Assignment IV Parabolic Reflector in Reception

EE4725 Quasi Optical Systems

Petar V. Peshev, p.v.peshev@student.tudelft.nl
Department of Electrical Engineering, Mathematics, and Computer Science,
Delft University of Technology, Delft, The Netherlands

Abstract

In this assignment, a parabolic reflector, with f-number $f_{\#} = 2$, has a circular feed with diameter $D_{feed} = 4\lambda$. Moreover, the diameter of the reflector is $D_r = 100\lambda$. Finally, the frequency of operation is $f = 50\ GHz$. The performance of this antenna geometry in reception is analyzed for an incident plane wave with amplitude $E_0^{PW} = 1$ V/m.

I. Simulations

The scripts used to simulate the lens antenna are in the GIT repository Assignment IV Repository; and the library developed and used in the scripts is in quasi-optics-library. To run the simulations either place the script and library repositories in the same parent folder or change the library path in the simulation scripts.

II. Reception GO Field, Received Power, and Efficiency at Broadside Incidence

The reception GO field is

$$\vec{e}_{rx}^{GO} = \vec{e}_{rx}^{GO}(\vec{r}', \Delta\vec{k}_{\rho,i} = 0)e^{-j\vec{k}_\rho \cdot \Delta\vec{k}_{\rho,i} F(1+\delta_n(\theta))/k_0}, \tag{1}$$

where $\vec{k}_\rho$ is the projection of the wave vector on the equivalent surface, $\vec{k}_{\rho,i}$ is the projection of the incident wave vector, F is the focal distance, k_0 is the propagation constant of the wave in free space, $\delta_n(\theta)$ quantifies the phase variation and is

$$\delta_n(\theta) = \frac{1-\cos\theta}{1+\cos\theta}, \tag{2}$$

and $\vec{e}_{rx}^{GO}(\vec{r}', \Delta\vec{k}_{\rho,i} = 0)$ is the GO field at the center of the equivalent FO sphere's azimuth at $\Delta\vec{k}_{\rho,i} = 0$ and is

$$\vec{e}_{rx}^{GO}(\vec{r}', \Delta\vec{k}_{\rho,i} = 0) = -\frac{2}{1+\cos\theta}E_0^{PW}(\sin\phi\hat{\theta} + \cos\phi\hat{\phi}), \tag{3}$$

where E_0^{PW} is the amplitude of the incident plane wave. The elevation angle domain is defined by maximum elevation angle and is given by

$$\theta_0 = 2\tan^{-1}(\frac{D_r}{4F}), \tag{4}$$

where D_r is diameter of the parabolic reflector. The $\vec{V}_{rx}^{GO}(\vec{r}', \Delta\vec{k}_{\rho,i})$ is

$$\vec{V}_{rx}^{GO}(\vec{r}', \Delta\vec{k}_{\rho,i}) = \vec{e}_{rx}^{GO}\frac{R}{e^{jkR}}, \tag{5}$$

where the field's amplitude is scaled to the center of the equivalent FO sphere, and the phase shift is cancelled (the exponential term is positive, because the wave is propagating towards the center of the equivalent FO sphere).

The transmission $\vec{V}_{TX}^{A}(\theta, \phi)$ field is

$$\vec{V}_{TX}^{A}(\theta, \phi) = \vec{E}_{TX}^{A}(\theta, \phi)\frac{R}{e^{-jkR}}, \tag{6}$$

where $\vec{E}_{TX}^{A}$ is the radiated far-field by the feed and the wave is propagating outwards (therefore the negative term in the exponential). The parabolic antenna has a circular feed with uniform current distribution, thus the electric far-field of the feed is

$$\vec{E}_{TX}^{A} = A_{feed}\frac{J_1(\frac{k_0\sin\theta D_{feed}}{2})}{\frac{k_0\sin\theta D_{feed}}{2}}(\cos\phi\hat{\phi} + \cos\theta\sin\phi\hat{\theta})\frac{e^{-jkR}}{R}, \tag{7}$$

where A_{feed} is the feed's area and J_1 is the Bessel function of first kind. Therefore, the $\vec{V}_{TX}^{A}(\theta, \phi)$ field is

$$\vec{V}_{TX}^{A}(\theta, \phi) = A_{feed}\frac{J_1(\frac{k_0\sin\theta D_{feed}}{2})}{\frac{k_0\sin\theta D_{feed}}{2}}(\cos\phi\hat{\phi} + \cos\theta\sin\phi\hat{\theta}). \tag{8}$$

