MATLAB Instruction - Far-field evaluation

Petar V. Peshev

Table of Contents

Initialization (Spectral Methods Library and Light Speed)

```
close all;
clear;
clc;

if ~exist([pwd() '\figures'], 'dir')
    mkdir('figures');
end

if ~exist([pwd() '\workspaces'], 'dir')
    mkdir('workspaces');
end

addpath('../spectral-methods-library');
c = physconst('LightSpeed');
```

Farfield evaluated using SGF

Theory

The far field radiated by an electric current can be evaluated asmptotically in spectral domain as follows:

$$\vec{E}^{far}(\vec{\boldsymbol{r}}) = jk_{zs}\tilde{\tilde{G}}_{2D}^{ej}\left(k_{xs}, k_{ys}, z, z'\right)\vec{J}\left(k_{xs}, k_{ys}\right)e^{jk_{zs}|z-z'|}\frac{e^{-jkr}}{2\pi r}$$

where $(x', y', z')$is the source position while (x, y, z) is the observation position,

$k_{xs} = k\sin\theta\cos\phi, k_{ys} = k\sin\theta\sin\phi, k_{zs} = k\cos\theta$ rerepresent the dominant plane wave, $\vec{J}(k_{xs}, k_{ys})$ is the FT of the current distribution, and $\tilde{G}^{ej}$ is the dyadic SGF for electric currents:

$$\tilde{\boldsymbol{G}}^{ej} = \begin{bmatrix} -\frac{v_{TM}k_x^2 + v_{TE}k_y^2}{k_\rho^2} & \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} & -\frac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} \\ \varsigma\frac{k_x}{k}i_{TM} & \varsigma\frac{k_y}{k}i_{TM} \end{bmatrix}$$

where $v_{TM}, v_{TE}, i_{TM}, i_{TE}$ are obtained from the transmission line solution of the stratification.

Directivity

The directivity of a farfield can be obtained as:

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{P_{rad}/(4\pi)}$$

where $U(\theta, \phi)$ is the radiation intensity:

$$U(\theta, \phi) = \frac{\left|\vec{E}^{far}(r, \theta, \phi)\right|^2}{2\zeta}r^2$$

and P_{rad} is the total radiated power:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U(\theta, \phi)sin\theta d\theta d\phi$$

Note that here we only consider the power integrated over the upper hemisphere. For antennas with symmetric patterns such as a free-space dipole, we can simply x2 outside the function.

MATLAB Implementation

To evaluate the farfield radiated by a current, we should follow the listed steps:

(1) Solve the transmission line model of the straitification. **(2)** Calculate the dyadic SGF. **(3)** Obtain the FT of the current distribution. **(4)** Use Eq.1 to calculate the farfield. **(5)** Evaluate the directivity and radiated power from the farfield.

The above steps are applicable for any source so it would be convenient to build separate routines for these steps, and then reuse them for different applications. Today you should learn how to build these routines for different stratifications:

(1) [v_TM, v_TE, i_TM, i_TE] = **txline_xxxx**(k0, er, h, kro, z);

(2) ej_SGF = **EJ_SGF**(er, k, kx, ky, v_TM, v_TE, i_TM, i_TE);

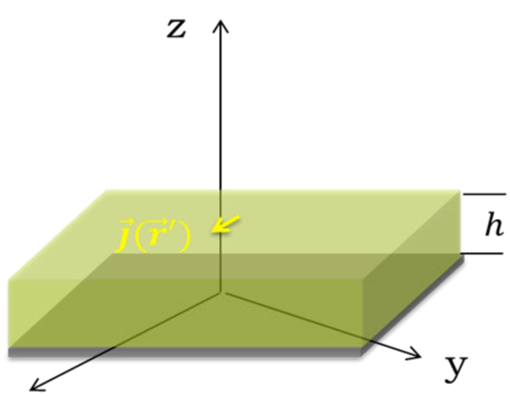
(3) Jx = **FTCurrent**(k, er, kx, ky, l, w)

(4) [Eth, Eph] = **farfield**(k, R_FF, TH, PH, kz, Gxx, Gyx, Gzx, Jx)

(5) [Dir, Prad] = **Directivity**(E_tot, Theta, dth, dph, er, r)

Q1 (3 points): Grounded Dielectric Slab

Evaluate the far field radiated by an electric dipole with $L = W = 0.25$ mm placed on the top of a grounded slab with thickness $h = 0.8$ mm and $\varepsilon_r = 12$.

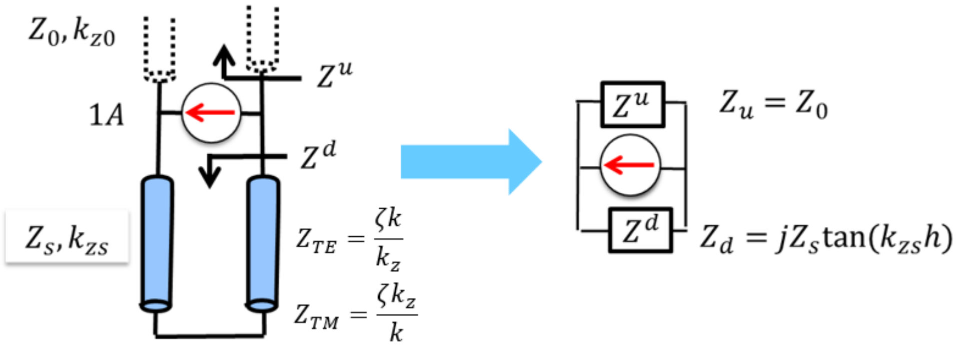


1. Far field along the main planes at 28 GHz

Calculate the far field in $\phi = 0^\circ/90^\circ$ planes by considering the radiation in spectral domain.

Step 1: Solution of the equivalent transmission line

The stratification can be modelled as a transmission line, where the current generator represents the electric current:



After solving the circuit, one can obtain the voltage and current solutions in the air:

$$V_0(z) = \frac{Z_u Z_d}{Z_u + Z_d} e^{jk_{z0}h} e^{-jk_{z0}z} \quad I_0(z) = \frac{1}{Z_0} V_0(z)$$

Write a routine to calculate TE/TM solutions: [vtm, vte, itm, ite] = **txline_groundslab**(k0, er, h, krho, z)

```
% Constant
L  = 0.25e-3;
W  = 0.25e-3;
h  = 0.8e-3;
er = 12;
f  = 28e9;
lambda = 3e8 / f;
k0 = 2 * pi / lambda;

% FF parameters
R_FF = 1;
phi  = (eps:2:360) * pi / 180;
theta = linspace(eps, 90 - 0.1, 90) * pi / 180;
[TH, PH] = meshgrid(theta, phi);
Z    = R_FF * cos(TH);

KX   = k0 * sin(TH) .* cos(PH);
KY   = k0 * sin(TH) .* sin(PH);
KZ   = k0 * cos(TH);
KRHO = sqrt(KX.^2 + KY.^2);

% calculate voltages and currents
[vte, ite, vtm, itm] = stratified_media(k0, KRHO, Z, 'GroundSlab', h, er);
```

Step 2: Calculate the Dyadic SGF

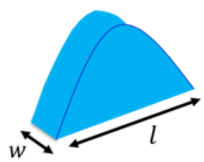
Build the Dyad for electric current $\tilde{G}^{ej}$: ej_SGF = **EJ_SGF**(er, k, kx, ky, v_TM, v_TE, i_TM, i_TE, flag);

$$\tilde{G}^{ej} = \begin{bmatrix} -\frac{v_{TM}k_x^2 + v_{TE}k_y^2}{k_\rho^2} & \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ \frac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} & -\frac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} \\ \varsigma \frac{k_x}{k} i_{TM} & \varsigma \frac{k_y}{k} i_{TM} \end{bmatrix}$$

```
% calculate Green's function
SGF = spectral_gf(1, k0, KX, KY, vtm, vte, itm, ite, 'E', 'J');
```

