Discrete Mathematics

24 December 2020 09:00

What is Discrete Mathematics?

Discrete Mathematics is mathematics that deals with discrete objects. Discrete objects are those which are those separated from (not connected or distinct from) each other. Discrete Mathematics is also called as finite mathematics. Integers, rational numbers, people, house, etc. are all discrete objects. On the other hand real numbers are not discrete as between any two different real numbers there is another real number different from each of them. So they are packed without any gaps and can not be separated from their immediate neighbors. In that sense they are not discrete.

Bachelor of Engineering
Subject Code: 3140708
Semester – IV
Subject Name: Discrete Mathematics

Type of course: Undergraduate

Prerequisite : Algebra, Logic

Rationale : This course introduces the basic concepts of discrete mathematics in the field of computer science. It covers sets, logic, functions, relations, graph theory and algebraic structures. These basic concepts of sets, logic functions and graph theory are applied to Boolean Algebra and logic networks, while the advanced concepts of functions and algebraic structures are applied to finite state machines and coding theory.

Teaching and Examination Scheme:

Teaching Scheme			Credits	Examination Marks				Total
L	T	P	C	Theory Marks		Practical Marks		
				ESE(E)	PA (M)	ESE(V)	PA(I)	
3	2	0	5	70	30	0	0	100

Contents:

Sr. No.	Content	Total Hrs.	% weightage
01	**Set Theory**: Basic Concepts of Set Theory: Definitions, Inclusion, Equality of Sets, Cartesian product, The Power Set, Some operations on Sets, Venn Diagrams, Some Basic Set Identities **Functions**: Introduction & definition, Co-domain, range, image, value of a function; Examples, surjective, injective, bijective; examples; Composition of functions, examples; Inverse function, Identity map, condition of a function to be invertible, examples; Inverse of composite functions, Properties of Composition of functions; **Counting:** The Basics of Counting, The Pigeonhole Principle, Permutations and Combinations, Binomial Coefficients, Generalized Permutations and Combinations, Generating Permutations and Combinations	**06**	**12%**
02	**Propositional Logic**: Definition, Statements & Notation, Truth Values, Connectives, Statement Formulas & Truth Tables, Well-formed Formulas, Tautologies, Equivalence of Formulas, Duality Law, Tautological Implications, Examples **Predicate Logic:** Definition of Predicates; Statement functions, Variables, Quantifiers, Predicate Formulas, Free & Bound Variables; The Universe of Discourse, Examples, Valid Formulas & Equivalences, Examples	**06**	**13%**
03	**Relations**: Definition, Binary Relation, Representation, Domain, Range, Universal Relation, Void Relation, Union, Intersection, and Complement Operations on Relations, Properties of Binary Relations in a Set: Reflexive, Symmetric, Transitive, Anti-symmetric Relations, Relation Matrix and Graph of a Relation; Partition and Covering of a Set, Equivalence Relation, Equivalence Classes, Compatibility Relation, Maximum Compatibility Block, Composite Relation, Converse of a Relation, Transitive Closure of a Relation R in Set X **Partial Ordering**: Definition, Examples, Simple or Linear Ordering, Totally Ordered Set (Chain), Frequently Used Partially Ordered Relations, Representation of Partially Ordered Sets, Hesse Diagrams, Least & Greatest Members, Minimal & Maximal Members, Least Upper Bound (Supremum), Greatest Lower Bound (infimum), Well-ordered Partially Ordered Sets (Posets). Lattice as Posets, complete, distributive	**10**	**25%**

