

CS 241 Homework X

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Theorem 1 (There are Infinite Primes). *Let P be the set of all prime numbers.*
 $\forall n \in \mathbb{N}, |P| > n$.

Proof. Suppose that there were a finite number of primes such that $|P| = n$. Consider the product of all prime numbers

$$k = p_1 p_2 p_3 \dots p_n. \tag{1}$$

Clearly k is divisible by all prime numbers. Now consider the number $k + 1$. Since k is divisible by all primes, $k + 1$ cannot be divisible by any primes, which means that $k + 1$ is prime. However $k + 1$ is not in our set P , which contradicts our claim that P contained all primes. \therefore there must be infinite number of primes. Q.E.D.