Step 3: Calculate the FT of the current distribution

For dipole antennas, we can approximate the spatial current distribution using the current distribution along an open circuited transmission line. This distribution can be expressed in general as follows:



$$\vec{J}_x(x,y) = l(x)t(y)\,\hat{x} \xrightarrow{\text{FT}} \vec{J}_x^{FT}(k_x,k_y) = L(k_x)T(k_y)$$

$$t(y) = \frac{1}{w} rect(y,w) \qquad T(k_y) = sinc(\frac{k_y w}{2})$$

$$k_{eq} = \frac{k_1 + k_2}{2} \qquad l(x) = \frac{sin\left(k_{eq}\left(\frac{l}{2} - |x|\right)\right)}{sin(k_{eq}\frac{l}{2})} \qquad L(k_x) = \frac{2k_{eq}\left(cos\left(\frac{k_x l}{2}\right) - cos\left(\frac{k_{eq} l}{2}\right)\right)}{(k_{eq}^2 - k_x^2)\, sin\left(k_{eq}\frac{l}{2}\right)}$$

Write a routine to calculate the FT of currents: Jx = **FTCurrent**(k, er, kx, ky, l, w, flag_current)

```
% calculate FT of current distribution
k_comp = NaN( [size(KX, 1, 2), 3] );
k_comp(:, :, 1) = KX;
k_comp(:, :, 2) = KY;
k_comp(:, :, 3) = KZ;
Jft = ft_current(k0, k_comp, W, L, er, 'dipole', 'x');
Jx = Jft(:, :, 1);
```

Step 4: Evaluate far fields

$$\vec{E}^{far}(\vec{\boldsymbol{r}}) = jk_{zs}\tilde{\bar{G}}_{2D}^{ej}(k_{xs},k_{ys},z,z')\vec{J}(k_{xs},k_{ys})e^{jk_{zs}|z-z'|}\frac{e^{-jkr}}{2\pi r}$$

Calculate the FF in Eth and Eph and plot the total field:

[Eth, Eph] = **farfield**(k, R_FF, TH, PH, kz, Gxx, Gyx, Gzx, Jx);

```
% calculate far field
sph_grid = NaN( [size(TH, 1, 2), 2] );
sph_grid(:, :, 1) = TH;
sph_grid(:, :, 2) = PH;
E = farfield(k0, R_FF, sph_grid, KZ, Z, SGF, Jft, h);
```

```
E_tot = total_field(E);
```

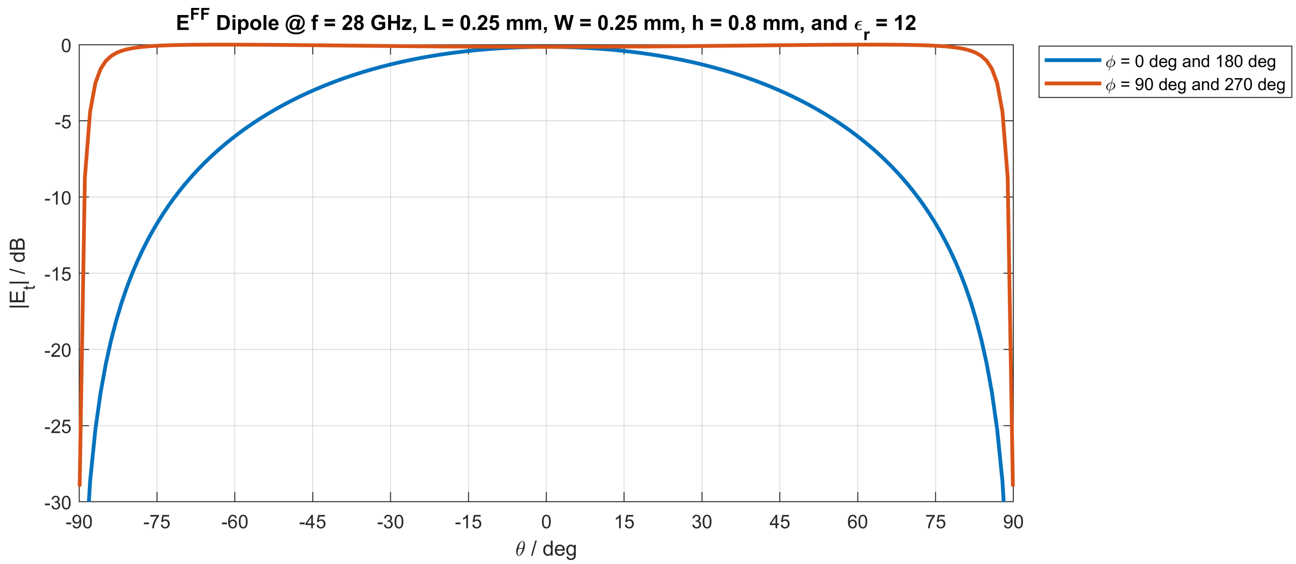
Step 5: Directivity

```
[~, ~, power] = directivity(1, E, sph_grid, R_FF);
power
```

```
power = 0.1019
```

Step 6: Plot far field

```
E_tot_norm = norm_magnitude(E_tot, 'dB');
theta_plot = NaN(1, length(theta) * 2);
theta_plot(1 : length(theta)) = - fliplr(theta) * 180 / pi;
theta_plot(length(theta) + 1 : end) = theta * 180 / pi;
phi0_plot = NaN(1, length(theta) * 2);
phi0_plot(1 : length(theta)) = fliplr(E_tot_norm(91, :));
phi0_plot(length(theta) + 1 : end) = E_tot_norm(1, :);
phi90_plot = NaN(1, length(theta) * 2);
phi90_plot(1 : length(theta)) = fliplr(E_tot_norm(136, :));
phi90_plot(length(theta) + 1 : end) = E_tot_norm(46, :);
figure('Position', [250 250 1050 400]);
plot(theta_plot, phi0_plot, 'LineWidth', 2.0, ...
    'DisplayName', '\phi = 0 deg and 180 deg');
hold on;
plot(theta_plot, phi90_plot, 'LineWidth', 2.0, ...
    'DisplayName', '\phi = 90 deg and 270 deg');
grid on;
xticks(-90 : 15 : 90);
xlim([-90 90]);
ylim([-30 0]);
legend show;
legend('location', 'bestoutside');
xlabel('\theta / deg');
ylabel('|E_{t}| / dB');
title(['E^{FF} Dipole @ f = ' num2str(f * 1e-9) ' GHz, L = ' ...
    num2str(L * 1e3) ' mm, W = ' num2str(W * 1e3) ' mm, h = ' num2str(h * 1e3) ...
    ' mm, and \epsilon_{r} = ' num2str(er)]);
```




```
saveas(gcf, 'figures\E_field_grounded_dielectric.fig');
save('workspaces\Q1_1.mat');
```

2. Plot the radiated power from 10 GHz to 40 GHz normalized to the radiated power that the same dipole will radiate in free space

Calculate the FF as a function of frequency for both stratified and free-space cases. Radiated power can be obtained by integrating the FF pattern: [Dir, Prad] = **Directivity**(E_tot, Theta, dth, dph, er, r);

Dipole in free space: 2* $P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U(\theta,\phi) sin\theta d\theta d\phi$, outside the function.

Dipole in grounded slab: $P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U(\theta,\phi) sin\theta d\theta d\phi$, radiation exists only on the upper hemisphere.