	modular and complemented lattices Boolean and pseudo Boolean lattices. (Definitions and simple examples only) **Recurrence Relation:** Introduction, Recursion, Recurrence Relation, Solving, Recurrence Relation		
04	**Algebraic Structures:** Algebraic structures with one binary operation- Semigroup, Monoid, Group, Subgroup, normal subgroup, group Permutations, Coset, homomorphic subgroups, Lagrange's theorem, Congruence relation and quotient structures. Algebraic structures (Definitions and simple examples only) with two binary operation- Ring, Integral domain and field.	**10**	**25%**
05	**Graphs:** Introduction, definition, examples; Nodes, edges, adjacent nodes, directed and undirected edge, Directed graph, undirected graph, examples; Initiating and terminating nodes, Loop (sling), Distinct edges, Parallel edges, Multi-graph, simple graph, weighted graphs, examples, Isolated nodes, Null graph; Isomorphic graphs, examples; Degree, Indegree, out-degree, total degree of a node, examples; Subgraphs: definition, examples; Converse (reversal or directional dual) of a digraph, examples; Path: Definition, Paths of a given graph, length of path, examples; Simple path (edge simple), elementary path (node simple), examples; Cycle (circuit), elementary cycle, examples; **Reachability:** Definition, geodesic, distance, examples; Properties of reachability, the triangle inequality; Reachable set of a given node, examples, Node base, examples; **Connectedness:** Definition, weakly connected, strongly connected, unilaterally connected, examples; Strong, weak, and unilateral components of a graph, examples, Applications to represent Resource allocation status of an operating system, and detection and correction of deadlocks; Matrix representation of graph: Definition, Adjacency matrix, boolean (or bit) matrix, examples; Determine number of paths of length n through Adjacency matrix, examples; Path (Reachability) matrix of a graph, examples; Warshall's algorithm to produce Path matrix, Flowchart. **Trees:** Definition, branch nodes, leaf (terminal) nodes, root, examples; Different representations of a tree, examples; Binary tree, m-ary tree, Full (or complete) binary tree, examples; Converting any m-ary tree to a binary tree, examples; Representation of a binary tree: Linked-list; Tree traversal: Pre-order, in-order, post-order traversal, examples, algorithms; Applications of List structures and graphs	**10**	**25%**

Reference Books:

1. J. P. Tremblay and R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, Tata McGraw-Hill,1997.
2. S. Lipschutz and M. L. Lipson, Schaum's Outline of Theory and Problems of Discrete Mathematics, 2^{nd} Ed., Tata McGraw-Hill,1999.
3. K. H. Rosen, Discrete Mathematics and its applications, Tata McGraw-Hill, 6th Ed., 2007.
4. David Liben-Nowell, Discrete Mathematics for Computer Science, Wiley publication, July 2017.
5. Eric Gossett, Discrete Mathematics with Proof, 2nd Edition,Wiley publication, July 2009.

Suggested Specification table with Marks (Theory):					
R Level	U Level	A Level	N Level	E Level	C Level
10	20	20	10	10	

Legends: R: Remembrance; U: Understanding; A: Application, N: Analyze and E: Evaluate C: Create and above Levels (Revised Bloom's Taxonomy).

Course Outcomes:

After Completion of this course students will be able

Sr. No.	Course Outcomes	Weightage in %
1	Understand the basic principles of sets and operations in sets and apply counting principles to determine probabilities, domain and range of a function, identify one-to- one functions, perform the composition of functions and apply the properties of functions to application problems.	12%
2	Write an argument using logical notation and determine if the argument is or is not valid. To simplify and evaluate basic logic statements including compound statements, implications, inverses, converses, and contra positives using truth tables and the properties of logic. To express a logic sentence in terms of predicates, quantifiers, and logical connectives.	13%
3	Apply relations and to determine their properties. Be familiar with recurrence relations	25%
4	Use the properties of algebraic structures.	25%
5	Interpret different traversal methods for trees and graphs. Model problems in Computer Science using graphs and trees.	25%

List of Open Source Software/learning website: NPTEL Discrete Mathematics lectures

CO1	Employ basic principles of set theory and functions for problem solution.

CO2	Solve logical statements in terms of predicates, quantifiers and connectives.
CO3	Determine the types of relation using its properties.
CO4	Exhibit the properties of algebraic structures relevant to computer science.
CO5	Solve the problems of computer science using graph theory.

Chapter-1 Set Theory

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❖ INTRODUCTION

- ✓ The theory of set was developed by German mathematician Georg Cantor (1845-1918). He first encountered sets while working on 'problems on trigonometric series'.
- ✓ The concept of set serves as a fundamental part of the present day mathematics. Today this concept is being used in almost every branch of mathematics. Sets are used to define the concepts of relation and function. The study of geometry, sequences, probability, etc. requires knowledge of sets. In this unit, we discuss basic definitions and operation involving sets.

