CS 241 Homework 1

Your name

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Theorem 1 (There are Infinite Primes)**.** *Let P be the set of all prime numbers.* $\forall n \in \mathbb{N}$, $|P| > n$.

Proof. Suppose that there were a finite number of primes such that $|P| = n$. Consider the product of all prime numbers

$$k = p_1 p_2 p_3 \dots p_n. \tag{1}$$

Clearly k is divisible by all prime numbers. Now consider the number $k + 1$. Since k is divisible by all primes, $k+1$ cannot be divisible by any primes, which means that $k+1$ is prime. However $k+1$ is not in our set P, which contradicts our claim that P contained all primes. $\therefore$ there must be infinite number of primes. Q.E.D.