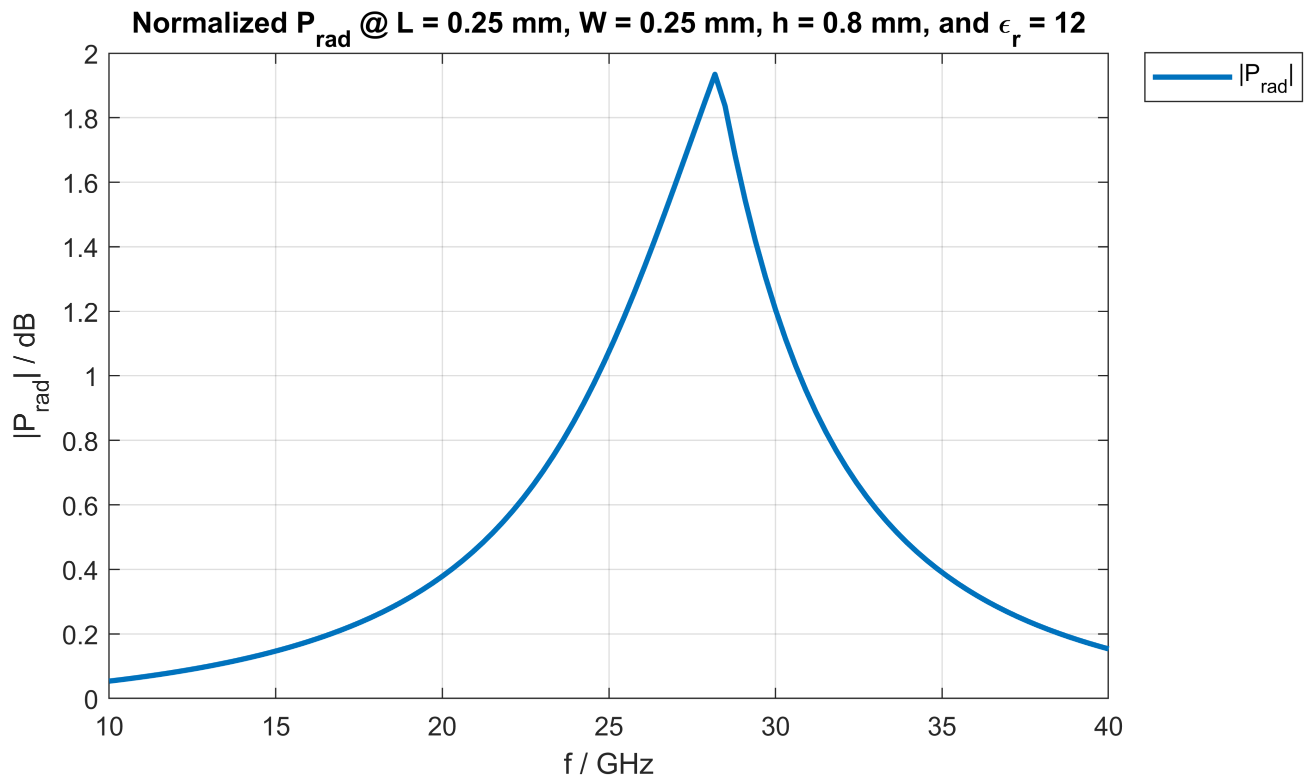
Step 1: Radiated power by dipole on dielectric normalized to power of dipole in free space

```
f = linspace(10, 40, 100) * 1e9;
lambda = 3e8 ./ f;
k0 = 2 * pi ./ lambda;

norm_power = NaN(1, length(f));
for idx = 1 : 1 : length(f)
    KX   = k0(idx) * sin(TH) .* cos(PH);
    KY   = k0(idx) * sin(TH) .* sin(PH);
    KZ   = k0(idx) * cos(TH);
    KRHO = sqrt(KX.^2 + KY.^2);

    k_comp = NaN( [size(KX, 1, 2), 3] );
    k_comp(:, :, 1) = KX;
    k_comp(:, :, 2) = KY;
```

```
    k_comp(:, :, 3) = KZ;

    % Radiated power by dipole in free space 2
    [fs_vte, fs_ite, fs_vtm, fs_itm] = tx_fs(k0(idx), KRHO, Z, 'GroundSlab', h, 1);
    FS_SGF = spectral_gf(1, k0(idx), KX, KY, fs_vtm, fs_vte, fs_itm, fs_ite, 'E', 'J');
    FS_Jft = ft_current(k0(idx), k_comp, W, L, 1, 'dipole', 'x');
    FS_E = farfield(k0(idx), R_FF, sph_grid, KZ, Z, FS_SGF, FS_Jft, h);
    [~, ~, fs_power] = directivity(1, FS_E, sph_grid, R_FF);
    fs_power = 2 * fs_power;

    % Radiated power by dipole on dielectric
    [vte, ite, vtm, itm] = stratified_media(k0(idx), KRHO, Z, 'GroundSlab', h, er);
    SGF = spectral_gf(1, k0(idx), KX, KY, vtm, vte, itm, ite, 'E', 'J');
    Jft = ft_current(k0(idx), k_comp, W, L, er, 'dipole', 'x');
    E = farfield(k0(idx), R_FF, sph_grid, KZ, Z, SGF, Jft, h);
    % Power radiated
    [~, ~, power] = directivity(1, E, sph_grid, R_FF);

    % Normalized power
    norm_power(idx) = power / fs_power;
end
```

Step 2: Plot normalized power

```
figure('Position', [250 250 750 400]);
plot(f * 1e-9, (norm_power), 'LineWidth', 2.0, 'DisplayName', '|P_{rad}|');
grid on;
xlim([min(f * 1e-9) max(f * 1e-9)]);
legend show;
legend('location', 'bestoutside');
xlabel('f / GHz');
ylabel('|P_{rad}| / dB');
title(['Normalized P_{rad} @ L = ' num2str(L * 1e3) ' mm, W = ' num2str(W * 1e3) ' mm, ' ...
    'h = ' num2str(h * 1e3) ' mm, and \epsilon_{r} = ' num2str(er)]);
```




```
saveas(gcf, 'figures\normalized_Prad.fig');
save('workspaces\Q1_2.mat');
```

3. For which thickness is this normalized power of the dipole the highest? Why?

Find this value in **2**, and represent it in terms of wavelength in dielectric.

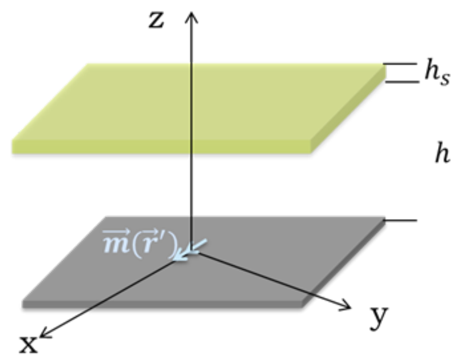
```
max_idx = find(norm_power == max(norm_power));
h_opt = h / lambda(max_idx)
```

```
h_opt = 0.0752
```

Q2 (4 points): Standard Leaky-wave antenna

Calculate the far field radiated by a magnetic current in the presence of the stratification shown in the figure.