❖ SET

- ✓ A set is collection of well-defined objects.
- ✓ Each of the objects in the set is called an element of the set.
- ✓ Elements of a set are usually denoted by lower case letter (a, b, c …). While sets are denoted by capital letters (A, B, C …).
- ✓ The symbol '∈ (is belongs to)' indicates the membership in a set. While the symbol '∉ (is not belongs to)' is used to indicate that an element is not in the set.
- ✓ For example, if A = {1, 2, 3, a, b} then we can write that a ∈ A, 1 ∈ A but 4 ∉ A.

Examples:
(1) Library is a set of books
(2) Classroom is a set of chairs
(3) Class is a set of students
(4) Family is a set of persons
(5) Set of all straight lines in a 2-D plane
(6) Set of all triangles in a 2-D plane

Some special sets:
- ❖ **N =set of all natural numbers = {1,2,3,4,…}**
- ❖ **Z = set of all integers = {…,-3,-2,-1,0,1,2,3,…}**
- ❖ **Q = set of all rational numbers =** $\{x\,;\ x = \frac{p}{q}, p, q \in Z \text{ and } q \neq 0\}$
- ❖ **R = set of all real numbers =** $\{x\,; x = a + b\sqrt{p}\,, \text{where } a\,, b \in Q \text{ and } p \in Q_+\}$
- ❖ **C = set of all complex numbers =** $\{\,x\,; x = a + ib, a, b \in R \text{ and } i = \sqrt{-1}\}$

❖ REPRESENTATION OF SETS

- ✓ **Listing method (or Tabular form or Roster method):** In listing method, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }. For example, the set of all natural numbers less than 6 is described in listing method as {1, 2, 3, 4, 5}.
- ✓ **Property method (or Set-builder form or Set Selector method):** In property method, all the elements of a set possess a single common property which is not possessed any element outside the set. A set S characterized by a property p may be written as $S = \{x\,; p(x)\}$
  For example, the set of all natural number between 0 & 6 described in property method as
  $$\{x;\ x \text{ is a natural number } \&\ 0 < x < 6\}$$

Ex:- ① $A = \{x \,/\, x^2-4=0\,;\ x \in \mathbb{R}\} = \{2,-2\}$

$x^2-4=0 \Rightarrow x^2=4 \Rightarrow x = \pm 2 \in \mathbb{R}$

② $B = \{x \,/\, 4 \le x \le 11\,;\ x \in \mathbb{N}\} = \{4,5,6,7,8,9,10,11\}$

$= \{x \,/\, 4 \le x \le 11\,;\ x \in \mathbb{R}\} = [4,11]$

③ $C = \{x \,/\, x^3+1=0\,;\ x \in \mathbb{R}\} = \{-1\}$

Subsets: Let A and B be two sets. If each elements of set A is an element of set B then A is called subset of set B and it is denoted by $A \subset B$

❖ SOME DEFINITIONS

- ✓ **Empty set** (null set) does not contain any element and It is denoted as **{ }** or **ϕ**.
- ✓ A set which contain at least one element is called **Non-empty set**.
- ✓ A set which contain exactly one element is called **Singleton set**.

- ✓ A set which contain finite number of elements is called **finite set**.
- ✓ A set which contain infinite number of elements is called **infinite set**.
- ✓ Two set A and B are said to be **equal** if they have exactly the same elements and we write A=B. Otherwise, the sets are said to be **unequal** and we write A≠B.
- ✓ In any discussion in set theory, there always happens to be a set that contains all set under consideration i.e. it is a super set of each of the given sets. Such a set is called the universal set and is denoted by U.

  Remark: if A is a subset of B then B is a superset of A.

❖ Ordered Pair:

Let A and B be two sets. Let $a \in A\ and\ b \in B$. Then ***(a,b)*** denotes ordered pair and ***a*** is known as forst co-ordinate of the ordered pair ***(a,b)*** and ***b*** is known as second co-ordinate of the ordered pair ***(a,b).***

❖ Cartesian Product:

Let A and B be two sets then $A \times B$ is called Cartesian product of A and B. $A \times B$ is a set of all distinct ordered pairs, in which first co-ordinate of ordered pair is from set A and second co-ordinate if from set B.

$$\boldsymbol{A \times B = \{(a,b); a \in A\ and\ b \in B\}}$$

Example: Let $\underline{A = \{1,2,3\}}$ and $\underline{B = \{a,b\}}$ then $A \times B = \{(a,b); a \in A\ and\ b \in B\}$

3 × 2 = 6 $\qquad = \{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$

Here, we can observe that Cartesian product is non commutative.

