Mathematical and Computational Logic
Example of Exam Subjects List

Claudia MUREȘAN
cmuresan@fmi.unibuc.ro, claudia.muresan@g.unibuc.ro, c.muresan@yahoo.com

University of Bucharest
Facultaty of Mathematics and Computer Science
Bucharest

2022–2023, Semester I

(Scores: maximum 11.5 points; grade: $\min\{10, \text{score}\})$

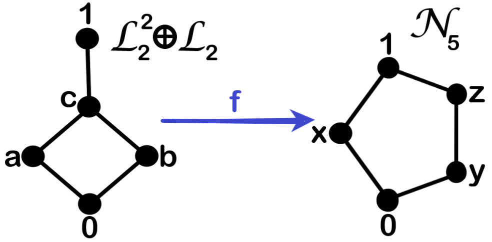
1 point ex officio;
3 points for the COLLECTIVE ASSIGNMENTS;
each of the two requirements of each exercise: **1.25 points**.

For the Prolog programming requirements, any predefined predicate, as well as any predicate written in any LAB LESSON or COLLECTIVE ASSIGNMENT can be used, using the *include* directive to include the knowledge bases *labNrlmcVer.pl* and *temeleNr.pl* into the current one, provided the **names of the auxiliary predicates** are written exactly as in the .PL FILES from the LAB CLASSES and the ENUNCIATIONS OF THE COLECTIVE ASSIGNMENTS. All the other auxiliary predicates necessary for defining the required predicates must be written in the exam paper.

Exercise (1)

Determine all bounded lattice morphisms from the ordinal sum $\mathcal{L}_2^2 \oplus \mathcal{L}_2$ of the four–element Boolean algebra with the two–element chain to the pentagon ($\mathcal{N}_5$):

1. mathematically;
2. with a unary Prolog predicate *morfL2xL2plusL2laN5*($-$*ListofMorphisms*).



Exercise (2)

Let V be the set of the propositional variables, E the set of the formulas and T the set of the formal theorems of classical propositional logic.
Let $p, q \in V$, $\alpha, \beta \in E$ and $\theta \in T$.
Prove that, if $\{\alpha, p\} \vdash (\theta \to q) \leftrightarrow \beta$, then $\vdash (\alpha \wedge \beta) \to (p \to q)$:

1. mathematically;
2. with a nullary Prolog predicate *demExercLogProp*.

Exercise (3)

Consider the first order signature $\tau = (1; 2; \emptyset)$, the unary operation symbol f and the binary relation symbol R, a set $A = \{a, b, c\}$ with $|A| = 3$ and the first order algebraic structure of type τ: $\mathcal{A} = (A, f^{\mathcal{A}}, R^{\mathcal{A}})$, having the set reduct A,

$f^{\mathcal{A}} : A \to A$ defined by the table: $\begin{array}{c|ccc} u & a & b & c \\ \hline f^{\mathcal{A}}(u) & b & c & a \end{array}$ and $R^{\mathcal{A}} \subseteq A^2$ such that

$R^{\mathcal{A}}$ is the transitive closure of the binary relation $\{(a, b), (b, a), (b, c)\}$ on A, as well as two distinct variables $x, y \in Var$.
Determine whether $\mathcal{A} \vDash \forall x \forall y \, [f(x) = y \to R(x, y)]$:

1. mathematically;
2. with a nullary Prolog predicate *verifAlgSatFormula*.