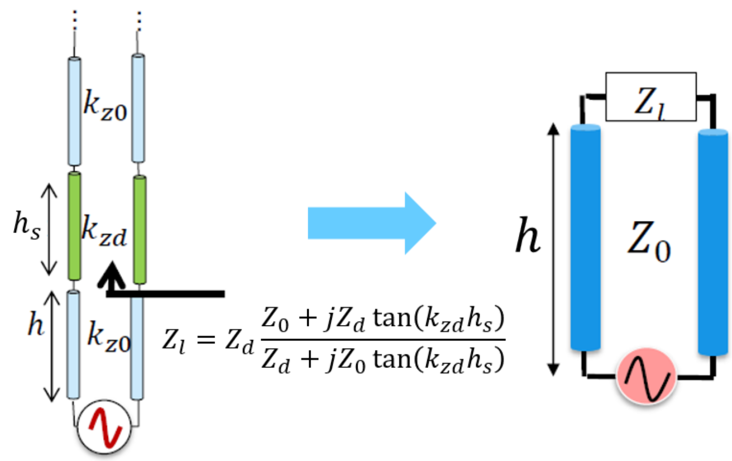
Consider $h = 5.4$mm, $h_s = 0.77$mm, $\varepsilon_r = 12$, and a half-wavelength magnetic dipole with $W = \lambda/20$.



1. Far field along main planes at 26, 28, and 30 GHz.

Calculate the far field in $\phi = 0^{\circ}/90^{\circ}$ planes at **three** frequency points.

Step 1: Solution of the equivalent transmission line



Write a routine to calculate TE/TM solutions: [vtm, vte, itm, ite] = **txline_superstrate**(k0, er, h, hs, krho, z)

Step 2: Calculate the Dyadic SGF

Build the Dyad for magnetic current $\tilde{G}^{em}$: em_SGF = **EM_SGF**(er, k, kx, ky, v_TM, v_TE, i_TM, i_TE, flag);

$$\tilde{\boldsymbol{G}}^{em} = \begin{bmatrix} \dfrac{(v_{TM} - v_{TE})k_x k_y}{k_\rho^2} & -\dfrac{v_{TE}k_y^2 + v_{TM}k_x^2}{k_\rho^2} \\ \dfrac{v_{TE}k_x^2 + v_{TM}k_y^2}{k_\rho^2} & \dfrac{(v_{TE} - v_{TM})k_x k_y}{k_\rho^2} \\ -\varsigma \dfrac{k_y}{k} i_{TM} & \varsigma \dfrac{k_x}{k} i_{TM} \end{bmatrix}$$

Step 3: Calculate the FT of the current distribution

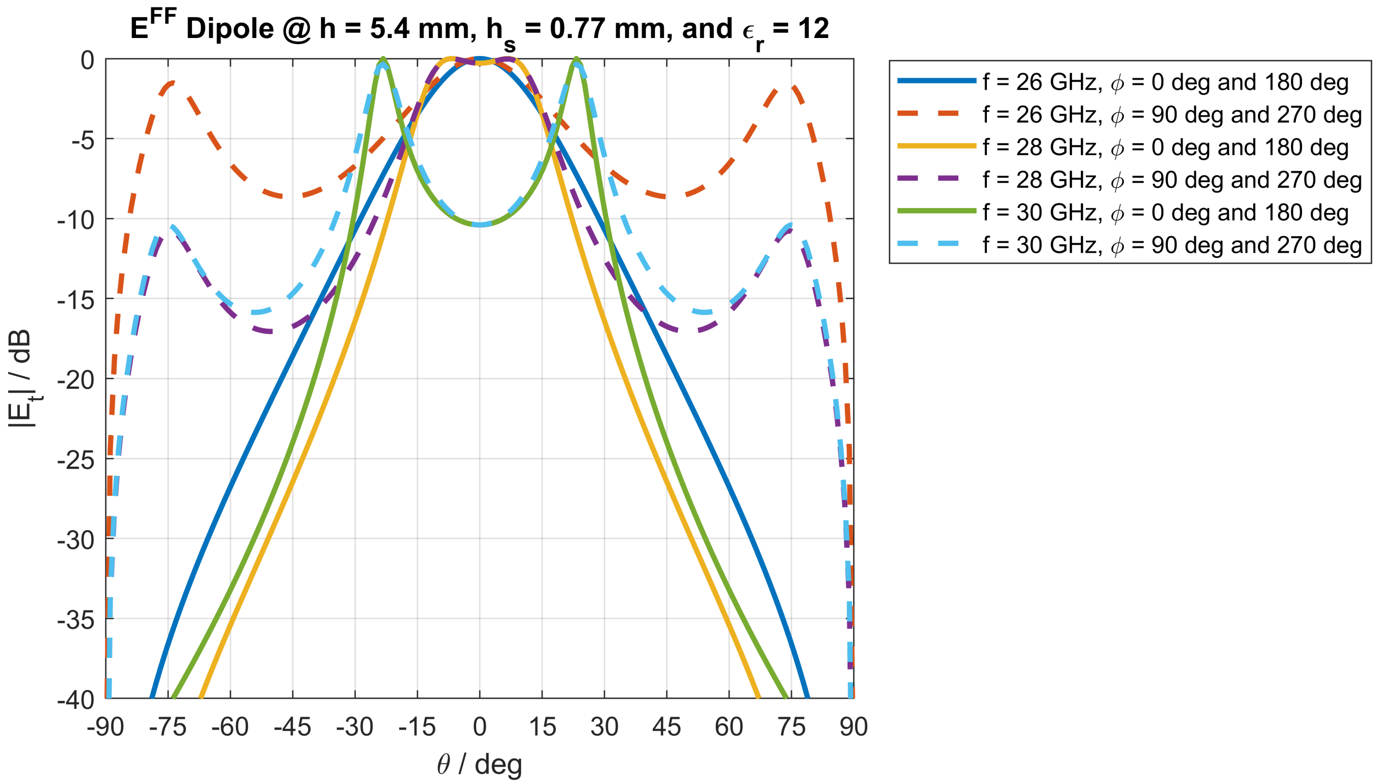
Same routine as the previous case.