Similarly, if A, B, C are three sets then $A \times B \times C$ is a set of all distinct ordered triple.

$$\boldsymbol{A \times B \times C = \{(a,b,c); a \in A, b \in B\ and\ c \in C\}}$$

Also, in this way

$\boldsymbol{A_1 \times A_2 \times A_3 \times \cdots \times A_n = \{(a_1, a_2, a_3, \ldots, a_n); a_i \in A_i, \forall i = 1,2,3,\ldots,n\}}$

❖ Remarks:

(1) If A contains 3 elements and B contains 4 elements then $A \times B$ contains $3 \times 4 = 12$ elements.

In general, if A contains m elements and B contains n elements then $A \times B$ contains $m \times n = mn$ elements.

(2) $A \times B \times C = \{(a,b,c); a \in A, b \in B\ and\ c \in C\}$

$A \times (B \times C) = \{(a,(b,c)); a \in A, b \in B\ and\ c \in C\}$

$(A \times B) \times C = \{((a,b),c); a \in A, b \in B\ and\ c \in C\}$

Thus, all these $A \times B \times C, A \times (B \times C)\ and\ (A \times B) \times C$ are different.

Theorem: If A, B, C are sets then,

(1) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(2) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

(3) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(4) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Proof: (1) Let $(a,b) \in A \times (B \cap C)$

$\Leftrightarrow a \in A\ and\ b \in B \cap C$

$\Leftrightarrow a \in A\ and\ (b \in B\ and\ b \in C)$

$\Leftrightarrow (a \in A\ and\ b \in B)\ and\ (a \in A\ and\ b \in C)$

$\Leftrightarrow (a,b) \in A \times B\ and\ (a,b) \in A \times C$

$\Leftrightarrow (a,b) \in (A \times B) \cap (A \times C)$

Therefore, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(3) Let $(a,b) \in A \times (B \cup C)$

$\Leftrightarrow a \in A\ and\ b \in B \cup C$

$\Leftrightarrow a \in A\ and\ (b \in B\ or\ b \in C)$

$\Leftrightarrow (a \in A\ and\ b \in B)\ or\ (a \in A\ and\ b \in C)$

$\Leftrightarrow (a,b) \in A \times B\ or\ (a,b) \in A \times C$

$\Leftrightarrow (a,b) \in (A \times B) \cup (A \times C)$

Therefore, $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Remarks: (1) If $\mathbb{R}$ is the set of all real numbers then $\mathbb{R} \times \mathbb{R}$ we represent by $\mathbb{R}^2$ and $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$ by $\mathbb{R}^3$.

(2) $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$ is called the Euclidean plane.

⊙ **Definition (Class of sets or Family of sets):**

If the elements of a set are sets themselves, then such a set is called as class of sets or family of sets.

⊙ **Definition(Power Set):**

If S is any set, then set of all subsets of S is called power set of S and is denoted by $\boldsymbol{P(S)}$. Symbolically $P(S) = \{X : X \subseteq S\}$.

It means, if $X \in P(S) \Rightarrow X \subseteq S$. Further, $\phi \in P(S)$ & $S \in P(S)$.

For example, if $A = \{a, b, c\}$ then $P(A) = \{\phi, A, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$

Theorem :- If a finite set S has n elements, then power set of S has 2^n elements.

Proof :- let us form all the subsets of S. ϕ is a subset of every set. Taking 1 element at a time, ${}^nC_1$ subsets can be formed out of S. Similarly, taking 2 elements at a time, ${}^nC_2$ subsets will be formed.

In general, taking r elements at a time, ${}^nC_r$ subsets can be formed This way total number of subsets will be

$$= 1 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_r + \cdots + {}^nC_n .$$

$$= (1+1)^n \qquad \left(\because (1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_r x^r + \cdots + {}^nC_n x^n \right)$$

$$= 2^n$$