The total reception GO field V_{RX}^{GO} for an incident plane wave with $E_i^{PW} = 1$ V/m, incident at $\theta_i = -0°$ and $\phi_i = 0°$, and transmission A field V_{TX}^{A} are plotted in Fig.1 as a function of the elevation angle θ in the valid domain. While it is obvious that the transmission A field exhibits maximum at broadside ($\theta = 0°$ and $\phi = 0°$), the GO field exhibits maximum at the edges of the parabolic reflector and minimum at broadside. Yet, the GO field at broadside direction is only -0.13 dB below the maximum GO field. Furthermore, the received power, at the load, of a parabolic antenna is the dot product between the GO field and the A field, therefore as the A field exhibits maximum at broadside and its magnitude decreases much faster than the magnitude increase of the GO field, then maximum reception is expected at broadside ($\theta = 0°$ and $\phi = 0°$).

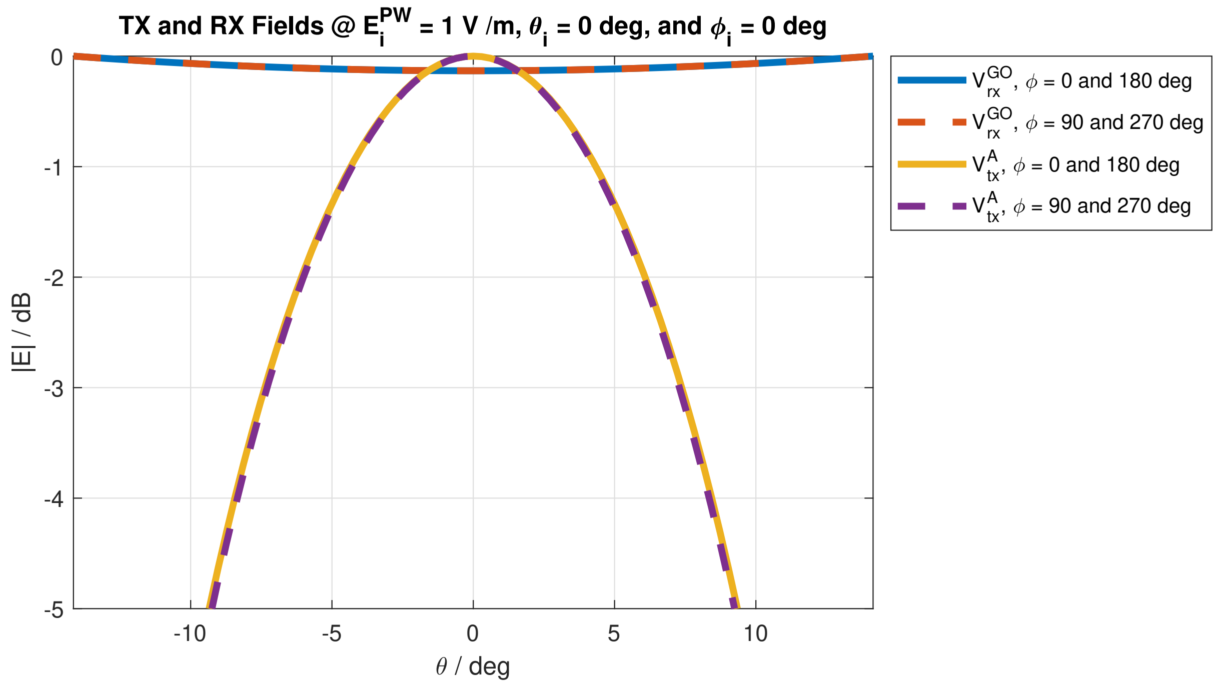

Fig. 1. Reception GO and transmission A fields, in main planes $\phi = 0°$ and $180°$ and $\phi = 90°$ and $270°$ as a function of the elevation angle, for a parabolic reflector and circular feed with diameter $D_f = 4\lambda$, reflector's diameter $D_r = 100\lambda$, and operating at $f = 50$ GHz; the incident wave is at broadside, $\theta_i = 0°$ and $\phi_i = 0°$ and has an amplitude $E_i^{PW} = 1$ V/m.