Step 4: Evaluate far fields

Same routine as the previous case. In fact, you can find $h = \frac{\lambda_0}{2}$, $h_s = \frac{\lambda_0}{4\sqrt{\varepsilon_r}}$ are the resonant conditions.

```
clear;

% Constant
h  = 5.4e-3;
hs = 0.77e-3;
er = 12;
f  = [26 28 30] * 1e9;
lambda = 3e8 ./ f;
L  = lambda / 2;
W  = lambda / 20;
k0 = 2 * pi ./ lambda;

% FF parameters
R_FF = 1;
phi  = (eps:2:360) * pi / 180;
theta = linspace(eps, 90 - 0.1, 90) * pi / 180;
[TH, PH] = meshgrid(theta, phi);
Z    = R_FF * cos(TH);

sph_grid = NaN( [size(TH, 1, 2), 2] );
sph_grid(:, :, 1) = TH;
sph_grid(:, :, 2) = PH;

E = NaN( [size(sph_grid, 1, 2), 3, length(f)] );
E_tot = NaN( [size(sph_grid, 1, 2), length(f)] );
for idx = 1 : 1 : length(f)
    KX   = k0(idx) * sin(TH) .* cos(PH);
    KY   = k0(idx) * sin(TH) .* sin(PH);
    KZ   = k0(idx) * cos(TH);
    KRHO = sqrt(KX.^2 + KY.^2);

    k_comp = NaN( [size(KX, 1, 2), 3] );
    k_comp(:, :, 1) = KX;
    k_comp(:, :, 2) = KY;
    k_comp(:, :, 3) = KZ;

    % calculate voltages and currents
    [vte, ite, vtm, itm] = stratified_media(k0(idx), KRHO, Z, 'Superstrate', h, hs, er);

    % calculate Green's function
    SGF = spectral_gf(1, k0(idx), KX, KY, vtm, vte, itm, ite, 'E', 'M');

    % calculate FT of current distribution
    Mx = ft_current(k0(idx), k_comp, W(idx), L(idx), 1, 'dipole', 'x');

    % calculate far field
```

```matlab
    E(:, :, :, idx) = farfield(k0(idx), R_FF, sph_grid, KZ, Z, SGF, Mx);
    E_tot(:, :, idx) = total_field(E(:, :, :, idx));
end

theta_plot = NaN(1, length(theta) * 2);
theta_plot(1 : length(theta)) = - fliplr(theta) * 180 / pi;
theta_plot(length(theta) + 1 : end) = theta * 180 / pi;
figure('Position', [250 250 750 400]);
for idx = 1 : 1 : length(f)
    E_tot_norm = norm_magnitude(E_tot, 'dB');
    phi0_plot = NaN(1, length(theta) * 2);
    phi0_plot(1 : length(theta)) = fliplr(E_tot_norm(91, :, idx));
    phi0_plot(length(theta) + 1 : end) = E_tot_norm(1, :, idx);
    phi90_plot = NaN(1, length(theta) * 2);
    phi90_plot(1 : length(theta)) = fliplr(E_tot_norm(136, :, idx));
    phi90_plot(length(theta) + 1 : end) = E_tot_norm(46, :, idx);
    plot(theta_plot, phi0_plot, 'LineWidth', 2.0, 'DisplayName', ...
        ['f = ' num2str(f(idx) * 1e-9) ' GHz, \phi = 0 deg and 180 deg']);
    hold on;
    plot(theta_plot, phi90_plot, '--', 'LineWidth', 2.0, 'DisplayName', ...
        ['f = ' num2str(f(idx) * 1e-9) ' GHz, \phi = 90 deg and 270 deg']);
    hold on;
end
hold off;
grid on;
xticks(-90 : 15 : 90);
xlim([-90 90]);
ylim([-40 0]);
legend show;
legend('location', 'bestoutside');
xlabel('\theta / deg');
ylabel('|E_{t}| / dB');
title(['E^{FF} Dipole @ h = ' num2str(h * 1e3) ' mm, h_{s} = ' ...
    num2str(hs * 1e3) ' mm, and \epsilon_{r} = ' num2str(er)]);
```




```
saveas(gcf, 'figures\E_field_superstrate.fig');
save('workspaces\Q2_1.mat');
```

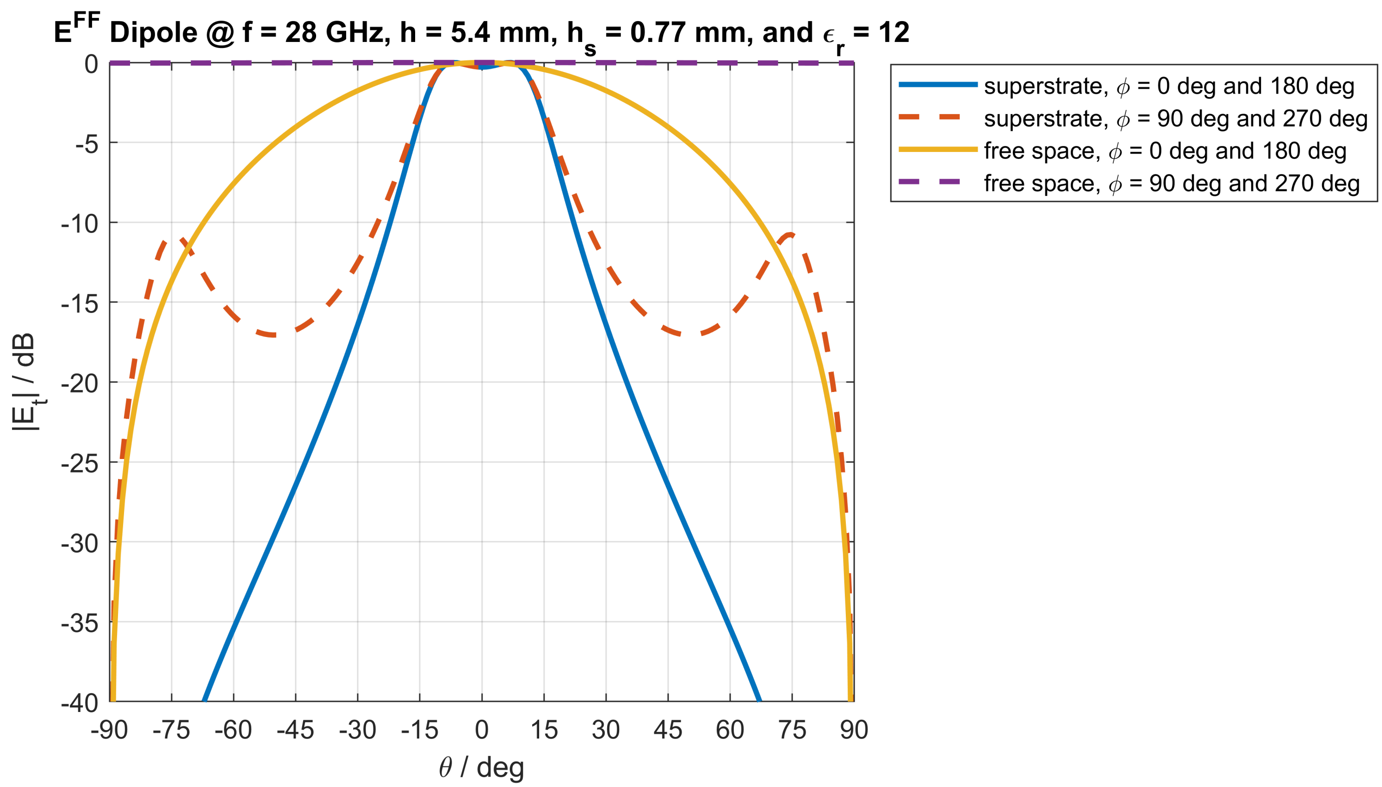
2. Compare the far field with that of the magnetic current radiating in free space at 28 GHz.

