CS 241 Homework 5

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Theorem 1. *Let $\approx$ be an equivalence relation on a finite set A. Suppose that*

$$f : A/\approx \to A \tag{1}$$

is a bijective function. Find and prove what $\approx$ is.

Proof. Suppose we have f^{-1} and are assigning pigeons to boxes.
Pigeons $= A$; Boxes $= A/\approx$
Since f is a bijection, also f^{-1} is.
So f^{-1} assigns pigeons to boxes, and there must be at least as many boxes as there are pigeons.
So, $\approx$ must be equal or else there will be fewer boxes than pigeons.

Q.E.D.

Theorem 2. *Let A be a finite set and $f : A \rightarrow A$. Define the sequence*

$$x_0, x_1, x_2, x_3 \ldots \tag{2}$$

to be

$$x_0 \in A, x_{n+1} = f(x_n). \tag{3}$$

There exists a value of N such that if $n > N$ then x_n is guaranteed to be inside of an infinitely repeating cycle of the sequence.

Proof. Since A is finite, $x_N = x_n$ for some n and N.
$x_{N+j} = x_{n+j}$ for all j and for some k, N= n+k.
Using induction, we can show that $x_{n+j} = x_{N+j} = x_{N+j+k} = x_{N+j+2k} = x_{N+j+3k} = \ldots$
Base Case: $x_{N+j} = x_{n+j+k}$
Inductive Case: $x_{n+j} = x_{n+j+mk}$ Q.E.D.