Progressive Archive in Adaptive jSO Algorithm

Petr Bujok
Department of Informatics and Computers
Faculty of Science, University of Ostrava
petr.bujok@osu.cz
ORCID: 0000-0003-2956-1226

***Abstract*—In this paper, a new variant of the adaptive variant Differential Evolution (DE) algorithm is proposed. The original variant of jSO uses an archive A for old successful individuals, which are inserted into random positions. A newly proposed variant of jSO uses a more progressive approach how to locate new individuals in the archive. When the archive is full, new individuals are inserted only into worse positions based on function value. Therefore, 50 % of better individuals in the archive are not replaced. This enhanced variant of jSOa increases the performance of the original jSO in 62 % of test problems CEC 2024.**

***Index Terms*—Differential evolution, jSO, archive, experiments, optimisation**

I. New jSO Variant With Progressive Archive

A newly proposed variant of jSO with a more progressive update of archive A is extended from the original jSO. Therefore, the steps of the original jSO are described before the newly employed approach is presented (see Algorithm 1). Initially, the population of N possible solutions is randomly located and evaluated by the objective function. The circle memories for F and *CR* parameters are allocated.

For each solution, mean values of F and *CR* parameters - M_F and M_{CR} are selected using a roulette wheel. The control parameters of jSO are generated by the standard Gauss (*CR*) and Cauchy (F) distributions using the mean values. Further, the values of control parameters F and *CR* are truncated to certain values based on the current step of the search process.

After the settings of the jSO control parameters, a mutation point is generated using a novel weighted mutation variant (1):

$$\vec{u}_i = \vec{x}_i + F_w(\vec{x}_{\mathrm{pBest}} - \vec{x}_i) + F(\vec{x}_{r_1} - \vec{x}_{r_2}), \qquad (1)$$

where $\vec{x}_i$ is the current point, $\vec{x}_{\mathrm{pBest}}$ is randomly selected point from $p \times N$ the best points of P, $\vec{x}_{r_1}$ is selected randomly from P, and $\vec{x}_{r_2}$ is selected randomly from population and archive, $P \cup A$. The newly introduced part is the newly used parameter F_w, computed using recommended rules (2):

$$F_w = \begin{cases} 0.7 \times F, & \mathit{FES} < 0.2 \times \mathit{maxFES} \\ 0.8 \times F, & \mathit{FES} < 0.4 \times \mathit{maxFES} \\ 1.2 \times F, & \mathit{otherwise}. \end{cases} \qquad (2)$$

The parameter controlling the portion of the best individuals to select $\vec{x}_{\mathrm{pBest}}$ point (p) is adapted during the search using the following formula (3):

$$p = \frac{p_{\max} - p_{\min}}{\mathit{maxFES}} \times \mathit{FES} + p_{\min}, \qquad (3)$$

Algorithm 1 jSO algorithm

```
archive A ← ∅
initialise population P = {x_1, x_2, ..., x_N}
set all values of M_F to 0.5
set all values of M_CR to 0.8
while stopping condition not reached do
  S_CR ← ∅, S_F ← ∅
  for i = 1, 2, ..., N do
    r_i ← select from [1, H] randomly
    if r_i = H then
      M_{F,r_i} ← 0.9
      M_{CR,r_i} ← 0.9
    end if
    if M_{CR,r_i} < 0 then
      CR_i ← 0
    else
      CR_i ← N_i(M_{CR,r_i}, 0.1)
    end if
    if g < 0.25 G_max then
      CR_i ← max(CR_i, 0.7)
    else if g < 0.5 G_max then
      CR_i ← max(CR_i, 0.6)
    end if
    F_i ← C_i(M_{F,r_i}, 0.1)
    if g < 0.6 G_max and F_i > 0.7 then
      F_i ← 0.7
    end if
    y_i ← current-to-pbest_w/1/bin
    compute f(y_i)
  end for
  for i = 1, 2, ..., N do
    if f(y_i) ≤ f(x_i) then
      x_i ← y_i
    end if
    if f(y_i) < f(x_i) then
      x_i → A, CR_i → S_CR, F_i → S_F
    end if
    update M_CR and M_F
    update population size
    update p
  end for
end while
```

where $p_{\max}$, $p_{\min}$ are input parameters, *maxFES* is the total number of function evaluation per run and *FES* is the current number of function evaluations. The authors of jSO recommended to use $p_{\max} = 0.25$, and $p_{\min} = 0.125$.

