can be defined as $g \circ f : A \to C$, $g \circ f(a) = g(f(a))$

$= g(b)$

The domain of $g \circ f$ is set A and



The domain of $g \circ f$ is set A and co-domain of $g \circ f$ is set C.

Ex:- Let $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 2x^3+5$

$g: \mathbb{R} \to \mathbb{R}$, $g(x) = \cos x$ then find $g \circ f$ & $f \circ g$

$g \circ f(x) = g(f(x)) = g(2x^3+5) = \cos(2x^3+5)$.

$f \circ g(x) = f(g(x)) = f(\cos x) = 2\cos^3 x + 5$

$f \circ g \neq g \circ f$

Ex:- Let $f: R \to R$, $f(x) = 2x^3+5$, $g: \mathbb{R} \to \mathbb{R}$, $g(x) = \cos x$,

$h: R \to R$, $h(x) = x^3 - 1$

Find $h \circ (g \circ f)$ and $(h \circ g) \circ f$. Are they equal?

Sol. $h \circ (g \circ f)(x) = h \circ [g(f(x))] = h \circ [g(2x^3+5)]$

$= h \circ [\cos(2x^3+5)] = h(\cos(2x^3+5))$

$= [\cos(2x^3+5)]^3 - 1 \rightarrow (1)$

$(h \circ g) \circ f(x) = (h \circ g)(2x^3+5) = h[g(2x^3+5)]$

$= h[\cos(2x^3+5)] = [\cos(2x^3+5)]^3 - 1 \rightarrow (2)$

From (1) & (2), $h \circ (g \circ f) = (h \circ g) \circ f$. Composition of function satisfies associative law

Ex:- Let f and g be two functions defined by $f(x) = 2x+1$ and $g(x) = x^2 - 2$. Then find (1) $g \circ f(4)$ (2) $f \circ g(4)$ (3) $g \circ f(a+2)$ (4) $f \circ g(a+2)$

Sol:- ① $g \circ f(4) = g(f(4)) = g(2(4)+1) = g(9) = (9)^2 - 2 = 81 - 2$
$= 79$

② $f \circ g(4) = f(g(4)) = f((4)^2 - 2) = f(14) = 2(14)+1 = 29$

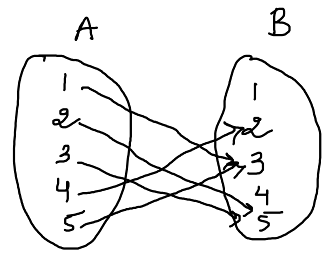
③ $g \circ f(a+2) = g(f(a+2)) = g(2(a+2)+1) = g(2a+5)$
$= (2a+5)^2 - 2$
$= 4a^2 + 20a + 25 - 2$
$= 4a^2 + 20a + 23$

④ $f \circ g(a+2) = f(g(a+2)) = f((a+2)^2 - 2) = f(a^2+4a+4-2)$
$= f(a^2+4a+2)$
$= 2(a^2+4a+2)+1 = 2a^2+8a+4+1$
$= 2a^2+8a+5$

Ex:- Represent the given function in:
(i) Graphical (ii) Tabular form and (iii) Matrix form

$f = \{(1,3), (2,5), (3,5), (4,2), (5,3)\}$

Sol:- ① Graphical representation



② Tabular form

f	1	2	3	4	5
1			✓		
2					✓
3					✓
4		✓			
5			✓		

③ matrix form

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Ex:- Let $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$, $\forall x \in \mathbb{R}$. Find:
① $g \circ f$, ② $f \circ g$, ③ $f \circ f$, ④ $h \circ g$, ⑤ $g \circ g$, ⑥ $f \circ h$,