The open-circuit voltage of the equivalent Thevenin circuit for the parabolic antenna in reception is

$$V_{OC}(\theta_i, \phi_i) = \frac{2}{\zeta} \int_0^{2\pi} \int_0^{\theta_0} \vec{V}_{TX}^{A}(\theta, \phi) \cdot \vec{V}_{RX}^{GO}(\vec{r}', \Delta\vec{k}_{\rho,i}) \sin\theta d\theta d\phi, \tag{9}$$

where ζ is the impedance of the medium at which the antenna is placed (in this case free space). The received power at the load is

$$P_{RX}(\theta_i, \phi_i) = \frac{|V_{OC}(\theta_i, \phi_i)|^2}{16 P_{rad}}, \tag{10}$$

where the antenna's active impedance $R_a = 2P_{rad}$ for a uniform current distribution with amplitude $I_0 = 1$ A, and it is evident that the reception power is dependent on the incident angle. The aperture efficiency is therefore given by the ration of the received power and the incident power due to the plane wave

$$\eta_{ap}^{RX}(\theta_i, \phi_i) = \frac{P_{RX}}{P_{inc}} = \frac{P_{RX}}{\frac{|E_0^{PW}|^2}{2\zeta} A_r}, \tag{11}$$

where A_r is the area of the reflector.

The received power by the parabolic reflector for an incident plane wave with amplitude $E_i^{PW} = 1$ V/m and its aperture efficiency are summarized in Table.II and are 269.65 μW and 71.96 % respectively. This efficiency menas that 71.96 % of the incident power is supplied to the load.

TABLE I
RECEIVED POWER AND EFFICIENCY

P_{rx} and η	
	Figure
P_{rx}	269.65 μW
η	71.96 %

III. Reception Pattern, Efficiency, Directivity, and Gain

The reception pattern represents the received power at the valid incident angles. Therefore, to evaluate the reception pattern, the received power at all valid incident angles is calculated.

The reception pattern in UV representation for the parabolic antenna with reflector diameter $D = 100\lambda$, feed diameter $D_{feed} = 4\lambda$, and f-number $f_{\#} = 2$ at $f = 50$ GHz is plotted in Fig.2; the amplitude of the incident plane wave is $E_i^{PW} = 1$ V/m. As previously stated, it is expected that the maximum reception is at broadside; from the plotted reception pattern, this assumption is confirmed. The maximum reception is at broadside ($\theta_i = 0°$ and $\phi_i = 0°$) and has a narrow 3dB beamwidth of approximately $0.5°$. The received power at angles above $\theta = 0.75°$ is below -40 dB. Therefore, the parabolic reflector must be aligned for good reception.

The received power and transmitted E-field are plotted at $\phi = 0°$ and $180°$ and $\phi = 90°$ and $270°$ in Fig.3 as a function of the incident and elevation angle. Compared to the transmitted field, the reception pattern has with $0.1°$ lower 3dB beamwidth, and with approximately 30 dB lower side-lobes. Therefore, the reception pattern exhibits approximately the same main-lobe to the transmitted field, but has lower side-lobes.

The maximum possible directivity in both reception and transmission is

$$\mathbb{D}_{max} = \frac{4\pi}{\lambda^2} A_r. \tag{12}$$

Furthermore, in reception, the directivity of the parabolic reflector is

$$\mathbb{D}_{RX} = \frac{4\pi}{\int_0^{2\pi} \int_0^{\theta_i^{max}} P_{RX}^{norm}(\theta_i, \phi_i) \sin\theta_i d\theta_i d\phi_i}, \tag{13}$$

where $P_{RX}^{norm}(\theta_i, \phi_i)$ is the normalized received power at every incident angle to the maximum received power. The gain of the antenna in reception is

$$\mathbb{G}(\theta_i, \phi_i) = \mathbb{D}_{max} \eta_{ap}^{RX}. \tag{14}$$

While for the antenna in transmission, the directivity is

$$\mathbb{D}_{TX} = \mathbb{D}_{max} \eta_{tap}^{TX}, \tag{15}$$

where η_{tap}^{TX} is the taper efficiency in transmission. The gain in transmission is

$$\mathbb{G} = \mathbb{D}_{max} \eta_{ap}^{TX}, \tag{16}$$

where η_{ap}^{TX} is the aperture efficiency in transmission (defined by the product of the taper and spillover efficiencies).

