$A, B \in Set$

$\{a, \{a, b\}\} = (a, b)$

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

$R \subseteq \underset{\ni a}{A} \times \underset{\ni b}{B}$ $\qquad (a, b) \in R \overset{not.}{\Longleftrightarrow} aRb$

$(\forall T \in Set)(\mathcal{P}(T) = \{S \in Set \mid S \subseteq T\})$

$\mathcal{P}(A \times B)$ $\qquad R^{-1} = \{(b, a) \mid (a, b) \in R\} \subseteq B \times A.$

$f : A \to B$

$f = (A, \underset{\subseteq A \times B}{G}, B), \ (\forall a \in A)(\exists!\, \underset{= f(a)}{b} \in B)(aGb)$

$G \equiv f \subseteq A \times B$

$f = \{(a, \underbrace{f(a)}_{\in B}) \mid a \in A\}$

$R \subseteq A \times B \to$ totală $\overset{def.}{\Longleftrightarrow} (\forall a \in A)(\exists b \in B)(aRb)$

$\downarrow$ $\quad \overset{not.}{\Longleftarrow} R : A \to B$

funcțională $\overset{def.}{\Longleftrightarrow} (\forall a \in A)(\exists$ cel mult un $b \in B)(aRb)$

$\Longleftrightarrow (\forall a \in A)\underbrace{(\forall b, c \in B)}_{\equiv (\forall b \in B)(\forall c \in B)}[(aRb$ și $aRc) \Rightarrow b = c]$

$R \to$ inj. $\Longleftrightarrow R^{-1} \to$ funcțională

$R \to$ surj. $\Longleftrightarrow R^{-1} \to$ totală.

$$f: A \to B \iff \begin{cases} f: A \xrightarrow{\circ} B \\ \text{și} \\ f \to \text{totală} \end{cases}$$

$$B^A = \{f \mid f: A \to B\}$$

$$M \simeq A \iff \exists f^{-1}: M \xrightarrow{\sim} A$$

$$|A| \overset{def}{=} \{M \in Set \mid A \simeq M\}$$

$$\Updownarrow \text{not.}$$

$$\exists f: A \xrightarrow{\sim} M$$

$$f: A \xrightarrow{\sim} B \overset{def}{\iff} f: A \to B,\ f \to \text{inj.},\ f \to \text{surj..}$$

$$\iff (\exists \underbrace{g}_{= f^{-1}: B \xrightarrow{\sim} A}: B \to A)(g \circ f = id_A \text{ și } f \circ g = id_B)$$

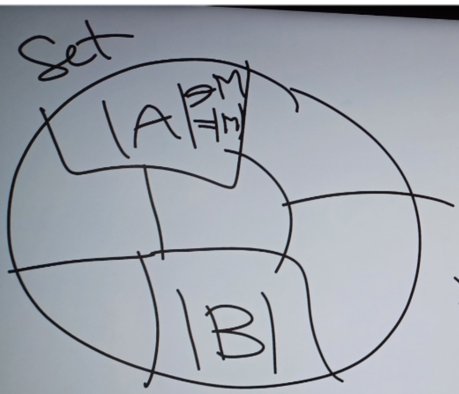
$$(\forall M \in Set)\big(id_M: M \xrightarrow{\sim} M,\ (\forall x \in M)(id_M(x) = x)\big)$$

$$f = \{(a, \underbrace{f(a)}_{=: b \in B}) \mid a \in A\}: A \xrightarrow{\sim} B$$

$$b \in B \Rightarrow f^{-1}(b) = f^{-1}(f(a)) = a$$

$$f^{-1} = \{(b, \underbrace{f^{-1}(b)}_{= a \in A}) \mid b \in B\}: B \xrightarrow{\sim} A$$

$$f^{-1} = \{(\underbrace{f(a)}_{=: b \in B}, \underbrace{a}_{f^{-1}(b)}) \mid a \in A\} = \{(b, f^{-1}(b)) \mid b \in B\}$$



$$|\emptyset| = \{M\in Set \mid M\simeq\emptyset\} = \{\emptyset\}$$

$$(\forall A\in Set\setminus\{\emptyset\})(|A|\notin Set).$$

$$(\forall A\in Set)(|A| \overset{def}{=} \{M\in Set \mid A\simeq M\})$$

$$(\forall A\in Set)(A \xrightarrow[\simeq]{id_A} A)$$

$$\Downarrow$$

$$A\simeq A \Rightarrow A\in|A| \Rightarrow |A|\neq\emptyset.$$

$$(\forall A,B\in Set)(A\simeq B \Leftrightarrow B\simeq A)$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$B\in|A| \qquad\qquad A\in|B|$$

