CS 241 Homework 3

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Theorem 1. *Let A,B be sets.* $A \triangle B = \phi$ *iff* $A = B$

Proof. The symmetric diffrence of A and B is
$A\Delta B = (A \cup B) - (A \cap B)$
$A\Delta B = (A - B) \cup (B - A)$

Suppose $A\Delta B =$ EMPTY
Suppose x is some variable and that x ∈ A.
I claim that $x \in B$. To suppose a contradiction that $x \notin B$.
So that $x \in A \cup B$ but $x \notin A \cap B$
So $x \in A\Delta B$, which results in $x \in$ EMPTY.
This results in a contradiction, so $x \in B$
So, $A \subseteq B$.

For the opposite direction:
Suppose $A\Delta B =$ EMPTY
Suppose x is some variable and that x ∈ B.
I clain that $x \in A$. To suppose a contradiction that $x \notin A$.
So that $x \in B \cup A$ but $x \notin B \cap A$
So $x \in B\Delta A$, which results in $x \in$ EMPTY.
This results in a contradiction, so $x \in A$
So, $B \subseteq A$.

Therefore: $A = B$. Q.E.D.

Theorem 2. *Let* $S_n = \{x \in \mathbb{R} | 0 \leq x \leq \frac{n-1}{n}\}$*. Then*

$$\bigcup_{n=1}^{\infty} S_n = [0,1)$$

Proof. $\bigcup_{n=1}^{\infty} S_n = \{x : \exists n \in N \text{ such that } x \in S_n\}$
Let $x \in \bigcup_{n=1}^{\infty} S_n$. Then for some $n, x \in \{x \in \mathbb{R} : 0 \leq x \leq \frac{n-1}{n}\}$.
Then $0 \leq x \leq \frac{n-1}{n}$. But $\frac{n-1}{n} < 1$.
So, $0 \leq x < 1$. And $x \in [0,1)$
This shows LHS $\subseteq RHS$.

Now suppose $x \in [0,1)$
Then $0 \leq x < 1$
I claim that there exists n, such that $x \leq \frac{n-1}{n}$.
For $x \leq 1- \in$ for some $\in > 0$

Then choose $n > 1\frac{1}{\in}$
$\in > \frac{1}{n}$
$\frac{1-1}{n} > 1- \in$
$\frac{n-1}{n} > 1- \in$

Then $\frac{n-1}{n} > 1- \in \leq x$ for this n.
So, $0 \leq x > \frac{n-1}{n}$.
Then $x \in \{x \in \mathbb{R} | 0 \leq x \leq \frac{n-1}{n}\}$.
Therefore $x \in \bigcup_{n=1}^{\infty} S_n = [0,1)$ Q.E.D.