The aperture efficiency, broadside (maximum) directivity, and gain for parabolic antenna in reception and transmission are summarized in Table.II. The aperture efficiency in reception and transmission are 71.76 % and 71.24 % respectively. Therefore, the aperture efficiency is comparably the same between reception and transmission (the small deviation might be the result of simulation tolerance). The broadside directivity of the antenna in reception and transmission are 49.17 dB and 49.16 dB respectively. The broadside gain of the antenna in reception and transmission are 48.50 dB and 48.47 dB respectively. Therefore, the directivity and gain of the antenna in reception and transmission are comparable (the small deviation is caused by simulation tolerance). The similarity between the aperture efficiency, broadside directivity, and broadside gain between the parabolic antenna in reception and transmission is the effect of the reciprocity principle, the antenna behaving in the same way when the observer and antenna places are interchanged.

TABLE II
RECEPTION AND TRANSMISSION APERTURE EFFICIENCY, BROADSIDE DIRECTIVITY, AND BROADSIDE GAIN

RX and TX Figures		
	RX	TX
η_{ap}	71.76 %	71.24 %
D	49.17 dB	49.16 dB
G	48.50 dB	48.47 dB

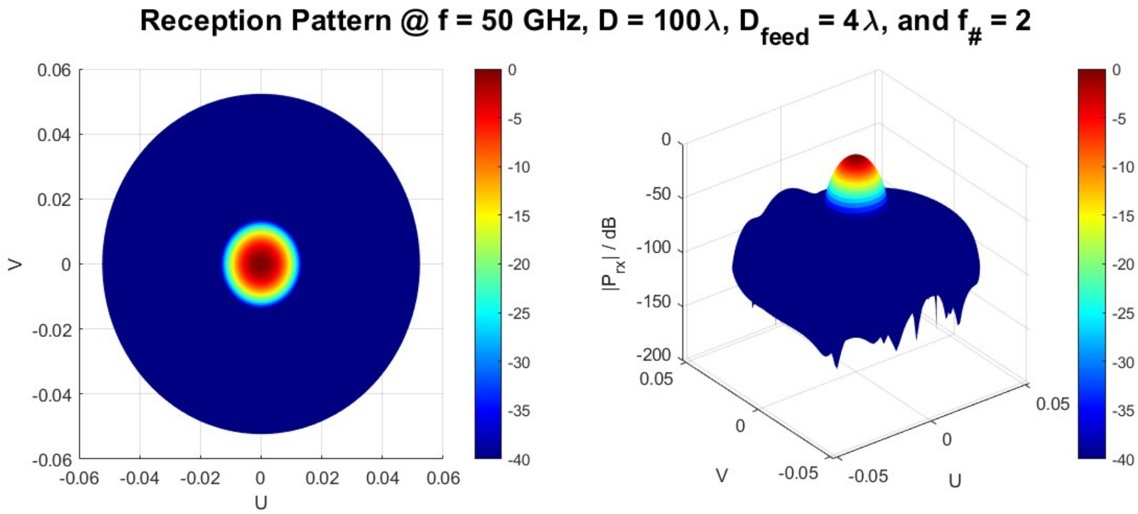

Fig. 2. Reception pattern for a parabolic reflector and circular feed with diameter $D_f = 4\lambda$, reflector's diameter $D_r = 100\lambda$, and operating at $f = 50$ GHz; the incident wave has an amplitude $E_i^{PW} = 1$ V/m. Visualized in UV coordinate.