$$(\forall A,B,C\in Set)(A\simeq B\simeq C \Rightarrow A\simeq C).$$

$\forall A,B\in Set$:

$$|A|\neq\emptyset$$

$$|A|\ni A$$

$$\bigcup_{A\in Set}\underbrace{\overset{A\,\in}{|A|}}_{\subseteq Set} = Set$$

$$A\simeq B \Rightarrow |A|=|B|$$

$$A\not\simeq B \Rightarrow |A|\cap|B|=\emptyset$$

$$|A|=|B| \Leftrightarrow A\simeq B$$

$$|A| \le |B| \overset{df.}{\iff} \exists i: A \hookrightarrow B$$

$$|A| < |B| \overset{df.}{\iff} (|A| \le |B| \text{ și } |A| \ne |B|)$$

$$\iff (\exists i: A \hookrightarrow B \text{ și } \nexists f: A \xrightarrow{\sim} B)$$

$$|A| + |B| = |A \amalg B| \qquad |A'| + |B'| = |A' \amalg B'|$$

$$|A| = |A'|, \forall A' \in |A| \qquad |B| = |B'|, \forall B' \in |B|$$

$$\mathbb{Q}_+ = \mathbb{Q} \cap [0;\infty)$$

$$f: \mathbb{N} \to \mathbb{Q}_+$$

$$f(0), f(1), \dots, f(n), \dots$$

$$\mathbb{N} \simeq \mathbb{Z} \simeq \mathbb{Q} \not\simeq \mathbb{R}$$

$$\aleph_0 = |\mathbb{N}| = |\mathbb{Z}| = |\mathbb{Q}| \underset{\ne}{\le} |\mathbb{R}| = 2^{\aleph_0} = |\mathcal{P}(\mathbb{N})|$$

$$(\forall T \in \text{set})\left(\mathcal{P}(T) \simeq \{0,1\}^T\right)$$

$$\Downarrow$$

$$|\mathcal{P}(T)| = |\{0,1\}^T| = |\{0,1\}|^{|T|} = 2^{|T|}$$

$$|\mathbb{Q}| \le |\mathbb{R}|$$

$$\mathbb{Q} \subset \mathbb{R}$$

$i: \mathbb{Q} \hookrightarrow \mathbb{R}, \ (\forall x \in \mathbb{Q})(i(x) = x).$

Pp. abs. $\exists f: \mathbb{N} \xrightarrow{\simeq} \mathbb{R}$, $\quad (\forall i,j \in \mathbb{N})(x_{ij} \in \overline{0,9})$

$f(0) = x_0 + 0,\overset{\neq y_0}{x_{00}}\, x_{01} \dots x_{0n} \dots \Rightarrow y \neq f(0)$

$f(1) = x_1 + 0,x_{10}\, \overset{\neq y_1}{x_{11}} \dots x_{1n} \dots \Rightarrow y \neq f(1)$

$\vdots$

$f(n) = x_n + 0,x_{n0}\, x_{n1} \dots \overset{\neq y_n}{x_{nn}} \dots \Rightarrow y \neq f(n)$

$x_n = [f(n)] = \max\{x \in \mathbb{Z} \mid x \leq f(n)\}$

$y = 0,y_0 y_1 \dots y_n \dots$

$(\forall i \in \mathbb{N})(y_i \in \overline{1,8})$

$\left.\begin{array}{l} \Rightarrow y \notin f(\mathbb{N}) \\ \quad\ \ \in \\ \quad\ \ \mathbb{R} \end{array}\right\} \Rightarrow$ Abs cu $f \to$ surj...