⑦ $f \circ h \circ g$, ⑧ $h \circ f$

Sol:- ① $g \circ f(x) = g(f(x)) = g(x+2) = (x+2)-2 = x$

② $f \circ g(x) = f(g(x)) = f(x-2) = (x-2)+2 = x$

$\Big\}\ f \circ g = g \circ f$

③ $f \circ f(x) = f(f(x)) = f(x+2) = (x+2)+2 = x+4$

④ $h \circ g(x) = h(g(x)) = h(x-2) = 3(x-2) = 3x-6$

⑤ $g \circ g(x) = g(g(x)) = g(x-2) = (x-2)-2 = x-4$

⑥ $f \circ h(x) = f(h(x)) = f(3x) = 3x+2$

⑦ $f \circ h \circ g(x) = f(h(g(x))) = f(h(x-2)) = f\big(3(x-2)\big) = f(3x-6)$

$= (3x-6)+2 = 3x-4$

⑧ $h \circ f(x) = h(f(x)) = h(x+2) = 3(x+2) = 3x+6$

✱ Some properties of composition of functions:-

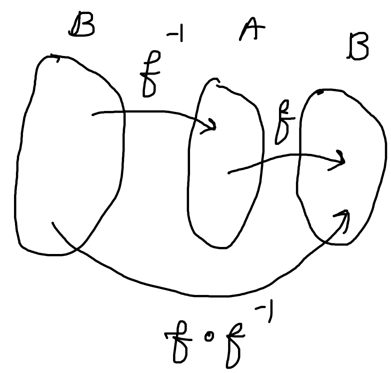
Theorem-1 :- If $f : A \to B$ be a one-one onto funⁿ. then

$f \circ f^{-1} = I_B$ and $f^{-1} \circ f = I_A$

where I_B and I_A are identity funⁿ. on set B and A respectively

Proof.- $f : A \to B$, one-one and onto

$\Rightarrow f^{-1}$ exists and $f^{-1} : B \to A$, one-one and onto

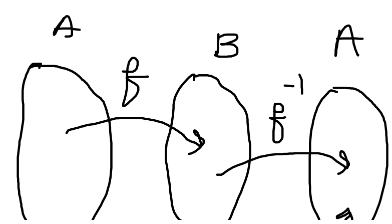


Now, $f \circ f^{-1} : B \to B$

$f \circ f^{-1}(b) = f(f^{-1}(b))$

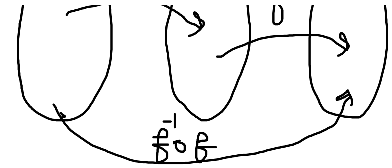
$= f(a) = b$, $\forall\, b \in B$

Hence, $f \circ f^{-1} = I_B$



Let $f^{-1} \circ f : A \to A$

$f^{-1} \circ f(a) = f^{-1}(f(a))$

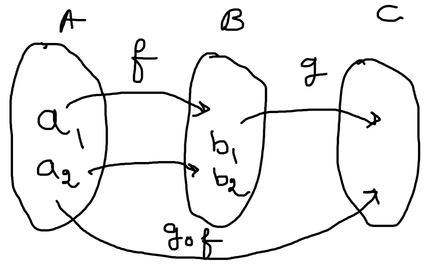


$$f^{-1}of(a) = f^{-1}(f(a))$$
$$= f^{-1}(b) = a \quad , \forall a \in A$$

Hence, $f^{-1}of = I_A$

Theorem-2 :- If $f: A \to B$ and $g: B \to C$ be two one-one onto functions then $gof : A \to C$ is also one-one onto and $(gof)^{-1} = f^{-1} o g^{-1}$

Proof:-



Let $gof(a_1) = gof(a_2) \Rightarrow g(f(a_1)) = g(f(a_2))$

$$\Rightarrow g(b_1) = g(b_2)$$
$$\Rightarrow b_1 = b_2 \quad (\because g \text{ is one-one})$$
$$\Rightarrow f(a_1) = f(a_2)$$
$$\Rightarrow a_1 = a_2 \quad (\because f \text{ is one-one})$$

So, gof is one-one function

Since g is onto mapping from B to C, $\exists\, b \in B \ni g(b) = c, \forall c \in C$

Also f is onto mapping from A to B, $\exists\, a \in A \ni f(a) = b, \forall b \in B$

$\forall c \in C, \quad gof(a) = g(f(a)) = g(b) = c$

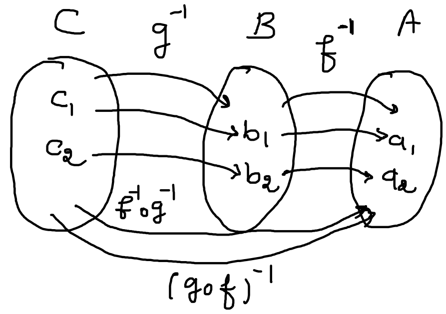
Hence, gof is onto function Therefore, gof is one-one and onto function and so, inverse of gof is exist.