Solve a new transmission line for free space case.

```
clear;

% Constant
h  = 5.4e-3;
hs = 0.77e-3;
er = 12;
f  = 28 * 1e9;
lambda = 3e8 / f;
L  = lambda / 2;
W  = lambda / 20;
k0 = 2 * pi / lambda;

% FF parameters
R_FF = 1;
phi   = (eps:2:360) * pi / 180;
theta = linspace(eps, 90 - 0.1, 90) * pi / 180;
[TH, PH] = meshgrid(theta, phi);
Z     = R_FF * cos(TH);
```

```
sph_grid = NaN( [size(TH, 1, 2), 2] );
sph_grid(:, :, 1) = TH;
sph_grid(:, :, 2) = PH;

KX   = k0 * sin(TH) .* cos(PH);
KY   = k0 * sin(TH) .* sin(PH);
KZ   = k0 * cos(TH);
KRHO = sqrt(KX.^2 + KY.^2);

k_comp = NaN( [size(KX, 1, 2), 3] );
k_comp(:, :, 1) = KX;
k_comp(:, :, 2) = KY;
k_comp(:, :, 3) = KZ;

% Radiated power by dipole in free space
FS_SGF = dyadic_sgf(1, k0, k_comp, 'E', 'J');
FS_Mft = zeros( [size(sph_grid, 1, 2), 3] );
FS_Mftxy = ft_current(k0, k_comp, W, L, 1, 'dipole', 'x');
FS_Mft(:, :, 1 : 2) = FS_Mftxy;
FS_E = fs_farfield(k0, R_FF, sph_grid, k_comp(:, :, 3), FS_SGF, FS_Mft);
E_fs_tot = total_field(FS_E);

% calculate voltages and currents
[vte, ite, vtm, itm] = stratified_media(k0, KRHO, Z, 'Superstrate', h, hs, er);

% calculate Green's function
SGF = spectral_gf(1, k0, KX, KY, vtm, vte, itm, ite, 'E', 'M');

% calculate FT of current distribution
Mx = ft_current(k0, k_comp, W, L, 1, 'dipole', 'x');

% calculate far field
E = farfield(k0, R_FF, sph_grid, KZ, Z, SGF, Mx);
E_tot = total_field(E);

theta_plot = NaN(1, length(theta) * 2);
theta_plot(1 : length(theta)) = - fliplr(theta) * 180 / pi;
theta_plot(length(theta) + 1 : end) = theta * 180 / pi;
E_tot_norm = norm_magnitude(E_tot, 'dB');
phi0_plot = NaN(1, length(theta) * 2);
phi0_plot(1 : length(theta)) = fliplr(E_tot_norm(91, :));
phi0_plot(length(theta) + 1 : end) = E_tot_norm(1, :);
phi90_plot = NaN(1, length(theta) * 2);
phi90_plot(1 : length(theta)) = fliplr(E_tot_norm(136, :));
phi90_plot(length(theta) + 1 : end) = E_tot_norm(46, :);
E_fs_tot_norm = norm_magnitude(E_fs_tot, 'dB');
phi0_plot_fs = NaN(1, length(theta) * 2);
phi0_plot_fs(1 : length(theta)) = fliplr(E_fs_tot_norm(91, :));
phi0_plot_fs(length(theta) + 1 : end) = E_fs_tot_norm(1, :);
phi90_plot_fs = NaN(1, length(theta) * 2);
```

```
phi90_plot_fs(1 : length(theta)) = fliplr(E_fs_tot_norm(136, :));
phi90_plot_fs(length(theta) + 1 : end) = E_fs_tot_norm(46, :);
figure('Position', [250 250 750 400]);
plot(theta_plot, phi0_plot, 'LineWidth', 2.0, ...
    'DisplayName', 'superstrate, \phi = 0 deg and 180 deg');
hold on;
plot(theta_plot, phi90_plot, '--', 'LineWidth', 2.0, ...
    'DisplayName', 'superstrate, \phi = 90 deg and 270 deg');
hold on;
plot(theta_plot, phi0_plot_fs, 'LineWidth', 2.0, ...
    'DisplayName', 'free space, \phi = 0 deg and 180 deg');
hold on;
plot(theta_plot, phi90_plot_fs, '--', 'LineWidth', 2.0, ...
    'DisplayName', 'free space, \phi = 90 deg and 270 deg');
grid on;
xticks(-90 : 15 : 90);
xlim([-90 90]);
ylim([-40 0]);
legend show;
legend('location', 'bestoutside');
xlabel('\theta / deg');
ylabel('|E_{t}| / dB');
title(['E^{FF} Dipole @ f = ' num2str(f * 1e-9) ' GHz, h = ' ...
    num2str(h * 1e3) ' mm, h_{s} = ' num2str(hs * 1e3) ...
    ' mm, and \epsilon_{r} = ' num2str(er)]);
```



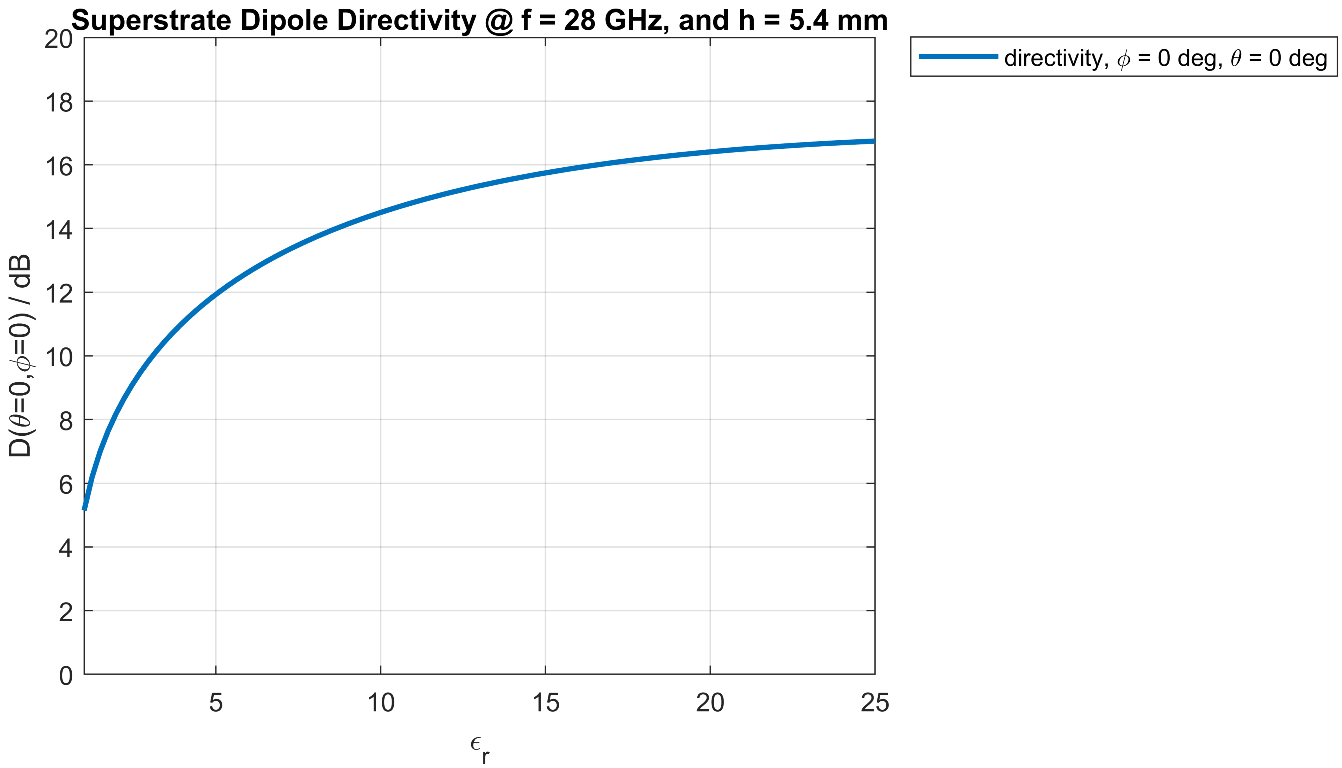

```
saveas(gcf, 'figures\E_field_superstrate_and_free_space.fig');
save('workspaces\Q2_2.mat');
```

3. Directivity $(\text{at } \theta = 0^\circ)$ **as a function of relative permittivity at 28 GHz. Consider a variation** (ε_r) **from 1 to 25, with the dielectric thickness** $h_s = \dfrac{\lambda_0}{4\sqrt{\varepsilon_r}}$**.**

Calculate the directivity at $\theta = 0^\circ$ as a function of ε_r.

```
clear;

% Constant
h  = 5.4e-3;
er = linspace(1, 25, 100);
f  = 28 * 1e9;
lambda = 3e8 / f;
L  = lambda / 2;
W  = lambda / 20;
k0 = 2 * pi / lambda;
hs = lambda ./ (4 * sqrt(er));

% FF parameters
R_FF = 1;
phi   = (eps:2:360) * pi / 180;
theta = linspace(eps, 90 - 0.1, 90) * pi / 180;
[TH, PH] = meshgrid(theta, phi);
Z     = R_FF * cos(TH);

sph_grid = NaN( [size(TH, 1, 2), 2] );
sph_grid(:, :, 1) = TH;
sph_grid(:, :, 2) = PH;

KX   = k0 * sin(TH) .* cos(PH);
KY   = k0 * sin(TH) .* sin(PH);
KZ   = k0 * cos(TH);
KRHO = sqrt(KX.^2 + KY.^2);

k_comp = NaN( [size(KX, 1, 2), 3] );
k_comp(:, :, 1) = KX;
k_comp(:, :, 2) = KY;
k_comp(:, :, 3) = KZ;

dir = NaN(1, length(er));
for idx = 1 : 1 : length(er)

    % calculate voltages and currents
    [vte, ite, vtm, itm] = stratified_media(k0, KRHO, Z, 'Superstrate', h, hs(idx), er(idx));

    % calculate Green's function
    SGF = spectral_gf(1, k0, KX, KY, vtm, vte, itm, ite, 'E', 'M');
```