After mutation, the new trial solution point $\vec{y}_i^{'}$ is generated using the current point $\vec{x}_i$ and the mutation point $\vec{u}_i$ in the binomial crossover. The new trial point is evaluated, and it replaces the old parent solution $\vec{x}_i$ only if $f(\vec{y}_i) \leq f(\vec{x}_i)$. In this case, the old solution is lost in the original DE algorithm. The variant of jSO uses an archive of old good solutions A where the parent solutions replaced by the trial solutions are stored. When the archive is full, the new outperformed parent individuals are located in randomly selected positions.

A. Progressive Update of Archive individuals in jSO

A newly proposed approach introduces a more progressive approach to store outperformed parent individuals in the archive of the jSO algorithm. At the moment when the archive is full, the archive individuals are ordered based on the functional values to divide the archive into two parts - better and worse individuals. The newly outperformed parent solution is inserted in the worse part of individuals to store the better solutions and refresh the worse individuals in the archive. It is possible to insert the newly outperformed parent point in the archive also if it is worse than the old solution of the archive.

This approach is very simple; it does not increase the complexity of the original jSO algorithm. The newly proposed method is called jSOa in the following text.

II. Experiments

In the experiment, 29 CEC 2024 problems were used. The functions are described in [1], including the experimental settings required for the competition. The source code of the functions is also available on the website given in the report [1]. The test functions CEC 2024 are divided into four categories based on their difficulty:

- unimodal functions - simple problems: $F1, F3$,
- multimodal functions - with many local minima: $F4 - F10$,
- hybrid functions - difficult, considered as the real-world problems: $F11 - F20$,
- composition functions - very difficult, composed of several different functions: $F21 - F30$.

The *jSOa* algorithm is implemented in Matlab 2020b, and this environment was also used for the experiments. All computations were carried out on a standard PC with Windows 11, Intel(R) Core(TM)i7-4790 CPU 3.6 GHz, 16 GB RAM.

Our tests were carried out at four levels of dimension, $D = 10, 30, 50\,100$, where 25 independent runs per each test function were performed. The function-error value is computed as the difference between the function value of the best-searched point and the known best function value of each test problem. The run of the algorithm stops if the prescribed amount of function evaluation $\mathit{MaxFES} = D \times 10^4$ is reached. The population size of all algorithms in the comparison are initialised at the same value $N_{\text{init}} = 25 \times \log(D) \times \sqrt{D}$ as recommended by the authors of jSO. The size of the archive A is set to $2.6 \times N$, where N is the current population size in jSO.