$(gof)^{-1} : C \to A$ is also one-one and onto

C g^{-1} B f^{-1} A

-1 .

$(g \circ f) : C \to A$ is also one-one and onto



$f^{-1} o g^{-1} : C \to A$

$f^{-1} : B \to A, \quad g^{-1} : C \to B$

$$
\begin{aligned}
(g \circ f)(a) &= g(f(a)) \\
&= g(b) \\
&= c \quad \Rightarrow \quad (g o f)(a) = c
\end{aligned}
$$

$$\Rightarrow \underline{(gof)^{-1}} \left[(g \circ f)(a) \right] = (gof)^{-1}(c)$$

$$\Rightarrow a = (gof)^{-1}(c), \quad \forall c \in C \quad \to (1)$$

$$(f^{-1} \circ g^{-1})(c) = f^{-1}(g^{-1}(c)) = f^{-1}(b) = a, \quad \forall c \in C$$

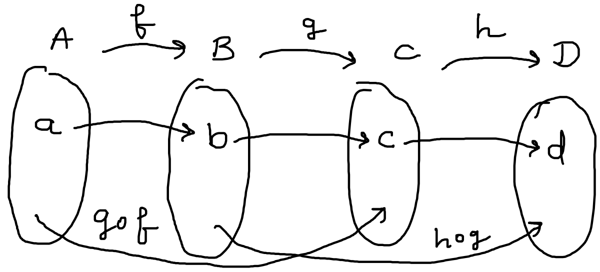
$$\therefore (f^{-1} o g^{-1})(c) = a \quad \to (2)$$

Hence, $(g o f)^{-1} = f^{-1} o g^{-1}$

Theorem-3:- composition of function satisfies associative property

Let $f : A \to B$, $g : B \to C$ and $h : C \to D$ then $(h o g) o f = h o (g o f)$

Proof:-



$h o g : B \to D, \quad g o f : A \to C$

$(h o g) o f : A \to D, \quad h o (g o f) = A \to D$

$(h\circ g)\circ f\,(a) = h\circ g\,[f(a)] = h\circ g\,(b) = h\,[g(b)] = h(c) = d, \quad \forall a \in A \rightarrow (1)$

$h\circ(g\circ f)\,(a) = h\circ[g(f(a))] = h\circ[g(b)] = h(c) = d, \forall a \in A \rightarrow (2)$

From (1) & (2), $h\circ(g\circ f) = (h\circ g)\circ f$

* Comparability of sets :-

If A and B are two sets and if $A \subset B$ or $B \subset A$, then A and B are said to be comparable. If A is not subset of B and B is not subset of A i.e. $A \not\subset B$ and $B \not\subset A$ then A and B are said to be incomparable sets

Ex.:- (1) $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

(2) Let $A = \{1, 2, 3\}$ and $B = \{x, y, z\}$ then $A \not\subset B$ and $B \not\subset A$

so, A and B are incomparable sets

* one-one correspondence :-

Let A and B be two sets. If $\exists$ a function $f : A \to B$ such that f is one-one and onto function then it is known as A and B in one-one correspondence and it is denoted by $A \leftrightarrow B$.

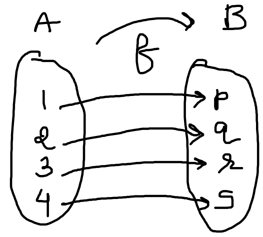
If A and B are in one-one correspondence then $|A| = |B|$.