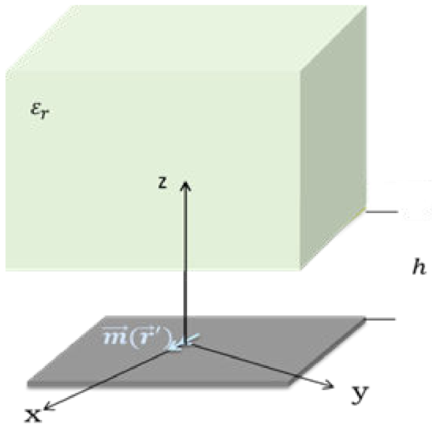
```
    % calculate FT of current distribution
    Mx = ft_current(k0, k_comp, W, L, 1, 'dipole', 'x');

    % calculate far field
    E = farfield(k0, R_FF, sph_grid, KZ, Z, SGF, Mx);

    [dir_all, ~, ~] = directivity(1, E, sph_grid, R_FF);
    dir(idx) = dir_all(1, 1);
end

figure('Position', [250 250 750 400]);
plot(er, 10 * log10(dir), 'LineWidth', 2.0, 'DisplayName', 'directivity, \phi = 0 deg, \theta =
grid on;
xlim([min(er) max(er)]);
ylim([0 20]);
legend show;
legend('location', 'bestoutside');
xlabel('\epsilon_{r}');
ylabel('D(\theta=0,\phi=0) / dB');
title(['Superstrate Dipole Directivity @ f = ' num2str(f * 1e-9) ' GHz, and h = ' num2str(h * 1
```




```
saveas(gcf, 'figures\directivity_superstrate.fig');
save('workspaces\Q2_3.mat');
```

Q3 (3 points): Resonant Leaky-wave antenna

Calculate the far field radiated by a magnetic current into an infinite medium in the presence of the stratification shown in the figure. Consider $h = 5.4$ mm, a frequency of 28 GHz, and a half-wavelength magnetic dipole with $W = \lambda/20$.



1. Far field along main planes when $\varepsilon_r = 12$.

Note that the far field is now radiated inside a dielectric medium. The spectral parameters should be modified:

$$k_{xs} = k_d \sin\theta \cos\phi, k_{ys} = k_d \sin\theta \sin\phi, k_{zs} = k_d \cos\theta$$

Step 1: Solution of the equivalent transmission line

```
clear;

% Constant
h  = 5.4e-3;
er = 12;
f  = 28 * 1e9;
lambda = 3e8 / f;
L  = lambda / 2;
W  = lambda / 20;
k0 = 2 * pi / lambda;

% FF parameters
R_FF = 1;
phi  = (eps:2:360) * pi / 180;
theta = linspace(eps, 90 - 0.1, 90) * pi / 180;
[TH, PH] = meshgrid(theta, phi);
Z    = R_FF * cos(TH);

sph_grid = NaN( [size(TH, 1, 2), 2] );
sph_grid(:, :, 1) = TH;
sph_grid(:, :, 2) = PH;

KX   = k0 * sin(TH) .* cos(PH);
KY   = k0 * sin(TH) .* sin(PH);
KRHO = sqrt(KX.^2 + KY.^2);
```

```
k_comp = NaN( [size(KX, 1, 2), 2] );
k_comp(:, :, 1) = KX;
k_comp(:, :, 2) = KY;

k = k0 * sqrt(er);
KZ = k * cos(TH);

% calculate voltages and currents
[vte, ite, vtm, itm] = stratified_media(k0, KRHO * sqrt(er), Z, 'SemiInfiniteSuperstrate', h, e
```

Step 2: Calculate the Dyadic SGF

```
% calculate Green's function
SGF = spectral_gf(er, k0, KX, KY, vtm, vte, itm, ite, 'E', 'M');
```

Step 3: Calculate the FT of the current distribution

```
% calculate FT of current distribution
Mx = ft_current(k0, k_comp * sqrt(er), W, L, 1, 'dipole', 'x');
```

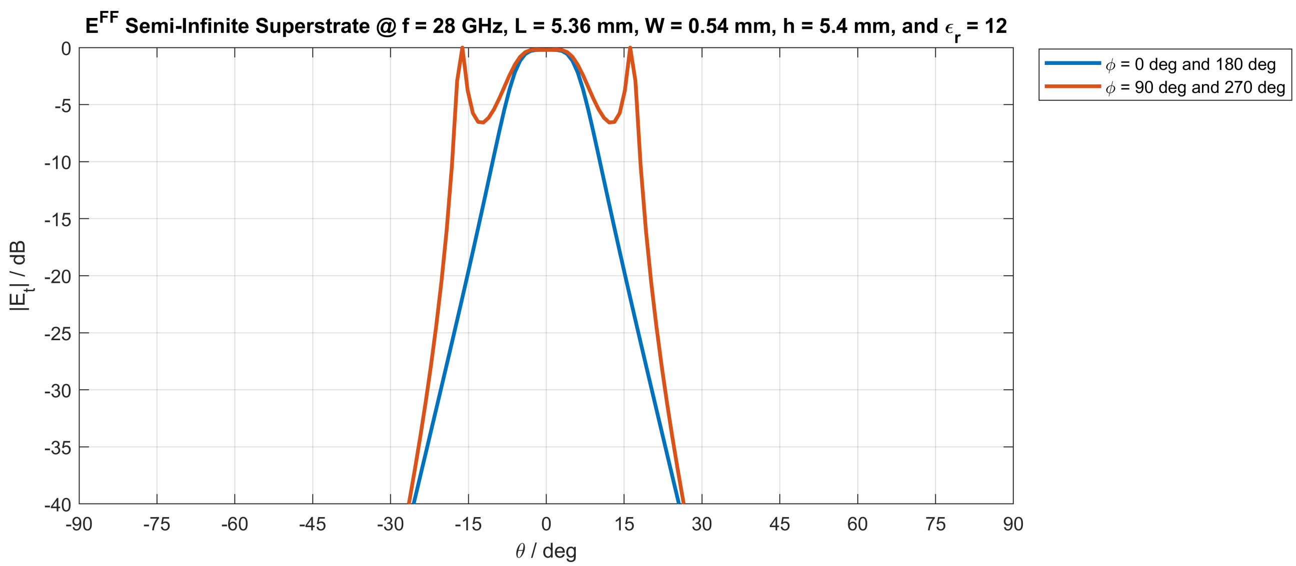
Step 4: Evaluate far fields

```
% calculate far field
E = farfield(k, R_FF, sph_grid, KZ, Z, SGF, Mx);
E_tot = total_field(E);
```

