CS 241 Homework 4

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Theorem 1. *Let n be an integer such that $n \geq 2$. Either n is prime, or n can be expressed as a unique product of primes.*

Proof. Let $P(n)$ where n is either prime or can be expressed as a unique product of primes
$P(n)$ holds all integers where $n \geq 2$

Base: $P(2)$ is true becuase 2 is a number.

Induction: Let $P(k)$ be true for all integers k with $2 \leq k \leq n$
Consider $n+1$. It is either prime or divisible by some number between 2 and n.
If $n+1$ is prime, then $P(n+1)$ is true.
If $n+1$ is divisible by some number, then suppose $n+1 = k * m$ where k and m are integers between 2 and n.
As a result, $P(k)$ and $P(m)$ are true by the Inductive Hypothesis.
Then k and m are a product of primes or are prime.
So $k = p_1 * \ldots * p_r$ and $m = q_1 * \ldots q_s$ where p_i and q_i are prime, and $r, s \geq 1$.
So $n+1 = k * m = p_1 * \ldots * p_r * q_1 * \ldots * Q_s$ so $n+1$ is a product of primes.
Therefore, $P(n)$ is true for all $n \geq 2$ Q.E.D.

Theorem 2. *Let* $A \subseteq \mathbb{N}$ *and* $a \neq \phi$. $\exists a_{min} \in A | \forall a \neq a_{min} \in A, a_{min} < a$

Proof. Base: $n = 1$. If $1 \in A$, then 1 is the smallest element.

Induction: Assume $P(1), \ldots, P(k), \ldots, P(n)$ and $P(n+1)$
Suppose set $A \subset N$ where A is non empty and contains $n+1$ as an element.
So either some number from $1, \ldots, n \in A$ or not.
If k is in A where $1 \leq k \leq n$ then A is true by the Inductive Hypothesis
As a result, A has a minimum element.

Q.E.D.