* Cardinally Equivalent Sets .-

Two sets A and B whose numbers can be placed in one-one correspondence are said to be cardinally equivalent or equinumerous

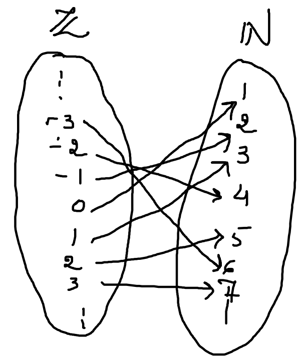
In other words, if $\exists$ a funⁿ. $f : A \to B$ which is bijective then A and B are equivalent. They are denoted by $A \sim B$.

Ex:- $A = \{1, 2, 3, 4\}$, $B = \{p, q, r, s\}$



So, $f : A \to B$ is bijection. so, $A \sim B$ i.e. $|A| = 4 = |B|$.

Ex:- Is $\mathbb{N} \sim \mathbb{Z}$?



sol. $f : \mathbb{Z} \to \mathbb{N}$, $f(n) = \begin{cases} -2n \ ; & n < 0 \\ 2n+1 \ ; & n \geq 0 \end{cases}$

Here, f is one-one and onto

$f(-3) = 6$, $f(-2) = 4$,
$f(-1) = 2$, $f(0) = 1$
$f(1) = 3$, $f(2) = 5$, $f(3) = 7$

suppose $a, b \in \mathbb{Z}$

$f(a) = f(b)$, $a, b \geq 0$
$a, b < 0$

① if $a, b \geq 0 \Rightarrow f(a) = f(b) \Rightarrow -2a = -2b \Rightarrow a = b$

② if $a, b < 0 \Rightarrow f(a) = f(b) \Rightarrow 2a+1 = 2b+1 \Rightarrow a = b$

Thus, f is one-one function

[suppose $a \neq b \Rightarrow 2a \neq 2b \Rightarrow -2a \neq -2b \Rightarrow f(a) \neq f(b)$]

Suppose for $b \in \mathbb{N}$ $\exists$ $a \in \mathbb{Z}$ $\ni$ $f(a) = b$

① if $a < 0$ then $f(a) = b \Rightarrow -2a = b \Rightarrow a = \frac{-b}{2}$

Thus, $f(a) = f\left(-\frac{b}{2}\right) = -2\left(-\frac{b}{2}\right) = b$

② if $a \geq 0$ then $f(a) = b \Rightarrow 2a+1 = b \Rightarrow a = \frac{b-1}{2}$

Thus, $f(a) = f\left(\frac{b-1}{2}\right) = 2\left(\frac{b-1}{2}\right)+1 = b$

Thus, f is onto function

Hence, $\mathbb{N} \sim \mathbb{Z}$ $\quad [\,|\mathbb{N}| = |\mathbb{Z}|\,]$

Ex:- Is $\mathbb{Z}\times\mathbb{Z} \sim \mathbb{N}\times\mathbb{N}$?

Sol:- $g((a,b)) = (f(a), f(b))$, $f: \mathbb{Z}\to\mathbb{N}$, $f(a) = \begin{cases} -2a, & a<0 \\ 2a+1, & a\geq 0 \end{cases}$

This is one-one and onto function as f is one-one and onto

Ex:- Is $\mathbb{Z} \sim \mathbb{Q}$?

Sol:- Define $f: \mathbb{Q} \to T$, $T = \{(p,q) \,/\, p\in\mathbb{Z},\ q\in\mathbb{N},\ \gcd(p,q)=1\}$

$$f\left(\frac{p}{q}\right) = (p,q),\ \forall\, \frac{p}{q} \in \mathbb{Q}$$

Here, f is one-one and onto $\therefore\ \mathbb{Q} \sim T$

Now, $\mathbb{Z}\times\{1\} \subseteq T \subseteq \mathbb{Z}\times\mathbb{N}$

Now, $g: \mathbb{Z} \to \mathbb{Z}\times\{1\}$, $h: \mathbb{Z}\times\{1\} \to T$, $k: T \to \mathbb{Z}\times\mathbb{N}$

$g(n) = (n,1)$ $\quad h(n,1) = (n,1)$ $\quad k(a,b) \to (a, f(b))$

Here, g, h and k are one-one and onto function.