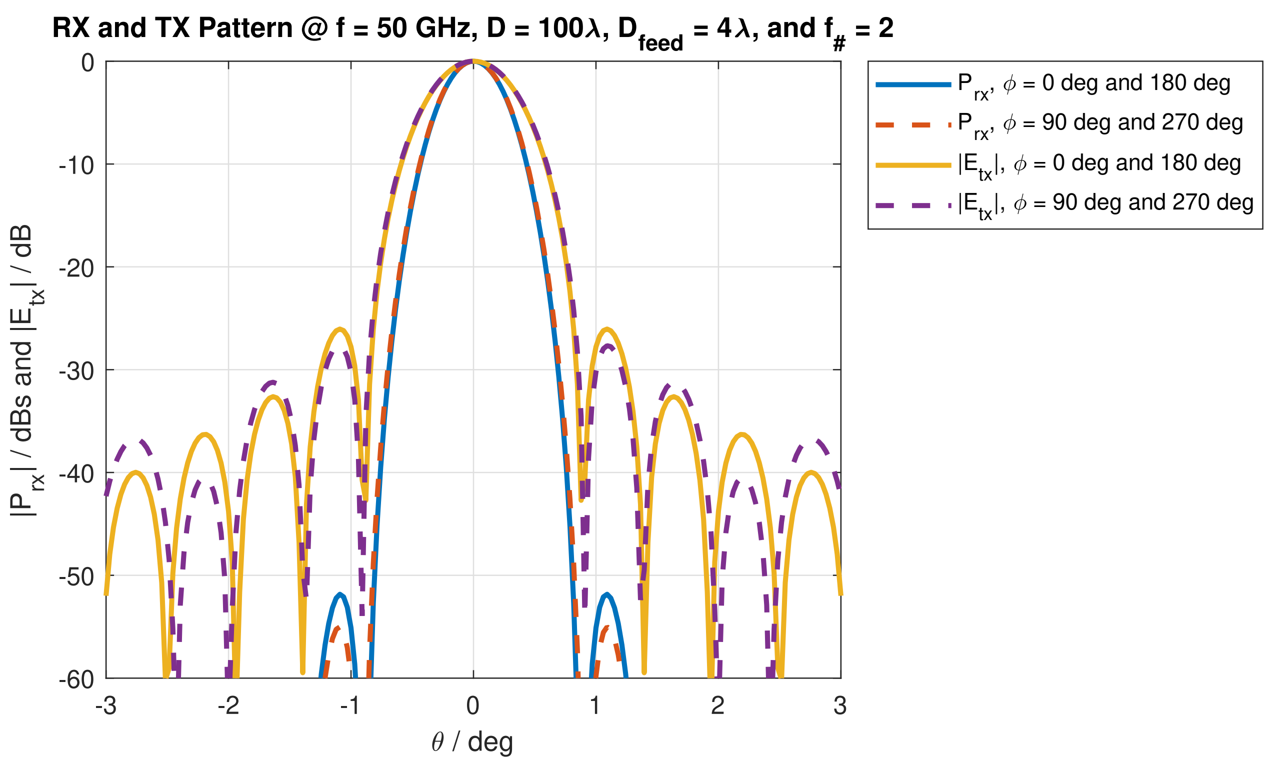

Fig. 3. Reception and transmission pattern, in main planes $\phi = 0°$ and $180°$ and $\phi = 90°$ and $270°$ as a function of the incident and elevation angles respectively, for a parabolic reflector and circular feed with diameter $D_f = 4\lambda$, reflector's diameter $D_r = 100\lambda$, and operating at $f = 50$ GHz; the incident wave has an amplitude $E_i^{PW} = 1$ V/m.

IV. Displaced Feed in Focal Plane

The displacement of the feed adds an additional phase term to the transmission A field

$$\vec{V}_{TX}^{A}(\Delta\vec{d}, \theta, \phi) = \vec{V}_{TX}^{A}(\Delta\vec{d} = 0, \theta, \phi) e^{j\vec{k}_\rho \cdot \Delta\vec{d}}, \tag{17}$$

where $\vec{V}_{TX}^{A}(\Delta\vec{d} = 0, \theta, \phi)$ is the transmission A field with no displaced feed as defined in Eq.8 and $e^{j\vec{k}_\rho \cdot \Delta\vec{d}}$ is the phase term added by the displacement of the feed with $\Delta\vec{d}$ in a certain direction defined by its vector components. Using the transmission A field with displaced feed, the reception power (and pattern) for an antenna with displaced feed is evaluated following the same procedures for the reception GO field, open circuit voltage of the Thevenin equivalent circuit, and received power at the load as for the antenna with centered feed.

The reception pattern for a displaced feed by $d_x = 4\lambda$ in UV representation for the parabolic antenna with reflector diameter $D = 100\lambda$, feed diameter $D_{feed} = 4\lambda$, and f-number $f_\# = 2$ at $f = 50$ GHz is plotted in Fig.4; the amplitude of the incident plane wave is $E_i^{PW} = 1$ V/m. The main lobe is similar to the one of the centered feed but displaced and pointing in the direction of $\theta = 1.12°$.

The reception patterns at $\phi = 0°$ and $180°$ for displaced feed by $d_x = 4\lambda$ and for centered feed with the same parameters are plotted in Fig.4 as a function of the incident angle. As previously stated, the main lobe and side-lobes are equivalent between the centered and displaced feed, however, the pointing direction is displaced to $\theta = 1.12°$.

