Chapter 3

What is a fractal?

In this chapter we give a definition of fractal, and introduce one of its properties, the self-similarity. Finally we present some examples of fractal objects.

Once we have defined the space of fractals $((\mathbb{H}(\mathbb{X}), d_{\mathbb{H}})$, we can define a fractal.

Definition 3.0.2. Let (X, d) be a metric space. We say that a fractal is a subset of $((\mathbb{H}(\mathbb{X}), d_{\mathbb{H}})$. In particular, is a fixed point of a contractive function on $((\mathbb{H}(\mathbb{X}), d_{\mathbb{H}})$.

A fractal is a geometric object that is repeated at ever smaller scales to produce irregular shapes that cannot be represented by classical geometry. We say that they are self-similar.

An object is said to be self-similar if it looks "roughly" the same on any scale. The Sierpinski triangle in Figure 3.1 is an example of a self-similar fractal. If we zoom the red triangle we see that it is similar to the first one. This occurs in all scales.

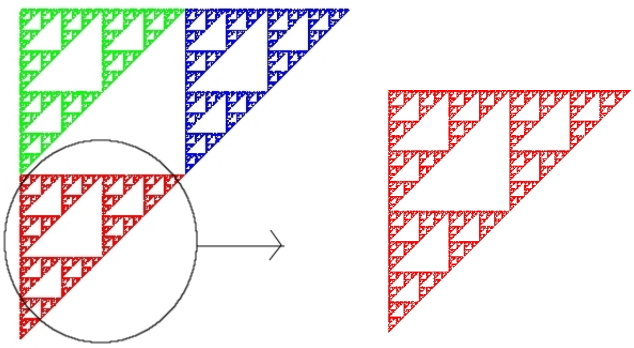

Figure 3.1: The Sierpinski triangle is self-similar.

In chapter 4 we will see that a fractal is invariant under certain tranformations of X.

In the following subsections we are going to introduce specific examples of fractals.

3.1 The Cantor Set

The cantor set is generated by beginning with a segment (usually of length 1) and removing the open middle third of this segment. The process of removing the open middle third of each remaining segment is repeated for each of the new segments.
Figure 3.2 shows the first five stages in this generation.

Figure 3.2: First 4 stages in Cantor set generation.

3.2 Koch curve

The Koch curve is another well known fractal. To construct it begin with a straight line. Divide it into three equal segments and replace the middle segment by the two sides of an equilateral triangle of the same length as the segment being removed. Now repeat the same construction for each of the new four segments. Continue these interations.

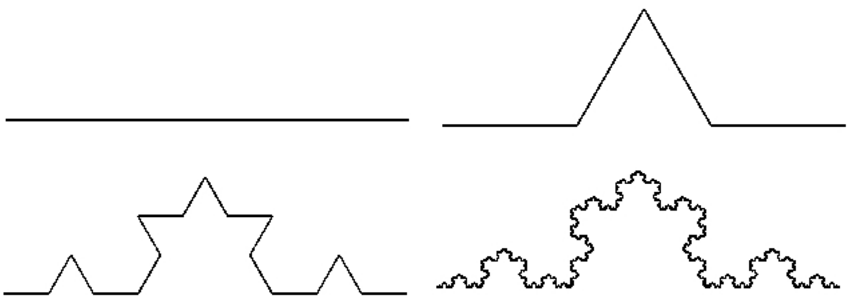

Figure 3.3: Stage 0, 1, 2 and 9 of the Koch curve.

3.3 Sierpinski triangle

Without a doubt, Sierpinski's Triangle is at the same time one of the most interesting fractals and one of the most simple to construct.

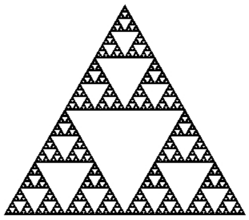

Figure 3.4: Sierpinski triangle

One simple way of generating the Sierpinski Triangle in Figure 3.4 is to begin with a triangle. Connect the midpoints of each side to form four separate triangles, and cut out the triangle in the center. For each of the three remaining triangles, perform this same act. Iterate infinitely. The firsts iterations of the sierpinski triangle are presented in Figure 3.5.

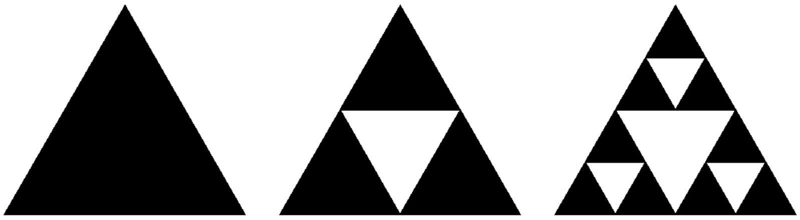

Figure 3.5: Stages 0, 1 and 2 of the Sierpinski triangle.

3.4 Other examples

In this subsection we show other examples of fractals.

3.4.1 Self-portrait fractal

Here we have a fractal constructed applying repeatedly the affine transformations seen in section 1.2.

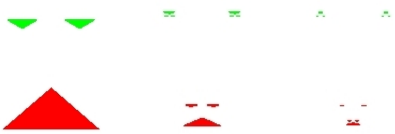

Figure 3.6: Stages 1, 2 and 3 of the self-portrait fractal.

3.4.2 Sierpinski carpet and Sierpinski pentagon

The Sierpinksi carpet is a generalization of the Cantor set to two dimensions. The construction of the Sierpinski carpet begins with a square. The square is cut into 9 congruent subsquares in a 3-by-3 grid, and the central subsquare is removed. The same procedure is then applied recursively to the remaining 8 subsquares, and infinitum.

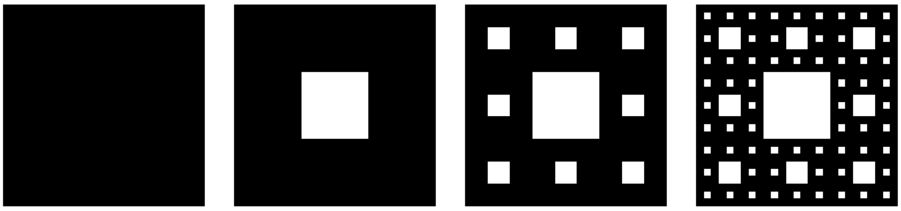

Figure 3.7: Firsts iterations of the Sierpinski Carpet.

The sierpinski pentagon, a fractal with 5-fold simmetry, is formed starting with a pentagon and using similar rules that in the sierpinski triangle but for pentagons.

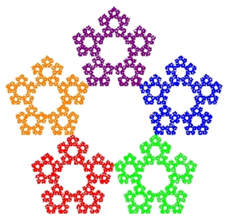

Figure 3.8: Sierpinski pentagon

3.4.3 Peano curve

The Peano curve is created by iterations of a curve. The limit of the Peano curve is a space-filling curve, whose range contains the entire 2-dimensional unit square. In Figure 3.9 we can see the firts 3 iterations of the curve. We will explore more this curve in chapter 5.2.

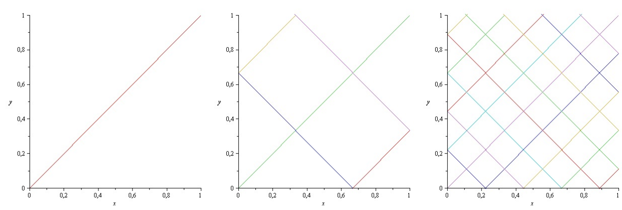

Figure 3.9: 3 iterations of the Peano curve construction.