III. Results

TABLE I
BASIC CHARACTERISTICS OF JSOA, $D = 10$

Fun	min	max	median	mean	SD
1	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	2.98E+00	1.99E+00	2.03E+00	7.31E-01
6	0	1.14E-13	0	5.46E-14	5.80E-14
7	10.8893	13.0551	11.963	11.98666	0.554072
8	0	3.97984	1.98992	1.950121	0.929813
9	0	0	0	0	0
10	0.187363	1.30E+02	6.95483	1.74E+01	3.29E+01
11	0	0	0	0	0
12	2.08E-01	120.154	0.416286	9.970332	33.13042
13	0	5.39E+00	4.83714	3.80E+00	2.08E+00
14	0.00E+00	2.27E-13	0	3.64E-14	8.51E-14
15	0.000523	0.5	0.471777	0.348918	0.207297
16	0.000689	0.708124	0.272581	0.328383	0.1954
17	0.056954	1.47346	0.397721	0.520482	0.402452
18	0.000106	5.00E-01	0.142704	2.17E-01	2.23E-01
19	0	0.029352	2.50E-12	0.008959	0.010493
20	0	0.312173	0	0.012487	0.062435
21	100	205.291	202.397	153.788	52.74632
22	100	100	100	100	0
23	300	304.135	300	301.3629	1.746531
24	100	332.193	329.544	292.8325	85.90924
25	397.743	443.357	398.009	410.6832	20.78669
26	300	300	300	300	0
27	389.006	389.518	389.006	389.2313	0.259391
28	300	611.822	300	346.5214	108.9052
29	228.625	239.351	234.794	234.4264	2.838202
30	394.501	394.667	394.502	394.5145	0.032763

IV. Conclusion

The new variant of a successful jSO optimiser with a progressive archive approach was introduced and evaluated by a set of 29 test problems CEC 2024. The results of the Wilcoxon rank-sum tests show that replacing worse individuals in the archive of the old good parent solution substantially increases the performance of the jSO algorithm. The efficiency of the jSOa variant is lower for lower dimension $D = 10$, and it increases for higher dimensions. The new jSOa algorithm is better in 62 % of test problems compared to the median values, and it is worse in 29 % of the test problems. Using the Wilcoxon rank-sum test, the jSOa performs significantly better in 19 % of test problems, and it is worse in 3 % of the cases. The approach is very simple, it does not increase the complexity of the original jSO. Moreover, this approach can be used in any optimisation where an archive of old solutions is employed. The aforementioned results promised further research into the enhanced mechanisms of adaptive DE variants.

TABLE IV
BASIC CHARACTERISTICS OF JSOA, $D = 100$

Fun	min	max	median	mean	SD
1	6.82E-13	1.87E-09	1.09E-10	3.53E-10	5.05E-10
3	1.21E-10	9.98E-09	1.86E-09	2.71E-09	2.67E-09
4	9.06E+01	212.277	192.068	194.3091	23.69604
5	2.30E+01	61.8196	4.40E+01	4.45E+01	8.006467
6	9.37E-08	6.03E-06	1.72E-06	2.17E-06	1.50E-06
7	1.33E+02	165.489	1.48E+02	147.9765	8.345096
8	3.19E+01	6.30E+01	4.51E+01	4.57E+01	7.60187
9	1.14E-13	1.14E-13	1.14E-13	1.14E-13	0
10	8273.66	10420.2	9623.37	9499.556	578.3018
11	47.5279	161.508	88.3205	96.07919	34.60926
12	6531.1	28775.9	11768.4	13940.85	5793.004
13	109.163	276.353	197.979	193.3524	44.09903
14	45.3358	69.8102	57.8118	57.31728	7.214057
15	99.2398	232.411	167.799	166.4265	40.13612
16	1382.36	2459.54	1841.99	1874.871	337.6773
17	933.663	1725.38	1321.67	1323.099	219.4104
18	125.864	208.794	160.641	168.4092	26.75587
19	63.4428	117.607	86.9284	90.51551	15.33583
20	892.586	1838.93	1326.38	1343.666	224.6785
21	258.56	280.459	271.32	270.2358	6.364778
22	9347.35	11401.5	10360.7	10242.65	557.0251
23	534.088	567.317	555.183	555.8448	8.972217
24	893.984	920.684	908.049	907.7644	7.343302
25	637.418	773.533	707.671	723.0916	40.2414
26	3029.87	3494.26	3302.35	3276.804	117.1983
27	531.118	611.771	575.786	574.2104	19.51622
28	478.472	576.861	518.742	519.5048	23.72568
29	1100.31	1770.89	1418.9	1426.518	208.2236
30	2128.74	3028.16	2496.02	2526.256	204.5526