Step 5: Plot far field

```
E_tot_norm = norm_magnitude(E_tot, 'dB');
theta_plot = NaN(1, length(theta) * 2);
theta_plot(1 : length(theta)) = - fliplr(theta) * 180 / pi;
theta_plot(length(theta) + 1 : end) = theta * 180 / pi;
phi0_plot = NaN(1, length(theta) * 2);
phi0_plot(1 : length(theta)) = fliplr(E_tot_norm(91, :));
phi0_plot(length(theta) + 1 : end) = E_tot_norm(1, :);
phi90_plot = NaN(1, length(theta) * 2);
phi90_plot(1 : length(theta)) = fliplr(E_tot_norm(136, :));
phi90_plot(length(theta) + 1 : end) = E_tot_norm(46, :);
figure('Position', [250 250 1050 400]);
plot(theta_plot, phi0_plot, 'LineWidth', 2.0, ...
    'DisplayName', '\phi = 0 deg and 180 deg');
hold on;
plot(theta_plot, phi90_plot, 'LineWidth', 2.0, ...
    'DisplayName', '\phi = 90 deg and 270 deg');
grid on;
xticks(-90 : 15 : 90);
xlim([-90 90]);
ylim([-40 0]);
```

```
legend show;
legend('location', 'bestoutside');
xlabel('\theta / deg');
ylabel('|E_{t}| / dB');
title(['E^{FF} Semi-Infinite Superstrate @ f = ' num2str(f * 1e-9) ' GHz, L = ' ...
    num2str(round(L * 1e3, 2)) ' mm, W = ' num2str(round(W * 1e3, 2)) ' mm, h = ' num2str(h * 1
    ' mm, and \epsilon_{r} = ' num2str(er)]);
```

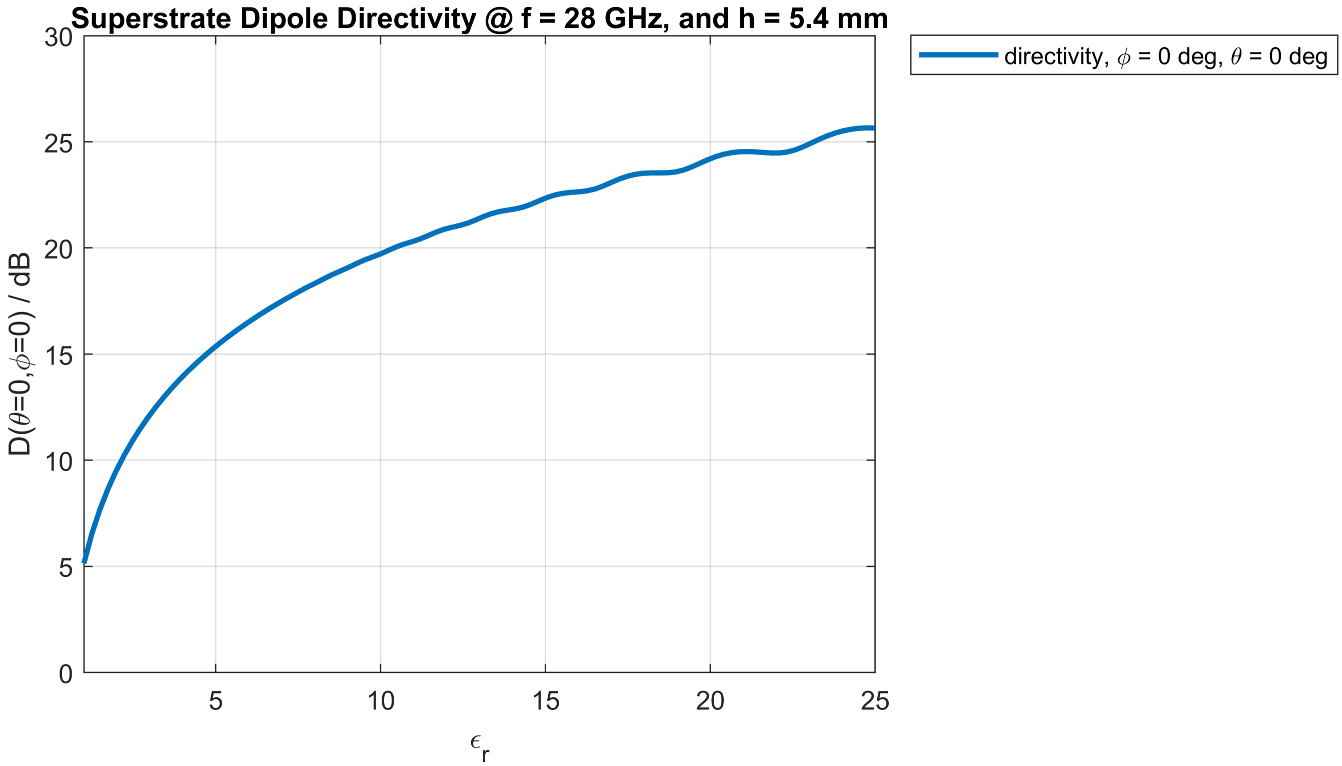



```
saveas(gcf, 'figures\E_field_semi_infinite_superstrate.fig');
save('workspaces\Q3_1.mat');
```

2. Directivity $(\text{at } \theta = 0^{\circ})$ as a function of relative permittivity at 28 GHz. Consider a variation (ε_r) from 1 to 25.

```
clear;

% Constant
h  = 5.4e-3;
er = linspace(1, 25, 100);
f  = 28 * 1e9;
lambda = 3e8 / f;
L  = lambda / 2;
W  = lambda / 20;
k0 = 2 * pi / lambda;

% FF parameters
R_FF = 1;
phi  = (eps:2:360) * pi / 180;
theta = linspace(eps, 90 - 0.1, 90) * pi / 180;
[TH, PH] = meshgrid(theta, phi);
Z    = R_FF * cos(TH);

sph_grid = NaN( [size(TH, 1, 2), 2] );
```

```
sph_grid(:, :, 1) = TH;
sph_grid(:, :, 2) = PH;

KX   = k0 * sin(TH) .* cos(PH);
KY   = k0 * sin(TH) .* sin(PH);
KRHO = sqrt(KX.^2 + KY.^2);

k_comp = NaN( [size(KX, 1, 2), 2] );
k_comp(:, :, 1) = KX;
k_comp(:, :, 2) = KY;

dir = NaN(1, length(er));
for idx = 1 : 1 : length(er)
    k = k0 * sqrt(er(idx));
    KZ   = k * cos(TH);

    % calculate voltages and currents

    [vte, ite, vtm, itm] = stratified_media(k0, KRHO * sqrt(er(idx)), Z, 'SemiInfiniteSuperstra

    % calculate Green's function
    SGF = spectral_gf(er(idx), k0, KX, KY, vtm, vte, itm, ite, 'E', 'M');

    % calculate FT of current distribution
    Mx = ft_current(k0, k_comp * sqrt(er(idx)), W, L, 1, 'dipole', 'x');
%     Mx = ft_current(k0, k_comp, W, L, 1, 'dipole', 'x');

    % calculate far field
    E = farfield(k, R_FF, sph_grid, KZ, Z, SGF, Mx);

    [dir_all, ~, ~] = directivity(er(idx), E, sph_grid, R_FF);
    dir(idx) = dir_all(1, 1);
end

figure('Position', [250 250 750 400]);
plot(er, 10 * log10(dir), 'LineWidth', 2.0, 'DisplayName', 'directivity, \phi = 0 deg, \theta =
grid on;
xlim([min(er) max(er)]);
ylim([0 30]);
legend show;
legend('location', 'bestoutside');
xlabel('\epsilon_{r}');
ylabel('D(\theta=0,\phi=0) / dB');
title(['Superstrate Dipole Directivity @ f = ' num2str(f * 1e-9) ' GHz, and h = ' num2str(h * 1
```




```
saveas(gcf, 'figures\directivity_semi_infinite_superstrate.fig');
save('workspaces\Q3_2.mat');
```

3. Compare the achieved directivity with that from the previous example.