Interdependent dynamics of awareness and epidemic spreading on multiplex networks

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INTRODUCTION

- Epidemic spreading models the processes of diffusion of diseases through a population.
- When a disease breaks out, information about the presence of this illness (awareness) is generated and spread throughout the population.
- These two processes may take place through different kind of interactions (in different networks)
  - E.g. awareness spreads through microblogs and the disease spreads through a network of physical contacts
- People aware of a disease can take measures to reduce their susceptibility.
  - Vaccination, use of face masks, hand sanitizer…

The onset and the incidence of the epidemics is affected by the awareness

UAU-SIS MODEL

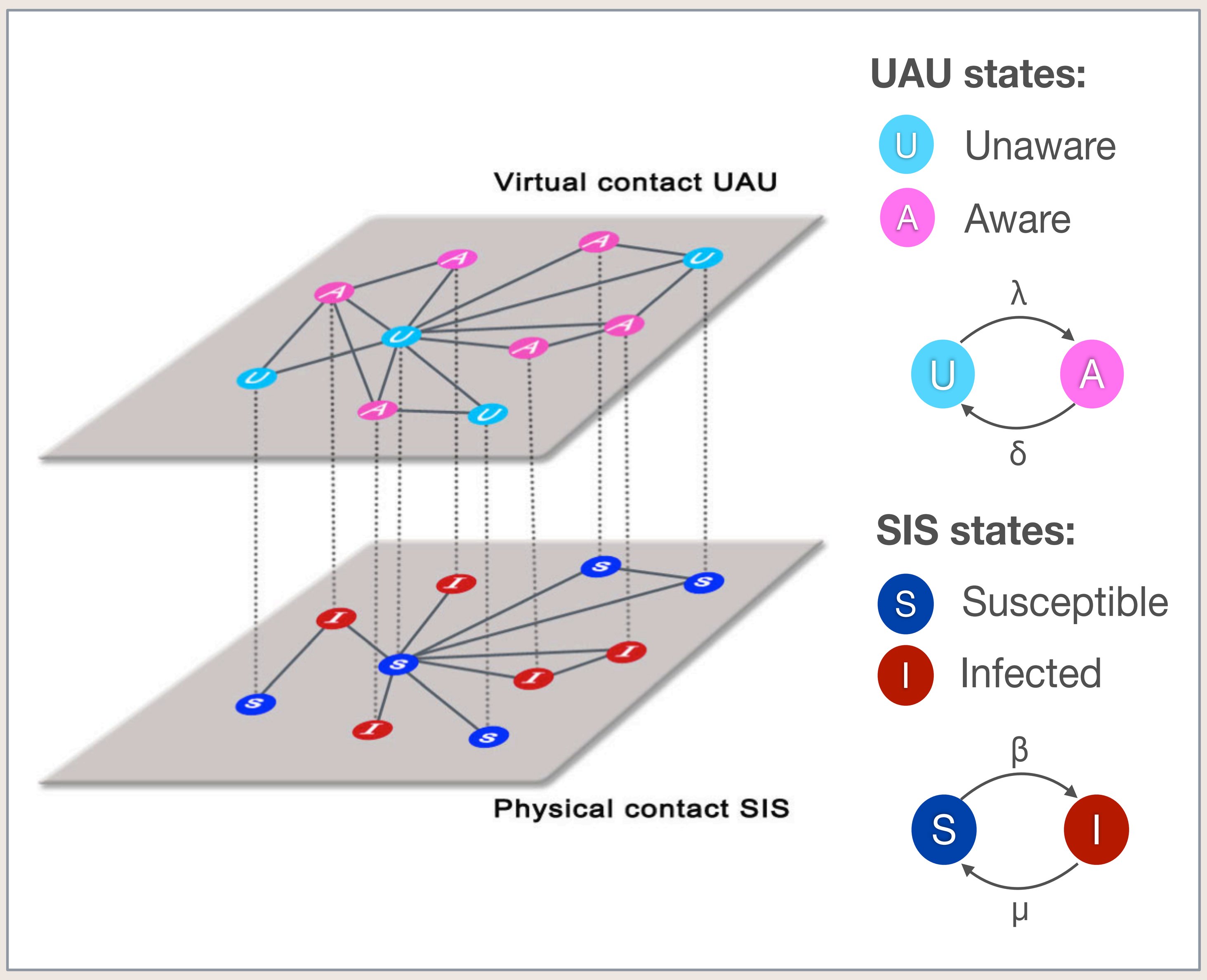

Fig.1. *Sketch of the UAU-SIS model*

- The UAU-SIS model is an abstraction of the process of spreading of information *simultaneously* with spreading of epidemics, where:
  - Each node has a state for each layer