The aperture efficiency and pointing direction of the antennas with centered and displaced feed are summarized in Table.III. The aperture efficiency of the centered and displaced feed are 71.66 % and 71.76 % respectively. The slightly lower aperture efficiency is caused by the slightly lower received power due to the displaced edges in the displaced feed antenna. Furthermore, the pointing direction of the displaced feed is $\theta_{dir} = 1.12°$.

TABLE III
RECEPTION APERTURE EFFICIENCY AND POINTING DIRECTION FOR DISPLACED AND CENTERED FEED

Displaced and Centered		
	$d_x = 4\lambda$	$d_x = 0$ (centered)
η_{ap}	71.66 %	71.76 %
θ_{dir}	1.12°	0°
ϕ_{dir}	0°	0°

The reception pattern at $\phi = 0°$ and $180°$ for displaced feeds with changing diameter by $d_x = D_{feed} = 2\lambda f_\#$, $d_x = D_{feed} = \lambda f_\#$, and $d_x = D_{feed} = 0.5\lambda f_\#$ as well as for the centered case (previously discussed) are plotted in Fig.6. As the feed's diameter decreases, the side-lobes of the reception pattern increase. Furthermore, the pointing directions are $1.12°$, $0.58°$, and $0.27°$ for $d_x = 2\lambda f_\#$, $d_x = \lambda f_\#$, and $d_x = 0.5\lambda f_\#$ respectively.

The aperture efficiency and pointing direction of the displaced feeds with $d_x = D_{feed} = 2\lambda f_\#$, $d_x = D_{feed} = \lambda f_\#$, and $d_x = D_{feed} = 0.5\lambda f_\#$ are summarized in Table.IV. The aperture efficiency is 71.66 %, 47.25 %, and 15.37 % for the feeds with diameters $D_{feed} = 2\lambda f_\#$, $D_{feed} = \lambda$, and $D_{feed} = 0.5\lambda$ respectively. The dominant cause of decrease in the aperture efficiency as the diameter of feeds decreases is caused by this decrease in the diameter. The decrease in the feed's diameter causes slightly narrow main lobe and increased side lobes; this increase in the side lobes and narrow side lobe are the main consequence to the aperture efficiency decrease. The pointing directions are $\theta_{dir} = 1.12°$, $\theta_{dir} = 0.58°$, and $\theta_{dir} - 0.27°$ for the $d_x = D_{feed} = 2\lambda f_\#$, $d_x = D_{feed} = \lambda f_\#$, and $d_x = D_{feed} = 0.5\lambda f_\#$ respectively (in all cases the $\phi_{dir} = 0°$, because the feed is displaced only in the x-axis).

TABLE IV
RECEPTION APERTURE EFFICIENCY AND POINTING DIRECTION FOR DISPLACED FEEDS

Displaced			
	$d_x = 4\lambda$ and $D_{feed} = 4\lambda$	$d_x = 2\lambda$ and $D_{feed} = 2\lambda$	$d_x = \lambda$ and $D_{feed} = \lambda$
η_{ap}	71.66 %	47.25 %	15.73 %
θ_{dir}	1.12°	0.58°	0.27°
ϕ_{dir}	0°	0°	0°

