CS 241 Homework 5

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Instructions

- The learning objective for this assignment is to ensure that you understand the underlying material, and to provide you with practice in solving problems of this kind.
- Only a few of these problems will be graded. You do not know beforehand which ones these are; hence, you should provide solutions to all the problems.
- Make your solutions concise and formal.
- Please type the assignment if your handwriting is difficult to read. Otherwise scan to pdf
- You are encouraged to work on problems in groups. List the people you worked in groups with please. If you used an online resource i.e. Math-Stack Exchange, link the question asked or resource such that I can check you understand rather then brain-off copying
- Note that you must write your solutions by yourself, in your own words. Individual submissions
- **Remember that proving a statement is false counts as proving the statement. To prove a statement you do not need to prove it true**

Problems

Extra Credit: All assignments must be uploaded to Canvas as a pdf. Should you type up your homework in Latex, I will give you +2 extra credit points (equivalent to 10%). Template file will be provided if you want

Problem 5.1. *Let $\approx$ be an equivalence relation on a finite set A. Suppose that*

$$f : A/\approx \to A \tag{1}$$

is a bijective function. Find and prove what $\approx$ is.

Prove the following theorem.

Theorem 5.1. *Let A be a finite set and $f : A \to A$. Define the sequence*

$$x_0, x_1, x_2, x_3 \ldots \tag{2}$$

to be

$$x_0 \in A, x_{n+1} = f(x_n). \tag{3}$$

There exists a value of N such that if $n > N$ then x_n is guaranteed to be inside of an infinitely repeating cycle of the sequence.

Note: For example, if the sequence was

$$1, 2, 3, 6, 5, 7, 6, 5, 7, 6, 5, 7, \ldots, 6, 5, 7, \ldots$$

then in this case if $n > 2$ you have entered into an infinitely repeating cycle. You need to prove that for any arbitrary finite set and any arbitrary function on that set, that every sequence will eventually have an infinitely repeating cycle.