TABLE V
MEAN VALUES OF JSOA AND JSO WITH RESULTS OF THE WILCOXON RANK-SUM TESTS, $D = 10$

Fun	jSO	sign.	jSOa
1	0	≈	0
3	0	≈	0
4	0	≈	0
5	1.99E+00	6.74E-01	2.03E+00
6	7.28E-14	2.64E-01	5.46E-14
7	1.21E+01	4.97E-01	1.20E+01
8	2.11E+00	5.06E-01	1.95E+00
9	0	≈	0
10	26.75115	0.907227	17.39475
11	9.09E-15	0.337055	0
12	15.3147	0.854784	9.970332
13	3.825212	0.87435	3.798012
14	2.73E-14	0.698509	3.64E-14
15	0.264498	0.145397	0.348918
16	0.380916	0.726901	0.328383
17	0.53515	0.969045	0.520482
18	0.290768	0.443376	0.216754
19	0.012056	0.289968	8.96E-03
20	0	0.337055	0.012487
21	149.631	0.835687	153.788
22	100	≈	100
23	301.6558	0.500693	301.3629
24	311.5011	0.137271	292.8325
25	408.8677	0.580893	410.6832
26	300	≈	300
27	3.89E+02	7.85E-01	3.89E+02
28	4.42E+02	1.55E-02	**3.47E+02**
29	2.35E+02	6.14E-01	2.34E+02
30	3.31E+04	4.04E-01	3.95E+02

TABLE VI
MEAN VALUES OF JSOA AND JSO WITH RESULTS OF THE WILCOXON RANK-SUM TESTS, $D = 30$

Fun	jSO	sign.	jSOa
1	9.66E-15	0.050575	5.68E-15
3	4.77E-14	5.74E-01	4.32E-14
4	5.86E+01	≈	5.86E+01
5	1.12E+01	2.40E-01	1.08E+01
6	6.84E-09	0.819724	2.74E-09
7	4.21E+01	4.97E-01	4.17E+01
8	11.38485	0.66168	11.1702
9	4.55E-15	0.571444	9.09E-15
10	**1278.249**	0.029771	1437.121
11	8.577837	0.697967	15.07073
12	250.5509	0.528277	221.8658
13	19.09217	0.449223	19.01671
14	16.74014	0.509448	16.36046
15	3.660004	0.341737	3.162764
16	37.15934	0.484863	43.96345
17	31.0951	0.130172	34.18114
18	20.52259	0.00167	**19.20896**
19	5.12033	0.953583	5.013231
20	27.38717	0.922715	28.16902
21	212.3881	0.020947	**211.0444**
22	100	≈	100
23	357.0464	0.011657	**353.7243**
24	430.4798	0.022048	**428.6836**
25	3.87E+02	2.86E-01	3.87E+02
26	9.68E+02	2.83E-02	**9.47E+02**
27	5.05E+02	4.08E-03	**4.99E+02**
28	3.04E+02	1.00E+00	3.05E+02
29	4.30E+02	8.08E-02	4.28E+02
30	1.98E+03	0.022042	**1.96E+03**

TABLE VII
MEAN VALUES OF JSOA AND JSO WITH RESULTS OF THE WILCOXON RANK-SUM TESTS, $D = 50$

Fun	jSO	sign.	jSOa
1	6.54E-14	5.65E-07	**3.52E-14**
3	1.57E-13	1.05E-03	**1.16E-13**
4	3.48E+01	0.422759	5.37E+01
5	20.97433	0.586937	21.65092
6	5.20E-08	9.84E-01	1.15E-08
7	7.33E+01	0.697974	7.25E+01
8	21.79043	0.785894	21.57034
9	6.37E-14	≈	6.37E-14
10	2993.602	0.415118	3055.127
11	28.72246	0.365629	27.92883
12	1907.216	0.509448	1842.178
13	51.23089	0.023199	**34.23463**
14	23.41872	0.341737	23.87574
15	24.45275	0.077453	23.24126
16	350.5859	0.415118	412.9931
17	281.2902	0.785899	272.271
18	25.07339	0.14561	24.31945
19	10.4773	0.277231	9.861877
20	128.2331	0.393257	114.1555
21	222.439	0.322396	223.2138
22	1.88E+03	4.89E-01	2.02E+03
23	4.39E+02	2.07E-01	4.37E+02
24	516.5796	0.000846	**512.2685**
25	4.81E+02	1.10E-01	4.81E+02
26	1.21E+03	6.25E-02	1.18E+03
27	518.1559	0.000113	**506.4695**
28	4.59E+02	3.37E-01	4.61E+02
29	3.50E+02	2.52E-01	3.54E+02
30	620086	0.297148	609742.7