  - Four combinations: AS, AI, US, UI
- UAU-SIS dynamical interaction:
  - An infected individual in the SIS layer immediately becomes aware in the UAU layer
  - A node which is aware in the UAU layer takes measures against the disease and has less chances to become infected.

General case: $\beta^U = \beta$

$\beta^A = \gamma\beta, \quad 0 \leq \gamma \leq 1$

Simplest case: $\gamma = 0$

MICROSCOPIC MARKOV CHAIN APPROACH EQUATIONS

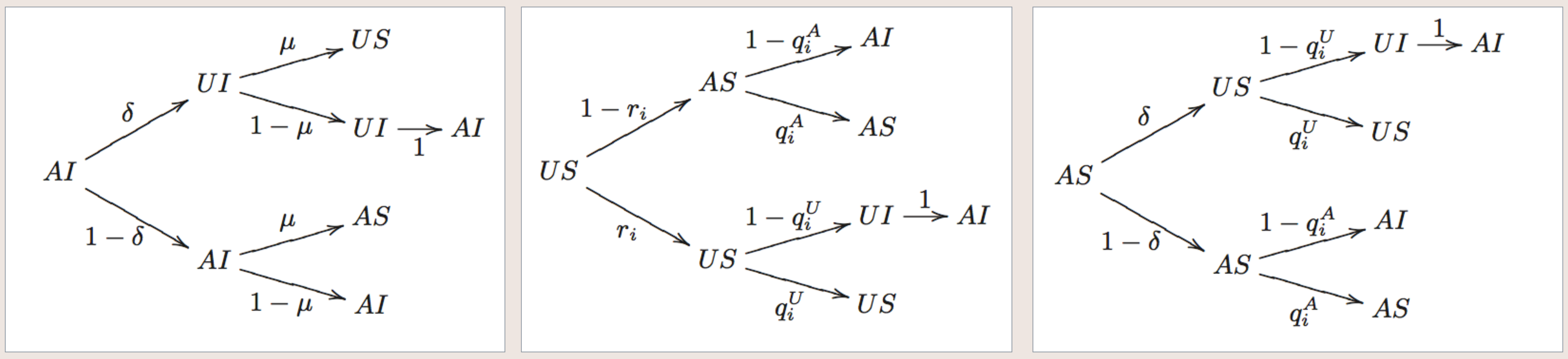

Fig. 2. *Transition probability trees for the three states of the UAU-SIS dynamics at each time step*

- Prob. node i not being infected by any neighbor if he is unaware:

$$q_i^{\mathrm{U}}(t) = \prod_j (1 - b_{ji} p_j^{\mathrm{AI}}(t) \beta^{\mathrm{U}})$$

- Prob. node i not being infected by any neighbor if he is aware:

$$q_i^{\mathrm{A}}(t) = \prod_j (1 - b_{ji} p_j^{\mathrm{AI}}(t) \beta^{\mathrm{A}})$$

- Prob. node i not being informed by any neighbor:

$$r_i(t) = \prod_j (1 - a_{ji} p_j^{\mathrm{A}}(t) \lambda)$$

MMCA Equations:

- They describe the probability for each node to be in each possible state at time t

$$p_i^{\mathrm{US}}(t+1) = p_i^{\mathrm{AI}}(t)\delta\mu + p_i^{\mathrm{US}}(t) r_i(t) q_i^{\mathrm{U}}(t) + p_i^{\mathrm{AS}} \delta q_i^{\mathrm{U}}(t)$$

$$p_i^{\mathrm{AS}}(t+1) = p_i^{\mathrm{AI}}(t)(1-\delta)\mu + p_i^{\mathrm{US}}(1 - r_i(t)) q_i^{\mathrm{A}}(t) + p_i^{\mathrm{AS}}(t)(1-\delta) q_i^{\mathrm{A}}(t)$$

$$p_i^{\mathrm{AI}}(t+1) = p_i^{\mathrm{AI}}(t)(1-\mu) + p_i^{\mathrm{US}} \left[(1 - r_i(t))(1 - q_i^{\mathrm{A}}(t)) + r_i(t)(1 - q_i^{\mathrm{U}}(t))\right] + p_i^{\mathrm{AS}}(t) \left[\delta(1 - q_i^{\mathrm{U}}(t)) + (1-\delta)(1 - q_i^{\mathrm{A}}(t))\right]$$

RESULTS

- SIS network: 1000 nodes power-law distributed with exponent 2.5
- UAU network: same network with 400 additional links
- We have performed extensive Monte Carlo simulations for fixed δ and μ for all values of λ and β.

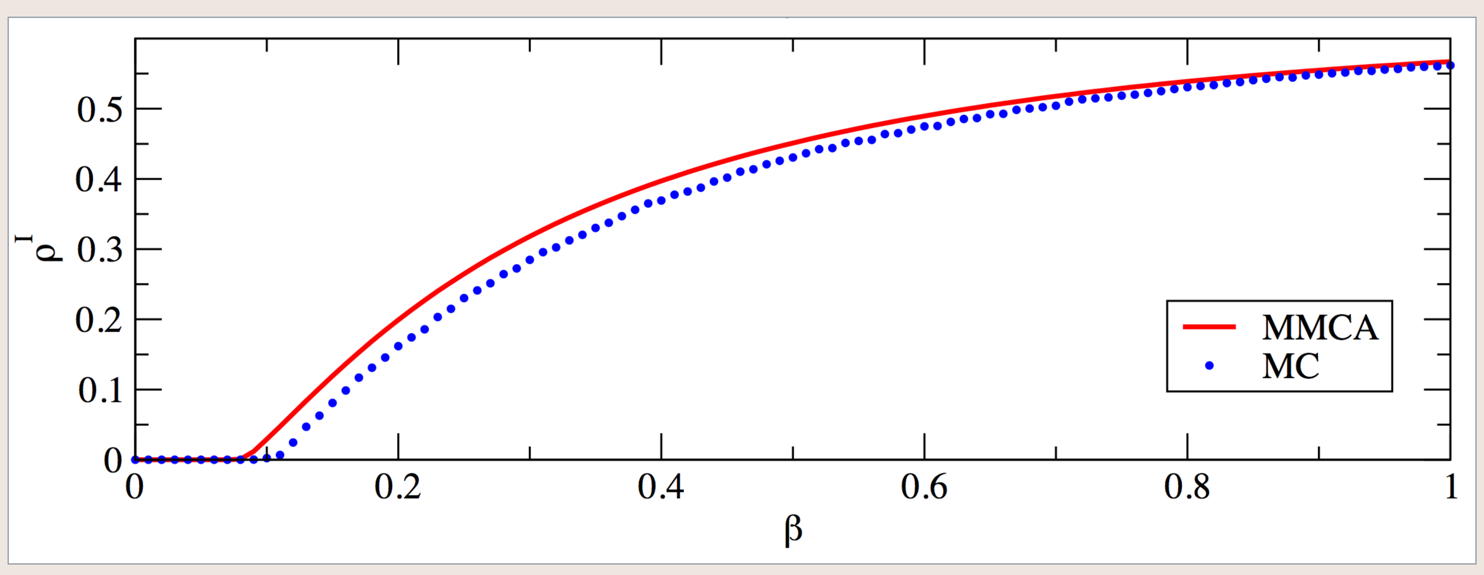

Fig. 3. *Fraction of infected nodes in the steady state as a function of infectivity rate β. For the UAU: infectivity rate is λ=0.15, and recovery rate δ = 0.6. For the SIS: the recovery rate is μ = 0.4 and the initial fraction of infected nodes is ρ(0) = 0.2. The relative error between MC and MMCA is less than 2.5%*