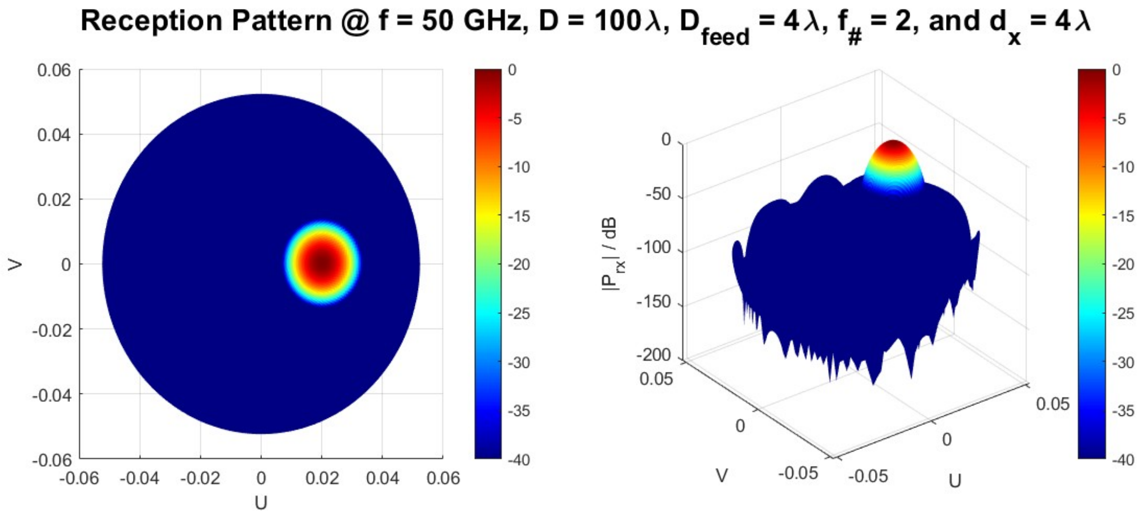

Fig. 4. Reception pattern for a parabolic reflector and displaced by $d_x = 4\lambda$ circular feed with diameter $D_f = 4\lambda$, reflector's diameter $D_r = 100\lambda$, and operating at $f = 50$ GHz; the incident wave has an amplitude $E_i^{PW} - 1$ V/m. Visualized in UV coordinates.

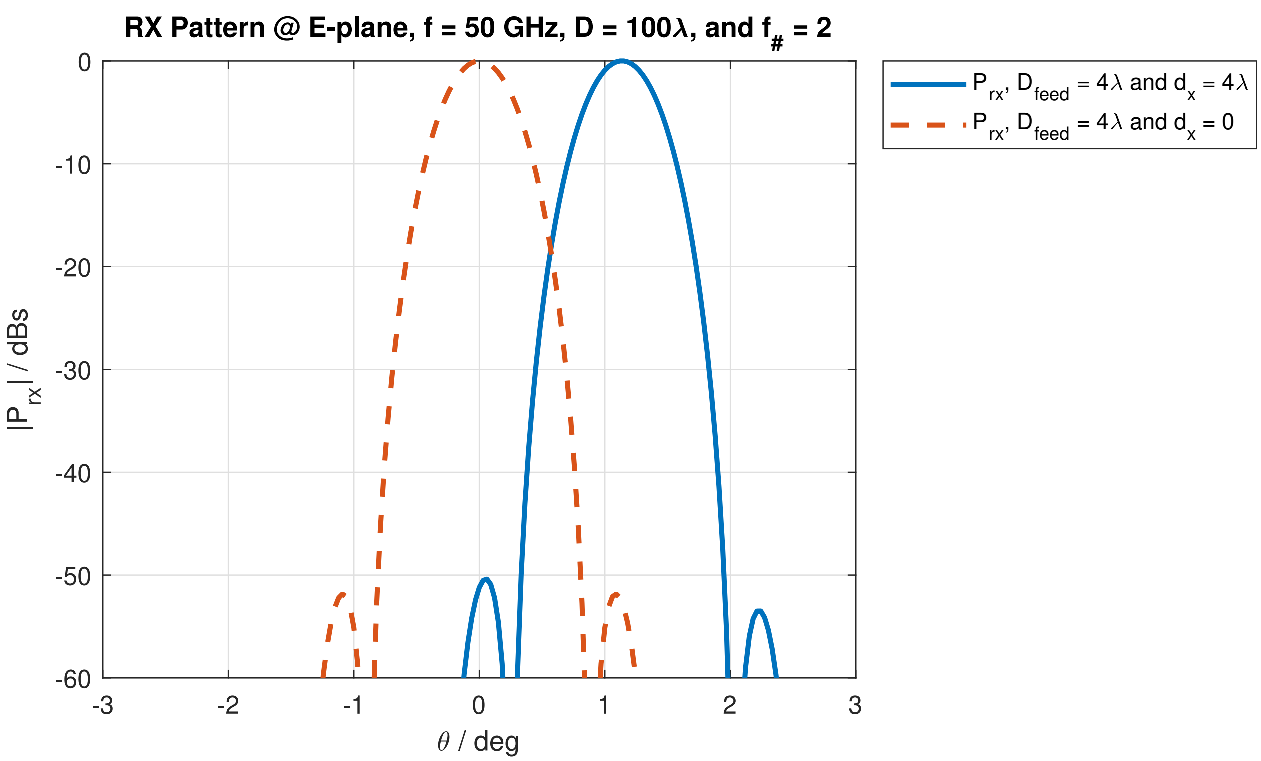

Fig. 5. Reception pattern for a parabolic reflector with displaced by $d_x = 4\lambda$ and centered circular feed with diameter $D_f = 4\lambda$, in main planes $\phi = 0°$ and $180°$ and $\phi = 90°$ and $270°$ as a function of the incident angle; the reflector's diameter is $D_r = 100\lambda$, and it operates at $f = 50$ GHz; the incident wave has an amplitude $E_i^{PW} = 1$ V/m.

