M and N are two distinct m-matchings in G (i)

(a) If $M \setminus N = \emptyset$, then $M \subsetneq N$, contradicting (i).
If $N \setminus M = \emptyset$, then $N \subsetneq M$, contradicting (i).
Therefore we know that both $M \setminus N$ and $N \setminus M$ are non-empty. $(a1)$
Let's assume that there exists an edge $e \in M \setminus N$ that is not adjacent to any edge in $N \setminus M$.
Then the set $N' = N \cup \{e\}$ is a matching in G that includes N, contradicting (i).
Therefore we know that every edge in $M \setminus N$ is adjacent to at least one edge in $N \setminus M$. $(a2)$

From $(a1)$ and $(a2)$ we know that φ is well-defined.

(b)
Since $\varphi(M \setminus N) \subset N \setminus M \Rightarrow |\varphi(M \setminus N)| \leq |N \setminus M| \Rightarrow 2|\varphi(M \setminus N)| \leq 2|N \setminus M|$ $(b1)$
For $\forall\ f \in N \setminus M$ we will consider $C_f = \{e \in M \setminus N | \varphi(e) = f\}$
let $cnt : N \setminus M \to \mathbb{N}$ be a function defined in the following way:
$\forall f \in N \setminus M$, $cnt(f) = |C_f|$
Naturally, we have $\sum_{f \in \varphi(N \setminus M)} cnt(f) = |M \setminus N|$ $(b2)$
Any edge $f \in N \setminus M$ is adjacent to at most 2 edges in $M \setminus N$, thus $cnt(f) \leq 2\ \forall f \in N \setminus M$.
Why is this true?
Take a random edge $f = (u, v)$ in $N \setminus M$, since both M and N are matchings, there is at most one edge in $M \setminus N$ adjacent to u and at most one edge in $M \setminus N$ adjacent to v, and since an edge $e \in M \setminus N$ can be adjacent to $f \iff$ it is adjacent to u or v, there are at most two edges in $M \setminus N$ adjacent to f.
Therefore we have that $\sum_{f \in \varphi(N \setminus M)} cnt(f) \leq 2|\varphi(N \setminus M)|$
Which, together with $(b2)$, gives us $|M \setminus N| \leq 2|\varphi(N \setminus M)|$ $(b3)$

From $(b1)$ and $(b3)$ we have $|M \setminus N| \leq 2|N \setminus M|$ and $|N \setminus M| \leq 2|M \setminus N|$

(c)
We know that $|A \setminus B| = |A| - |A \cap B|$ for any two sets A and B
From (b) we know that: $|M \setminus N| \leq 2|N \setminus M|$
$\Rightarrow |M| - |M \cap N| \leq 2(|N| - |M \cap N|)$
By adding $|M \cap N|$ to both sides we have:
$|M| \leq 2|N| - |M \cap N|$
And since $|M \cap N| \geq 0$ we have:

$|M| \leq 2|N|$