TABLE II
BASIC CHARACTERISTICS OF JSOA, $D = 30$

Fun	min	max	median	mean	SD
1	0	1.42E-14	0	5.68E-15	7.11E-15
3	0	5.68E-14	5.68E-14	4.32E-14	2.48E-14
4	5.86E+01	5.86E+01	5.86E+01	5.86E+01	0
5	5.96975	1.39E+01	1.09E+01	1.08E+01	1.80E+00
6	1.14E-13	3.42E-08	1.14E-13	2.74E-09	9.47E-09
7	35.7333	49.4485	41.9991	41.73943	3.00416
8	7.95967	1.69E+01	10.9446	1.12E+01	1.99E+00
9	0	1.14E-13	0	9.09E-15	3.15E-14
10	952.868	1943.97	1408.73	1437.121	260.8179
11	2.12E-05	63.959	5.03045	15.07073	23.73379
12	3.93315	622.323	139.032	221.8658	180.2231
13	7.57682	23.1013	18.5728	19.01671	3.416751
14	0.065422	23.823	21.7599	16.36046	9.047931
15	0.532021	6.19504	2.9159	3.162764	2.162506
16	8.34245	165.391	25.2335	43.96345	45.89018
17	22.5677	43.257	35.359	34.18114	6.321498
18	0.620108	21.7909	20.5996	19.20896	5.356961
19	2.38885	7.19191	4.99597	5.013231	1.174222
20	8.99207	41.0492	30.4499	28.16902	7.781739
21	207.562	213.772	211.199	211.0444	1.703539
22	100	100	100	100	0
23	347.18	362.534	353.361	353.7243	4.459579
24	424.282	432.11	429.14	428.6836	2.249486
25	386.688	386.718	386.695	386.6978	0.00778
26	868.22	1047.95	945.421	947.1272	39.21903
27	484.625	509.602	498.73	499.0251	7.057737
28	300	413.975	300	304.559	22.795
29	365.143	442.659	429.637	428.1	14.55697
30	1941.62	1987.92	1969.75	1963.319	11.34524

TABLE III
BASIC CHARACTERISTICS OF JSOA, $D = 50$

Fun	min	max	median	mean	SD
1	2.84E-14	5.68E-14	2.84E-14	3.52E-14	8.33E-15
3	5.68E-14	1.71E-13	1.14E-13	1.16E-13	2.58E-14
4	0.603363	1.42E+02	2.85E+01	5.37E+01	5.10E+01
5	15.926	3.28E+01	2.19E+01	2.17E+01	3.76E+00
6	0	4.79E-08	2.27E-13	1.15E-08	2.09E-08
7	63.225	8.05E+01	73.2106	7.25E+01	3.81E+00
8	13.9294	2.70E+01	2.12E+01	2.16E+01	3.02E+00
9	0	1.14E-13	1.14E-13	6.37E-14	5.76E-14
10	2283.76	4026.34	3045.65	3055.127	353.0983
11	21.3186	33.1323	29.1525	27.92883	3.183197
12	1176.08	2502.12	1874.62	1842.178	368.5819
13	5.65498	51.8057	42.4076	34.23463	15.65656
14	22.3186	27.1443	23.8588	23.87574	0.935271
15	19.1107	27.9985	22.9371	23.24126	2.192637
16	133.684	826.418	407.364	412.9931	149.0083
17	77.8382	460.372	259.083	272.271	101.9506
18	20.5114	28.7967	24.2157	24.31945	1.877672
19	5.7708	13.5595	10.4087	9.861877	1.960145
20	46.2472	303.033	95.5593	114.1555	67.65573
21	215.774	229.966	223.297	223.2138	3.322049
22	100	4142.97	2916.32	2024.849	1763.861
23	427.271	449.515	436.163	437.0896	6.049041
24	504.414	521.455	512.109	512.2685	4.530173
25	480.231	491.837	480.243	480.7101	2.318147
26	1102.9	1329.34	1182.3	1180.312	47.52694
27	492.12	526.024	505.813	506.4695	8.71E+00
28	458.849	507.695	458.849	460.8028	9.7692
29	323.75	384.709	355.644	354.3049	16.67917
30	579411	665338	608409	609742.7	27783.98

