# CS 241 Homework 2

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**Theorem 1.** $\forall e_1, e_2, e_1 * e_2$ *is even*

**Negated Theorem 1.** $\exists e_1, e_2, e_1 * e_2$ is odd

*Proof.* Prove P:
$e_1 = 2n$ for some integer n
$e_1 = 2m$ for some integer m
$e_1 * e_2 = 2n * 2m = 2(2nm) = 2k$ for some integer k
Clearly the product is even due to being divisible by 2

Q.E.D.

**Theorem 2.** *Prove* $\bar{P}$*:*
$\exists d, e | e^d$ *is odd*

**Negated Theorem 1.** $\forall d, e | e^d$ is even

*Proof.* $e^d = e * e * e \ldots e$ d times
which is a product containing 2 as a factor, so it is even by definition. Q.E.D.

**Theorem 3.** $\forall a, b \exists c | a^2 * b^2 = c^2$

**Negated Theorem 1.** $\exists a, b \forall c | a^2 * b^2 = c^2$

*Proof.* Prove P:
$a^2 * b^2 = (a * b)^2 = c^2$
$c = a * b$ so c exists

Q.E.D.

**Theorem 4.** $\exists c \forall a, b | a^2 * b^2 = c^2$

**Negated Theorem 1.** $\forall c \exists a, b | a^2 * b^2 = c^2$

*Proof.* Prove $\bar{P}$:
$a^2 * b^2 = (a * b)^2 = c^2$
$ab = c$ so a and b exist

Q.E.D.