- From the MMCA equations we can calculate the dependence $\beta_c(\lambda)$
- The line of critical points separates two phases
- There is a *metacritical* point that denotes the beginning of the dependence

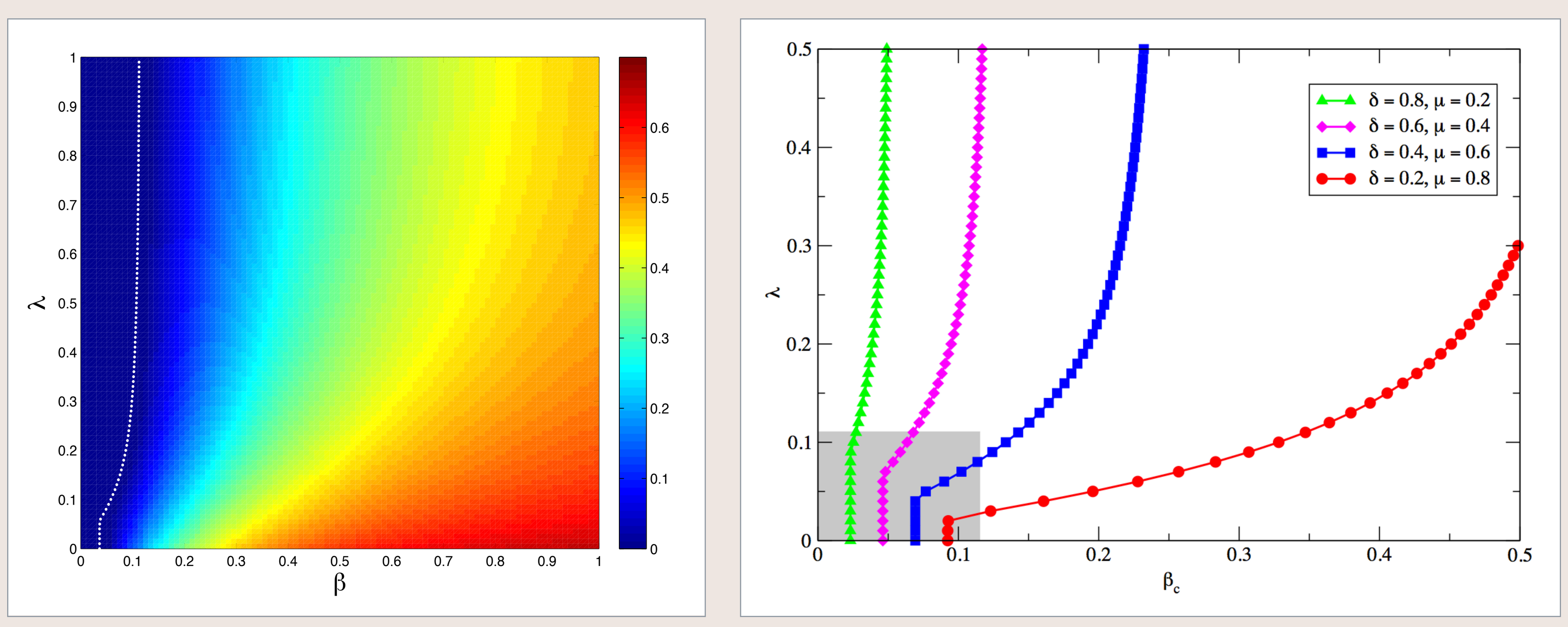

Fig. 4. *Left: Final fraction of infected nodes as a function of β and λ. The dotted line contains all the critical points β_c. Right: Curves of critical points for different values of δ and μ. Both plots refer to the same networks as in Fig. 3.*

Reference: Dynamical interplay between awareness and epidemic spreading in multiplex networks. C. Granell, S. Gómez and A. Arenas, Physical Review Letters, 111, 128701 (2013)