CS 241 Homework 1

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Problem 1. *On January 22nd 2020, I turned 22 years old. Prove that for any person, there is exactly one year in which they turn x years old on the x day of a month.*

Answer 1. Let

$$x \epsilon X = [1, 31] \cap Z = [1, 2, 3, \ldots, 31] \tag{1}$$

be the day of the month on which I was born. Let Let P(x)="I will turn x years old on day x" Suppose P(x) and P(y) are both true with x!=y. Then I have two diffrent birthdays, the x'th and the y'th, which is impossible. Therefore if P(x) is true then it is true for a unique x. My age in years takes on all possible values in X, therfore there exists x such that P(x) is true.

Problem 2. *Clearly the following proof must be incorrect, where and what is the error?*

Answer 1.

$$\frac{d}{dx}[x + x + x + \ldots + x] xtimes \tag{2}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \tag{3}$$

$$x + x + x + \ldots + x = \sum_{i=1}^{x} x \tag{4}$$

$$\lim_{h \to 0} = \frac{\sum_{i=1}^{x+h}(x + h) - \sum_{i=1}^{x}(x)}{h} \tag{5}$$

$$\lim_{h \to 0} = \frac{\sum_{i=1}^{x+h}(x) + \sum_{i=1}^{x+h}(h) + \sum_{i=1}^{x}(x)}{h} \tag{6}$$

$$\lim_{h \to 0} = \frac{\sum_{i=1}^{x}(x) + \sum_{i=x+1}^{x+h}(x) + \sum_{i=1}^{x+h}(h) - \sum_{i=1}^{x}(x)}{h} \tag{7}$$

$$\lim_{h \to 0} = \frac{\sum_{i=x+1}^{x+h}(x) + \sum_{i=1}^{x+h}(h)}{h} \tag{8}$$

$$\lim_{h \to 0} = \frac{(x + x + x + \ldots + x + x) + (h + h + h + \ldots + h + h + h)}{h} x{+}h{-}xtimes, x{+}htimes \tag{9}$$

$$\lim_{h \to 0} = \frac{(x + x + x + \ldots + x + x) + (h + h + h + \ldots + h + h + h)}{h} htimes, x{+}htimes \tag{10}$$

$$\lim_{h \to 0} = \frac{h * x + (x + h) * h}{h} \tag{11}$$

$$\lim_{h \to 0} = \frac{h(x + x + h)}{h} \tag{12}$$

$$\lim_{h \to 0} = \frac{h(x+x+h)}{h} \tag{13}$$

$$\lim_{h \to 0} = (x+x+h) \tag{14}$$

$$= x+x+0 \tag{15}$$

$$2x = 2x \tag{16}$$

Proof. Clearly $1 = 2$ is is false due it containing diffrent numbers and inbetween steps 2, and 3 forgetting the limit definiton of a derivitve.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Q.E.D.