$\Rightarrow \mathbb{N} \not\sim \mathbb{R}$

$f: \mathbb{N} \to \mathbb{R}$

$\equiv$

$(x_n)_{n \in \mathbb{N}} \subseteq \mathbb{R}$

$(\forall n \in \mathbb{N})(x_n = f(n))$

$\forall I \in Set$

$f: I \to \mathbb{R}$

$\equiv$

$(x_i)_{i \in I} \subseteq \mathbb{R}$

$(\forall i \in I)(f(i) \in \mathbb{R})$

$T \in Set \Rightarrow \mathcal{P}(T) \in Set \not\ni Set$

$f : \mathbb{N} \to \cancel{Set}\ \mathcal{P}(T)$

$\equiv (A_n)_{n\in\mathbb{N}} \subset \cancel{Set}$

$(\forall n \in \mathbb{N})(A_n = f(n))$

$\forall I \in Set,$

$f : I \to \cancel{Set}\ \mathcal{P}(T)$

$\equiv (A_i)_{i\in I} \subset \cancel{Set}$

$(\forall i \in I)(A_i = f(i))$

Fie $I \in Set$ și $(A_i)_{i\in I} \subset Set$.

$\equiv f : I \to Set,$

$(\forall i \in I)(A_i = f(i))$.

$$\bigcup_{i\in \emptyset} A_i = \{x \mid \underbrace{(\exists i \in \emptyset)(x \in A_i)}_{\Leftrightarrow (\exists i)(\overbrace{i\in\emptyset}^{F\,\forall i} \text{ și } x\in A_i)}\} = \emptyset$$

F

$F\ \forall i$

$(A_i)_{i\in I} \subseteq \mathcal{P}(T)$

$$\bigcap_{i\in\emptyset} A_i = \{x \in T \mid \underbrace{(\forall i \in \emptyset)(x \in A_i)}_{\Leftrightarrow (\forall i)(i\in\emptyset \Rightarrow x \in A_i)}\} = T$$

$F\ \forall i$

$A\ \forall i$

A

$$\prod_{i\in \cancel{J}\,\emptyset} A_i = \{(a_i)_{i\in \cancel{J}\,\emptyset} \mid (\forall i\in \cancel{J}\,\emptyset)(a_i\in A_i)\} =$$

$$(a_i)_{i\in J} \cong f: \cancel{J}\,\emptyset \to \bigcup_{i\in \cancel{J}\,\emptyset} A_i = \emptyset, \quad (\forall i\in J)(a_i = f(i))$$

$$\emptyset^{\emptyset} = \{(\emptyset,\emptyset,\emptyset)\}$$

$$= \{f \mid f: \cancel{J}\,\emptyset \to \bigcup_{i\in \cancel{J}\,\emptyset} A_i,\ \underbrace{(\forall i\in \cancel{J}\,\emptyset)(f(i)\in A_i)}_{\Leftrightarrow (\forall i)(i\in\emptyset \Rightarrow f(i)\in A_i)}\}$$

$$= \emptyset^{\emptyset} = \{(\emptyset,\emptyset,\emptyset)\}$$

$$\prod_{i\in J} A_i = \{f \mid f: J\to \bigcup_{i\in J} A_i,\ (\forall i\in J)(f(i)\in A_i)\}.$$

$A_i = A \in Set$, $\bigcup_{i\in J} A_i = A$, $J \neq \emptyset$, $A_i = A$

$$= \{f \mid f: J \to A\} = A^J.$$

$(G, \circ, {}^{-1}, e)$, $e \in G$

$\circ : \underbrace{G\times G}_{=G^2} \to G$

$e: G^0 \to G$, $G^0 = \{(\emptyset,\emptyset,\emptyset)\} = \{*\}$, $G \ni e = e(*)$

${}^{-1} : G \to G$, $G = G^1$

$$(L, \overset{2}{\vee}, \overset{2}{\wedge}, \overset{2}{\leq}, \overset{0}{0}, \overset{0}{1})$$

$$(A, \overset{2}{\vee}, \overset{2}{\wedge}, \overset{2}{\leq}, \overset{1}{'}, \overset{0}{0}, \overset{0}{1})$$

$J, A, B \in Set$

$f, g: A \to B$

$(f_i)_{i\in J} \subseteq B^A$

$\Updownarrow$

$(\forall i\in J)(f_i : A \to B)$

$\leq$, $+$, max

$f \leq g \overset{def}{\Leftrightarrow} (\forall x\in A)(f(x)\leq g(x))$

$f+g: A\to B\ (\forall x\in A)((f+g)(x) = f(x)+g(x))$

$\max\{f_i \mid i\in J\}: A\to B\ (\forall x\in A)((\max\{f_i\mid i\in J\})(x) := \max\{f_i(x)\mid i\in J\})$