$\mathbb{Z} \sim \mathbb{Z}\times\{1\} \sim \mathbb{Z}\times\mathbb{N} \sim T$ $\quad (A\sim B,\ B\sim C \Rightarrow A\sim C)$

$\therefore\ \mathbb{Z}\sim T$ but $T\sim\mathbb{Q}$

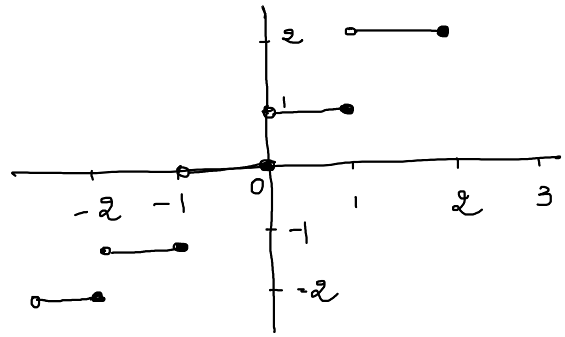
$\Rightarrow \mathbb{Z}\sim\mathbb{Q}$ $\quad [\,|\mathbb{Z}| = |\mathbb{Q}|\,]$

* Ceiling Function:-

✱ Ceiling Function:-

ceiling function is a function which maps x to the least integer greater than or equal to x and it is denoted by $\lceil x \rceil$. $f: \mathbb{R} \to \mathbb{Z}$, $f(x) = \lceil x \rceil$

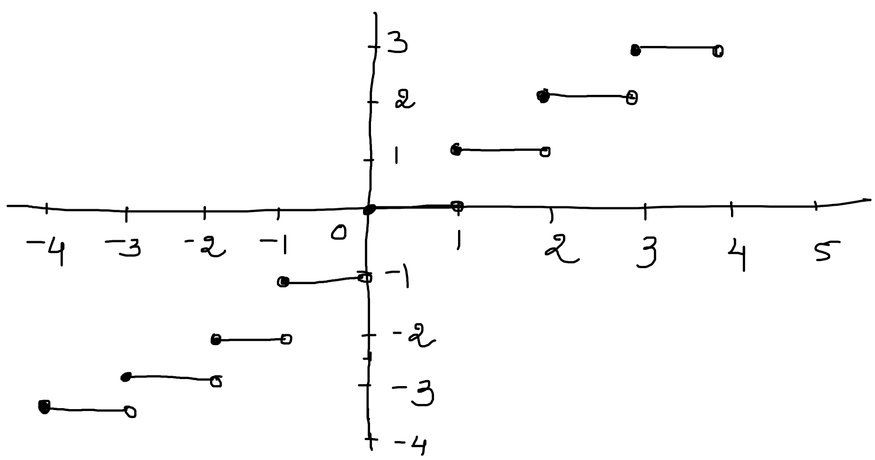
Eg:- $f(2.5) = \lceil 2.5 \rceil = 3$



✱ Floor Function:-

The floor function is the function that takes as a input a real number x and gives as output the greatest integer less than or equal to x. It is denoted by $\lfloor x \rfloor$

ex:- $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \lfloor x \rfloor$, $f(2.5) = 2$



✱ Integral Part or Integer part function:-

The integral part or integer part of number x is denoted by $[x]$ is defined as

$$[x] = \begin{cases} \lfloor x \rfloor ; & x \text{ is non negative} \\ \lceil x \rceil ; & \text{otherwise} \end{cases}$$

$[x]$ is defined:

$$[x] = \begin{cases} \lfloor x \rfloor \ ; & x \text{ is non negative} \\ \lceil x \rceil \ ; & \text{otherwise} \end{cases}$$

In words, this is the integer that has the largest absolute value less than or equal to the absolute value of x.

Fractional part function:-

The fractional part function is denoted by $\{x\}$ and is defined as $\{x\} = x - \lfloor x \rfloor$

Here, we can observe that $0 \le \{x\} < 1$

$f : \mathbb{R} \to [0,1)$, $f(x) = \{x\}$ then f is onto

Ex:-

x	$\lfloor x \rfloor$	$\lceil x \rceil$	$\{x\}$	$[x]$
2	2	2	0	2
2.4	2	3	0.4	2
2.9	2	3	0.9	2
-2.7	-3	-2	0.3	-2
-2	-2	-2	0	-2