

# CS 241 Homework 3

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**Theorem 1.** *Let  $A, B$  be sets.  $A \Delta B = \emptyset$  iff  $A = B$*

*Proof.* The symmetric difference of  $A$  and  $B$  is

$$A \Delta B = (A \cup B) - (A \cap B)$$

$$A \Delta B = (A - B) \cup (B - A)$$

Suppose  $A \Delta B = \text{EMPTY}$

Suppose  $x$  is some variable and that  $x \in A$ .

I claim that  $x \in B$ . To suppose a contradiction that  $x \notin B$ .

So that  $x \in A \cup B$  but  $x \notin A \cap B$

So  $x \in A \Delta B$ , which results in  $x \in \text{EMPTY}$ .

This results in a contradiction, so  $x \in B$

So,  $A \subseteq B$ .

For the opposite direction:

Suppose  $A \Delta B = \text{EMPTY}$

Suppose  $x$  is some variable and that  $x \in B$ .

I claim that  $x \in A$ . To suppose a contradiction that  $x \notin A$ .

So that  $x \in B \cup A$  but  $x \notin B \cap A$

So  $x \in B \Delta A$ , which results in  $x \in \text{EMPTY}$ .

This results in a contradiction, so  $x \in A$

So,  $B \subseteq A$ .

Therefore:  $A = B$ .

Q.E.D.

**Theorem 2.** Let  $S_n = \{x \in \mathbb{R} | 0 \leq x \leq \frac{n-1}{n}\}$ . Then

$$\bigcup_{n=1}^{\infty} S_n = [0, 1)$$

*Proof.*  $\bigcup_{n=1}^{\infty} S_n = \{x : \exists n \in \mathbb{N} \text{ such that } x \in S_n\}$

Let  $x \in \bigcup_{n=1}^{\infty} S_n$ . Then for some  $n$ ,  $x \in \{x \in \mathbb{R} : 0 \leq x \leq \frac{n-1}{n}\}$ .

Then  $0 \leq x \leq \frac{n-1}{n}$ . But  $\frac{n-1}{n} < 1$ .

So,  $0 \leq x < 1$ . And  $x \in [0, 1)$

This shows  $LHS \subseteq RHS$ .

Now suppose  $x \in [0, 1)$

Then  $0 \leq x < 1$

I claim that there exists  $n$ , such that  $x \leq \frac{n-1}{n}$ .

For  $x \leq 1 - \epsilon$  for some  $\epsilon > 0$

Then choose  $n > \frac{1}{\epsilon}$

$\epsilon > \frac{1}{n}$

$\frac{1-\epsilon}{n} > 1 - \epsilon$

$\frac{n-1}{n} > 1 - \epsilon$

Then  $\frac{n-1}{n} > 1 - \epsilon \leq x$  for this  $n$ .

So,  $0 \leq x < \frac{n-1}{n}$ .

Then  $x \in \{x \in \mathbb{R} | 0 \leq x \leq \frac{n-1}{n}\}$ .

So  $x \in \bigcup_{n=1}^{\infty} S_n = [0, 1)$

Q.E.D.