TABLE VIII
MEAN VALUES OF JSOA AND JSO WITH RESULTS OF THE WILCOXON RANK-SUM TESTS, $D = 100$

Fun	jSO	sign.	jSOa
1	1.19E-08	2.00E-06	**3.53E-10**
3	7.47E-08	7.38E-09	**2.71E-09**
4	2.00E+02	9.22E-02	1.94E+02
5	4.46E+01	0.907322	4.45E+01
6	3.86E-05	0.148275	2.17E-06
7	147.015	0.830991	147.9765
8	43.3726	0.252305	45.73394
9	1.14E-13	≈	1.14E-13
10	**8992.786**	0.013007	9499.556
11	106.37	0.33681	96.07919
12	18161.11	0.018887	**13940.85**
13	278.8107	0.000331	**193.3524**
14	67.45224	0.005866	**57.31728**
15	196.3393	0.111602	166.4265
16	1759.578	0.268743	1874.871
17	**1163.729**	0.024403	1323.099
18	189.7944	0.047806	**168.4092**
19	101.208	0.107303	90.51551
20	1228.883	6.53E-02	1343.666
21	269.585	0.669479	270.2358
22	9910.666	0.087738	10242.65
23	565.8174	4.13E-04	**555.8448**
24	922.1768	2.21E-07	**907.7644**
25	718.4125	0.853753	723.0916
26	3352.596	0.045663	**3276.804**
27	596.7078	3.58E-05	574.2104
28	522.0698	0.433769	519.5048
29	1481.483	0.303785	1426.518
30	2394.615	0.00832	2526.256

TABLE IX
NUMBER OF BETTER, SIMILAR, AND WORSE RESULTS OF JSOA COMPARED WITH JSO

D	jSOa	≈	jSO
10	16	6	7
30	19	2	8
50	19	1	9
100	18	1	10
Σ	72	10	34

TABLE X
NUMBER OF SIGNIFICANTLY BETTER, SIMILAR, AND WORSE RESULTS OF JSOA COMPARED WITH JSO

D	jSOa	≈	jSO
10	1	28	0
30	7	21	1
50	4	25	0
100	10	16	3
Σ	22	90	4

TABLE XI
TIME COMPLEXITY OF THE PROPOSED JSOA

D	T_1	T_2	$(T_2 - T_1)/T_1$
10	0.0266	0.4497	15.9060
30	0.0758	1.5192	19.0422
50	2.0245	0.1564	11.9463
100	0.5399	9.2385	16.1116

REFERENCES

[1] K. Qiao, X. Wen, X. Ban, P. Chen, K. V. Price, P. N. Suganthan, J. Liang, G. Wu, and C. Yue, "Evaluation criteria for cec 2024 competition and special session on numerical optimization considering accuracy and speed," Computational Intelligence Laboratory, Zhengzhou University, Zhengzhou China And Technical Report, Nanyang Technological University, Singapore, Tech. Rep., 2023, github.com/P-N-Suganthan.