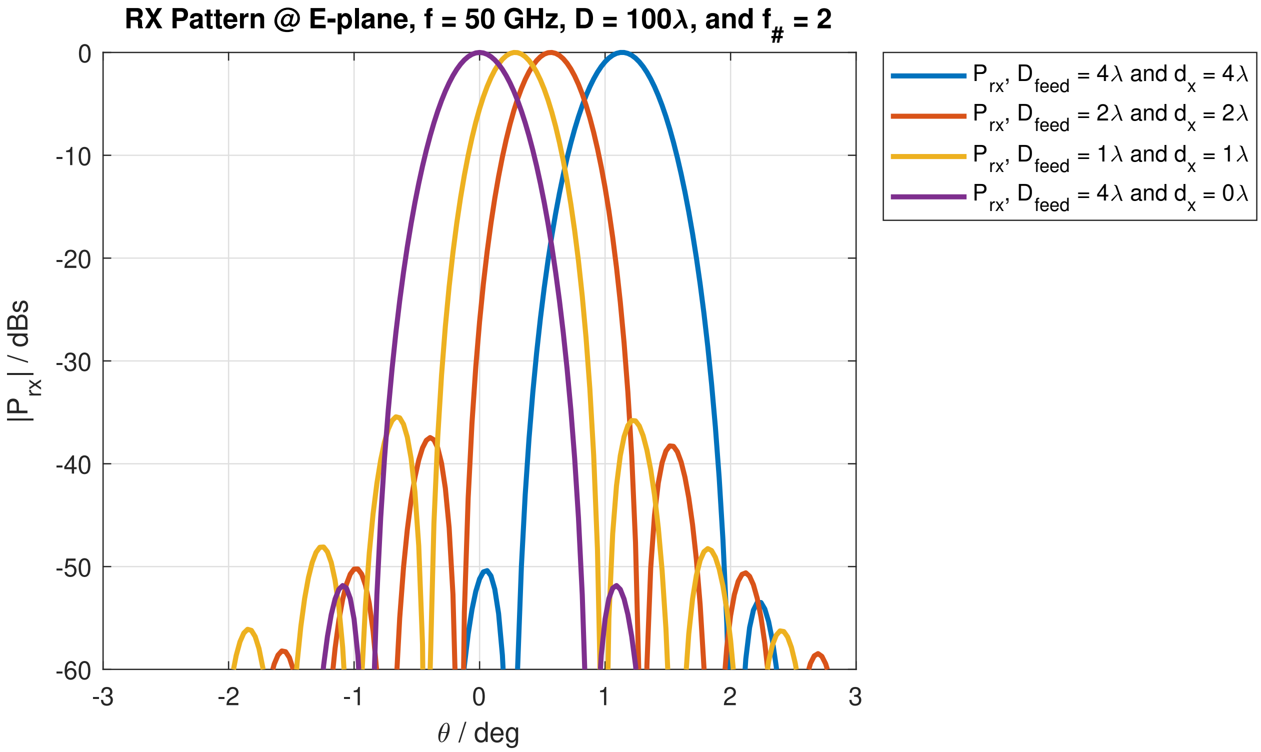

Fig. 6. Reception pattern for a parabolic reflector with displaced by $d_x = 4\lambda$, $d_x = 2\lambda$, $d_x = \lambda$ and centered circular feed with diameters $D_f = 4\lambda$, $D_f = 2\lambda$, $D_f = \lambda$, and $D_f = 4\lambda$ respectively, in main planes $\phi = 0°$ and $180°$ and $\phi = 90°$ and $270°$ as a function of the incident angle; the reflector's diameter is $D_r = 100\lambda$, and it operates at $f = 50$ GHz; the incident wave has an amplitude $E_i^{PW} = 1$ V/m.